Class Notes for

Physics 2

(including AP® Physics 2)

in Plain English

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This is a set of class notes that can be used for an algebra-based, second-year high school Physics 2 course at the honors or AP® level. This hardcopy is provided so that you can fully participate in class discussions without having to worry about writing everything down.

While a significant amount of detail is included in these notes, they are intended as a supplement to textbooks, classroom discussions, experiments and activities. These class notes and any textbook discussion of the same topics are intended to be complementary. In some cases, the notes and the textbook differ in method or presentation, but the physics is the same. There may be errors and/or omissions in any textbook. There are almost certainly errors and omissions in these notes, despite my best efforts to make them clear, correct, and complete.

Topics

The AP® curriculum is, of course, set by the College Board. My decision was to have the same units in the honors course and the AP® course. However, the honors course has more flexibility with regard to pacing, difficulty, and topics.

Topics that are part of the curriculum for either the honors or AP® course but not both are marked in the left margin as follows:

honors only
(not AP®)

AP® (only)

Topics that are not otherwise marked should be assumed to apply to both courses.

The first two units (*Laboratory & Measurement* and *Mathematics*) are repeated from the Physics 1 notes, so these notes can be used without having to refer to them.

About the Homework Problems

The homework problems include a mixture of easy and challenging problems. *The process of making yourself smarter involves challenging yourself, even if you are not sure how to proceed.* By spending at least 10 minutes attempting each problem, you build neural connections between what you have learned and what you are trying to do. Even if you are not able to get the answer, when we go over those problems in class, you will reinforce the neural connections that led in the correct direction.

Answers to most problems are provided so you can check your work and see if you are on the right track. Do not simply write those answers down in order to receive credit for work you did not do. This will give you a false sense of confidence, and will actively prevent you from using the problems to make yourself smarter. *You have been warned.*

Using These Notes

As we discuss topics in class, you will want to add your own notes to these. If you have purchased this copy, you are encouraged to write directly in it, just as you would write in your own notebook. If this copy was issued to you by the school and you intend to return it at the end of the year, you will need to write your supplemental notes on separate paper. If you do this, be sure to write down page numbers in your notes, to make cross-referencing easier.

You should bring these notes to class every day, because lectures and discussions will follow these notes, which will also be projected onto the SMART board.

Features

These notes, and the course they accompany, are designed to follow both the 2016 Massachusetts Curriculum Frameworks, which are based on the Next Generation Science Standards (NGSS), and the AP® Physics 1 curriculum. (Note that the AP® learning objectives are the ones from 2014.) The notes also utilize strategies from the following popular teaching methods:

- Each topic includes Mastery Objectives and Success Criteria. These are based on the *Studying Skillful Teaching* course, from Research for Better Teaching (RBT), and are in "Students will be able to…" language.
- AP® topics include Learning Objectives from the College Board.
- Each topic includes Language Objectives and Tier 2 vocabulary words for English Learners, based on the Massachusetts Rethinking Equity and Teaching for English Language Learners (RETELL) course.
- Notes are organized in Cornell notes format as recommended by Keys To Literacy.
- Problems in problem sets are designated "Must Do" (M), "Should Do" (S) and "Aspire to Do" (A), as recommended by the Modern Classrooms Project (MCP).

Conventions

Some of the conventions in these notes are different from conventions in some physics textbooks. Although some of these are controversial and may incur the ire of other physics teachers, here is an explanation of my reasoning:

- When working sample problems, the units are left out of the algebra until the end. While I agree that there are good reasons for keeping the units to show the dimensional analysis, many students confuse units for variables, *e.g.,* confusing the unit "m" (meters) with the variable "*m*" (mass).
- Problems are worked using $g = 10\,\frac{\text{m}}{\text{s}^2} = 10\,\frac{\text{N}}{\text{kg}}$. This is because many students are not adept with algebra, and have trouble seeing where a problem is going once they take out their calculators. With simpler numbers, students have an easier time following the physics.
- Vector quantities are denoted with arrows as well as boldface, *e.g.,* $\vec{\boldsymbol{v}}, \vec{\boldsymbol{d}}, \vec{\boldsymbol{F}}_g$. This is to help students keep track of which quantities are vectors and which are scalars. In some cases, this results in equations that are nonsensical as vector expressions, such as $\vec{\boldsymbol{v}}^2 - \vec{\boldsymbol{v}}_o^{\,2} = 2\vec{\boldsymbol{a}}\vec{\boldsymbol{d}}$. (A vector can't be "squared", and multiplying $\vec{\boldsymbol{a}}$ by $\vec{\boldsymbol{d}}$ would have to be either $\vec{\boldsymbol{a}} \bullet \vec{\boldsymbol{d}}$ or $\vec{\boldsymbol{a}} \times \vec{\boldsymbol{d}}$.) It is good to point this out to students when they encounter these expressions, but in my opinion the benefits of keeping the vector notation even where it results in an incorrect vector expression outweigh the drawbacks.
- Forces are denoted as the variable $\vec{\boldsymbol{F}}$ with a subscript, e.g., $\vec{\boldsymbol{F}}_g, \vec{\boldsymbol{F}}_f, \vec{\boldsymbol{F}}_N, \vec{\boldsymbol{F}}_T$, *etc.* instead of $m\vec{\boldsymbol{g}}, \vec{\boldsymbol{f}}, \vec{\boldsymbol{N}}, \vec{\boldsymbol{T}}$, *etc.* This is to reinforce the connection between a quantity (force), a single variable $(\vec{\boldsymbol{F}})$, and a unit.
- Average velocity is denoted $\vec{\boldsymbol{v}}_{ave.}$ instead of $\overline{\vec{\boldsymbol{v}}}$. I have found that using the subscript "ave." helps students remember that average velocity is different from initial and final velocity.

- The variable V is used for electric potential. Voltage (potential difference) is denoted by ΔV. Although $\Delta V = IR$ is different from how the equation looks in most physics texts, it is useful to teach circuits starting with electric potential, and it is useful to maintain the distinction between absolute electric potential (V) and potential difference (ΔV). This is also how the College Board represents electric potential *vs.* voltage on AP® Physics exams.
- Equations are typeset on one line when practical. While there are very good reasons for teaching $\vec{\boldsymbol{a}} = \frac{\vec{\boldsymbol{F}}_{net}}{m}$ rather than $\vec{\boldsymbol{F}}_{net} = m\vec{\boldsymbol{a}}$ and $I = \frac{\Delta V}{R}$ rather than $\Delta V = IR$, students' difficulty in solving for a variable in the denominator often causes more problems than does their lack of understanding of which are the independent and dependent variables.

Learning Progression

There are several categories of understandings and skills that simultaneously build on themselves throughout this course:

Content

The sequence of topics starts with preliminaries—laboratory and then mathematical skills—in case a student is taking this course without having taken physics 1. The content topics are:

- Fluids & Fluid Mechanics
- Heat & Thermodynamics
- Electricity & Magnetism
- Waves, Light & Optics
- Special Relativity
- Quantum, Particle, Atomic & Nuclear Physics

Problem-Solving

This course builds on the problem-solving skills from physics 1. The topics in this course require more high-level thinking to decide what the situation is for each problem, and which equation(s) apply.

Laboratory

This course continues the experimental design lessons learned in physics 1. Because the topics require more specialized equipment, more time will be spent teaching students to use the equipment and giving them sufficient time to practice with it.

Scientific Discourse

In this course, the causal relationships between quantities are significantly more complex than in physics 1. Students need to continue to be given opportunities to explain these relationships throughout the course, both orally and in writing.

Acknowledgements

These notes would not have been possible without the assistance of many people. It would be impossible to include everyone, but I would particularly like to thank:

- Every student I have ever taught, for helping me learn how to teach, and how to explain and convey challenging concepts.
- The physics teachers I have worked with over the years who have generously shared their time, expertise, and materials. In particular, Mark Greenman, who has taught multiple courses on teaching physics and who, as the PhysTEC Teacher in Residence at Boston University, organizes a monthly meeting for Boston-area physics teachers to share laboratory activities and demonstrations; Barbara Watson, whose AP® Physics 1 and AP® Physics 2 Summer Institutes I attended, and with whom I have had numerous conversations about the teaching of physics, particularly at the AP® level; and Eva Sacharuk, who met with me weekly during my first year teaching physics to share numerous demonstrations, experiments and activities that she collected over her many decades in the classroom.
- Every teacher I have worked with, for their kind words, sympathetic listening, helpful advice and suggestions, and other contributions great and small that have helped me to enjoy and become competent at the profession of teaching.
- The department heads, principals and curriculum directors I have worked with, for mentoring me, encouraging me, allowing me to develop my own teaching style, and putting up with my experiments, activities and apparatus that place students physically at the center of a physics concept. In particular: Mark Greenman, Marilyn Hurwitz, Scott Gordon, Barbara Osterfield, Wendell Cerne, John Graceffa, Maura Walsh, Lauren Mezzetti, Jill Joyce, Tom Strangie, and Anastasia Mower.
- Everyone else who has shared their insights, stories, and experiences in physics, many of which are reflected in some way in these notes.

I am reminded of Sir Isaac Newton's famous quote, *"If I have seen further it is because I have stood on the shoulders of giants."*

About the Author

Jeff Bigler is a physics teacher at Lynn English High School in Lynn, Massachusetts. He has degrees from MIT in chemical engineering and biology, and is a National Board certified teacher in Science–Adolescence and Young Adulthood. He worked in biotech and IT prior to starting his teaching career in 2003. He has taught both physics and chemistry at all levels from conceptual to AP®.

He is married and has two adult daughters. His hobbies are music and Morris dancing.

Errata

As is the case in just about any large publication, these notes undoubtedly contain errors despite my efforts to find and correct them all.

Known errata for these notes are listed at:
https://www.mrbigler.com/Physics-2/Notes-Physics-2-errata.shtml

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MA Curriculum Frameworks for Physics

Standard	Topics	Chapters
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HS-PS2-1	Newton's 2nd ($F_{net} = ma$), motion graphs; ramps, friction, normal force, gravity, magnetic force	—
HS-PS2-2	conservation of momentum	—
HS-PS2-3	lab: reduce impulse in a collision	—
HS-PS2-4	gravitation & coulomb's law including relative changes	6
HS-PS2-5	electromagnetism: current produces magnetic field & vice-versa, including examples	8
HS-PS2-9(MA)	Ohm's Law, circuit diagrams, evaluate series & parallel circuits for ΔV, I or R.	7
HS-PS2-10(MA)	free-body diagrams, algebraic expressions & Newton's laws to predict acceleration for 1-D motion, including motion graphs	—
HS-PS3-1	conservation of energy including thermal, kinetic, gravitational, magnetic or electrical including gravitational & electric fields	5
HS-PS3-2	energy can be motion of particles or stored in fields. kinetic → thermal, evaporation/condensation, gravitational potential energy, electric fields	4, 6
HS-PS3-3	lab: build a device that converts energy from one form to another.	—
HS-PS3-4a	zero law of thermodynamics (heat flow & thermal equilibrium)	5
HS-PS3-5	behavior of charges or magnets attracting & repelling	6, 8
HS-PS4-1	waves: $v = f\lambda$ & T = 1/f, EM waves traveling through space or a medium vs. mechanical waves in a medium	9, 10
HS-PS4-3	EM radiation is both wave & particle. Qualitative behavior of resonance, interference, diffraction, refraction, photoelectric effect and wave vs. particle model for both	10, 11
HS-PS4-5	Devices use waves and wave interactions with matter, such as solar cells, medical imaging, cell phones, wi-fi	10

MA Science Practices

Practice	Description
SP1	Asking questions.
SP2	Developing & using models.
SP3	Planning & carrying out investigations.
SP4	Analyzing & interpreting data.
SP5	Using mathematics & computational thinking.
SP6	Constructing explanations.
SP7	Engaging in argument from evidence.
SP8	Obtaining, evaluating and communicating information.

AP® Physics 2 Big Ideas & Learning Objectives

These are current as of 2015.

ELECTRIC FORCE, FIELD AND POTENTIAL

BIG IDEA 1: Objects and systems have properties such as mass and charge. Systems may have internal structure.
1.B.1.1: The student is able to make claims about natural phenomena based on conservation of electric charge. [**SP 6.4**]
1.B.1.2: The student is able to make predictions, using the conservation of electric charge, about the sign and relative quantity of net charge of objects or systems after various charging processes, including conservation of charge in simple circuits. [**SP 6.4, 7.2**]
1.B.2.2: The student is able to make a qualitative prediction about the distribution of positive and negative electric charges within neutral systems as they undergo various processes. [**SP 6.4, 7.2**]
1.B.2.3: The student is able to challenge claims that polarization of electric charge or separation of charge must result in a net charge on the object. [**SP6.1**]
1.B.3.1: The student is able to challenge the claim that an electric charge smaller than the elementary charge has been isolated. [**SP 1.5, 6.1, 7.2**]

BIG IDEA 2: Fields existing in space can be used to explain interactions.
2.C.1.1: The student is able to predict the direction and the magnitude of the force exerted on an object with an electric charge *q* placed in an electric field *E* using the mathematical model of the relation between an electric force and an electric field: $\vec{F} = q\vec{E}$; a vector relation.[**SP 6.4, 7.2**]
2.C.1.2: The student is able to calculate any one of the variables — electric force, electric charge, and electric field — at a point given the values and sign or direction of the other two quantities.[**SP 2.2**]
2.C.2.1: The student is able to qualitatively and semi-quantitatively apply the vector relationship between the electric field and the net electric charge creating that field. [**SP 2.2, 6.4**]
2.C.3.1: The student is able to explain the inverse square dependence of the electric field surrounding a spherically symmetric electrically charged object. [**SP 6.2**]
2.C.4.1: The student is able to distinguish the characteristics that differ between monopole fields (gravitational field of spherical mass and electrical field due to single point charge) and dipole fields (electric dipole field and magnetic field) and make claims about the spatial behavior of the fields using qualitative or semiquantitative arguments based on vector addition of fields due to each point source, including identifying the locations and signs of sources from a vector diagram of the field. [**SP 2.2, 6.4, 7.2**]
2.C.4.2: The student is able to apply mathematical routines to determine the magnitude and direction of the electric field at specified points in the vicinity of a small set (2–4) of point charges, and express the results in terms of magnitude and direction of the field in a visual representation by drawing field vectors of appropriate length and direction at the specified points. [**SP 1.4, 2.2**]

BIG IDEA 2: Fields existing in space can be used to explain interactions.
2.C.5.1: The student is able to create representations of the magnitude and direction of the electric field at various distances (small compared to plate size) from two electrically charged plates of equal magnitude and opposite signs, and is able to recognize that the assumption of uniform field is not appropriate near edges of plates. **[SP 1.1, 2.2**]
2.C.5.2: The student is able to calculate the magnitude and determine the direction of the electric field between two electrically charged parallel plates, given the charge of each plate, or the electric potential difference and plate separation. **[SP 2.2**]
2.C.5.3: The student is able to represent the motion of an electrically charged particle in the uniform field between two oppositely charged plates and express the connection of this motion to projectile motion of an object with mass in the Earth's gravitational field. **[SP 1.1, 2.2, 7.1**]
2.E.1.1: The student is able to construct or interpret visual representations of the isolines of equal gravitational potential energy per unit mass and refer to each line as a gravitational equipotential. **[SP 1.4, 6.4, 7.2**]
2.E.2.1: The student is able to determine the structure of isolines of electric potential by constructing them in a given electric field. **[SP 6.4, 7.2**]
2.E.2.2: The student is able to predict the structure of isolines of electric potential by constructing them in a given electric field and make connections between these isolines and those found in a gravitational field. **[SP 6.4, 7.2**]
2.E.2.3: The student is able to qualitatively use the concept of isolines to construct isolines of electric potential in an electric field and determine the effect of that field on electrically charged objects. **[SP 1.4**]
2.E.3.1: The student is able to apply mathematical routines to calculate the average value of the magnitude of the electric field in a region from a description of the electric potential in that region using the displacement along the line on which the difference in potential is evaluated. **[SP 2.2**]
2.E.3.2: The student is able to apply the concept of the isoline representation of electric potential for a given electric charge distribution to predict the average value of the electric field in the region. **[SP 1.4, 6.4]**

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.A.2.1: The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **[SP 1.1**]
3.A.3.2: The student is able to challenge a claim that an object can exert a force on itself. **[SP 6.1**]
3.A.3.3: The student is able to describe a force as an interaction between two objects and identify both objects for any force. **[SP 1.4]**
3.A.3.4: The student is able to make claims about the force on an object due to the presence of other objects with the same property: mass, electric charge. **[SP 6.1, 6.4]**
3.A.4.1: The student is able to construct explanations of physical situations involving the interaction of bodies using Newton's third law and the representation of action-reaction pairs of forces. **[SP 1.4, 6.2]**

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.A.4.2: The student is able to use Newton's third law to make claims and predictions about the action-reaction pairs of forces when two objects interact. **[SP 6.4, 7.2]**
3.A.4.3: The student is able to analyze situations involving interactions among several objects by using free-body diagrams that include the application of Newton's third law to identify forces. **[SP 1.4]**
3.B.1.3: The student is able to reexpress a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. **[SP 1.5, 2.2]**
3.B.1.4: The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations. **[SP 6.4, 7.2]**
3.B.2.1: The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. **[SP 1.1, 1.4, 2.2]**
3.C.2.1: The student is able to use Coulomb's law qualitatively and quantitatively to make predictions about the interaction between two electric point charges. **[SP 2.2, 6.4]**
3.C.2.2: The student is able to connect the concepts of gravitational force and electric force to compare similarities and differences between the forces. **[SP 7.2]**
3.C.2.3: The student is able to use mathematics to describe the electric force that results from the interaction of several separated point charges (generally 2 to 4 point charges, though more are permitted in situations of high symmetry). **[SP 2.2]**
3.G.1.2: The student is able to connect the strength of the gravitational force between two objects to the spatial scale of the situation and the masses of the objects involved and compare that strength to other types of forces. **[SP 7.1]**
3.G.2.1: The student is able to connect the strength of electromagnetic forces with the spatial scale of the situation, the magnitude of the electric charges, and the motion of the electrically charged objects involved. [**SP 7.1**]

BIG IDEA 4: Interactions between systems can result in changes in those systems.
4.E.3.1: The student is able to make predictions about the redistribution of charge during charging by friction, conduction, and induction. **[SP 6.4]**
4.E.3.2: The student is able to make predictions about the redistribution of charge caused by the electric field due to other systems, resulting in charged or polarized objects. **[SP 6.4, 7.2]**
4.E.3.3: The student is able to construct a representation of the distribution of fixed and mobile charge in insulators and conductors. **[SP 1.1, 1.4, 6.4]**
4.E.3.4: The student is able to construct a representation of the distribution of fixed and mobile charge in insulators and conductors that predicts charge distribution in processes involving induction or conduction. **[SP 1.1, 1.4, 6.4]**
4.E.3.5: The student is able to plan and/or analyze the results of experiments in which electric charge rearrangement occurs by electrostatic induction, or is able to refine a scientific question relating to such an experiment by identifying anomalies in a data set or procedure. **[SP 3.2, 4.1, 4.2, 5.1, 5.3]**

BIG IDEA 5: Changes that occur as a result of interactions are constrained by conservation laws.
5.A.2.1: The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. **[SP 6.4, 7.2**]
5.B.2.1: The student is able to calculate the expected behavior of a system using the object model (i.e., by ignoring changes in internal structure) to analyze a situation. Then, when the model fails, the student can justify the use of conservation of energy principles to calculate the change in internal energy due to changes in internal structure because the object is actually a system. **[SP 1.4, 2.1**]
5.C.2.1: The student is able to predict electric charges on objects within a system by application of the principle of charge conservation within a system. **[SP 6.4]**
5.C.2.2: The student is able to design a plan to collect data on the electrical charging of objects and electric charge induction on neutral objects and qualitatively analyze that data. **[SP 4.2, 5.1]**
5.C.2.3: The student is able to justify the selection of data relevant to an investigation of the electrical charging of objects and electric charge induction on neutral objects. **[SP 4.1]**

ELECTRIC CIRCUITS

BIG IDEA 1: Objects and systems have properties such as mass and charge. Systems may have internal structure.
1.E.2.1: The student is able to choose and justify the selection of data needed to determine resistivity for a given material. **[SP 4.1**]

BIG IDEA 4: Interactions between systems can result in changes in those systems.
4.E.4.1: The student is able to make predictions about the properties of resistors and/or capacitors when placed in a simple circuit, based on the geometry of the circuit element and supported by scientific theories and mathematical relationships. **[SP 2.2, 6.4]**
4.E.4.2: The student is able to design a plan for the collection of data to determine the effect of changing the geometry and/or materials on the resistance or capacitance of a circuit element and relate results to the basic properties of resistors and capacitors. **[SP 4.1, 4.2]**
4.E.4.3: The student is able to analyze data to determine the effect of changing the geometry and/or materials on the resistance or capacitance of a circuit element and relate results to the basic properties of resistors and capacitors. **[SP 5.1]**
4.E.5.1: The student is able to make and justify a quantitative prediction of the effect of a change in values or arrangements of one or two circuit elements on the currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel. **[SP 2.2, 6.4]**
4.E.5.2: The student is able to make and justify a qualitative prediction of the effect of a change in values or arrangements of one or two circuit elements on currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel. **[SP 6.1, 6.4]**

BIG IDEA 4: Interactions between systems can result in changes in those systems.
4.E.5.3: The student is able to plan data collection strategies and perform data analysis to examine the values of currents and potential differences in an electric circuit that is modified by changing or rearranging circuit elements, including sources of emf, resistors, and capacitors. **[SP 2.2, 4.2, 5.1**]

BIG IDEA 5: Changes that occur as a result of interactions are constrained by conservation laws.
5.B.9.4: The student is able to analyze experimental data including an analysis of experimental uncertainty that will demonstrate the validity of Kirchhoff's loop rule.**[SP 5.1]**
5.B.9.5: The student is able to use conservation of energy principles (Kirchhoff's loop rule) to describe and make predictions regarding electrical potential difference, charge, and current in steady-state circuits composed of various combinations of resistors and capacitors. **[SP 6.4]**
5.B.9.6: The student is able to mathematically express the changes in electric potential energy of a loop in a multiloop electrical circuit and justify this expression using the principle of the conservation of energy. **[SP 2.1, 2.2]**
5.B.9.7: The student is able to refine and analyze a scientific question for an experiment using Kirchhoff's Loop rule for circuits that includes determination of internal resistance of the battery and analysis of a non-ohmic resistor. **[SP 4.1, 4.2, 5.1, 5.3]**
5.B.9.8: The student is able to translate between graphical and symbolic representations of experimental data describing relationships among power, current, and potential difference across a resistor. **[SP 1.5]**
5.C.3.4: The student is able to predict or explain current values in series and parallel arrangements of resistors and other branching circuits using Kirchhoff's junction rule and relate the rule to the law of charge conservation. **[SP 6.4, 7.2]**
5.C.3.5: The student is able to determine missing values and direction of electric current in branches of a circuit with resistors and NO capacitors from values and directions of current in other branches of the circuit through appropriate selection of nodes and application of the junction rule. **[SP 1.4, 2.2]**
5.C.3.6: The student is able to determine missing values and direction of electric current in branches of a circuit with both resistors and capacitors from values and directions of current in other branches of the circuit through appropriate selection of nodes and application of the junction rule. **[SP 1.4, 2.2]**
5.C.3.7: The student is able to determine missing values, direction of electric current, charge of capacitors at steady state, and potential differences within a circuit with resistors and capacitors from values and directions of current in other branches of the circuit. **[SP 1.4, 2.2**]

MAGNETISM AND ELECTROMAGNETIC INDUCTION

BIG IDEA 2: Fields existing in space can be used to explain interactions.
2.C.4.1: The student is able to distinguish the characteristics that differ between monopole fields (gravitational field of spherical mass and electrical field due to single point charge) and dipole fields (electric dipole field and magnetic field) and make claims about the spatial behavior of the fields using qualitative or semiquantitative arguments based on vector addition of fields due to each point source, including identifying the locations and signs of sources from a vector diagram of the field. **[SP 2.2, 6.4, 7.2**]

BIG IDEA 2: Fields existing in space can be used to explain interactions.
2.D.1.1: The student is able to apply mathematical routines to express the force exerted on a moving charged object by a magnetic field. **[SP 2.2**]
2.D.2.1: The student is able to create a verbal or visual representation of a magnetic field around a long straight wire or a pair of parallel wires. **[SP 1.1**]
2.D.3.1: The student is able to describe the orientation of a magnetic dipole placed in a magnetic field in general and the particular cases of a compass in the magnetic field of the Earth and iron filings surrounding a bar magnet. **[SP 1.2**]
2.D.4.1: The student is able to use the representation of magnetic domains to qualitatively analyze the magnetic behavior of a bar magnet composed of ferromagnetic material. **[SP 1.4**]

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.A.2.1: The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **[SP 1.1**]
3.A.3.2: The student is able to challenge a claim that an object can exert a force on itself. **[SP 6.1**]
3.A.3.3: The student is able to describe a force as an interaction between two objects and identify both objects for any force. **[SP 1.4]**
3.A.4.1: The student is able to construct explanations of physical situations involving the interaction of bodies using Newton's third law and the representation of action-reaction pairs of forces. **[SP 1.4, 6.2]**
3.A.4.2: The student is able to use Newton's third law to make claims and predictions about the action-reaction pairs of forces when two objects interact. **[SP 6.4, 7.2]**
3.A.4.3: The student is able to analyze situations involving interactions among several objects by using free-body diagrams that include the application of Newton's third law to identify forces. **[SP 1.4]**
3.C.3.1: The student is able to use right-hand rules to analyze a situation involving a current-carrying conductor and a moving electrically charged object to determine the direction of the magnetic force exerted on the charged object due to the magnetic field created by the current-carrying conductor. **[SP 1.4]**
3.C.3.2: The student is able to plan a data collection strategy appropriate to an investigation of the direction of the force on a moving electrically charged object caused by a current in a wire in the context of a specific set of equipment and instruments and analyze the resulting data to arrive at a conclusion. **[SP 4.2, 5.1]**

BIG IDEA 4: Interactions between systems can result in changes in those systems.
4.E.1.1: The student is able to use representations and models to qualitatively describe the magnetic properties of some materials that can be affected by magnetic properties of other objects in the system. **[SP 1.1, 1.4, 2.2**]
4.E.2.1: The student is able to construct an explanation of the function of a simple electromagnetic device in which an induced emf is produced by a changing magnetic flux through an area defined by a current loop (i.e., a simple microphone or generator) or of the effect on behavior of a device in which an induced emf is produced by a constant magnetic field through a changing area. **[SP 6.4**]

THERMODYNAMICS

BIG IDEA 1: Objects and systems have properties such as mass and charge. Systems may have internal structure.
1.E.3.1: The student is able to design an experiment and analyze data from it to examine thermal conductivity. **[SP 4.1, 4.2, 5.1]**

BIG IDEA 4: Interactions between systems can result in changes in those systems.
4.C.3.1: The student is able to make predictions about the direction of energy transfer due to temperature differences based on interactions at the microscopic level. **[SP 6.4]**

BIG IDEA 5: Changes that occur as a result of interactions are constrained by conservation laws.
5.A.2.1: The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. **[SP 6.4, 7.2]**
5.B.4.1: The student is able to describe and make predictions about the internal energy of systems. **[SP 6.4, 7.2]**
5.B.4.2: The student is able to calculate changes in kinetic energy and potential energy of a system, using information from representations of that system. **[SP 1.4, 2.1, 2.2]**
5.B.5.4: The student is able to make claims about the interaction between a system and its environment in which the environment exerts a force on the system, thus doing work on the system and changing the energy of the system (kinetic energy plus potential energy). **[SP 6.4, 7.2]**
5.B.5.5: The student is able to predict and calculate the energy transfer to (i.e., the work done on) an object or system from information about a force exerted on the object or system through a distance. **[SP 2.2, 6.4]**
5.B.5.6: The student is able to design an experiment and analyze graphical data in which interpretations of the area under a pressure-volume curve are needed to determine the work done on or by the object or system. **[SP 4.2, 5.1]**
5.B.6.1: The student is able to describe the models that represent processes by which energy can be transferred between a system and its environment because of differences in temperature: conduction, convection, and radiation. **[SP 1.2]**
5.B.7.1: The student is able to predict qualitative changes in the internal energy of a thermodynamic system involving transfer of energy due to heat or work done and justify those predictions in terms of conservation of energy principles. **[SP 6.4, 7.2]**
5.B.7.2: The student is able to create a plot of pressure versus volume for a thermodynamic process from given data. **[SP 1.1]**
5.B.7.3: The student is able to use a plot of pressure versus volume for a thermodynamic process to make calculations of internal energy changes, heat, or work, based upon conservation of energy principles (i.e., the first law of thermodynamics). **[SP 1.1, 1.4, 2.2]**

BIG IDEA 7: The mathematics of probability can be used to describe the behavior of complex systems and to interpret the behavior of quantum mechanical systems.
7.A.1.1: The student is able to make claims about how the pressure of an ideal gas is connected to the force exerted by molecules on the walls of the container, and how changes in pressure affect the thermal equilibrium of the system. **[SP 6.4, 7.2]**
7.A.1.2: Treating a gas molecule as an object (i.e., ignoring its internal structure), the student is able to analyze qualitatively the collisions with a container wall and determine the cause of pressure, and at thermal equilibrium, to quantitatively calculate the pressure, force, or area for a thermodynamic problem given two of the variables. **[SP 1.4, 2.2]**
7.A.2.1: The student is able to qualitatively connect the average of all kinetic energies of molecules in a system to the temperature of the system. **[SP 7.1]**
7.A.2.2: The student is able to connect the statistical distribution of microscopic kinetic energies of molecules to the macroscopic temperature of the system and to relate this to thermodynamic processes. **[SP 7.1]**
7.A.3.1: The student is able to extrapolate from pressure and temperature or volume and temperature data to make the prediction that there is a temperature at which the pressure or volume extrapolates to zero. **[SP 6.4, 7.2]**
7.A.3.2: The student is able to design a plan for collecting data to determine the relationships between pressure, volume, and temperature, and amount of an ideal gas, and to refine a scientific question concerning a proposed incorrect relationship between the variables. **[SP 3.2, 4.2]**
7.A.3.3: The student is able to analyze graphical representations of macroscopic variables for an ideal gas to determine the relationships between these variables and to ultimately determine the ideal gas law $PV = nRT$. **[SP 5.1]**
7.B.1.1: The student is able to extrapolate from pressure and temperature or volume and temperature data to make the prediction that there is a temperature at which the pressure or volume extrapolates to zero. **[SP 6.4, 7.2]**
7.B.2.1: The student is able to connect qualitatively the second law of thermodynamics in terms of the state function called entropy and how it (entropy) behaves in reversible and irreversible processes. **[SP 7.1]**

FLUIDS

BIG IDEA 1: Objects and systems have properties such as mass and charge. Systems may have internal structure.
1.E.1.1: The student is able to predict the densities, differences in densities, or changes in densities under different conditions for natural phenomena and design an investigation to verify the prediction. **[SP 4.2, 6.4**]
1.E.1.2: The student is able to select from experimental data the information necessary to determine the density of an object and/or compare densities of several objects. **[SP 4.1, 6.4**]

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.C.4.1: The student is able to make claims about various contact forces between objects based on the microscopic cause of those forces. **[SP 6.1**]

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.C.4.2: The student is able to explain contact forces (tension, friction, normal, buoyant, spring) as arising from interatomic electric forces and that they therefore have certain directions. **[SP 6.2**]

BIG IDEA 5: Changes that occur as a result of interactions are constrained by conservation laws.
5.B.10.1: The student is able to use Bernoulli's equation to make calculations related to a moving fluid. **[SP 2.2**]
5.B.10.2: The student is able to use Bernoulli's equation and/or the relationship between force and pressure to make calculations related to a moving fluid. **[SP 2.2**]
5.B.10.3: The student is able to use Bernoulli's equation and the continuity equation to make calculations related to a moving fluid. **[SP 2.2**]
5.B.10.4: The student is able to construct an explanation of Bernoulli's equation in terms of the conservation of energy. **[SP 6.2**]
5.F.1.1: The student is able to make calculations of quantities related to flow of a fluid, using mass conservation principles (the continuity equation). **[SP 2.1, 2.2, 7.2**]

GEOMETRIC AND PHYSICAL OPTICS

BIG IDEA 6: Waves can transfer energy and momentum from one location to another without the permanent transfer of mass and serve as a mathematical model for the description of other phenomena.
6.A.1.2: The student is able to describe representations of transverse and longitudinal waves. **[SP 1.2]**
6.A.1.3: The student is able to analyze data (or a visual representation) to identify patterns that indicate that a particular mechanical wave is polarized and construct an explanation of the fact that the wave must have a vibration perpendicular to the direction of energy propagation. **[SP 5.1, 6.2]**
6.A.2.2: The student is able to contrast mechanical and electromagnetic waves in terms of the need for a medium in wave propagation. **[SP 6.4, 7.2**]
6.B.3.1: The student is able to construct an equation relating the wavelength and amplitude of a wave from a graphical representation of the electric or magnetic field value as a function of position at a given time instant and vice versa, or construct an equation relating the frequency or period and amplitude of a wave from a graphical representation of the electric or magnetic field value at a given position as a function of time and vice versa. **[SP 1.5**]
6.C.1.1: The student is able to make claims and predictions about the net disturbance that occurs when two waves overlap. Examples should include standing waves. **[SP 6.4, 7.2]**
6.C.1.2: The student is able to construct representations to graphically analyze situations in which two waves overlap over time using the principle of superposition. **[SP 1.4]**
6.C.2.1: The student is able to make claims about the diffraction pattern produced when a wave passes through a small opening, and to qualitatively apply the wave model to quantities that describe the generation of a diffraction pattern when a wave passes through an opening whose dimensions are comparable to the wavelength of the wave. **[SP 1.4, 6.4, 7.2**]

BIG IDEA 6: Waves can transfer energy and momentum from one location to another without the permanent transfer of mass and serve as a mathematical model for the description of other phenomena.
6.C.3.1: The student is able to qualitatively apply the wave model to quantities that describe the generation of interference patterns to make predictions about interference patterns that form when waves pass through a set of openings whose spacing and widths are small compared to the wavelength of the waves. **[SP 1.4, 6.4**]
6.C.4.1: The student is able to predict and explain, using representations and models, the ability or inability of waves to transfer energy around corners and behind obstacles in terms of the diffraction property of waves in situations involving various kinds of wave phenomena, including sound and light. **[SP 6.4, 7.2**]
6.E.1.1: The student is able to make claims using connections across concepts about the behavior of light as the wave travels from one medium into another, as some is transmitted, some is reflected, and some is absorbed. **[SP 6.4, 7.2**]
6.E.2.1: The student is able to make predictions about the locations of object and image relative to the location of a reflecting surface. The prediction should be based on the model of specular reflection with all angles measured relative to the normal to the surface. **[SP 6.4, 7.2**]
6.E.3.1: The student is able to describe models of light traveling across a boundary from one transparent material to another when the speed of propagation changes, causing a change in the path of the light ray at the boundary of the two media. **[SP 1.1, 1.4]**
6.E.3.2: The student is able to plan data collection strategies as well as perform data analysis and evaluation of the evidence for finding the relationship between the angle of incidence and the angle of refraction for light crossing boundaries from one transparent material to another (Snell's law). **[SP 4.1, 5.1, 5.2, 5.3]**
6.E.3.3: The student is able to make claims and predictions about path changes for light traveling across a boundary from one transparent material to another at non-normal angles resulting from changes in the speed of propagation. **[SP 6.4, 7.2]**
6.E.4.1: The student is able to plan data collection strategies, and perform data analysis and evaluation of evidence about the formation of images due to reflection of light from curved spherical mirrors. **[SP 3.2, 4.1, 5.1, 5.2, 5.3]**
6.E.4.2: The student is able to use quantitative and qualitative representations and models to analyze situations and solve problems about image formation occurring due to the reflection of light from surfaces. **[SP 1.4, 2.2]**
6.E.5.1: The student is able to use quantitative and qualitative representations and models to analyze situations and solve problems about image formation occurring due to the refraction of light through thin lenses. **[SSP 1.4, 2.2]**
6.E.5.2: The student is able to plan data collection strategies, perform data analysis and evaluation of evidence, and refine scientific questions about the formation of images due to refraction for thin lenses. **[SP 3.2, 4.1, 5.1, 5.2, 5.3]**
6.F.1.1: The student is able to make qualitative comparisons of the wavelengths of types of electromagnetic radiation. **[SP 6.4, 7.2**]
6.F.2.1: The student is able to describe representations and models of electromagnetic waves that explain the transmission of energy when no medium is present. **[SP 1.1**]

QUANTUM PHYSICS, ATOMIC AND NUCLEAR PHYSICS

BIG IDEA 1: Objects and systems have properties such as mass and charge. Systems may have internal structure.
1.A.2.1: The student is able to construct representations of the differences between a fundamental particle and a system composed of fundamental particles and to relate this to the properties and scales of the systems being investigated. [**SP 1.1, 7.1**]
1.A.4.1: The student is able to construct representations of the energy-level structure of an electron in an atom and to relate this to the properties and scales of the systems being investigated. [**SP 1.1, 7.1**]
1.C.4.1: The student is able to articulate the reasons that the theory of conservation of mass was replaced by the theory of conservation of mass-energy. [**SP 6.3**]
1.D.1.1: The student is able to explain why classical mechanics cannot describe all properties of objects by articulating the reasons that classical mechanics must be refined and an alternative explanation developed when classical particles display wave properties. [**SP 6.3**]
1.D.3.1: The student is able to articulate the reasons that classical mechanics must be replaced by special relativity to describe the experimental results and theoretical predictions that show that the properties of space and time are not absolute. [Students will be expected to recognize situations in which nonrelativistic classical physics breaks down and to explain how relativity addresses that breakdown, but students will not be expected to know in which of two reference frames a given series of events corresponds to a greater or lesser time interval, or a greater or lesser spatial distance; they will just need to know that observers in the two reference frames can “disagree” about some time and distance intervals.] [**SP 6.3, 7.1**]

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.G.3.1: The student is able to identify the strong force as the force that is responsible for holding the nucleus together. [**SP 7.2**]

BIG IDEA 4: Interactions between systems can result in changes in those systems.
4.C.4.1: The student is able to apply mathematical routines to describe the relationship between mass and energy and apply this concept across domains of scale. [**SP 2.2, 2.3, 7.2**]

BIG IDEA 5: Changes that occur as a result of interactions are constrained by conservation laws.
5.B.8.1: The student is able to describe emission or absorption spectra associated with electronic or nuclear transitions as transitions between allowed energy states of the atom in terms of the principle of energy conservation, including characterization of the frequency of radiation emitted or absorbed. [**SP 1.2, 7.2**]
5.B.11.1: The student is able to apply conservation of mass and conservation of energy concepts to a natural phenomenon and use the equation $E = mc^2$ to make a related calculation. [**SP 2.2, 7.2**]
5.C.1.1: The student is able to analyze electric charge conservation for nuclear and elementary particle reactions and make predictions related to such reactions based upon conservation of charge. [**SP 6.4, 7.2**]

BIG IDEA 5: Changes that occur as a result of interactions are constrained by conservation laws.
5.D.1.6: The student is able to make predictions of the dynamical properties of a system undergoing a collision by application of the principle of linear momentum conservation and the principle of the conservation of energy in situations in which an elastic collision may also be assumed. **[SP 6.4]**
5.D.1.7: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values. **[SP 2.1, 2.2]**
5.D.2.5: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum as the appropriate solution method for an inelastic collision, recognize that there is a common final velocity for the colliding objects in the totally inelastic case, solve for missing variables, and calculate their values. **[SP 2.1, 2.2]**
5.D.2.6: The student is able to apply the conservation of linear momentum to a closed system of objects involved in an inelastic collision to predict the change in kinetic energy. **[SP 6.4, 7.2]**
5.D.3.2: The student is able to make predictions about the velocity of the center of mass for interactions within a defined one-dimensional system. **[SP 6.4]**
5.D.3.3: The student is able to make predictions about the velocity of the center of mass for interactions within a defined two-dimensional system. **[SP 6.4]**
5.G.1.1: The student is able to apply conservation of nucleon number and conservation of electric charge to make predictions about nuclear reactions and decays such as fission, fusion, alpha decay, beta decay, or gamma decay. **[SP 6.4**]

BIG IDEA 6: Waves can transfer energy and momentum from one location to another without the permanent transfer of mass and serve as a mathematical model for the description of other phenomena.
6.F.3.1: The student is able to support the photon model of radiant energy with evidence provided by the photoelectric effect. **[SP 6.4**]
6.F.4.1: The student is able to select a model of radiant energy that is appropriate to the spatial or temporal scale of an interaction with matter. **[SP 6.4, 7.1**]
6.G.1.1: The student is able to make predictions about using the scale of the problem to determine at what regimes a particle or wave model is more appropriate. **[SP 6.4, 7.1**]
6.G.2.1: The student is able to articulate the evidence supporting the claim that a wave model of matter is appropriate to explain the diffraction of matter interacting with a crystal, given conditions where a particle of matter has momentum corresponding to a de Broglie wavelength smaller than the separation between adjacent atoms in the crystal. **[SP 6.1]**
6.G.2.2: The student is able to predict the dependence of major features of a diffraction pattern (e.g., spacing between interference maxima), based upon the particle speed and de Broglie wavelength of electrons in an electron beam interacting with a crystal. (de Broglie wavelength need not be given, so students may need to obtain it.) **[SP 6.4]**

BIG IDEA 7: The mathematics of probability can be used to describe the behavior of complex systems and to interpret the behavior of quantum mechanical systems.
7.C.1.1: The student is able to use a graphical wave function representation of a particle to predict qualitatively the probability of finding a particle in a specific spatial region. **[SP 1.4**]
7.C.2.1: The student is able to use a standing wave model in which an electron orbit circumference is an integer multiple of the de Broglie wavelength to give a qualitative explanation that accounts for the existence of specific allowed energy states of an electron in an atom. **[SP 1.4**]
7.C.3.1: The student is able to predict the number of radioactive nuclei remaining in a sample after a certain period of time, and also predict the missing species (alpha, beta, gamma) in a radioactive decay. **[SP 6.4**]
7.C.4.1: The student is able to construct or interpret representations of transitions between atomic energy states involving the emission and absorption of photons. [For questions addressing stimulated emission, students will not be expected to recall the details of the process, such as the fact that the emitted photons have the same frequency and phase as the incident photon; but given a representation of the process, students are expected to make inferences such as figuring out from energy conservation that since the atom loses energy in the process, the emitted photons taken together must carry more energy than the incident photon.] **[SP 1.1, 1.2**]

Cornell (Two-Column) Notes

Unit: Introduction

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Use the Cornell note-taking system to take effective notes, or add to existing notes.

Success Criteria:

- Notes are in two columns with appropriate main ideas on the left and details on the right.
- Bottom section includes summary and/or other important points.

Language Objectives:

- Understand and describe how Cornell notes are different from other forms of note-taking.

Tier 2 Vocabulary: N/A

Notes:

The Cornell note-taking system was developed in the 1950s at Cornell University. Besides being a useful system for note-taking in general, it is an especially useful system for interacting with someone else's notes (such as these) in order to get more out of them.

The main features of Cornell notes are:

1. The main section of the page is for the details of what actually gets covered in class.
2. The left section (Cornell notes call for 2½ inches, though I have shrunk it to 2 inches) is for "big ideas"—the organizational headings that help you organize these notes and find details that you are looking for. These have been left blank for you to add throughout the year, because the process of deciding what is important is a key element of understanding and remembering.
3. Cornell notes call for the bottom section (2 inches) to be used for a 1–2 sentence summary of the page in your own words. This is always a good idea, but you may also choose to use that space for other things you want to remember that aren't in these notes.

Use this space for summary and/or additional notes:

How to Get Nothing Worthwhile Out Of These Notes

Because this book serves as a combination of your textbook and a set of notes, you may be tempted to sleep through class because "it's all in the book," and then use these notes look up how to do the homework problems when you get confused. If you do this, you will learn very little physics, and you will find this class to be both frustrating and boring.

How to Get the Most Out Of These Notes

These notes are provided so you can pay attention and participate in class without having to worry about writing everything down. However, because active listening, participation and note-taking improve your ability to understand and remember, it is important that you interact with these notes and the discussion.

The "Big Ideas" column on the left of each page has been deliberately left blank. This is to give you the opportunity to go through your notes and categorize each section according to the big ideas it contains. Doing this throughout the year will help you keep the information organized in your brain—it's a lot easier to remember things when your brain has a place to put them!

If we discuss something in class that you want to remember, *mark or highlight it in the notes*! If we discuss an alternative way to think about something that works well for you, *write it in*! You paid for these notes—don't be afraid to use them!

There is a summary section at the bottom of each page. Utilize it! If you can summarize something, you understand it; if you understand something, it is much easier to remember.

Use this space for summary and/or additional notes:

Reading & Taking Notes from a Textbook

Unit: Introduction

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Use information from the organization of a textbook to take well-organized notes.

Success Criteria:

- Section headings from text are represented as main ideas.
- All information in section summary is represented in notes.
- Notes include page numbers.

Language Objectives:

- Understand and be able to describe the strategies presented in this section.

Tier 2 Vocabulary: N/A

Notes:

If you read a textbook the way you would read a novel, you probably won't remember much of what you read. Before you can understand anything, your brain needs enough context to know how to file the information. This is what Albert Einstein was talking about when he said, "It is the theory which decides what we are able to observe."

When you read a section of a textbook, you need to create some context in your brain, and then add a few observations to solidify the context before reading in detail.

Use this space for summary and/or additional notes:

René Descartes described this process in 1644 in the preface to his *Principles of Philosophy*:

> "I should also have added a word of advice regarding the manner of reading this work, which is, that I should wish the reader at first go over the whole of it, as he would a romance, without greatly straining his attention, or tarrying at the difficulties he may perhaps meet with, and that afterwards, if they seem to him to merit a more careful examination, and he feels a desire to know their causes, he may read it a second time, in order to observe the connection of my reasonings; but that he must not then give it up in despair, although he may not everywhere sufficiently discover the connection of the proof, or understand all the reasonings—it being only necessary to mark with a pen the places where the difficulties occur, and continue reading without interruption to the end; then, if he does not grudge to take up the book a third time, I am confident that he will find in a fresh perusal the solution of most of the difficulties he will have marked before; and that, if any remain, their solution will in the end be found in another reading."

Use this space for summary and/or additional notes:

Descartes is advocating reading the text four times, but it is not necessary to do a thorough reading each time. It is indeed useful to make four passes over the text, but each one should add a new level of understanding, and three of the four are quick and require minimal effort.

The following 4-step system takes about the same amount of time that you're probably used to spending on reading and taking notes, but it will likely make a tremendous difference in how much you understand and how much you remember.

1. Make a Cornell notes template. Copy the title/heading of each section as a big idea in the left column. (If the author has taken the trouble to organize the textbook, you should take advantage of it!) *Write the page numbers next to the headings so you will know where to go if you need to look up details in the textbook.* For each big idea, leave about ¼ to ½ page of space for the details. (Don't do anything else yet.) This process should take only about 1–2 minutes.
2. Do not write anything else yet! Look through the section for pictures, graphs, and tables. Take a moment to look at each one of these—if the author gave them space in the textbook, they must be important. Also read over (but don't try to answer) the homework questions/problems at the end of the section—these illustrate what the author thinks you should be able to do once you know the content. This process should take about 10–15 minutes.
3. Actually read the text, one section at a time. For each section, jot down keywords and sentence fragments that remind you of the key ideas about the text, and the pictures and questions/problems from step 2 above. Remember that you are not allowed to write more than the ¼ to ½ page allotted. (You don't need to write out the details—those are in the book, which you already have!) This is the time-consuming step, though it is probably less time-consuming than what you're used to doing.
4. Read the summary at the end of the chapter or section—this is what the author thinks you should know now that you've finished the reading. If there's anything you don't recognize, go back, look it up, and add it to your notes. This process should take about 5–10 minutes.

For a high school textbook, you shouldn't need to use more than about one side of a sheet of paper per 5 pages of reading!

Use this space for summary and/or additional notes:

Taking Notes on Math Problems

Unit: Introduction

MA Curriculum Frameworks (2016): SP5

AP® Physics 2 Learning Objectives: SP5

Mastery Objective(s): (Students will be able to…)

- Take notes on math problems that both show and explain the steps.

Success Criteria:

- Notes show the order of the steps, from start to finish.
- A reason or explanation is indicated for each step.

Language Objectives:

- Be able to describe and explain the process of taking notes on a math problem.

Tier 2 Vocabulary: N/A

Notes:

If you were to copy down a math problem and look at it a few days or weeks later, chances are you'll recognize the problem, but you won't remember how you solved it.

Solving a math problem is a process. For notes to be useful, they need to describe the process as it happens, not just the final result.

If you want to take good notes on how to solve a problem, you need your notes to show what you did at each step.

Use this space for summary and/or additional notes:

For example, consider the following physics problem:

> A 25 kg cart is accelerated from rest to a velocity of $3.5\,\frac{\text{m}}{\text{s}}$ over an interval of 1.5 s. Find the net force applied to the cart.

The solved problem looks like this:

$v_o = 0$

> A $\overset{m}{\underline{25\text{ kg}}}$ cart is accelerated <u>from rest</u> to a velocity of $\overset{v}{\underline{3.5\,\frac{\text{m}}{\text{s}}}}$ over an interval of $\overset{t}{\underline{1.5\text{ s}}}$. Find the $\overset{\boxed{F_{net}}}{\underline{\text{net force}}}$ applied to the cart.

$$\begin{aligned} F_{net} &= ma & v - v_o &= at \\ F_{net} &= 25a & 3.5 - 0 &= (a)(1.5) \\ F_{net} &= (25)(5.5) & 3.5 &= 1.5a \\ F_{net} &= 138.\overline{8}\text{ N} & a &= 5.5\,\tfrac{\text{m}}{\text{s}^2} \end{aligned}$$

This looks nice, and it's the right answer. But if you look at it now (or look back at it in a month), you won't know what you did. The quickest and easiest way to fix this is to number the steps and add a couple of words of description:

(1) Label quantities

> A $\overset{m}{\underline{25\text{ kg}}}$ cart is accelerated $\overset{v_o\,=\,0}{\underline{\text{from rest}}}$ to a velocity of $\overset{v}{\underline{3.5\,\frac{\text{m}}{\text{s}}}}$ over an interval of $\overset{t}{\underline{1.5\text{ s}}}$. Find the $\overset{\boxed{F_{net}}}{\underline{\text{net force}}}$ applied to the cart.

(2) Equation with desired quantity

(3) Need another equation for *a*

(4) Solve for *a*

(5) Substitute *a* into 1st equation

$$\begin{aligned} F_{net} &= ma \longrightarrow & v - v_o &= at \\ F_{net} &= 25a & 3.5 - 0 &= (a)(1.5) \\ F_{net} &= (25)(5.5) \longleftarrow & 3.5 &= 1.5a \\ F_{net} &= 138.\overline{8}\text{ N} & a &= 5.5\,\tfrac{\text{m}}{\text{s}^2} \end{aligned}$$

(6) Apply unit

The math is exactly the same as above, but notice that the annotated problem includes two features:

- Steps are numbered, so you can see what order the steps were in.
- Each step has a short descriptive phrase so you know exactly what was done and why.

Use this space for summary and/or additional notes:

Introduction: Laboratory & Measurement

Unit: Laboratory & Measurement

Topics covered in this chapter:

The purpose of this chapter is to teach skills necessary for designing and carrying out laboratory experiments, recording data, and writing summaries of the experiment in different formats.

- *The Scientific Method* describes scientific thinking and how it applies to physics and to this course.
- *The AP® Physics Science Practices* lists & describes the scientific practices that are required by the College Board for an AP® Physics course.
- *Designing & Performing Experiments* discusses strategies for coming up with your own experiments and carrying them out.
- *Random vs. Systematic Error, Uncertainty & Error Analysis,* and *Significant Figures* discuss techniques for estimating how closely measured data can quantitatively predict an outcome.
- *Graphical Solutions (Linearization)* discusses strategies for turning a relationship into a linear equation and using the slope of a best-fit line to represent the quantity of interest.
- *Keeping a Laboratory Notebook, Internal Laboratory Reports,* and *Formal Laboratory Reports* discuss ways in which you might record and communicate (write up) your laboratory experiments.

Calculating uncertainty (instead of relying on significant figures) is a new and challenging skill that will be used in lab write-ups throughout the year.

Use this space for summary and/or additional notes:

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

This chapter addresses the following MA science and engineering practices:

Practice 1: Asking Questions and Defining Problems

Practice 2: Developing and Using Models

Practice 3: Planning and Carrying Out Investigations

Practice 4: Analyzing and Interpreting Data

Practice 6: Constructing Explanations and Designing Solutions

Practice 7: Engaging in Argument from Evidence

Practice 8: Obtaining, Evaluating, and Communicating Information

AP®

AP® Physics 2 Learning Objectives & Science Practices:

This chapter addresses the following AP Physics 1 science practices:

SP 4.1 The student can justify the selection of the kind of data needed to answer a particular scientific question.

SP 4.2 The student can design a plan for collecting data to answer a particular scientific question.

SP 4.3 The student can collect data to answer a particular scientific question.

SP 4.4 The student can evaluate sources of data to answer a particular scientific question.

SP 5.1 The student can analyze data to identify patterns or relationships.

SP 5.2 The student can refine observations and measurements based on data analysis.

SP 5.3 The student can evaluate the evidence provided by data sets in relation to a particular scientific question.

Skills learned & applied in this chapter:

- Designing laboratory experiments
- Estimating uncertainty in measurements
- Propagating uncertainty through calculations
- Writing up lab experiments

Use this space for summary and/or additional notes:

The Scientific Method

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP1, SP2, SP6, SP7

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain how the scientific method can be applied to a problem or question.

Success Criteria:

- Steps in a specific process are connected in consistent and logical ways.
- Explanation correctly uses appropriate vocabulary.

Tier 2 Vocabulary: theory, model, claim, law, peer

Language Objectives:

- Understand and correctly use terms relating to the scientific method, such as "peer review".

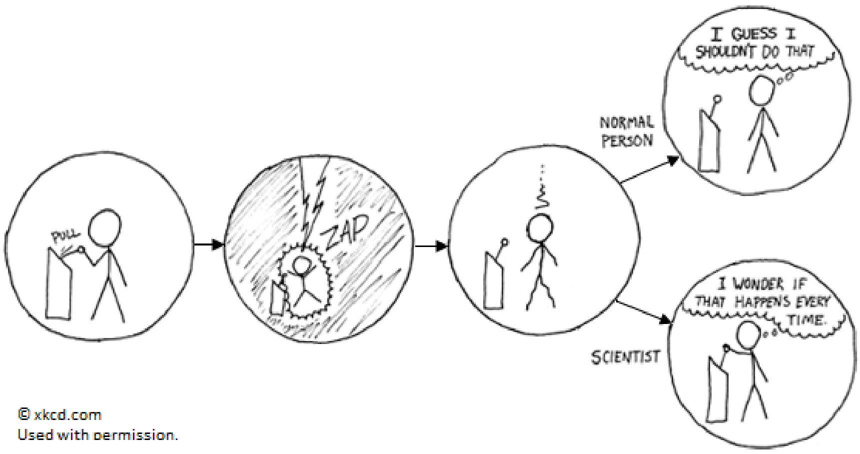
Tier 2 Vocabulary: N/A

Notes:

The scientific method is a fancy name for "figure out what happens by trying it."

In the middle ages, "scientists" were called "philosophers." These were church scholars who decided what was "correct" by a combination of observing the world around them and then arguing and debating with each other about the mechanisms and causes.

During the Renaissance, scientists like Galileo Galilei and Leonardo da Vinci started using experiments instead of argument to decide what really happens in the world.

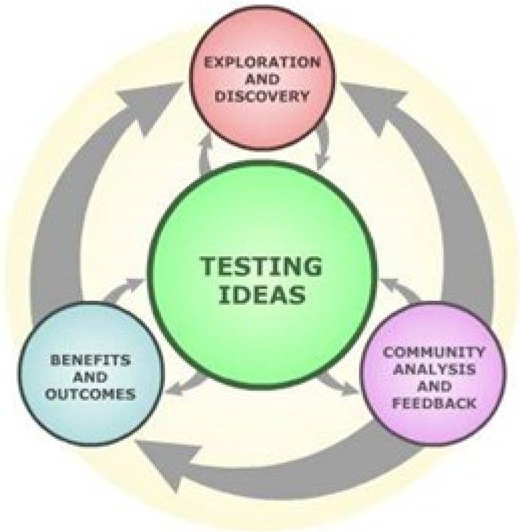


Use this space for summary and/or additional notes:

A Mindset, Not a Recipe

The scientific method is a mindset, which basically amounts to "let nature speak". Despite what you may have been taught elsewhere, the scientific method does not have specific "steps," and does not necessarily require a hypothesis.

The scientific method looks more like a map, with testing ideas (experimentation) at the center:

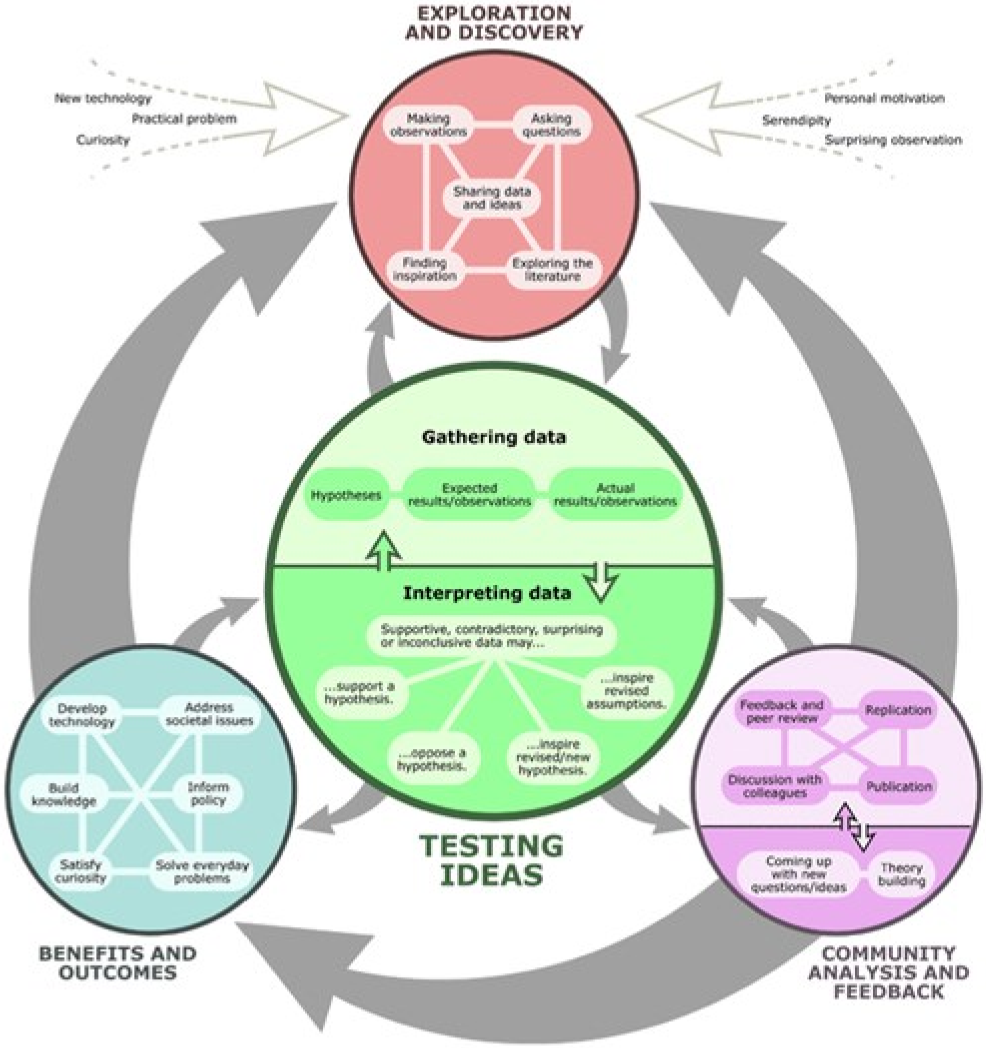


from the *Understanding Science* website[1]

[1] Understanding Science. 2018. University of California Museum of Paleontology. 1 July 2018 http://www.understandingscience.org. Used with permission.

Use this space for summary and/or additional notes:

Each of the circles in the above diagram is a broad area that contains many processes:



from the *Understanding Science* website

Use this space for summary and/or additional notes:

When scientists conclude something interesting that they think is important and want to share, they state it in the form of a ***claim***, which states that something happens, under what conditions it happens, and in some cases gives a possible explanation.

Before a claim is taken seriously, the original scientist and any others who are interested try everything they can think of to disprove the claim. If the claim holds up despite many attempts to disprove it, the claim gains support.

peer review: the process by which scientists scrutinize, evaluate and attempt to disprove each other's claims.

If a claim has gained widespread support among the scientific community and can be used to predict the outcomes of experiments (and it has *never* been disproven), it might eventually become a theory or a law.

theory: a claim that has never been disproven, that gives an explanation for a set of observations, and that can be used to predict the outcomes of experiments.

model: a way of viewing a set of concepts and their relationships to one another. A model is one type of theory.

law: a claim that has never been disproven and that can be used to predict the outcomes of experiments, but that does not attempt to model or explain the observations.

Note that the word "theory" in science has a different meaning from the word "theory" in everyday language. In science, a theory is a model that:

- *has never failed* to explain a collection of related observations
- *has never failed* to successfully predict the outcomes of related experiments

For example, the theory of evolution *has never failed* to explain the process of changes in organisms caused by factors that affect the survivability of the species.

If a repeatable experiment contradicts a theory, and the experiment passes the peer review process, the theory is deemed to be wrong. If the theory is wrong, it must either be modified to explain the new results, or discarded completely.

Use this space for summary and/or additional notes:

Theories *vs.* Natural Laws

The terms "theory" and "law" developed organically over many centuries, so any definition of either term must acknowledge that common usage, both within and outside of the scientific community, will not always be consistent with the definitions.

Nevertheless, the following rules of thumb may be useful:

A *theory* is a model that attempts to explain *why* or *how* something happens. A *law* simply describes or quantifies what happens without attempting to provide an explanation. Theories and laws can both be used to predict the outcomes of related experiments.

> For example, the *Law of Gravity* states that objects attract other objects based on their masses and distances from each other. It is a law and not a theory because the Law of Gravity does not explain *why* masses attract each other.
>
> *Atomic Theory* states that matter is made of atoms, and that those atoms are themselves made up of smaller particles. The interactions between these particles are used to explain certain properties of the substances. This is a theory because we cannot see atoms or prove that they exist. However, the model gives an explanation for *why* substances have the properties that they do.

A theory cannot become a law for the same reasons that a definition cannot become a measurement, and a postulate cannot become a theorem.

Use this space for summary and/or additional notes:

AP®

The AP® Physics Science Practices

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP1, SP2, SP3, SP4, SP5, SP6, SP7

AP® Physics 2 Learning Objectives: SP1, SP2, SP3, SP4, SP5, SP6, SP7

Mastery Objective(s): (Students will be able to…)

- Describe what the College Board and the State of Massachusetts want you to know about how science is done.

Tier 2 Vocabulary: data, claim, justify

Language Objectives:

- Explain the what the student is expected to do for each of the AP® Science Practices.

Tier 2 Vocabulary: practice, pose, model

Notes:

The College Board has described the scientific method in practical terms, dividing them into seven Science Practices that students are expected to learn in AP Physics 1.

Science Practice 1: The student can use representations and models to communicate scientific phenomena and solve scientific problems.

A model is any mental concept that can explain and predict how something looks, works, is organized, or behaves. Atomic theory is an example of a model: matter is made of atoms, which are made of protons, neutrons, and electrons. The number, location, behavior and interactions of these sub-atomic particles explains and predicts how different types of matter behave.

1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.

1.2 The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.

1.3 The student can *refine representations and models* of natural or man-made phenomena and systems in the domain.

1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

1.5 The student can express key elements of natural phenomena across multiple representations in the domain.

Use this space for summary and/or additional notes:

AP®

Science Practice 2: The student can use mathematics appropriately.

Physics is the representation of mathematics in nature. It is impossible to understand physics without a solid understand of mathematics and how it relates to physics. For AP Physics 1, this means having an intuitive feel for how algebra works, and how it can be used to relate quantities or functions to each other. If you are the type of student who solves algebra problems via memorized procedures, you may struggle to develop the kind of mathematical understanding that is necessary in AP Physics 1.

2.1 The student can justify the selection of a mathematical routine to solve problems.

2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

2.3 The student *can estimate numerically quantities* that describe natural phenomena.

Science Practice 3: The student can engage in scientific questioning to extend thinking or to guide investigations within the context of the AP course.

Ultimately, the answer to almost any scientific question is “maybe” or “it depends”. Scientists pose questions to understand not just what happens, but the extent to which it happens, the causes, and the limits beyond which outside factors become dominant.

3.1 The student can pose scientific questions.

3.2 The student can refine scientific questions.

3.3 The student can evaluate scientific questions.

Science Practice 4: The student can plan and implement data collection strategies in relation to a particular scientific question.

Scientists do not “prove” things. Mathematicians and lawyers prove that something must be true. Scientists collect data in order to evaluate what happens under specific conditions, in order to determine what is likely true, based on the information available. Data collection is important, because the more and better the data, the more scientists can determine from it.

4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.

4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

4.3 The student can *collect data* to answer a particular scientific question.

Use this space for summary and/or additional notes:

4.4 The student can *evaluate sources of data* to answer a particular scientific question.

Science Practice 5: The student can perform data analysis and evaluation of evidence.

Just as data collection is important, analyzing data and being able to draw meaningful conclusions is the other crucial step to understanding natural phenomena. Scientists need to be able to recognize patterns that actually exist within the data, and to be free from the bias that comes from expecting a particular result beforehand.

5.1 The student can *analyze data* to identify patterns or relationships.

5.2 The student can *refine observations and measurements* based on data analysis.

5.3 The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.

Science Practice 6: The student can work with scientific explanations and theories.

In science, there are no "correct" answers, only claims and explanations. A scientific claim is any statement that is believed to be true. In order to be accepted, a claim must be verifiable based on evidence, and any claim or explanation must be able to make successful predictions, which are also testable. Science does not prove claims to be universally true or false; science provides supporting evidence. Other scientists will accept or believe a claim provided that there is sufficient evidence to support it, and no evidence that directly contradicts it.

6.1 The student can justify claims with evidence.

6.2 The student can *construct explanations of phenomena based on evidence* produced through scientific practices.

6.3 The student can articulate the reasons that scientific explanations and theories are refined or replaced.

6.4 The student can make *claims and predictions about natural phenomena* based on scientific theories and models.

6.5 The student can evaluate alternative scientific explanations.

Use this space for summary and/or additional notes:

AP®

Science Practice 7: The student is able to connect and relate knowledge across various scales, concepts, and representations in and across domains.

If a scientific principle is true in one domain, scientists must be able to consider that principle in other domains and apply their understanding from the one domain to the other. For example, conservation of momentum is believed by physicists to be universally true on every scale and in every domain, and it has implications in the contexts of laboratory-scale experiments, quantum mechanical behaviors at the atomic and sub-atomic levels, and special relativity.

7.1 The student can *connect phenomena and models* across spatial and temporal scales.

7.2 The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

Use this space for summary and/or additional notes:

Designing & Performing Experiments

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP1, SP3, SP8

AP® Physics 2 Learning Objectives: SP 1, SP2, SP3, SP4, SP5, SP6, SP7

MA Curriculum Frameworks (2006): N/A

Mastery Objective(s): (Students will be able to…)

- Create a plan and procedure to answer a question through experimentation.

Success Criteria:

- Experimental Design utilizes backward design.
- Experimental Design uses logical steps to connect the desired answer or quantity to quantities that can be observed or measured.
- Procedure gives enough detail to set up experiment.
- Procedure establishes values of control and independent variables.
- Procedure explains how to measure dependent variables.

Tier 2 Vocabulary: inquiry, independent, dependent, control

Language Objectives:

- Understand and correctly use the terms "dependent variable" and "independent variable."
- Understand and be able to describe the strategies presented in this section.

Tier 2 Vocabulary: design, perform, control, independent (variable), dependent (variable)

Notes:

If your experience in science classes is like that of most high school students, you have always done "experiments" that were devised, planned down to the finest detail, painstakingly written out, and debugged before you ever saw them. You learned to faithfully follow the directions, and as long as everything that happened matched the instructions, you knew that the "experiment" must have come out right.

If someone asked you immediately after the "experiment" what you just did or what its significance was, you had no answers for them. When it was time to do the analysis, you followed the steps in the handout. When it was time to write the lab report, you had to frantically read and re-read the procedure in the hope of understanding enough of what the "experiment" was about to write something intelligible.

This is not how science is supposed to work.

Use this space for summary and/or additional notes:

In an actual scientific experiment, you would start with an objective, purpose or goal. You would figure out what you needed to know, do, and/or measure in order to achieve that objective. Then you would set up your experiment, observing, doing and measuring the things that you decided upon. Once you had your results, you would figure out what those results told you about what you needed to know. At that point, you would draw some conclusions about how well the experiment worked, and what to do next.

That is precisely how experiments work in this course. You and your lab group will design every experiment that you perform. You will be given an objective or goal and a general idea of how to go about achieving it. You and your lab group (with help) will decide the specifics of what to do, what to measure (and how to measure it), and how to make sure you are getting good results. The education "buzzword" for this is ***inquiry-based experiments***.

Types of Experiments

There are many ways to categorize experiments. For the purpose of this discussion, we will categorize them as either qualitative experiments or quantitative experiments.

Qualitative Experiments

If you are trying to cause something to happen, observe whether or not something happens, or determine the conditions under which something happens, you are performing a qualitative experiment. Your experimental design section needs to address:

- What it is that you are trying to observe or measure.
- If something needs to happen, what you will do to try to make it happen.
- How you will observe it.
- How you will determine whether or not the thing you were looking for actually happened.

Often, determining whether or not the thing happened is the most challenging part. For example, in atomic & particle physics (as was also the case in chemistry), what "happens" involves atoms and sub-atomic particles that are too small to see. For example, you might detect radioactive decay by using a Geiger counter to detect charged particles that are emitted.

Use this space for summary and/or additional notes:

Quantitative Experiments

If you are trying to determine the extent to which something happens, your experiment almost certainly involves measurements and calculations. Your experimental design section needs to address:

- What it is that you are trying to measure.
- If something needs to happen, what you will do to try to make it happen.
- What you can actually measure, and how to connect it to the quantities of interest.
- How to set up your experimental conditions so the quantities that you will measure are within measurable limits.
- How to calculate and interpret the quantities of interest based on your results.

What to Control and What to Measure

In every experiment, there are some quantities that you need to keep constant, some that you need to change, and some that you need to observe. These are called ***control variables***, ***independent variables***, and ***dependent variables***.

<u>control variables</u>: conditions that are being kept constant. These are usually parameters that could be independent variables in a different experiment, but are being kept constant so they do not affect the relationship between the variables that you are testing in this experiment. For example, if you are dropping a ball from different heights to find out how long it takes to hit the ground, you want to make sure the wind is the same speed and direction for each trial, so wind does not affect the outcome of the experiment. This means wind speed and direction are *control* variables.

<u>independent variables</u>: the conditions you are setting up. These are the parameters that you specify when you set up the experiment. You are choosing the values for these variables, so they are *independent* of what happens in the experiment. For example, if you are dropping a ball from different heights to find out how long it takes to hit the ground, you are choosing the heights before the experiment begins, so height is the *independent* variable.

<u>dependent variable</u>: the things that happen during the experiment. These are the quantities that you won't know the values for until you measure them, because they are *dependent* on what happens in the experiment. For example, if you are dropping a ball from different heights to find out how long it takes to hit the ground, the times depend on what happens after you let go of the ball. This means time is the *dependent* variable.

Use this space for summary and/or additional notes:

If someone asks what your independent, dependent and control variables are, the question simply means:

- "What did you vary on purpose (independent variables)?"
- "What did you measure (dependent variables)?"
- "What did you keep the same for each trial (control variables)?"

Variables in Qualitative Experiments

If the goal of your experiment is to find out ***whether or not*** something happens at all, you need to set up a situation in which the phenomenon you want to observe can either happen or not, and then observe whether or not it does. The only hard part is making sure the conditions of your experiment don't bias whether the phenomenon happens or not.

If you want to find out ***under what conditions*** something happens, what you're really testing is whether or not it happens under different sets of conditions that you can test. In this case, you need to test three situations:

1. A situation in which you are sure the thing will happen, to make sure you can observe it. This is your **positive control**.
2. A situation in which you sure the thing cannot happen, to make sure your experiment can produce a situation in which it doesn't happen and you can observe its absence. This is your **negative control**.
3. A condition or situation that you want to test to see whether or not the thing happens. The condition is your **independent variable**, and whether or not the thing happens is your **dependent variable**.

Variables in Quantitative Experiments

If the goal of your experiment is to quantify (find a numerical relationship for) the extent to which something happens (the dependent variable), you need to figure out a set of conditions that enable you to measure the thing that happens. Once you know that, you need to figure out how much you can change the parameter you want to test (the independent variable) and still be able to measure the result. This gives you the highest and lowest values of your independent variable. Then perform the experiment using a range of values for the independent value that cover the range from the lowest to the highest (or *vice-versa*).

For quantitative experiments, a good rule of thumb is the **8 & 10 rule**: you should have at least 8 data points, and the range from the highest to the lowest values tested should span at least a factor of 10.

Use this space for summary and/or additional notes:

Letting the Physics Design the Experiment

Most high school physics experiments are relatively simple to understand, set up and execute—much more so than in chemistry or biology. This makes physics well-suited for teaching you how to design experiments.

Determining what to measure usually means determining what you need to know and then figuring out how to get there starting from *quantities that you can measure*.

For a quantitative experiment, if you have a mathematical formula that includes the quantity you want to measure, you need to find the values of the other quantities in the equation.

For example, suppose you need to calculate the force of friction that brings a sliding object to a stop. If we design the experiment so that there are no other horizontal forces, friction will be the net force. We can then calculate force from the equation for Newton's Second Law:

$$F_f = F_{net} = \underline{\boldsymbol{m}}a$$

In order to use this equation to calculate force, we need to know:

- **<u>mass</u>**: we can measure this directly, using a balance. (*Note that* ***<u>m</u>*** *is underlined because we can measure it directly, which means we don't need to pursue another equation to calculate it.*)
- **<u>acceleration</u>**: we could measure this with an accelerometer, but we do not have one in the lab. This means we will need to find the acceleration some other way.

Because we need to *calculate* acceleration rather than measuring it, that means we need to expand our experiment in order to get the necessary data to do so. Instead of just measuring force and acceleration, we now need to:

1. Measure the mass.
2. *Perform an experiment* in which we apply the force and collect enough information to *determine the acceleration*.
3. Calculate the force on the object, using the mass and the acceleration.

Use this space for summary and/or additional notes:

In order to determine the acceleration, we need another equation. We can use:

$$\underline{\boldsymbol{v}} = v_o + a\underline{\boldsymbol{t}}$$

This means in order to calculate acceleration, we need to know:

- **<u>final velocity</u> (*v*)**: the force is being applied until the object is at rest (stopped), so the final velocity $v = 0$. (*Underlined because we have designed the experiment in a way that we know its value.*)
- **<u>initial velocity</u> (v_o)**: not known; we need to either measure or calculate this.
- **<u>time</u> (*t*)**: we can measure this directly with a stopwatch. (*Underlined because we can measure it directly.*)

Now we need to expand our experiment further, in order to calculate v_o. We can calculate the initial velocity from the equation:

$$\boldsymbol{v}_{ave.} = \frac{\underline{\boldsymbol{d}}}{\underline{\boldsymbol{t}}} = \frac{v_o + \cancelto{0}{\underline{\boldsymbol{v}}}}{2}$$

We have already figured out how to measure $\underline{\boldsymbol{t}}$, and we set up the experiment so that $\underline{\boldsymbol{v}} = 0$ at the end. This means that to calculate v_o, the only quantities we need to measure are:

- **<u>time</u> (*t*)**: as noted above, we can measure this directly with a stopwatch. (*Underlined because we can measure it directly.*)

- **<u>displacement</u> (*d*)**: the change in the object's position. We can measure this with a meter stick or tape measure. (*Underlined because we can measure it.*)

Notice that every quantity is now expressed in terms of quantities that we know or can measure, or quantities we can calculate, so we're all set. We simply need to set up an experiment to measure the underlined quantities.

Use this space for summary and/or additional notes:

To facilitate this approach, it is often helpful to use a table. For the above experiment, such a table might look like the following:

Desired Variable	**Equation**	**Description/ Explanation**	**Fixed Control Variable(s) or Constants**	**Quantities to be Measured**	**Quantities to be Calculated (Still Needed)**
$\vec{F}_f$	$\vec{F}_f = \vec{F}_{net}$	Set up experiment so other forces cancel	—	—	$\vec{F}_{net}$
$\vec{F}_{net}$	$\vec{F}_{net} = m\vec{a}$	Newton's 2nd Law	—	m	$\vec{a}$
$\vec{a}$	$\vec{v} - \vec{v}_o = \vec{a}t$	Kinematics equation	$\vec{v} = 0$	t	$\vec{v}_o$
$\vec{v}_o$	$\frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$	Kinematics equation	$\vec{v} = 0$	$\vec{d}$, t	—

In this table, we started with the quantity we want to determine ($\vec{F}_f$). We found an equation that contains it ($\vec{F}_f = \vec{F}_{net}$). In that equation, $\vec{F}_{net}$ is neither a fixed control variable nor a constant and we cannot measure it, so it is "still needed" and becomes the start of a new row in the table.

This process continues until every quantity that is needed is either a Fixed quantity (control variable or constant) or can be measured, and there are no quantities that are still needed.

- Notice that every variable in the equation is either the desired variable, or it appears in one of the three columns on the right.
- Notice that when we get to the third row, the equation contains a control variable that is designed into the experiment ($\vec{v} = 0$ because the object stops at the end), a quantity that can be measured (t, using a stopwatch), and a quantity that is still needed ($\vec{v}_o$).
- Notice that every quantity that you need to measure appears in the "Quantities to be Measured" column.
- Notice that your experimental conditions need to account for the control variables in the "Fixed Control Variable(s) or Constants" column.
- Notice that your calculations are simply the entire "Equation" column, starting at the bottom and working your way back to the top.

Use this space for summary and/or additional notes:

Procedure

Looking at the "Quantities to be Measured" column, we have determined that we need to measure mass (*m*), time (*t*) and distance (*d*). This makes sense because the objective is to determine the force of friction, which means we need to measure the distance that the object travels and the time that it spends moving while it is sliding.

Our procedure is therefore to (a) make sure the event that we are trying to measure happens, (b) measure everything in the "Measured Variable(s)" column, and (c) set up the experiment so that everything in the "Control Variable(s) & Constants" column has the appropriate values:

1. Measure the **mass** of the object.
2. Determine a way to measure the **displacement** of the object. (A simple way would be to mark a starting line and use a tape measure.)
3. Determine a way to measure the **time** that the object spends moving. (A simple way would be to start a stopwatch when the object crosses the starting line and stop the stopwatch when the object comes to rest.)
4. Get the object moving.
5. Allow the object to slide until it stops (**final velocity** = 0), measuring time and displacement as determined above.
6. Repeat the experiment, using different masses based on the **8 & 10 rule**—take at least **8 data points**, varying the mass over at least a **factor of 10**.

Data

We need to make sure we have recorded the measurements (including uncertainties, which are addressed in the Uncertainty & Error Analysis topic, starting on page 53) of every quantity we need in order to calculate our result. In this experiment, we need measurements for **mass**, **displacement** and **time**.

Use this space for summary and/or additional notes:

Analysis

Most of your analysis is your calculations. Start from the bottom of your experimental design table and work upward.

$$\frac{d}{t}=\frac{v_o+\cancel{v}^{\,0}}{2}$$

In this experiment that means start with:

$$v_o=\frac{2d}{t}$$

The reason we needed this equation was to find v_o, so we need to rearrange it to:

(We are allowed to use *d* and *t* in the equation because we measured them.)

$$\cancel{v}^{\,0}=v_o+at$$

Now we go to the equation above it in our experimental design and substitute our expression for v_o into it:

$$0=\frac{2d}{t}+at$$

The purpose of this equation was to find acceleration, so we need to rearrange it to:

$$a=\frac{-2d}{t^2}$$

(We can drop the negative sign because we are only interested in the magnitude of the acceleration.)

Our last equation is $F_f=F_{net}=ma$. If we are interested only in finding one value of F_f, we can just substitute and solve:

$$F_{net}=ma=m\left(-\frac{2d}{t^2}\right)=\frac{-2md}{t^2}$$

However, we will get a much better answer if we plot a graph relating each of our values of mass (remember the 8 & 10 rule) to the resulting acceleration and calculate the force using the graph. This process is described in detail in the "Graphical Solutions (Linearization)" section, starting on page 73.

Use this space for summary and/or additional notes:

Generalized Approach

The generalized approach to experimental design is therefore:

Experimental Design

1. Find an equation that contains the quantity you want to find.
2. Using a table to organize your information, work your way from that equation through related equations until every quantity in every equation is either something you can calculate or something you can measure.

Procedure

3. Determine how to measure each of the quantities that you need (dependent variables). Decide what your starting conditions need to be (independent variables) and measure any that are needed, and figure out what you need to keep constant (control variables).

Data & Observations

4. Set up your experiment and do a test run. *This means you need to perform the calculations for your test run before doing the rest of the experiment,* in case you need to modify your procedure. You will be extremely frustrated if you finish your experiment and go home, only to find out at 2:00 am the night before the write-up is due that it didn't work.
5. Take and record your measurements and other data.
6. Remember to record the uncertainty for every quantity that you measure. (See the "Uncertainty & Error Analysis" section, starting on page 53.)

Analysis

7. Calculate the results. Whenever possible, apply the **8 & 10 rule** and calculate your answer graphically.

AP® If you are taking one of the AP® Physics exams, you should answer the experimental design question by writing the Experimental Design, Procedure, and Analysis sections above.

Use this space for summary and/or additional notes:

honors (not AP®)

Random *vs.* Systematic Error

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP3

AP® Physics 2 Learning Objectives: SP4

Mastery Objective(s): (Students will be able to…)

- Correctly use the terms "random error" and "systematic error" in a scientific context.
- Explain the difference between random and systematic errors.

Success Criteria:

- Be able to recognize situations as accurate/inaccurate and/or precise/imprecise.

Tier 2 Vocabulary: accurate, precise

Language Objectives:

- Be able to describe the difference between random errors and systematic errors.

Tier 2 Vocabulary: random, systematic, accurate, precise

Notes:

Science relies on making and interpreting measurements, and the accuracy and precision of these measurements affect what you can conclude from them.

Random vs. Systematic Errors

<u>random errors</u>: are natural uncertainties in measurements because of the limits of precision of the equipment used. Random errors are assumed to be distributed around the actual value, without bias in either direction.

<u>systematic errors</u>: occur from specific problems in your equipment or your procedure. Systematic errors are often biased in one direction more than another, and can be difficult to identify.

Use this space for summary and/or additional notes:

"Accuracy" *vs.* "Precision"

honors (not AP®)

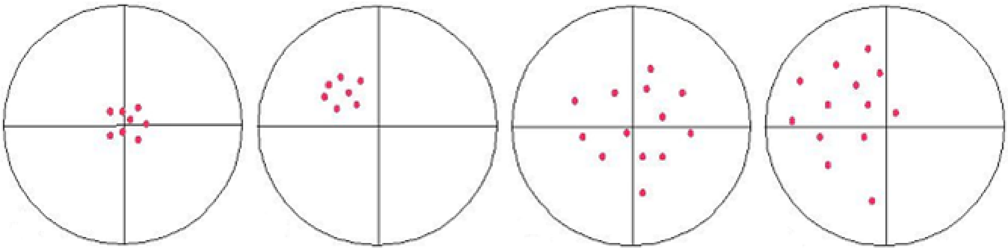
The words "accuracy" and "precision" are not used in science because these words are often used as synonyms in everyday English. However, because some high school science teachers insist on using the terms, their usual meanings are:

accuracy: the amount of systematic error in a measurement. A measurement is said to be accurate if it has low systematic error.

precision: either how finely a measurement was made or the amount of random error in a set of measurements. A single measurement is said to be precise if it was measured within a small fraction of its total value. A group of measurements is said to be precise if the amount of random error is small (the measurements are close to each other).

Examples:

Suppose the following drawings represent arrows shot at a target.



low random error low systematic error	low random error high systematic error	high random error low systematic error	high random error high systematic error

The first set has *low random error* because the points are close to each other. It has *low systematic error* because the points are approximately equally distributed about the expected value.

The second set has *low random error* because the points are close to each other. However, it has *high systematic error* because the points are centered on a point that is noticeably far from the expected value.

The third set has *low systematic error* because the points are approximately equally distributed around the expected value. However, it has *high random error* because the points are not close to each other.

The fourth set has *high random error* because the points are not close to each other. It has *high systematic error* because the points are centered on a point that is noticeably far from the expected value.

Use this space for summary and/or additional notes:

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For another example, suppose a teacher is 55 years old, and two of their classes estimate their age.

High Systematic Error

The first class's estimates are 72, 73, 77, and 78 years old. These measurements have low random error because they are close together, but high systematic error (because the average is 75, which is far from the expected value of 55).

When there is a significant amount of systematic error, it often means there is some problem with the way the experiment was set up or performed (or a problem with the equipment) that caused all of the numbers to be off in the same direction.

In this example, the teacher may have gray hair and very wrinkled skin, and may appear much older than they actually are.

High Random Error

The second class's estimates are 10, 31, 77 and 98. This *set* of data has low systematic error (because the average is 54, which is close to the expected value), but high random error because the individual values are not close to each other.

When there is a significant amount of random error, it can also mean a problem with the way the experiment was set up or performed (or a problem with the equipment). However, it can also mean that the experiment is not actually measuring what the scientist thinks it is measuring.

If there is a lot of random error, it can look like there is no relationship between the independent variables and the dependent variables. If there is no relationship between the independent variables and the dependent variables, it can look like there is a lot of random error. Scientists must consider both possibilities.

In this example, the class may have not cared about providing valid numbers, or they may not have realized that the numbers they were guessing were supposed to be the age of a person.

Use this space for summary and/or additional notes:

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Uncertainty & Error Analysis

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP4

AP® Physics 2 Learning Objectives: SP5

Mastery Objective(s): (Students will be able to…)

- Determine the uncertainty of a measured or calculated value.

Success Criteria:

- Take analog measurements to one extra digit of precision.
- Correctly estimate measurement uncertainty.
- Correctly read and interpret stated uncertainty values.
- Correctly propagate uncertainty through calculations involving addition/subtraction and multiplication/division.

Tier 2 Vocabulary: uncertainty, error

Language Objectives:

- Understand and correctly use the terms "uncertainty" and "relative error."
- Correctly explain the process of estimating and propagating uncertainty.

Tier 2 Vocabulary: uncertainty, error

Notes:

In science, unlike mathematics, there is no such thing as an exact answer. Ultimately, every quantity is limited by the precision and accuracy of the measurements that it came from. If you can only measure a quantity to within 10 %, that means any calculation that is derived from that measurement can't be any better than ±10 %.

Error analysis is the practice of determining and communicating the causes and extents of uncertainty in your results. Error analysis involves understanding and following the uncertainty in your data, from the initial measurements to the final calculated and reported results.

Note that the word "error" in science has a different meaning from the word "error" in everyday language. In science, **"error" means "uncertainty."** If you report that you drive (2.4 ± 0.1) miles to school every day, you would say that this distance has an error of ±0.1 mile. This does not mean your car's odometer is wrong; it means that the actual distance *could be* 0.1 mile more or 0.1 mile less—*i.e.,* somewhere between 2.3 and 2.5 miles. ***When you are analyzing your results, <u>never</u> use the word "error" to mean mistakes that you might have made!***

Use this space for summary and/or additional notes:

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Uncertainty

The uncertainty or error of a measurement describes how close the actual value is likely to be to the measured value. For example, if a length was measured to be 22.3 cm, and the uncertainty was 0.3 cm (meaning that the measurement is only known to within ± 0.3 cm), we could represent this measurement in either of two ways:

22.3 ± 0.3 cm[2]   22.3(3) cm

The first of these states the variation (±) explicitly in cm (the actual unit). The second is shows the variation in the last digits shown.

What it means is that the true length is approximately 22.3 cm, and is statistically likely[3] to be somewhere between 22.0 cm and 22.6 cm.

Absolute Error

Absolute error (or absolute uncertainty) refers to the uncertainty in the actual measurement. For the measurement 22.3 ± 0.3 cm, the absolute error is ± 0.3 cm.

Relative Error

Relative error shows the error or uncertainty as a fraction of the total.

The formula for relative error is $\text{R.E.} = \dfrac{\text{uncertainty}}{\text{measured value}}$

For the measurement 22.3 ± 0.3 cm, the relative error would be 0.3 cm out of 22.3 cm. Mathematically, we express this as:

$$\text{R.E.} = \frac{0.3\,\cancel{\text{cm}}}{22.3\,\cancel{\text{cm}}} = 0.013$$

Note that relative error is dimensionless (does not have any units), because the numerator and denominator have the same units, which means the units cancel.

Percent Error

Percent error is relative error expressed as a percentage. You can turn relative error into percent error by multiplying by 100.

In the example above, the relative error of 0.013 would be 1.3 % error.

---

[2] The unit is assumed to apply to both the value and the uncertainty. The unit for the value and uncertainty should be the same. A value of 10.63 m ± 2 cm should be rewritten as 10.63 ± 0.02 m

[3] Statistically, the standard uncertainty is one standard deviation, which is discussed on the following page.

Use this space for summary and/or additional notes:

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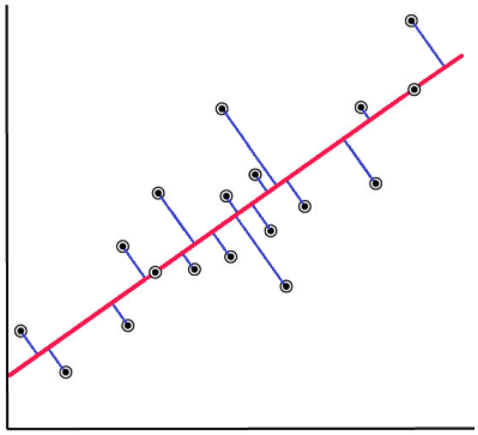
Best-Fit Lines & Standard Deviation

<u>best-fit line</u>: a line that represents the expected value of your dependent variable for values of your dependent variable. The best-fit line minimizes the total accumulated error (difference between each actual data point and the line).

<u>standard deviation</u> (σ): the average of how far each data point is from its expected value.

The standard deviation is calculated mathematically as the average difference between each data point and the value predicted by the best-fit line.

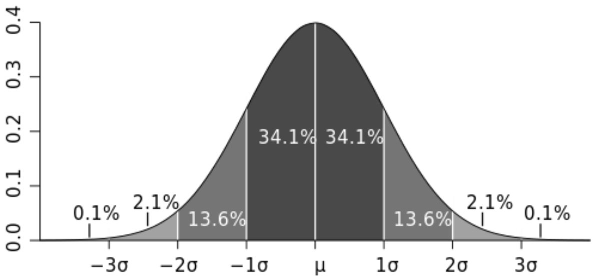


A small standard deviation means that most or all of the data points lie close to the best-fit line. A larger standard deviation means that on average, the data points lie farther from the line.

Unless otherwise stated, ***the standard deviation is the uncertainty (the "plus or minus") of a calculated quantity***. *E.g.,* a measurement of 25.0 cm with a standard deviation of 0.5 cm would be expressed as (25.0 ± 0.5) cm.

The expected distribution of values relative to the mean is called the Gaussian distribution (named after the German mathematician Carl Friedrich Gauss.)

It looks like a bell, and is often called a "bell curve".



Statistically, approximately 68 % of the measurements are expected to fall within one standard deviation of the mean, *i.e.,* within the standard uncertainty.

<u>correlation coëfficient</u> (R or R^2 value): a measure of how linear the data are—how well they approximate a straight line. In general, an R^2 value of less than 0.9 means that the data are not linear, there was a problem with one or more data points, or there was a problem with the entire experiment.

Use this space for summary and/or additional notes:

Uncertainty of Measurements

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If you have the ability to measure a quantity that is not changing (such as the mass or length of an object), you will get the same value every time you measure it. This means you have only one data point.

When you have only one data point, the uncertainty is the limit of how well you can measure it. This will be your best educated guess, based on how closely you think you actually measured the quantity. This means you need to take measurements as carefully and precisely as possible, because *every careless measurement needlessly increases the uncertainty of the result*.

Digital Measurements

For digital equipment, if the reading is *stable* (not changing), look up the published precision of the instrument in its user's manual. (For example, many balances used in high schools have a readability of 0.01 g but are only precise to within ± 0.02 g.) If there is no published value (or the manual is not available), assume the uncertainty is ± 1 in the last digit.

If the reading is *unstable* (changing), state the reading as the average of the highest and lowest values, and the uncertainty as the amount that you would need to add to or subtract from the average to obtain either of the extremes (but never less than the published uncertainty of the equipment).

Use this space for summary and/or additional notes:

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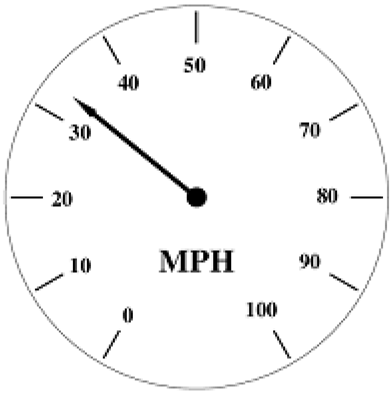
Analog Measurements

When making analog measurements, *always* estimate one extra digit beyond the finest markings on the equipment. For example, if you saw the speedometer on the left, you would imagine that each tick mark was divided into ten smaller tick marks like the one on the right.



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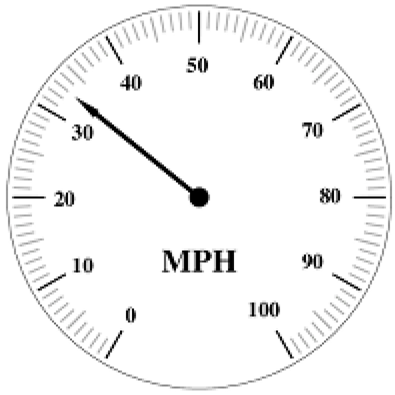

what you see:	what you visualize:
between 30 & 40 MPH	33 ± 1 MPH

Note that the ***measurement and uncertainty must be expressed to the same decimal place***.

Use this space for summary and/or additional notes:

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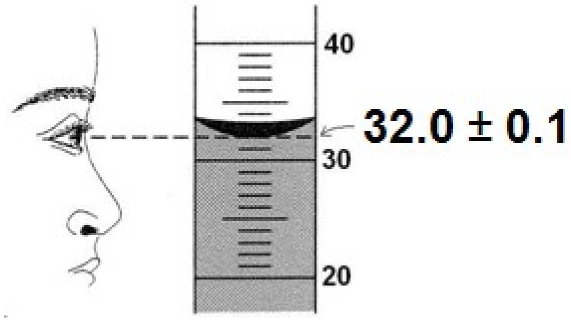
For example, in the diagram below, the graduated cylinder is marked in 1 mL increments. When measuring volume in this graduated cylinder[4], you would estimate and write down the volume to the nearest 0.1 mL, as shown:



In the above experiment, you *must* record the volume as:

32.0 ± 0.1 mL ← correct

32 ± 0.1 mL ← wrong

32 ± 1 mL ← inadequate

In other words, the zero at the end of 32.0 mL is *required*. It is necessary to show that *you measured the volume to the nearest tenth, not to the nearest one.*

When estimating, the uncertainty depends on how well you can see the markings, but you can usually assume that the estimated digit has an uncertainty of $\pm \frac{1}{10}$ of the finest markings on the equipment. Here are some examples:

Equipment	Typical Markings	Estimate To	Assumed Uncertainty
ruler	1 mm	0.1 mm	± 0.1 mm
25 mL graduated cylinder	0.2 mL	0.02 mL	± 0.02 mL
thermometer	1 °C	0.1 °C	± 0.1 °C

[4] Remember that for most liquids, which have a downward meniscus, volume is measured at the *bottom* of the meniscus.

Use this space for summary and/or additional notes:

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Propagating Uncertainty in Calculations

When you perform calculations using numbers that have uncertainty, you need to propagate the uncertainty through the calculation.

Addition & Subtraction

When quantities with uncertainties are added or subtracted, add the quantities to get the answer, then add the uncertainties to get the total uncertainty.

Sample Problem:

Q: A substance is being heated. You record the initial temperature as (23.0 ± 0.2) °C, and the final temperature as (84.4 ± 0.2) °C. You need to calculate the temperature change (ΔT) with its uncertainty to use in a later calculation. What is the temperature change?

A: To calculate ΔT, simply subtract:

$$\Delta T = T_{final} - T_{initial} = 84.4 - 23.0 = 61.4\,°\text{C}$$

To calculate the uncertainty, add the individual uncertainties (even though the quantities were subtracted):

$$u = 0.2 + 0.2 = 0.4\,°\text{C}$$

Report the value as: $\Delta T = (61.4 \pm 0.4)\,°\text{C}$

Multiplication & Division

Because most calculations that we will perform in physics involve multiplication and/or division, you can

For calculations involving multiplication and division, estimate the uncertainty of your calculated answer by adding the relative errors and applying the total relative error to your result.

1. Perform the calculation for the desired quantity.
2. Divide the uncertainty (the ±) for each quantity by its measured value to determine its relative error.

$$\text{R.E.} = \frac{\text{uncertainty}}{\text{measured value}}$$

3. Add up all of the relative errors to get the total relative error.
4. Multiply your calculated result by the total relative error to get its uncertainty (the ± amount).

Use this space for summary and/or additional notes:

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Note: ***Most of the calculations that you will perform in physics involve multiplication and/or division, so almost all of your uncertainty calculations throughout the course will use relative error.***

Exponents

Calculations that involve ***exponents*** use the same rule as for multiplication and division. If you think of exponents as multiplying a number by itself the indicated number of times, it means you would need to add the relative error of that number that many times.

In other words, when a value is raised to an exponent, multiply its relative error by the exponent.

Sample Problem:

Q: You want to determine the amount of heat released by a process. You use the heat from the reaction to heat up some water in an insulated container called a calorimeter. You will calculate the heat using the equation: $Q = mC\Delta T$.

Suppose you recorded the following data (including uncertainties):

- The mass of the water in the calorimeter is (24.8 ± 0.1) g.
- The temperature change of the water was (12.4± 0.2) °C.
- The specific heat capacity of water is $4.18 \frac{\text{J}}{\text{g}\bullet{}^{\circ}\text{C}}$. (This is a published value. The uncertainty of this value is so small that we can leave it out of our calculations.)

A: The heat released by the reaction is given by the equation:

$$Q = mC\Delta T$$
$$Q = (24.8)(4.18)(12.4)$$
$$Q = 1\,285.43\ \text{J}$$

The relative errors for the two quantities that we measured are:

- mass: $\dfrac{0.1}{24.8} = 0.004\,03$
- temperature change: $\dfrac{0.2}{12.4} = 0.016\,13$

The total relative error is 0.004 03 + 0.016 13 = 0.020 16

The uncertainty is therefore (0.020 16)(1 285.43) = ±25.92 J

(Note that the absolute uncertainty has the same units as the measurement.)

We would report the measurement as (1 285.43 ± 25.92) J.

Use this space for summary and/or additional notes:

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Rounding

In the example above, the uncertainty tells us that our actual result could be different from our calculated value by as much as 25.92 J.

However, we only estimated one digit (which happened to be the tenths place) when we took our measurements. This means we have only one digit of uncertainty. Because we can't report more precision than we actually have, we need to round the calculated uncertainty off, so that we have only one unrounded digit. This means we should report our uncertainty as ± 30 J.

It wouldn't make sense to report our answer as (1 285.43 ± 30) J. Think about that—if the *tens* digit could be different from our calculated value, there is no point in reporting the ones or tenths digits. So we need to round our calculated answer to the same place value as the uncertainty—the tens place.

This means our final, rounded answer should be (1 290 ± 30) J.

Use this space for summary and/or additional notes:

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Sample Problem #2:

Q: You need to find the density of a piece of metal. We measure its mass on a balance to be (24.75 ± 0.02) g. You measure its volume in a graduated cylinder using water displacement, and you find the volume to be (7.2 ± 0.1) mL. Calculate the density, including its uncertainty.

A: 1. Calculate the density.

$$\rho = \frac{m}{V} = \frac{24.75\text{ g}}{7.2\text{ mL}} = 3.4375\,\tfrac{\text{g}}{\text{mL}}$$

2. Calculate the relative errors of your two measurements:

$$R.E._{mass} = \frac{\text{uncertainty}}{\text{measured value}} = \frac{0.02}{24.75} = 0.000\,808$$

$$R.E._{volume} = \frac{0.1}{7.2} = 0.013\,889$$

3. Add the individual relative errors together to get the total R.E.:

$$R.E._{mass} + R.E._{volume} = R.E._{total}$$

$$0.000\,808 + 0.013\,889 = 0.014\,697$$

4. Multiply the total R.E. by the density to get the uncertainty:

$$3.4375 \times 0.014697 = 0.050521$$

Because you only estimated one decimal place of uncertainty, you need to round the uncertainty off to ± 0.05.

Because uncertainty is rounded to the hundredths place, you need to also round your answer to the hundredths place:

$$\rho = (3.44 \pm 0.05)\,\tfrac{\text{g}}{\text{mL}}$$

Use this space for summary and/or additional notes:

Honors (not AP®)

Homework Problems

Because the answers are provided, you must show sufficient work in order to receive credit.

1. **(M = Must Do)** In a 4 × 100 m relay race, the four runners' times were: (10.52 ± 0.02) s, (10.61 ± 0.01) s, (10.44 ± 0.03) s, and (10.21 ± 0.02) s. What was the team's (total) time for the event, including the uncertainty?

   Answer: (41.78 ± 0.08) s

2. **(M = Must Do)** A baseball pitcher threw a baseball for a distance of (18.44 ± 0.05) m in (0.52 ± 0.02) s.

   a. What was the velocity of the baseball in meters per second? (Divide the distance in meters by the time in seconds.)

   Answer: $35.46\,\frac{\text{m}}{\text{s}}$

   b. What are the relative errors of the distance and time? What is the total relative error?

   Answer: distance: 0.0027; time: 0.0385; total: 0.0412

   c. Calculate the uncertainty of the velocity of the baseball and express your answer as the velocity (from part a above) plus or minus the uncertainty that you just calculated, with correct rounding.

   Answer: $35.46\,\frac{\text{m}}{\text{s}} \pm 1.46\,\frac{\text{m}}{\text{s}}$ which rounds to $35\,\frac{\text{m}}{\text{s}} \pm 1\,\frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

Significant Figures

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: SP5

Mastery Objective(s): (Students will be able to…)

- Identify the significant figures in a number.
- Perform calculations and round the answer to the appropriate number of significant figures

Success Criteria:

- Be able to identify which digits in a number are significant.
- Be able to count the number of significant figures in a number.
- Be able to determine which places values will be significant in the answer when adding or subtracting.
- Be able to determine which digits will be significant in the answer when multiplying or dividing.
- Be able to round a calculated answer to the appropriate number of significant figures.

Tier 2 Vocabulary: significant, round

Language Objectives:

- Explain the concepts of significant figures and rounding.

Tier 2 Vocabulary: significant, round

Notes:

Because it would be tedious to calculate the uncertainty for every calculation in physics, we can use significant figures (or significant digits) as a simple way to estimate and represent the uncertainty.

Significant figures are based on the following approximations:

- All stated values are rounded off so that the uncertainty is only in the last unrounded digit.
- Assume that the uncertainty in the last unrounded digit is ±1.
- The results of calculations are rounded so that the uncertainty of the result is only in the last unrounded digit and is assumed to be ±1.

Use this space for summary and/or additional notes:

While these assumptions are often (though not always) the right order of magnitude, they rarely give a close enough approximation of the uncertainty to be useful. For this reason, ***significant figures are used as a convenience, and are used <u>only</u> when the uncertainty does not actually matter***.

If you need to express the uncertainty of a measured or calculated value, you must express the uncertainty separately from the measurement, as described in the previous section.

Therefore, when you take measurements and perform calculations in the laboratory, you will specifically state the measurements and their uncertainties. ***Never use significant figures in lab experiments!***

For homework problems and written tests, you will not be graded on your use of significant figures, but you may use them as a simple way to keep track of the approximate effects of uncertainty on your answers, if you wish.

The *only* reasons that significant figures are presented in these notes are:

2. If you are taking the AP® exam, you are expected to round your answers to an appropriate number of significant figures.
3. After a year of surviving the emotional trauma of significant figures in chemistry class, students expect to be required to use significant figures in physics and every science course afterwards. It is kinder to just say "[*sigh*] Yes, please do your best to round to the correct number of significant figures." than it is to say "Nobody actually uses significant figures. All that trauma was for nothing."

Every time you perform a calculation, you need to express your answer to enough digits that you're not introducing additional uncertainty. However, as long as that is true, feel free to round your answer off in order to omit digits that are one or more orders of magnitude smaller than the uncertainty.

In the example on page 61, we rounded the number 1 285.74 off to the tens place, resulting in the value of 1 290, because we couldn't show more precision than we actually had.

In the number 1 290, we would say that the first three digits are "significant", meaning that they are the part of the number that is not rounded off. The zero in the ones place is "insignificant," because the digit that was there was lost when we rounded.

Use this space for summary and/or additional notes:

significant figures (significant digits): the digits in a measured value or calculated result that are not rounded off. (Note that the terms "significant figures" and "significant digits" are used interchangeably.)

insignificant figures: the digits in a measured value or calculated result that were "lost" (became zeroes before a decimal point or were cut off after a decimal point) due to rounding.

Identifying the Significant Digits in a Number

The first significant digit is where the "measured" part of the number begins—the first digit that is not zero.

The last significant digit is the last "measured" digit—the last digit whose true value is known.

- If the number doesn't have a decimal point, the last significant digit will be the last digit that is not zero. (Anything after that has been rounded off.)

  Example: If we round the number 234 567 to the thousands place, we would get 235 000. (Note that because the digit after the "4" in the thousands place was 5 or greater, so we had to "round up".) In the rounded-off number, the first three digits (the 2, 3, and 5) are the significant digits, and the last three digits (the zeroes at the end) are the insignificant digits.
- If the number has a decimal point, the last significant digit will be the last digit shown. (Anything rounded after the decimal point gets chopped off.)

  Example: If we round the number 11.223 344 to the hundredths place, it would become 11.22. When we rounded the number off, we "chopped off" the extra digits.
- If the number is in scientific notation, it has a decimal point. Therefore, the above rules tell us (correctly) that all of the digits before the "times" sign are significant.

In the following numbers, the significant figures have been underlined:

- $\underline{13}\,000$
- $0.0\underline{275}$
- $0.0\underline{150}$
- $\underline{6\,804.305\,00}$
- $\underline{6.0} \times 10^{23}$
- $\underline{3400.}$ (note the decimal point at the end)

Digits that are not underlined are insignificant. Notice that only zeroes can ever be insignificant.

Use this space for summary and/or additional notes:

Mathematical Operations with Significant Figures

Addition & Subtraction

When adding or subtracting, calculate the total normally. Then identify the smallest place value where nothing is rounded. Round your answer to that place.

For example, consider the following problem.

problem:		"sig figs" equivalent:
123 000	± 1000	123 ???.????
0.0075	± 0.0001	0.0075
+ 1 650	± 10	+ 1 65?.????
124 650.0075	± 1010.0001	124 ???.????

(Check this digit for rounding)

Use this space for summary and/or additional notes:

In the first number (123 000), the hundreds, tens, and ones digit are zeros, presumably because the number was rounded to the nearest 1000. The second number (0.0075) is presumably rounded to the ten-thousandths place, and the number 1650 is presumably rounded to the tens place.

The first number has the largest uncertainty, so we need to round our answer to the thousands place to match, giving 125 000 ± 1 000.

A silly (but pedantically correct) example of addition with significant digits is:

$$100 + 37 = 100$$

Multiplication and Division

When multiplying or dividing, calculate the result normally. Then count the total *number* of significant digits in the values that you used in the calculation. Round your answer so that it has the same number of significant digits as the value that had the *fewest*.

Consider the problem:

$$34.52 \times 1.4$$

The answer (without taking significant digits into account) is $34.52 \times 1.4 = 48.328$

The number 1.4 has only two significant digits, so we need to round our answer so that it also has only two significant digits. This means we should round our answer to 48.

A silly (but pedantically correct) example of multiplication with significant digits is:

$$141 \times 1 = 100$$

Use this space for summary and/or additional notes:

Mixed Operations

For mixed operations, keep all of the digits until you're finished (so round-off errors don't accumulate), but keep track of the last significant digit in each step by putting a line over it (even if it's not a zero). Once you have your final answer, round it to the correct number of significant digits. Don't forget to use the correct order of operations (PEMDAS)!

For example:

$$137.4 \times 52 + 120 \times 1.77$$
$$(137.4 \times 52) + (120 \times 1.77)$$
$$7\,\bar{1}44.8 + 2\bar{1}2.4 = 7\,\bar{3}57.2 = 7\,400$$

Note that in the above example, ***we kept all of the digits and didn't round until the end***. This is to avoid introducing small rounding errors at each step, which can add up to enough to change the final answer. Notice how, if we had rounded off the numbers at each step, we would have gotten the wrong answer:

$$137.4 \times 52 + 120 \times 1.77$$
$$(137.4 \times 52) + (120 \times 1.77)$$
$$7\,\bar{1}00 + 2\bar{1}0 = 7\,\bar{3}10 = 7\,300 \longleftarrow$$ ☹

What to Do When Rounding Doesn't Give the Correct Number of Significant Figures

If you have a different number of significant digits from what the rounding shows, you can place a line over the last significant digit, or you can place the whole number in scientific notation. Both of the following have <u>four</u> significant digits, and both are equivalent to writing 13 000 ± 10

- $13\,0\bar{0}0$
- 1.300×10^4

Use this space for summary and/or additional notes:

When Not to Use Significant Figures

Significant figure rules only apply in situations where the numbers you are working with have a limited precision. This is usually the case when the numbers represent measurements. Exact numbers have infinite precision, and therefore have an infinite number of significant figures. Some examples of exact numbers are:

- Pure numbers, such as the ones you encounter in math class.
- Anything you can count. (*E.g.,* there are 24 people in the room. That means exactly 24 people, not 24.0 ± 0.1 people.)
- Whole-number exponents in formulas. (*E.g.,* the area of a circle is πr^2. The exponent "2" is a pure number.)
- Exact values. (*E.g.,* in the International System of Units, the speed of light is defined to be *exactly* $2.997\,924\,58 \times 10^8\,\frac{\text{m}}{\text{s}}$.)

You should also avoid significant figures any time the uncertainty is likely to be substantially different from what would be implied by the rules for significant figures, or any time you need to quantify the uncertainty more exactly.

Summary

Significant figures are a source of ongoing stress among physics students. To make matters simple, realize that few formulas in physics involve addition or subtraction, so you can usually just apply the rules for multiplication and division: look at each of the numbers you were given in the problem. Find the one that has the fewest significant figures, and round your final answer to the same number of significant figures.

If you have absolutely no clue what else to do, ***round to three significant figures and stop worrying***. You would have to measure quite carefully to have more than three significant figures in your original data, and three is usually enough significant figures to avoid unintended loss of precision, at least in a high school physics course. ☺

Use this space for summary and/or additional notes:

Homework Problems

1. **(M = Must Do)** For each of the following, Underline the significant figures in the number and Write the assumed uncertainty as ± the appropriate quantity.

   $\underline{57\,3}00 \pm 100$ ← Sample problem with correct answer.

   a. 13 500
   b. 26.0012
   c. 01902
   d. 0.000 000 025
   e. 320.
   f. 6.0×10^{-7}
   g. 150.00
   h. 10
   i. 0.005 310 0

2. **(M = Must Do)** Round off each of the following numbers as indicated and indicate the last significant digit if necessary.

   a. 13 500 to the nearest 1000
   b. 26.0012 to the nearest 0.1
   c. 1902 to the nearest 10
   d. 0.000 025 to the nearest 0.000 01
   e. 320. to the nearest 10
   f. 6.0×10^{-7} to the nearest 10^{-6}
   g. 150.00 to the nearest 100
   h. 10 to the nearest 100

Use this space for summary and/or additional notes:

3. **(S = Should Do)** Solve the following math problems and round your answer to the appropriate number of significant figures.

   a. $3\,521 \times 220$

   b. $13\,580.160 \div 113$

   c. $2.71828 + 22.4 - 8.31 - 62.4$

   d. $23.5 + 0.87 \times 6.02 - 105$ (Remember PEMDAS!)

Use this space for summary and/or additional notes:

Graphical Solutions (Linearization)

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP4, SP5

AP® Physics 2 Learning Objectives: SP2, SP5

Mastery Objective(s): (Students will be able to…)

- Use a graph to calculate the relationship between two variables.

Success Criteria:

- Graph has the independent variable on the *x*-axis and the dependent variable on the *y*-axis.
- Graph includes best-fit line that appears to minimize the total accumulated distance between the points and the line.
- Axes and best-fit line drawn with straightedge and divisions on axes are evenly spaced.
- Slope of line determined correctly (rise/run) and used correctly in calculation of desired result.

Tier 2 Vocabulary: plot, axes

Language Objectives:

- Explain why a best-fit line gives a better answer than calculating an average.
- Explain how the slope of the line relates to the desired quantity.

Tier 2 Vocabulary: best-fit, independent variable, dependent variable

Notes:

Most experiments in a high-school physics class involve finding a mathematical relationship between two quantities. While it is possible to simply measure each quantity once and calculate, or to measure several times and calculate the average, an approach that measures the relationship across a range of values will provide a better result.

Use this space for summary and/or additional notes:

A common way to achieve this is to manipulate equations and plot data such that the expected result is a straight line, and the experimental quantity is calculated by plotting a best-fit line and determining the slope. This means that you need to plot graphs ***accurately***, either on graph paper or using a computer or calculator. If you use graph paper:

- The data points need to be as close to their actual locations as you are capable of drawing.
- The best-fit line needs to be as close as you can practically get to its mathematically correct location.
- The best-fit line must be drawn with a straightedge.
- The slope needs to be calculated using the actual rise and run of points on the best-fit line.

Common mistakes:

- Axes are labeled unevenly. (The "skip" between divisions is not consistent.)
- There is a break in either or both axes, but the best-fit line is drawn through zero anyway.
- Points are not plotted exactly. (In this case, "close enough" usually isn't!)

As mentioned in the previous section, a good rule of thumb for quantitative experiments is the **8 & 10 rule**: you should have at least 8 data points, and the range from the highest to the lowest values tested should span at least a factor of 10.

Once you have your data points, arrange the equation into $y = mx + b$ form, such that the slope (or $^{1}/_{\text{slope}}$) is the quantity of interest. Then accurately plot your data and draw a best-fit line. The slope of this line will be the quantity of interest (or its reciprocal).

Use this space for summary and/or additional notes:

For example, suppose you wanted to calculate the spring constant of a spring by measuring the displacement caused by an applied force. You are given the following data:

Applied Force (N)	0.0	1.0	2.0	3.0	5.0
Displacement (m)	−0.01	0.05	0.16	0.20	0.34
Uncertainty (m)	± 0.06	± 0.06	± 0.06	± 0.06	± 0.06

The equation is $F_s = kx$, which is already in $y = mx + b$ form. However, we varied the force and measured the displacement, which means force is the independent variable (x-axis), and displacement is the dependent variable (y-axis). Therefore, we need to rearrange the equation to:

$$\begin{array}{ccccc} y & = & m & x & +b \\ \downarrow & & \downarrow & \downarrow & \downarrow \\ x & = & \left(\frac{1}{k}\right) & F_s & +0 \end{array}$$

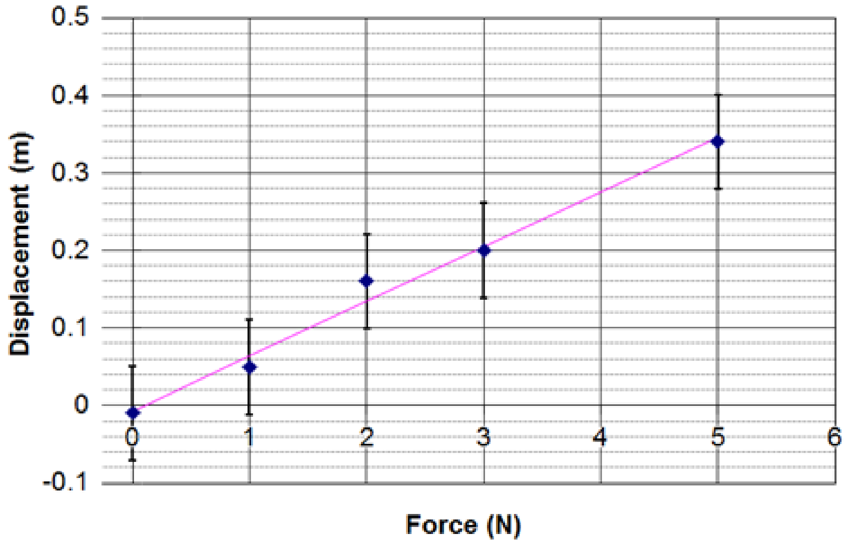
This means that if we plot a graph of all of our data points, a graph of F_s *vs.* x will have a slope of $\frac{1}{k}$.

You therefore need to:

1. Plot the data points, expressing the uncertainties as error bars.
2. Draw a best-fit line that passes through each error bar and minimizes the total accumulated distance away from each data point. (You can use linear regression, provided that the regression line actually passes through each error bar. If the line cannot pass through all of the error bars, you need to determine what the problem was with the outlier(s).) You may disregard a data point in your determination of the best-fit line *only* if you know *and can explain* the problem that caused it to be an outlier.

Use this space for summary and/or additional notes:

The plot looks like the following:



We calculate the slope using the actual rise (Δy) and run (Δx) from the graph. The best-fit line goes through the points (0, 0) and (3.0, 0.21). From these points, we would calculate the slope as:

$$m = \frac{\Delta y}{\Delta x} = \frac{0.21 - 0}{3.0 - 0} = 0.07$$

Because the slope is $\frac{1}{k}$, the spring constant is the reciprocal of the slope of the above graph. $\frac{1}{0.07} = 14\,\frac{\text{N}}{\text{m}}$ (rounded to two significant figures).

Use this space for summary and/or additional notes:

honors (not AP®)

Keeping a Laboratory Notebook

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP3, SP8

AP® Physics 2 Learning Objectives: SP4

Mastery Objective(s): (Students will be able to…)

- Determine which information to record in a laboratory notebook.
- Record information in a laboratory notebook according to practices used in industry.

Success Criteria:

- Record data accurately and correctly, with units and including estimated digits.
- Use the correct protocol for correcting mistakes.

Language Objectives:

- Understand and be able to describe the process for recording lab procedures and data.

Tier 2 Vocabulary: notebook, data

Notes:

A laboratory notebook serves two important purposes:

1. It is a legal record of what you did and when you did it.
2. It is a diary of exactly what you did, so you can look up the details later.

Your Notebook as an Official Record

Laboratory notebooks are kept by scientists in research laboratories and high tech companies. If a company or research institution needs to prove (perhaps in a court case) that you did a particular experiment on a particular date and got a particular set of results, your lab notebook is the primary evidence. While there is no right or wrong way for something to exist as a piece of evidence, the goal is for you to maintain a lab notebook that gives the best chance that it can be used to prove beyond a reasonable doubt exactly what you did, exactly when you did it, and exactly what happened.

Use this space for summary and/or additional notes:

honors (not AP®)

For companies that use laboratory notebooks in this way, there are a set of guidelines that exist to prevent mistakes that could compromise the integrity of the notebook. Details may vary somewhat from one company to another, but are probably similar to these, and the spirit of the rules is the same.

- All entries in a lab notebook must be hand-written in ink. (*This proves that you did not erase information.*)
- Your actual procedure and all data must be recorded directly into the notebook, not recorded elsewhere and copied in. (*This proves that you could not have made copy errors.*)
- All pages must be numbered consecutively, to show that no pages have been removed. If your notebook did not come with pre-numbered pages, you need to write the page number on each page before using it. (*This proves that no pages were removed.*) ***Never remove pages from a lab notebook for any reason.*** If you need to cross out an entire page, you may do so with a single large "X". If you do this, write a brief explanation of why you crossed out the page, and sign and date the cross-out.
- Start each experiment on a new page. (*This way, if you have to submit an experiment as evidence, you don't end up submitting parts of other experiments.*)
- Sign and date the bottom of the each page when you finish recording information on it. Make sure your supervisor witnesses each page within a few days of when you sign it. (*The legal date of an entry is the date it was witnessed. The date is important in patent claims.*)
- When crossing out an incorrect entry in a lab notebook, never obliterate it. Always cross it out with a single line through it, so that it is still possible to read the original mistake. (*This is to prove that it was a mistake, and you didn't change your data or observations. Erased or covered-up data is considered the same as faked or changed data in the scientific community.*) ***Never use "white-out" in a laboratory notebook.*** Any time you cross something out, write your initials and the date next to the change.
- ***Never, ever change data after the experiment is completed.*** Your data, right or wrong, is what you actually observed. Changing your data constitutes fraud, which is a form of cheating that is worse than plagiarism.
- ***Never change <u>anything</u> on a page you have already signed and dated.*** If you realize that an experiment was flawed, leave the bad data where it is and add a note that says "See page ____." with your initials and date next to the addendum. On the new page, refer back to the page number of the bad data and describe briefly what was wrong with it. Then, give the correct information and sign and date it as you would an experiment.

Use this space for summary and/or additional notes:

honors (not AP®)

Recording Your Procedure

Recording a procedure in a laboratory notebook is a challenging problem, because on the one hand, you need to have a legal record of what you did that is specific enough to be able to stand as evidence in court. On the other hand, you also need to be able to perform the experiment quickly and efficiently without stopping to write down every detail.

If your experiment is complicated and you need to plan your procedure ahead of time, you can record your intended procedure in your notebook before performing the experiment. Then all you need to do during the experiment is to note any differences between the intended procedure and what you actually did.

If the experiment is quick and simple, or if you suddenly think of something that you want to do immediately, without taking time to plan a procedure beforehand, you can jot down brief notes during the experiment for anything you may not remember, such as instrument settings and other information that is specific to the values of your independent variables. Then, as soon as possible after finishing the experiment, write down *all* of the details of the experiment. Include absolutely *everything*, including the make and model number of any major equipment that you used. Don't worry about presentation or whether the procedure is written in a way that would be easy for someone else to duplicate; concentrate on making sure the specifics are accurate and complete. The other niceties matter in reports, but not in a notebook.

Use this space for summary and/or additional notes:

honors (not AP®)

Recording Data

Here are some general rules for working with data. (Most of these are courtesy of Dr. John Denker, at http://www.av8n.com/physics/uncertainty.htm):

- Write something about what you did on the same page as the data, even if it is a very rough outline. Your procedure notes should not get in the way of actually performing the experiment, but there should be enough information to corroborate the detailed summary of the procedure that you will write afterwards. (Also, for evidence's sake, the sooner after the experiment that you write the detailed summary, the more weight it will carry in court.)
- Keep *all* of the raw data, whether you will use it or not.
- Don't discard a measurement, even if you think it is wrong. Record it anyway and put a "?" next to it. You can always choose not to use the data point in your calculations (as long as you give an explanation).
- Never erase or delete a measurement. The only time you should ever cross out recorded data is if you accidentally wrote down the wrong number.
- Record all digits. Never round off original data measurements. If the last digit is a zero, you must record it anyway!
- For analog readings (*e.g.,* ruler, graduated cylinder, thermometer), always estimate and record one extra digit.
- Always write down the units with each measurement!
- Record *every* quantity that will be used in a calculation, whether it is changing or not.
- Don't convert in your head before writing down a measurement. Record the original data in the units you actually measured it in, and convert in a separate step.

Use this space for summary and/or additional notes:

honors (not AP®)

Calculations

In general, calculations only need to be included in a laboratory notebook when they lead directly to another data point or another experiment. When this is the case, the calculation should be accompanied by a short statement of the conclusion drawn from it and the action taken. Calculations in support of the after-the-fact analysis of an experiment or set of experiments may be recorded in a laboratory notebook if you wish, or they may appear elsewhere.

Regardless of where calculations appear, you must:

- Use enough digits to avoid unintended loss of significance. (Don't introduce round-off errors in the middle of a calculation.) This usually means use at least two more "guard" digits beyond the number of "significant figures" you expect your answer to have.
- You may round for convenience only to the extent that you do not lose significance.
- Always calculate and express uncertainty separately from the measurement. Never rely on "sig figs" to express uncertainty. (In fact, you should never rely on "sig figs" at all!)
- Leave digits in the calculator between steps. (Don't round until the end.)
- When in doubt, keep plenty of "guard digits" (digits after the place where you think you will end up rounding).

Integrity of Data

Your data are your data. In classroom settings, people often get the idea that the goal is to report an uncertainty that reflects the difference between the measured value and the "correct" value. That idea certainly doesn't work in real life—if you knew the "correct" value you wouldn't need to make measurements!

In all cases—in the classroom and in real life—you need to determine the uncertainty of your own measurement by scrutinizing your own measurement procedures and your own analysis. Then you judge how well they agree. For example, we would say that the quantities 10 ± 2 and 11 ± 2 agree reasonably well, because there is considerable overlap between their probability distributions. However, 10 ± 0.2 does not agree with 11 ± 0.2, because there is no overlap.

If you get an impossible result or if your results disagree with well-established results, you should look for and comment on possible problems with your procedure and/or measurements that could have caused the differences you observed. You must *never* fudge your data to improve the agreement.

Use this space for summary and/or additional notes:

honors (not AP®)

Your Laboratory Notebook is *Not* a Report

Many high school students are taught that a laboratory notebook should be a journal-style book in which they must write perfect after-the-fact reports, but they are not allowed to change anything if they make a mistake. ***If you have been taught this, you need to unlearn it right now, because it's is false and damaging!***

A laboratory notebook was never meant to communicate your experiments to anyone else. A laboratory notebook is only your official signed and dated record of your procedure (what you did) and your data (what happened) at the exact instant that you took it and wrote it down. If anyone asks to see your laboratory notebook, they should not necessarily expect to understand anything in it without an explanation.

Of course, because it is your journal, your laboratory notebook *may* contain anything that you think is relevant. You may choose to include an explanation of the motivations for one or more experiments, the reasons you chose the procedure that you used, alternative procedures or experiments you may have considered, ideas for future experiments, *etc.* Or you may choose to record these things separately and cross-reference them to specific pages in your lab notebook.

Use this space for summary and/or additional notes:

Internal Laboratory Reports

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP3, SP8

AP® Physics 2 Learning Objectives: SP5

Success Criteria:

- The report has the correct sections in the correct order.
- Each section contains the appropriate information.

Language Objectives:

- Understand and be able to describe the sections of an internal laboratory report, and which information goes in each section.
- Write an internal laboratory report with the correct information in each section.

Tier 2 Vocabulary: N/A

Notes:

An internal laboratory report is written for co-workers, your boss, and other people in the company or research facility that you work for. It is usually a company confidential document that is shared internally, but not shared outside the company or facility.

Every lab you work in, whether in high school, college, research, or industry, will have its own internal report format. It is much more important to understand what *kinds* of information you need to report and what you will use it for than it is to get attached to any one format.

Most of the write-ups you will be required to do this year will be internal write-ups, as described in this section. The format we will use is based on the outline of the actual experiment.

AP®

Although lab reports are not specifically required for AP® Physics, each section of the internal laboratory report format described here is presented in a way that can be used directly in the "design an experiment" question.

Title & Date

Each experiment should have the title and date the experiment was performed written at the top. The title should be a descriptive sentence fragment (usually without a verb) that gives some information about the purpose of the experiment.

Use this space for summary and/or additional notes:

Objective

This should be a one or two-sentence description of what you are trying to determine or calculate by performing the experiment.

Experimental Design

This is the most important section in your report. This section needs to explain:

- What you were trying to observe or measure.
- If something needed to happen, how you made it happen. A flow chart can be useful for this
- Which aspects of the outcome you needed to observe or measure. (Note that you do not need to include the details of how to make the observations or measurements. That information will be included in your procedure.)

Qualitative Experiments

If you are trying to cause something to happen, observe whether or not something happens, or determine the conditions under which something happens, you are performing a qualitative experiment. Your experimental design section needs to explain:

- What you are trying to observe or measure.
- If something needs to happen, what you will do to try make it happen.
- How you will determine whether or not it has happened.
- How you will interpret your results.

Interpreting results is usually the challenging part. For example, in atomic & particle physics (as well as in chemistry), what "happens" involves atoms and electrons that are too small to see. You might detect radioactive decay by using a Geiger counter to detect the charged particles that are emitted.

As you define your experiment, you will need to pay attention to:

- Which conditions you need to keep constant (control variables)
- Which conditions you are changing intentionally (independent variables)
- Which outcomes you are observing or measuring (dependent variables)

Use this space for summary and/or additional notes:

Quantitative Experiments

If you are trying to determine the extent to which something happens, your experiment almost certainly involves measurements and calculations. Your experimental design section needs to explain:

- Your approach to solving the problem and/or gathering the data that you need.
- The specific quantities that you are going to vary (your independent variables).
- The specific quantities that you are going to keep constant (your control variables).
- The specific quantities that you are going to measure or observe (your dependent variables) .
- How you are going to calculate or interpret your results.

Use this space for summary and/or additional notes:

A good way to record this is to use a table like the following. For example, if you were writing up the experiment described in the section, “Designing & Performing Experiments” (starting on page 40), your experimental design table might include the following:

Desired Variable	Equation	Description/ Explanation	Fixed Control Variable(s) or Constants	Quantities to be Measured	Quantities to be Calculated (Still Needed)
$\vec{F}_f$	$\vec{F}_f = \vec{F}_{net}$	Set up experiment so other forces cancel	—	—	$\vec{F}_{net}$
$\vec{F}_{net}$	$\vec{F}_{net} = m\vec{a}$	Newton’s 2nd Law	—	m	$\vec{a}$
$\vec{a}$	$\vec{v} - \vec{v}_o = \vec{a}t$	Kinematics equation	$\vec{v} = 0$	t	$\vec{v}_o$
$\vec{v}_o$	$\frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$	Kinematics equation	$\vec{v} = 0$	$\vec{d}$, t	—

What needed to happen:

The object needed to slide from a starting point until it stops on its own due to friction.

Fixed Quantities:

- constants: none
- control variables: final velocity $\vec{v} = 0$

Measured Quantities:

- independent variables: none
- dependent variables: time (t) using a stopwatch; distance (d) using a meter stick or tape measure

Flow Chart:

measure mass of object → start object moving & start stopwatch → stop stopwatch when object stops moving (t) → measure distance object traveled from start (d)

Use this space for summary and/or additional notes:

Procedure

Your procedure is a detailed description of exactly what you did in order to take your measurements. You already identified what you needed to measure in your experimental design section. Your procedure will therefore be fairly brief and much easier to write. This section is where you give a detailed description of everything you need to do in order to take those data.

You need to include:

- A photograph or sketch of your apparatus, with *each component labeled* (with *both* dimensions *and* specifications), and details about how the components were connected. You need to do this even if the experiment is simple. The picture will serve to answer many questions about how you set up the experiment and most of the key equipment you used.
- A list of any significant equipment that is not labeled in your sketch or photograph. (You do not need to mention generic items like pencils and paper.)
- A narrative description of how you set up the experiment, referring to your sketch or photograph. Generic lab safety procedures and protective equipment may be assumed, but mention any unusual precautions that you needed to take.
- A descriptive list of your *control variables*, including their *values* and how you ensured that they remain constant.
- A descriptive list of your *independent variables*, including their *values* and how you set them.
- A descriptive list of your *dependent variables* and a step-by-step description of everything you did to measure their values. (Do not include the *values* of the dependent variables here—you will present those in your Data & Observations section.)
- Any significant things you did as part of the experiment besides the ones mentioned above.
- Do not start by saying "Gather the materials." This is assumed.

Use this space for summary and/or additional notes:

Data & Observations

This is a section in which you present all of your data. Be sure to record every quantity specified in your procedure, including quantities that are not changing (your control variables), quantities that are changing (your independent variables), and quantities you measured (your dependent variables). ***Remember to include the units!***

For a high school lab write-up, it is usually sufficient to present one or more data tables that include your measurements for each trial and the quantities that you calculated from them. However, if you have other data or observations that you recorded during the lab, they must be listed here.

You must also include estimates of the *uncertainty for every quantity that you measured*. You will also need to state the calculated uncertainty for the final quantity that your experiment is intended to determine.

Although calculated values are actually part of your analysis, it is often more convenient (and easier for the reader) to include them in your data table, even though the calculations will be presented in the next section.

Analysis

The analysis section is where you interpret your data. Your analysis should mirror your Experimental Design section (possibly in the same order, but more likely in reverse), with the goal of guiding the reader from your data to the quantity that you ultimately want to calculate or determine.

Your analysis needs to include:

- A narrative description (one or more paragraphs) of the outcome of the experiment, which guides the reader from your data through your calculations to the quantity you set out to determine.
- One (and only one) sample calculation for each separate equation that you used. For example, if you calculated acceleration for each of five data points, you would write down the formula, and then choose one set of data to plug in and show how you got the answer.
- Any calculated values that did not appear in the data table in your Data & Observations section

Use this space for summary and/or additional notes:

- If you need to do a graphical analysis, include a carefully-plotted graph showing the data points you took for your dependent *vs.* independent variables. Often, the quantity you are calculating will be the slope of this graph (or its reciprocal). The graph needs to show the region in which the slope is linear, because this is the range over which your experiment is valid. Note that ***any graphs you include in your write-up must be drawn accurately to scale, using graph paper, and using a ruler/straightedge wherever a straight line is needed***. (When an accurate graph is required, you will lose points if you include a freehand sketch instead.)

  It is acceptable to use a linear regression program to determine the slope. If you do this, you need to say so and give the correlation coëfficient. However, you still need to plot an accurate graph.

- Quantitative error analysis. In general, most quantities in a high school physics class are calculated from equations that use multiplication and division. Therefore, you need to:

  1. Determine the uncertainty of each your measurements.
  2. Calculate the relative error for each measurement.
  3. Combine your relative errors to get the total relative error for your calculated value(s).
  4. Multiply the total relative error by your calculated values to get the absolute uncertainty (±) for each one.

- Sources of uncertainty: this is a list of factors ***inherent in your procedure*** that limit how precise your answer can be. In general, you need a source of uncertainty for each measured quantity.

  Never include mistakes, especially mistakes you aren't sure whether or not you made! A statement like "We might have written down the wrong number." or "We might have done the calculations incorrectly." is really saying, "We might be stupid and you shouldn't believe anything else in this report." (Any "we might be stupid" statements will not count toward your required number of sources of uncertainty.)

  However, if a problem *actually occurred*, and if you *used that data point in your calculations anyway*, you need to explain what happened and why you were unable to fix the problem during the experiment, and you also need to calculate an estimate of the effects on your results.

Use this space for summary and/or additional notes:

Conclusion

Your conclusion should be worded similarly to your objective, but this time including your final calculated result(s) and uncertainty. You do not need to restate sources of uncertainty in your conclusions unless you believe they were significant enough to create some doubt about your results.

Your conclusion should also include 1–2 sentences describing ways the experiment could be improved. These should specifically address the sources of uncertainty that you listed in the analysis section above.

Summary

You can think of the sections of the report in pairs. For each pair, the first part describes the intent of the experiment, and the corresponding second part describes the result.

- **Objective**: describes the purpose of the experiment
  - **Experimental Design**: explains how the experiment was decided
    - **Procedure**: describes in detail how the data were acquired
    - **Data & Observations**: lists the data acquired via the procedure
  - **Analysis**: describes in detail what was learned from the experiment, including calculations and uncertainty.
- **Conclusions**: addresses how well the objective was achieved.

Use this space for summary and/or additional notes:

honors
(not AP®)

Formal Laboratory Reports

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP3, SP8

AP® Physics 2 Learning Objectives: SP5

Mastery Objective(s): (Students will be able to…)

- Write a formal (journal article-style) laboratory report that appropriately communicates all of the necessary information.

Success Criteria:

- The report has the correct sections in the correct order.
- Each section contains the appropriate information.
- The report contains an abstract that conveys the appropriate amount of information.

Tier 2 Vocabulary: abstract

Language Objectives:

- Understand and be able to describe the sections of a formal laboratory report, and which information goes in each section.
- Write a formal laboratory report with the correct information in each section.

Tier 2 Vocabulary: N/A

Notes:

A formal laboratory report serves the purpose of communicating the results of your experiment to other scientists outside of your laboratory or institution.

A formal report is a significant undertaking. In a research laboratory, you might submit as many as one or two articles to a scientific journal in a year. Some college professors require students to write their lab reports in journal article format.

The details of what to include are similar to the Internal Report format described in the previous section, except as noted below. The format of a formal journal article-style report is as follows:

Use this space for summary and/or additional notes:

honors (not AP®)

Abstract

This is the most important part of your report. It is a (maximum) 200-word executive summary of *everything* about your experiment—the procedure, results, analysis, and conclusions. In most scientific journals, the abstracts are searchable via the internet, so it needs to contain enough information to enable someone to find your abstract, and after reading it, to know enough about your experiment to determine whether or not to purchase a copy of the full article (which can sometimes cost $100 or more). It also needs to be short enough that the person doing the search won't just think "TL; DR" ("Too Long; Didn't Read") and move on to the next abstract.

Because the abstract is a complete summary, it is always best to wait to write it until you have already written the rest of your report.

Introduction

Your introduction is actually a mini research paper on its own, including citations. (For a high school lab report, it should be 1–3 pages; for scientific journals, 5–10 pages is not uncommon.) Your introduction needs to describe background information that another scientist might not know, plus all of the background information that specifically led to your experiment. Assume that your reader has a similar knowledge of physics as you, but does not know anything about this experiment. The introduction is usually the most time-consuming part of the report to write.

Materials and Methods

This section combines both the experimental design and procedure sections of an informal lab write-up. Unlike an informal write-up, the Materials and Methods section of a formal report is written in paragraph form, in the past tense, using the passive voice, and avoiding pronouns. As with the informal write-up, a labeled photograph or drawing of your apparatus is a necessary part of this section, but you need to *also* describe the set-up in the text.

Also unlike the informal write-up, your Materials and Methods section needs to give some *explanation* of your choices of the values used for your control and independent variables.

Use this space for summary and/or additional notes:

honors (not AP®)

Data and Observations

This section is similar to the same section in the lab notebook write-up, except that:

1. You should present only data you actually recorded/measured in this section. (Calculated values are presented in the Discussion section.)
2. You need to *introduce* the data table. (This means you need to describe the important things that someone should notice in the table first, and then say something like "Data are shown in Table 1.")

Note that all figures and tables in the report need to be numbered separately and consecutively.

Discussion

This section is similar to the Analysis section in the lab notebook write-up, but with some important differences.

As with the rest of the formal report, your discussion must be in paragraph form. Your discussion is essentially a long essay discussing your results and what they mean. You need to introduce and present a table with your calculated values and your uncertainty. After presenting the table, you should discuss the results, uncertainties, and sources of uncertainty in detail. If your results relate to other experiments, you need to discuss the relationship and include citations for those other experiments.

Your discussion needs to include each of the formulas that you used as part of your discussion and give the results of the calculations, but you do not need to show the intermediate step of substituting the numbers into the equation.

Conclusions

Your conclusions are written much like in the internal write-up. You need at least two paragraphs. In the first, restate your findings and summarize the significant sources of uncertainty. In the second paragraph, list and explain improvements and/or follow-up experiments that you suggest.

Works Cited

As with a research paper, you need to include a complete list of bibliography entries for the references you cited in your introduction and/or discussion sections.

Your ELA teachers probably require MLA-style citations; scientific papers typically use APA style. However, in a high school physics class, while it is important that you know which information needs to be cited and *what* information needs to go into each citation, you may use any format you like as long as you use it correctly and consistently.

Use this space for summary and/or additional notes:

Honors (not AP®)

Typesetting Superscripts and Subscripts

Because formal laboratory reports need to be typed, and because physics uses superscripts and subscripts extensively, it is important to know how to typeset superscripts and subscripts.

You can make use of the following shortcuts:

superscript: text that is raised above the line, such as the exponent "2" in $A = \pi r^2$.

In Google Docs, select the text, then hold down "Ctrl" and press the "." (period) key.

In Microsoft programs (such as Word) running on Windows, select the text, then hold down "Ctrl" and "Shift" and press the "+" key.

On a Macintosh, select the text, then hold down "Command" and "Control" and press the "+" key.

subscript: text that is lowered below the line, such as the "o" in $x = x_o + v_o t$.

In Google Docs, select the text, then hold down "Ctrl" and press the "," (comma) key.

In Microsoft programs (such as Word) running on Windows, select the text, then hold down "Ctrl" and press the "–" key.

On a Macintosh, select the text, then hold down "Command" and "Control" and press the "–" key.

Note that you will lose credit in laboratory reports if you don't use superscripts and subscripts correctly. For example, you will lose credit if you type *d* = *vot* + 1/2*at*^2 instead of $d = v_o t + \frac{1}{2} a t^2$.

Use this space for summary and/or additional notes:

Introduction: Mathematics

Unit: Mathematics

Topics covered in this chapter:

The purpose of this chapter is to familiarize you with mathematical concepts and skills that will be needed in physics.

- *Standard Assumptions in Physics* discusses what you can and cannot assume to be true in order to be able to solve the problems you will encounter in this class.
- *Solving Word Problems Systematically* discusses how to solve word problems, including determining which quantity and which variable apply to a number given in a problem based on the units, choosing an equation that applies to a problem, and substituting numbers from the problem into the equation.
- *Solving Problems Symbolically* discusses rearranging equations to solve for a particular variable before (or without) substituting values.
- *The International System of Units* and *Scientific Notation* briefly review skills that you are expected to remember from your middle school math and science classes.
- *Vectors*, *Vectors* vs. *Scalars in Physics*, and *Vector Multiplication* discuss the use and manipulation of vectors (quantities that have a direction) to represent quantities in physics.
- *Logarithms* is a review of the base 10 and natural logarithm functions.

Depending on your math background, some of the topics, such as trigonometry and vectors, may be unfamiliar. These topics may be taught, reviewed or skipped, depending on the needs of the students in the class.

Use this space for summary and/or additional notes:

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

This chapter addresses the following MA science and engineering practices:

Practice 4: Analyzing and Interpreting Data

Practice 5: Using Mathematics and Computational Thinking

Practice 8: Obtaining, Evaluating, and Communicating Information

AP®

AP® Physics 2 Learning Objectives & Science Practices:

SP 2.1: The student can *justify the selection of a mathematical routine* to solve problems.

SP 2.2: The student can *apply mathematical routines* to quantities that describe natural phenomena.

SP 2.3: The student can *estimate numerically quantities* that describe natural phenomena.

Skills learned & applied in this chapter:

- Identifying quantities in word problems and assigning them to variables
- Choosing a formula based on the quantities represented in a problem
- Using trigonometry to calculate the lengths of sides and angles of triangles
- Representing quantities as vectors
- Adding and subtracting vectors
- Multiplying vectors using the dot product and cross product

Prerequisite Skills:

These are the mathematical understandings that are necessary for Physics 1 that are taught in the MA Curriculum Frameworks for Mathematics.

- Construct and use tables and graphs to interpret data sets.
- Solve simple algebraic expressions.
- Perform basic statistical procedures to analyze the center and spread of data.
- Measure with accuracy and precision (*e.g.,* length, volume, mass, temperature, time)
- Convert within a unit (*e.g.,* centimeters to meters).
- Use common prefixes such as milli-, centi-, and kilo-.
- Use scientific notation, where appropriate.
- Use ratio and proportion to solve problems.

Fluency in all of these understandings is a prerequisite for this course. Students who lack this fluency may have difficulty passing the class.

Use this space for summary and/or additional notes:

Standard Assumptions in Physics

Unit: Mathematics

MA Curriculum Frameworks (2016): SP1, SP2

AP® Physics 2 Learning Objectives: SP 1.1, 1.2, 1.3, 1.4

Mastery Objective(s): (Students will be able to…)

- Make reasonable assumptions in order to be able to solve problems using the information given.

Success Criteria:

- Assumptions account for quantities that might affect the situation, but whose effects are either negligible.

Tier 2 Vocabulary: assumption

Language Objectives:

- Explain why we need to make assumptions in our everyday life.

Tier 2 Vocabulary: assumption

Notes:

Many of us have been told not to make assumptions. There is a popular expression that states that "when you assume, you make an ass of you and me":

ass|u|me

In science, particularly in physics, this adage is crippling. Assumptions are part of everyday life. When you cross the street, you assume that the speed of cars far away is slow enough for you to walk across without getting hit. When you eat your lunch, you assume that the food won't cause an allergic reaction. When you run down the hall and slide across the floor, you assume that the friction between your shoes and the floor will be enough to stop you before you crash into your friend.

assumption: something that is unstated but considered to be fact for the purpose of making a decision or solving a problem. Because it is impossible to measure and/or calculate everything that is going on in a typical physics or engineering problem, it is almost always necessary to make assumptions.

Use this space for summary and/or additional notes:

In a first-year physics course, in order to make problems and equations easier to understand and solve, we will often assume that certain quantities have a minimal effect on the problem, even in cases where this would not actually be true. The term used for these kinds of assumptions is “ideal”. Some of the ideal physics assumptions we will use include the following. Over the course of the year, you can make each of these assumptions unless you are explicitly told otherwise.

- Constants are constant and variables vary as described. This means that constants (such as acceleration due to gravity) have the same value in all parts of the problem, and variables change in the manner described by the relevant equation(s).
- Ideal machines and other objects that are not directly considered in the problem have negligible mass, inertia, and friction. (Note that these idealizations may change from problem-to-problem. A pulley may have negligible mass in one problem, but another pulley in another problem may have significant mass that needs to be considered as part of the problem.)
- If a problem does not give enough information to determine the effects of friction, you may assume that sliding (kinetic) friction between surfaces is negligible. In physics problems, ice is assumed to be frictionless unless you are explicitly told otherwise.
- If a problem does not mention air resistance and air resistance is not a central part of the problem, you may assume that friction due to air resistance is negligible.
- The mass of an object can often be assumed to exist at a single point in 3-dimensional space. (This assumption does not hold for problems where you need to calculate the center of mass, or torque problems where the way the mass is spread out is part of the problem.)
- All energy can be accounted for when energy is converted from one form to another. (This is always true, but in an ideal collision, energy lost to heat is usually assumed to be negligible.)
- Collisions between objects are assumed to be either perfectly elastic or perfectly inelastic, unless the problem states otherwise.
- The amount that solids and liquids expand or contract due to changes in temperature or pressure is negligible. (This will not be the case in problems involving thermal expansion.)
- Gas molecules do not interact when they collide or are forced together from pressure. (Real gases can form liquids and solids or participate in chemical reactions.)
- Electrical wires have negligible resistance.
- Physics students always do all of their homework. ☺

Use this space for summary and/or additional notes:

In some topics, a particular assumption may apply to some problems but not others. In these cases, the problem needs to make it clear whether or not you can make the relevant assumption. (For example, in the "forces" topic, some problems involve friction and others do not. A problem that does not involve friction might state that "a block slides across a frictionless surface.")

If you are not sure whether you can make a particular assumption, you should ask the teacher. If this is not practical (such as an open response problem on a standardized test), you should decide for yourself whether or not to make the assumption, and *explicitly state what you are assuming as part of your answer*.

Use this space for summary and/or additional notes:

Solving Word Problems Systematically

Unit: Mathematics

MA Curriculum Frameworks (2016): **SP1, SP5**

AP® Physics 2 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Assign (declare) variables in a word problem according to the conventions used in physics.
- Substitute values for variables in an equation.

Success Criteria:

- Variables match the quantities given and match the units.
- Quantities are substituted for the correct variables in the equation.

Tier 2 Vocabulary: **equation, variable**

Language Objectives:

- Describe the quantities used in physics, list their variables, and explain why that particular variable might have been chosen for the quantity.

Tier 2 Vocabulary: unit

Notes:

Math is a language. Like other languages, it has nouns (numbers), pronouns (variables), verbs (operations), and sentences (equations), all of which must follow certain rules of syntax and grammar.

This means that turning a word problem into an equation is translation from English to math.

Use this space for summary and/or additional notes:

Mathematical Operations

You have probably been taught translations for most of the common math operations:

word(s)	symbol	word(s)	symbol
of	×	percent (“per” + “cent”)	÷ 100
per, out of	÷		
is	=	change in *x*, difference in *x*	Δx

Identifying Variables

In science, almost every measurement must have a unit. These units are your key to what kind of quantity the numbers describe. Some common quantities in physics and their units are:

quantity	S.I. unit	variable	quantity	S.I. unit	variable
mass	kg	m	power	W	P^*
distance, length	m	d , ℓ	pressure	Pa	P^*
height	m	h	momentum	N·s	p^*
area	m^2	A	density	kg/m^3	ρ^*
acceleration	m/s^2	a	moles	mol	n
volume	m^3	V	temperature	K	T
velocity (speed)	m/s	v	heat	J	Q
time	s	t	electric charge	C	q , Q

*Note the subtle differences between uppercase “*P*”, lowercase “*p*”, and the Greek letter ρ (“rho”).

Any time you see a number in a word problem that has a unit that you recognize (such as one listed in this table), notice which quantity the unit is measuring and label the quantity with the appropriate variable.

Be especially careful with uppercase and lowercase letters. In physics, the same uppercase and lowercase letter may be used for completely different quantities.

Use this space for summary and/or additional notes:

Variable Substitution

Variable substitution simply means taking the numbers you have from the problem and substituting those numbers for the corresponding variable in an equation. A simple version of this is a density problem:

If you have the formula:

$\rho^{*} = \frac{m}{V}$ and you're given: $m = 12.3\text{ g}$ and $V = 2.8\text{ cm}^3$

simply substitute 12.3 g for m, and 2.8 cm^3 for V, giving:

$$\rho = \frac{12.3\,\text{g}}{2.8\,\text{cm}^3} = 4.4\,\tfrac{\text{g}}{\text{cm}^3}$$

Because variables and units both use letters, it is often easier to leave the units out when you substitute numbers for variables and then add them back in at the end:

$$\rho = \frac{12.3}{2.8} = 4.4\,\tfrac{\text{g}}{\text{cm}^3}$$

Many physics teachers disagree with this approach and insist on having students include the units with the number throughout the calculation. However, this can lead to confusion about which symbols are variables and which are units. For example, if a device applies a power of 150 W for a duration of 30 s and we wanted to find out the amount of work done, we would have:

$$P = \frac{W}{t}$$
$$150\,\text{W} = \frac{W}{30\,\text{s}}$$

The student would need to realize that the W on the left side of the equation is the unit "watts", and the *W* on the right side of the equation is the variable *W*, which stands for "work".

* Physicists use the Greek letter ρ ("rho") for density. Note that the Greek letter ρ is different from the Roman letter "p".

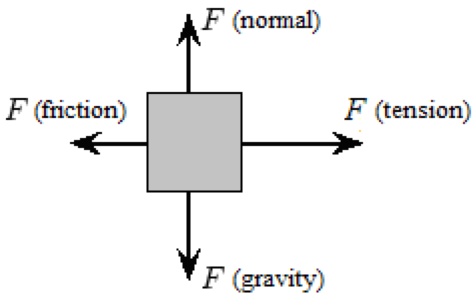
Use this space for summary and/or additional notes:

Subscripts

In physics, one problem can often have several instances of the same quantity. For example, consider a box with four forces on it:

1. The force of gravity, pulling downward.
2. The "normal" force of the table resisting gravity and holding the box up.
3. The tension force in the rope, pulling the box to the right.
4. The force of friction, resisting the motion of the box and pulling to the left.

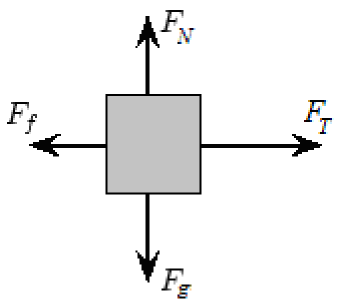
The variable for force is "*F*". There are four types of forces, which means "*F*" means four different things in this problem:



To make the diagram easier to read, we add subscripts to the variable "*F*". Note that in most cases, the subscript is the first letter of the word that describes the particular instance of the variable:

1. F_g is the force of gravity.
2. F_N is the normal force.
3. F_T is the tension in the rope.
4. F_f is friction.

This results in the following free-body diagram:



We use these same subscripts in the equations that relate to the problem. For example:

$$F_g = mg \quad \text{and} \quad F_f = \mu F_N$$

Use this space for summary and/or additional notes:

When writing variables with subscripts, be especially careful that the subscript looks like a subscript—***it needs to be smaller than the other letters and lowered slightly***. For example, when we write F_g, the variable is F (force) and the subscript $_g$ attached to it tells which kind of force it is (gravity). This might occur in the following equation:

$F_g = mg$ ← right ☺

It is important that the subscript $_g$ on the left does not get confused with the variable g on the right. Otherwise, the following error might occur:

$$Fg = mg$$
$$F\cancel{g} = m\cancel{g}$$ ← wrong! ☹
$$F = m$$

Another common use of subscripts is the subscript "o" to mean "initial". (Imagine that the "story problem" is shown as a video. When the slider is at the beginning of the video, the time is shown as 0, and the values of all of the variables at that time are shown with a subscript of o.)

For example, if an object is moving slowly at the beginning of a problem and then it speeds up, we need subscripts to distinguish between the initial velocity and the final velocity. Physicists do this by calling the initial velocity "v_o*" where the subscript "o" means "at time zero", *i.e.,* at the beginning of the problem. The final velocity is simply "v" without the zero.

* pronounced either "v-zero" or "v-naught"

Use this space for summary and/or additional notes:

The Problem-Solving Process

1. Identify the quantities in the problem, based on the units and any other information in the problem.
2. Assign the appropriate variables to those quantities.
3. Find an equation that relates all of the variables.
4. Use algebra to rearrange the equation to solve for the variable you're looking for. ("Undo PEMDAS.")
5. Substitute the values of the variables into the equation.
    a. If you have only one variable left, it should be the one you're looking for.
    b. If you have more than one variable left, repeat this sequence, finding another equation that uses one of the variables you have left, plus other quantities that you know.
6. Solve the equation(s), using algebra.
7. Apply the appropriate unit(s) to the result.

Use this space for summary and/or additional notes:

Sample Problem

A force of 30 N acts on an object with a mass of 1.5 kg. What is the acceleration of the object? (*mechanics/forces*)

We have units of N and kg, and we're looking for acceleration. We need to look these up in the *Appendix: Physics* Reference Tables in the appendix at the end of these notes.

From *Table C. Quantities, Variables and Units* on page 611 of the reference tables, we find:

Symbol	Unit	Quantity	Variable
N	newton	force	$\vec{F}$
kg	kilogram	mass	m
		acceleration	$\vec{a}$

We can label these quantities on the problem itself:

$\vec{F}$ m $\vec{a}$

A force of <u>30 N</u> acts on an object with a mass of <u>1.5 kg</u>. What is the <u>acceleration</u> of the object?

Now we know that we need an equation that relates the variables $\vec{F}$, *m*, and .
($\vec{F}$ and $\vec{a}$ *are* in boldface with an arrow above them because they are vector quantities. We will discuss vectors a little later in the course.)

Now that we have the variables, we find a formula that relates them. From the second formula box ("Forces") in *Table D. Mechanics Formulas and Equations* on page 612 of the reference tables, we find that:

$$\Delta V = \frac{W}{q} = \vec{E} \bullet \vec{d} \qquad \vec{F}_{net} = m\vec{a}$$

Rearranging to solve for $\vec{a}$ gives:

$$R = \frac{\rho L}{A}$$

$$\vec{F}_{net} = m\vec{a}$$

$$\frac{\vec{F}_{net}}{m} = \vec{a}$$

$$\frac{30}{1.5} = \vec{a} = 20$$

Again from *Table C. Quantities, Variables and Units*, we find that acceleration has units of meters per second squared, so our final answer is $20\ \frac{\text{m}}{\text{s}^2}$.

Use this space for summary and/or additional notes:

Homework Problems

To solve these problems, refer to your Appendix: Physics Reference Tables starting on page 609. To make the equations easier to find, the table and section of the table in your Physics Reference Tables where the equation can be found is given in parentheses.

1. What is the velocity of a car that travels 90. m in 4.5 s? (*mechanics/kinematics*)

   Answer: $20.\,\frac{m}{s}$

2. If a force of 100. N acts on a mass of 5.0 kg, what is its acceleration? (*mechanics/forces*)

   Answer: $20.\,\frac{m}{s^2}$

3. If the momentum of a block is *p* and its velocity is *v*, derive an expression for the mass, *m*, of the block.

   (*If you are not sure how to do this problem, do #4 below and use the steps to guide your algebra.*)

   (*mechanics/momentum*)

   Answer: $m = \frac{p}{v}$

Use this space for summary and/or additional notes:

4. If the momentum of a block is 18 N·s and its velocity is $3\,\frac{\text{m}}{\text{s}}$, what is the mass of the block?
   (You must start with the equations in your Physics Reference Tables. *You may only use the answer to question #3 above as a starting point if you have already solved that problem.*)
   (*mechanics/momentum*)

   Answer: 6 kg

5. What is the potential energy due to gravity of a 95 kg anvil that is about to fall off a 150 m cliff onto Wile E. Coyote's head?
   (*mechanics/energy, work & power*)

   Answer: 142 500 J

6. A 25 Ω resistor is placed in an electrical circuit with a voltage of 110 V. How much current flows through the resistor?
   (*electricity/circuits*)

   Answer: 4.4 A

Use this space for summary and/or additional notes:

7. What is the frequency of a wave that is traveling at a velocity of $300.\,\frac{\text{m}}{\text{s}}$ and has a wavelength of 10. m?
   (*waves/waves*)

   Answer: 30. Hz

8. What is the energy of a photon that has a frequency of 6×10^{15} Hz?
   (*atomic, particle & nuclear physics/energy*)

   Answer: 3.96×10^{-18} J

9. A piston with an area of $2.0\,\text{m}^2$ is compressed by a force of 10 000 N. What is the pressure applied by the piston?
   (*fluid mechanics/pressure*)

   Answer: 5 000 Pa

10. Derive an expression for the acceleration, *a*, of a car whose velocity changes from v_o to v in time t.

    (*If you are not sure how to do this problem, do #11 below and use the steps to guide your algebra.*)

    (*mechanics/kinematics*)

    Answer: $a = \dfrac{v - v_o}{t}$

Use this space for summary and/or additional notes:

11. What is the acceleration of a car whose velocity changes from $60.\,\frac{\text{m}}{\text{s}}$ to $80.\,\frac{\text{m}}{\text{s}}$ over a period of 5.0 s?

    Hint: v_o is the initial velocity and v is the final velocity.

    (*You must start with the equations in your* Physics Reference Tables. *You may only use the answer to question #10 above as a starting point if you have already solved that problem.*)

    (*mechanics/kinematics*)

    Answer: $4.0\,\frac{\text{m}}{\text{s}^2}$

12. If the normal force on an object is 100. N and the coëfficient of kinetic friction between the object and the surface it is sliding on is 0.35, what is the force of friction on the object as it slides along the surface? (*mechanics/forces*)

    Answer: 35 N

13. A 1200 W hair dryer is plugged into a electrical circuit with a voltage of 110 V. How much electric current flows through the hair dryer? (*electricity/circuits*)

    Answer: 10.9 A

14. A car has mass *m* and kinetic energy *K*. Derive an expression for its velocity, *v*. You may use your work from problem #15 below to guide your algebra.

    (*If you are not sure how to do this problem, do #11 above and use the steps to guide your algebra.*)

    (*mechanics/energy*)

Use this space for summary and/or additional notes:

Answer: $v=\sqrt{\frac{2K}{m}}$

15. A car has a mass of 1 200 kg and kinetic energy of 240 000 J. What is its velocity?

    (*You must start with the equations in your* Physics Reference Tables. *You may only use the answer to question #14 above as a starting point if you have already solved that problem.*)

    (*mechanics/energy*)

    Answer: $20.\,\frac{\text{m}}{\text{s}}$

16. What is the velocity of a photon (wave of light) through a block of clear plastic that has an index of refraction of 1.40?

    Hint: You will need to look up the index of refraction in your Physcis Reference Tables.

    (*waves/reflection & refraction*)

    Answer: $2.14\times10^{8}\,\frac{\text{m}}{\text{s}}$

17. If a pressure of 100 000 Pa is applied to a gas and the volume decreases by $0.05\,\text{m}^3$, how much work was done on the gas?

    (Note: represent the *change* in volume as ΔV.)

    (*fluid mechanics/work*)

    Answer: 5 000 J

Use this space for summary and/or additional notes:

18. If the distance from a mirror to an object is s_o and the distance from the mirror to the image is s_i, derive an expression for the distance from the lens to the focus, *f*. You may use your work from problem #17 to guide your algebra.

    (*If you are not sure how to do this problem, do #19 below and use the steps to guide your algebra.*)

    (*waves/mirrors & lenses*)

    Answer: $f = \frac{s_i + s_o}{s_i s_o}$

19. If the distance from a mirror to an object is 0.8 m and the distance from the mirror to the image is 0.6 m, what is the distance from the mirror to the focus?

    (*You must start with the equations in your* Physics Reference Tables. *You may only use the answer to question #18 above as a starting point if you have already solved that problem.*)

    (*waves/mirrors & lenses*)

    Answer: 0.343 m

20. What is the momentum of a photon that has a wavelength of 400 nm?
    (*Hint: you need to convert nanometers to meters.*)
    (*atomic, particle & nuclear physics/energy*)

    Answer: $1.65\times10^{-27}\ \mathrm{N\cdot s}$

Use this space for summary and/or additional notes:

Solving Equations Symbolically

Unit: Mathematics

MA Curriculum Frameworks (2016): SP5

AP® Physics 2 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Rearrange algebraic expressions to solve for any variable in the expression.

Success Criteria:

- Rearrangements are algebraically correct.

Tier 2 Vocabulary: equation, variable

Language Objectives:

- Describe how the rules of algebra are applied to expressions that contain only variables.

Tier 2 Vocabulary: N/A

Notes:

In solving physics problems, we are more often interested in the relationship between the quantities in the problem than we are in the numerical answer.

For example, suppose we are given a problem in which a person with a mass of 65 kg accelerates on a bicycle from rest $(0\,\frac{\text{m}}{\text{s}})$ to a velocity of $10\,\frac{\text{m}}{\text{s}}$ over a duration of 12 s and we wanted to know the force that was applied.

We could calculate acceleration as follows:

$$\begin{aligned}
&v - v_o = at \\
&10 - 0 = a(12) \\
&a = \frac{10}{12} = 0.8\overline{3}\,\frac{\text{m}}{\text{s}^2}
\end{aligned}$$

Then we could use Newton's second law:

$$\begin{aligned}
&F = ma \\
&F = (65)(0.8\overline{3}) = 54.2\,\text{N}
\end{aligned}$$

We have succeeded in answering the question. However, the question and the answer are of no consequence. Obtaining the correct answer shows that we can manipulate two related equations and come out with the correct number.

Use this space for summary and/or additional notes:

However, if instead we decided that we wanted to come up with an expression for force in terms of the quantities given (mass, initial and final velocities and time), we would need to rearrange the relevant equations to give an expression for force in terms of those quantities.

Just like algebra with numbers, rearranging an equation to solve for a variable is simply "undoing PEMDAS:"

1. "Undo" addition and subtraction by doing the opposing operation. If a variable is added, subtract it from both sides; if the variable is subtracted, then add it to both sides.

$$\begin{aligned} a+c&=b \\ -c&=-c \\ a&=b-c \end{aligned}$$

2. "Undo" multiplication and division by doing the opposing operation. If a variable is multiplied, divide both sides by it; if the variable is in the denominator, multiply both sides by it. *Note: whenever you have variables in the denominator that are on the same side of the equation as the variable you are solving for, always multiply both sides by it to clear the fraction.*

$$\frac{x\cancel{y}}{\cancel{y}}=\frac{z}{y}$$
$$x=\frac{z}{y}$$

$$\frac{n}{r}=s$$
$$\cancel{r}\cdot\frac{n}{\cancel{r}}=s\cdot r$$
$$\frac{n}{s}=\frac{\cancel{s}r}{\cancel{s}}$$
$$\frac{n}{s}=r$$

3. "Undo" exponents by taking the appropriate root of both sides. (Most often, the exponent will be 2, which means take the square root.) Similarly, you can "undo" roots by raising both sides to the appropriate power.

$$t^2=4ab$$
$$\sqrt{t^2}=\sqrt{4ab}$$
$$t=\sqrt{4}\cdot\sqrt{ab}=2\sqrt{ab}$$

4. When you are left with only parentheses and nothing outside of them, you can drop the parentheses, and then repeat steps 1–3 above until you have nothing left but the variable of interest.

Use this space for summary and/or additional notes:

Returning to the previous problem:

We know that $F = ma$. We are given m, but not a, which means we need to replace a with an expression that includes only the quantities given.

First, we find an expression that contains a:

$$v - v_o = at$$

We recognize that $v_o = 0$, and we use algebra to rearrange the rest of the equation so that a is on one side, and everything else is on the other side.

$$v - v_o = \underline{a}t$$
$$v - 0 = \underline{a}t$$
$$v = \underline{a}t$$
$$\underline{a} = \frac{v}{t}$$

Finally, we replace a in the first equation with $\frac{v}{t}$ from the second:

$$F = ma$$
$$F = (m)(\frac{v}{t})$$
$$F = \frac{mv}{t}$$

If the only thing we want to know is the value of F in one specific situation, we can substitute numbers at this point. However, we can also see from our final equation that increasing the mass or velocity will increase the numerator, which will increase the value of the fraction, which means the force would increase. We can also see that increasing the time would increase the denominator, which would decrease the value of the fraction, which means the force would decrease.

Solving the problem symbolically gives a relationship that holds true for all problems of this type in the natural world, instead of merely giving a number that answers a single pointless question. This is why the College Board and many college professors insist on symbolic solutions to equations.

Use this space for summary and/or additional notes:

Homework Problems

1. Given $a=2bc$ and $e=c^2d$, write an expression for *e* in terms of *a*, *b*, and *d*.

2. Given $w=\frac{3}{2}xy^2$ and $z=\frac{q}{y}$:

   a. Write an expression for *z* in terms of *q*, *w*, and *x*.

   b. If you wanted to maximize the value of the variable *z* in question #2 above, what adjustments could you make to the values of *q*, *w*, and *x*?

   c. Changing which of the variables *q*, *w*, or *x* would give the largest change in the value of *z*?

Use this space for summary and/or additional notes:

The International System of Units

Unit: Mathematics

MA Curriculum Frameworks (2016): SP5

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Use and convert between metric prefixes attached to units.

Success Criteria:

- Conversions between prefixes move the decimal point the correct number of places.
- Conversions between prefixes move the decimal point in the correct direction.
- The results of conversions have the correct answers with the correct units, including the prefixes.

Tier 2 Vocabulary: prefix

Language Objectives:

- Set up and solve problems relating to the concepts described in this section.

Tier 2 Vocabulary: unit, prefix, convert

Notes:

This section is intended to be a brief review. You learned to use the metric system and its prefixes in elementary school. Although you will learn many new S.I. units this year, ***you are expected to be able to fluently apply any metric prefix to any unit and be able to convert between prefixes in any problem you might encounter throughout the year****.*

A unit is a specifically defined measurement. Units describe both the type of measurement, and a base amount.

For example, 1 cm and 1 inch are both lengths. They are used to measure the same dimension, but the specific amounts are different. (In fact, 1 inch is exactly 2.54 cm.)

Every measurement is a number multiplied by its units. In algebra, the term "3x" means "3 times x". Similarly, the distance "75 m" means "75 times the distance 1 meter".

Use this space for summary and/or additional notes:

The number and the units are *__both__* *necessary to describe any measurement.* You *always* need to write the units. Saying that "12 is the same as 12 g" would be as ridiculous as saying "12 is the same as 12 × 3".

The International System (often called the metric system) is a set of units of measurement that is based on natural quantities (on Earth) and powers of 10.

The metric system has 7 fundamental "base" units:

Unit	**Quantity**
meter (m)	length
kilogram (kg)	mass
second (s)	time
Kelvin (K)	temperature
mole (mol)	amount of substance
ampere (A)	electric current
candela (cd)	intensity of light

All other S.I. units are combinations of one or more of these seven base units.

For example:

Velocity (speed) is a change in distance over a period of time, which would have units of distance/time (m/s).

Force is a mass subjected to an acceleration. Acceleration has units of distance/time^2 (m/s^2), and force has units of mass × acceleration. In the metric system this combination of units (kg·m/s^2) is called a Newton, which means:
$1\text{ N} \equiv 1\text{ kg}\cdot\text{m/s}^2$

(The symbol "≡" means "is identical to," whereas the symbol "=" means "is equivalent to".)

The S.I. base units are calculated from these seven definitions, after converting the derived units (joule, coulomb, hertz, lumen and watt) into the seven base units (second, meter, kilogram, ampere, kelvin, mole and candela).

Use this space for summary and/or additional notes:

Prefixes

The metric system uses prefixes to indicate multiplying a unit by a power of ten. Prefixes are defined for powers of ten from 10^{-24} to 10^{24}:

Factor		Prefix	Symbol
1 000 000 000 000 000 000 000 000	10^{24}	yotta	Y
1 000 000 000 000 000 000 000	10^{21}	zeta	Z
1 000 000 000 000 000 000	10^{18}	exa	E
1 000 000 000 000 000	10^{15}	peta	P
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto	h
10	10^{1}	deca	da
1	10^{0}	—	—
0.1	10^{-1}	deci	d
0.01	10^{-2}	centi	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p
0.000 000 000 000 001	10^{-15}	femto	f
0.000 000 000 000 000 001	10^{-18}	atto	a
0.000 000 000 000 000 000 001	10^{-21}	zepto	z
0.000 000 000 000 000 000 000 001	10^{-24}	yocto	y

↑ Move Decimal Point to the Left

↓ Move Decimal Point to the Right

Note that some of the prefixes skip by a factor of 10 and others skip by a factor of 10^3. This means ***you can't just count the steps in the table—you have to actually look at the exponents***.

The most commonly used prefixes are:

- mega (M) = 10^6 = 1 000 000
- kilo (k) = 10^3 = 1000
- centi (c) = $10^{-2} = \frac{1}{100}$ = 0.01
- milli (m) = $10^{-3} = \frac{1}{1\,000}$ = 0.001
- micro (μ) = $10^{-6} = \frac{1}{1\,000\,000}$ = 0.000 001

Use this space for summary and/or additional notes:

Any metric prefix is allowed with any metric unit. For example, "35 cm" means "35 × c × m" or "(35)($\frac{1}{100}$)(m)". If you multiply this out, you get 0.35 m.

Note that some units have two-letter abbreviations. *E.g.,* the unit symbol for pascal (a unit of pressure) is (Pa). Standard atmospheric pressure is 101 325 Pa. This same number could be written as 101.325 kPa or 0.101 325 MPa.

There is a popular geek joke based on the ancient Greek heroine Helen of Troy. She was said to have been the most beautiful woman in the world, and she was an inspiration to the entire Trojan fleet. She was described as having "the face that launched a thousand ships." Therefore a milliHelen must be the amount of beauty required to launch one ship.

Use this space for summary and/or additional notes:

Conversions

If you need to convert from one prefix to another, simply move the decimal point.

- Use the starting and ending powers of ten to determine the number of places to move the decimal point.
- When you convert, the actual measurement needs to stay the same. This means that if the prefix gets larger, the number needs to get smaller (move the decimal point to the left), and if the prefix gets smaller, the number needs to get larger (move the decimal point to the right).

Definitions

In order to have measurements be the same everywhere in the universe, any system of measurement needs to be based on some defined values. Starting in May 2019, instead of basing units on physical objects or laboratory measurements, all S.I. units are defined by specifying exact values for certain fundamental constants:

- The Planck constant, *h,* is exactly $6.626\,070\,15 \times 10^{-34}$ J·s
- The elementary charge, *e,* is exactly $1.602\,176\,634 \times 10^{-19}$ C
- The Boltzmann constant, *k,* is exactly $1.380\,649 \times 10^{-23}$ $J{\cdot}K^{-1}$
- The Avogadro constant, N_A, is exactly $6.022\,140\,76 \times 10^{23}$ mol^{-1}
- The speed of light, *c,* is exactly 299 792 458 $m{\cdot}s^{-1}$
- The ground state hyperfine splitting frequency of the caesium-133 atom, $\Delta\nu(^{133}Cs)_{hfs}$, is exactly 9 192 631 770 Hz
- The luminous efficacy, K_{cd}, of monochromatic radiation of frequency 540×10^{12} Hz is exactly 683 $lm{\cdot}W^{-1}$

The exact value of each of the base units is calculated from combinations of these fundamental constants, and every derived unit is calculated from combinations of base units.

Use this space for summary and/or additional notes:

The MKS vs. cgs Systems

Because physics heavily involves units that are derived from other units, it is important to make sure that all quantities are expressed in the appropriate units before applying formulas. (This is how we get around having to do factor-label unit-cancelling conversions—like you learned in chemistry—for every single physics problem.)

There are two measurement systems commonly used in physics. In the MKS, or "meter-kilogram-second" system, units are derived from the S.I. units of meters, kilograms, seconds, moles, Kelvins, amperes, and candelas. In the cgs, or "centimeter-gram-second" system, units are derived from the units of centimeters, grams, seconds, moles, Kelvins, amperes, and candelas. The following table shows some examples:

Quantity	MKS Unit	Base Units Equivalent	cgs Unit	Base Units Equivalent
force	newton (N)	$\frac{kg \cdot m}{s^2}$	dyne (dyn)	$\frac{g \cdot cm}{s^2}$
energy	joule (J)	$\frac{kg \cdot m^2}{s^2}$	erg	$\frac{g \cdot cm^2}{s^2}$
magnetic flux density	tesla (T)	$\frac{N}{A}$, $\frac{kg \cdot m}{A \cdot s^2}$	gauss (G)	$\frac{0.1\ dyn}{A}$, $\frac{0.1\ g \cdot cm}{A \cdot s^2}$

In general, because 1 kg = 1 000 g and 1 m = 100 cm, each MKS unit is 100 000 times the value of its corresponding cgs unit.

In this class, we will use exclusively MKS units. This means you have to learn only one set of derived units. However, you can see the importance, when you solve physics problems, of making sure all of the quantities are in MKS units before you plug them into a formula!

Use this space for summary and/or additional notes:

Formatting Rules for S.I. Units

- The value of a quantity is written as a number followed by a non-breaking space (representing multiplication) and a unit symbol; *e.g.,* 2.21 kg, 7.3×10^2 m^2, or 22 K. This rule explicitly includes the percent sign (*e.g.,* 10 %, not 10%) and the symbol for degrees of temperature (*e.g.,* 37 °C, not 37°C). (However, note that angle measurements in degrees are written next to the number without a space.)
- Units do not have a period at the end, except at the end of a sentence.
- A prefix is part of the unit and is attached to the beginning of a unit symbol without a space. Compound prefixes are not allowed.
- Symbols for derived units formed by multiplication are joined with a center dot (·) or a non-breaking space; *e.g.,* N·m or N m.
- Symbols for derived units formed by division are joined with a solidus (fraction line), or given as a negative exponent. *E.g.,* "meter per second" can be written $\frac{\text{m}}{\text{s}}$, m/s, m·s^{-1}, or m s^{-1}.
- The first letter of symbols for units derived from the name of a person is written in upper case; otherwise, they are written in lower case. *E.g.,* the unit of pressure is the pascal, which is named after Blaise Pascal, so its symbol is written "Pa" (note that "Pa" is a two-letter symbol). Conversely, the mole is not named after anyone, so the symbol for mole is written "mol". Note, however, that the symbol for liter is "L" rather than "l", because a lower case "l" is too easy to confuse with the number "1".
- A plural of a symbol must not be used; *e.g.,* 25 kg, not 25 kgs.
- Units and prefixes are case-sensitive. *E.g.,* the quantities 1 mW and 1 MW represent two different quantities (milliwatt and megawatt, respectively).
- The symbol for the decimal marker is either a point or comma on the line. In practice, the decimal point is used in most English-speaking countries and most of Asia, and the comma is used in most of Latin America and in continental European countries.
- Spaces should be used as a thousands separator (1 000 000) instead of commas (1,000,000) or periods (1.000.000), to reduce confusion resulting from the variation between these forms in different countries.
- Any line-break inside a number, inside a compound unit, or between number and unit should be avoided.

Use this space for summary and/or additional notes:

Homework Problems

Perform the following conversions.

1. 2.5 m = __________ cm

2. 18mL = __________ L

3. 68 kJ = __________ J

4. 6 500 mg = __________ kg

5. 101 kPa = __________ Pa

6. 325 ms = __________ s

Use this space for summary and/or additional notes:

Scientific Notation

Unit: Mathematics

MA Curriculum Frameworks (2016): SP5

AP® Physics 2 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Correctly use numbers in scientific notation in mathematical problems.

Success Criteria:

- Numbers are converted correctly to and from scientific notation.
- Numbers in scientific notation are correctly entered into a calculator.
- Math problems that include numbers in scientific notation are set up and solved correctly.

Language Objectives:

- Explain how numbers are represented in scientific notation, and what each part of the number represents.

Tier 2 Vocabulary: N/A

Notes:

This section is intended to be a brief review. You learned to use the scientific notation in elementary or middle school. ***You are expected to be able to fluently perform calculations that involve numbers in scientific notation, and to express the answer correctly in scientific notation when appropriate.***

Scientific notation is a way of writing a very large or very small number in compact form. The value is always written as a number between 1 and 10, multiplied by a power of ten.

For example, the number 1 000 would be written as 1×10^{3}. The number 0.000 075 would be written as 7.5×10^{-5}. The number 602 000 000 000 000 000 000 000 would be written as 6.02×10^{23}. The number 0.000 000 000 000 000 000 000 000 000 000 000 663 would be written as 6.63×10^{-34}.

Scientific notation is really just math with exponents, as shown by the following examples:

$$5.6 \times 10^{3} = 5.6 \times 1\,000 = 5\,600$$

$$2.17 \times 10^{-2} = 2.17 \times \frac{1}{10^{2}} = 2.17 \times \frac{1}{100} = \frac{2.17}{100} = 0.0217$$

Use this space for summary and/or additional notes:

Notice that if 10 is raised to a positive exponent means you're multiplying by a power of 10. This makes the number larger, which means the decimal point moves to the right. If 10 is raised to a negative exponent, you're actually dividing by a power of 10. This makes the number smaller, which means the decimal point moves to the left.

Significant figures are easy to use with scientific notation: all of the digits before the "×" sign are significant. The power of ten after the "×" sign represents the (insignificant) zeroes, which would be the rounded-off portion of the number. In fact, the mathematical term for the part of the number before the "×" sign is the *significand*.

Math with Scientific Notation

Because scientific notation is just a way of rewriting a number as a mathematical expression, all of the rules about how exponents work apply to scientific notation.

Adding & Subtracting: adjust one or both numbers so that the power of ten is the same, then add or subtract the significands.

$$\left(3.50\times10^{-6}\right)+\left(2.7\times10^{-7}\right)=\left(3.50\times10^{-6}\right)+\left(0.27\times10^{-6}\right)$$
$$=(3.50+0.27)\times10^{-6}=3.77\times10^{-6}$$

Multiplying & dividing: multiply or divide the significands. If multiplying, add the exponents. If dividing, subtract the exponents.

$$\frac{6.2\times10^{8}}{3.1\times10^{10}}=\frac{6.2}{3.1}\times10^{8-10}=2.0\times10^{-2}$$

Exponents: raise the significand to the exponent. Multiply the exponent of the power of ten by the exponent to which the number is raised.

$$\left(3.00\times10^{8}\right)^{2}=(3.00)^{2}\times\left(10^{8}\right)^{2}=9.00\times10^{(8\times2)}=9.00\times10^{16}$$

Use this space for summary and/or additional notes:

Using Scientific Notation on Your Calculator

Scientific calculators are designed to work with numbers in scientific notation. It's possible to can enter the number as a math problem (always use parentheses if you do this!) but math operations can introduce mistakes that are hard to catch.

Scientific calculators all have some kind of scientific notation button. The purpose of this button is to enter numbers directly into scientific notation and make sure the calculator stores them as a single number instead of a math equation. (This prevents you from making PEMDAS errors when working with numbers in scientific notation on your calculator.) On most Texas Instruments calculators, such as the TI-30 or TI-83, you would do the following:

What you type	What the calculator shows	What you would write
6.6 $\boxed{\text{EE}}$ −34	6.6E−34	6.6×10^{-34}
1.52 $\boxed{\text{EE}}$ 12	1.52E12	1.52×10^{12}
−4.81 $\boxed{\text{EE}}$ −7	−4.81E−7	-4.81×10^{-7}

On some calculators, the scientific notation button is labeled $\boxed{\text{EXP}}$ or $\boxed{\times 10^{x}}$ instead of $\boxed{\text{EE}}$.

Important notes:

- *Many high school students are afraid of the $\boxed{\text{EE}}$ button because it is unfamiliar. If you are afraid of your $\boxed{\text{EE}}$ button, you need to get over it and start using it anyway. However, if you insist on clinging to your phobia, you need to at least use parentheses around all numbers in scientific notation, in order to minimize the likelihood of PEMDAS errors in your calculations.*
- *Regardless of how you enter numbers in scientific notation into your calculator, always place parentheses around the denominator of fractions.*

$$\frac{2.75\times10^{3}}{5.00\times10^{-2}} \text{ becomes } \frac{2.75\times10^{3}}{(5.00\times10^{-2})}$$

- *You need to **write** answers using correct scientific notation. For example, if your calculator displays the number* 1.52E12*, you need to write* 1.52×10^{12} *(plus the appropriate unit, of course) in order to receive credit.*

Use this space for summary and/or additional notes:

Homework Problems

Convert each of the following between scientific and algebraic notation.

1. $2.65 \times 10^{9} =$

2. 387 000 000 =

3. $1.06 \times 10^{-7} =$

4. 0.000 000 065 =

Solve each of the following on a calculator that can do scientific notation.

5. $(2.8\times 10^{6})(1.4\times 10^{-2}) =$

   Answer: 3.9×10^{4}

6. $\dfrac{3.75\times 10^{8}}{1.25\times 10^{4}} =$

   Answer: 3.00×10^{4}

7. $\dfrac{1.2\times 10^{-3}}{5.0\times 10^{-1}} =$

   Answer: 2.4×10^{-3}

Use this space for summary and/or additional notes:

Vectors

Unit: Mathematics

MA Curriculum Frameworks (2016): SP5

AP® Physics 2 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Identify the magnitude and direction of a vector.
- Combine vectors graphically and calculate the magnitude and direction.

Success Criteria:

- Magnitude is calculated correctly (Pythagorean theorem).
- Direction is correct: angle (using trigonometry) or direction (*e.g.,* "south", "to the right", "in the negative direction", *etc.*)

Tier 2 Vocabulary: magnitude, direction

Language Objectives:

- Explain what a vector is and what its parts are.

Tier 2 Vocabulary: vector, sign, direction

Notes:

vector: a quantity that has both a magnitude (value) and a direction.

E.g., if you are walking $1\,\frac{\text{m}}{\text{s}}$ to the north, the magnitude is $1\,\frac{\text{m}}{\text{s}}$ and the direction is north.

scalar: a quantity that has a value but does not have a direction. (A scalar is what you think of as a "regular" number, including its unit.)

magnitude: the scalar part of a vector (*i.e.,* the number and its units, but without the direction). If you have a force of 25 N to the east, the magnitude of the force is 25 N.

The mathematical operation of taking the magnitude of a vector is represented by two double vertical bars (like double absolute value bars) around the vector. For example, if $\vec{\boldsymbol{F}}$ is 25 N to the east, then $\|\vec{\boldsymbol{F}}\| = 25\,\text{N}$. However, to make typesetting easier, it is common to use regular absolute value bars instead, *e.g.,* $|\vec{F}| = 25\,\text{N}$.

resultant: a vector that is the result of a mathematical operation (such as the addition of two vectors).

Use this space for summary and/or additional notes:

Variables that represent vectors are traditionally typeset in ***bold Italics***. Vector variables may also optionally have an arrow above the letter:

$$\boldsymbol{J}, \vec{\boldsymbol{F}}, \boldsymbol{v}$$

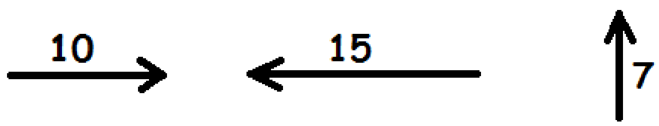
Variables that represent scalars are traditionally typeset in *plain Italics:*

$$V, t, \lambda$$

Note that a variable that represents only the magnitude of a vector quantity is generally typeset as if it were a scalar:

> For example, suppose $\vec{\boldsymbol{F}}$ is a vector representing a force of 25 N to the east. (Notice that the vector includes the magnitude or amount ***and*** the direction.)
>
> The magnitude would be 25 N, and would be represented by the variable *F*.

Vectors are represented graphically using arrows. The length of the arrow represents the magnitude of the vector, and the direction of the arrow represents the direction of the vector:

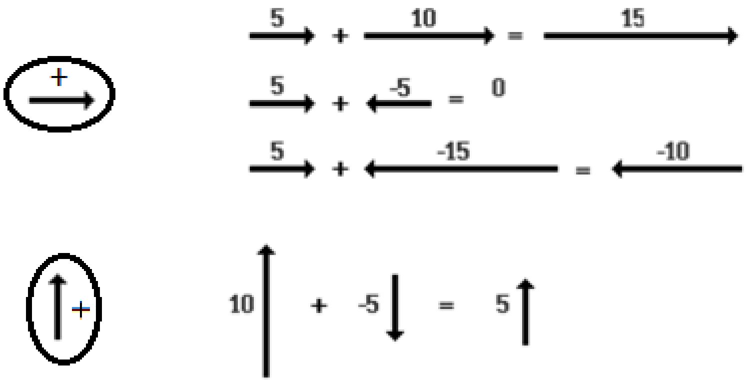


magnitude 10	magnitude 15	magnitude 7
direction: "to the right", 0°	direction: "to the left", +180°	direction: "up", +90°

Use this space for summary and/or additional notes:

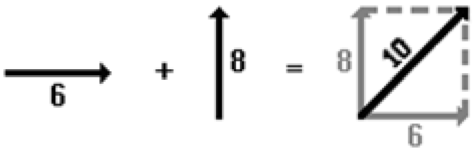
Adding & Subtracting Vectors

If the vectors have the same direction or opposite directions, the resultant is easy to envision:



If the vectors are not in the same direction, we move them so they start from the same place and complete the parallelogram. If they are perpendicular, we can add them by doing the following:

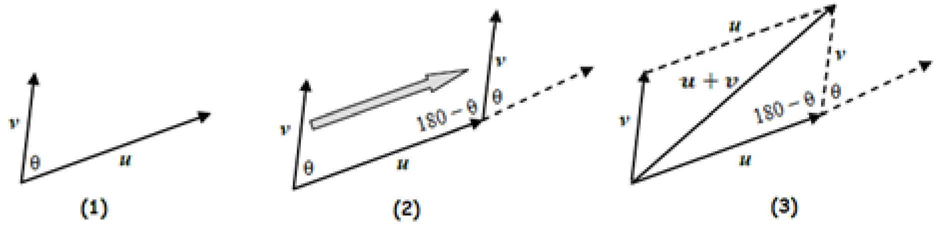
1. Translate (slide) the vectors so that they are either tip-to-tail or tail-to-tail.*
2. Calculate the length of the resultant by completing the rectangle and using the Pythagorean theorem:



$(6^2 + 8^2 = 10^2)$

Note that the sum of these two vectors has a magnitude (length) of 10, not 14.

The same process applies to adding vectors that are not perpendicular:

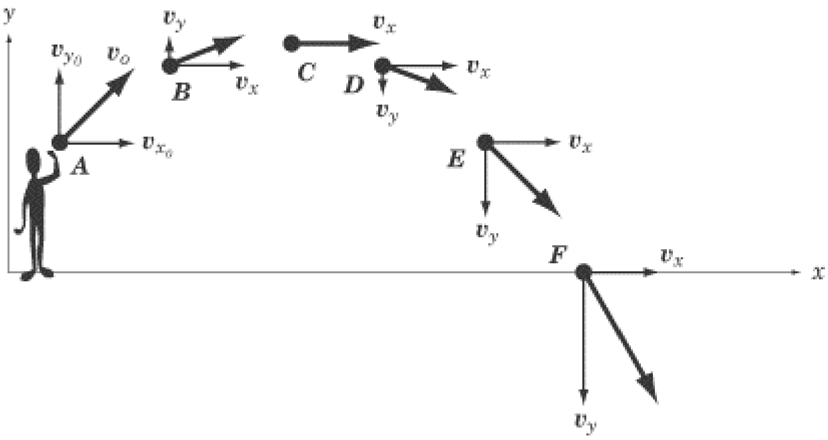


However, the trigonometry needed for these calculations is beyond the scope of this course.

* In this section, examples are shown translating vectors tail-to-tail and completing the parallelogram. While this does not always result in the best representation of the physics involved, it is less confusing for students to keep the procedure consistent when they are first learning.

Use this space for summary and/or additional notes:

One type of physics problem that commonly uses vectors is two-dimensional projectile motion. If the motion of the projectile is represented by a vector, $\vec{\boldsymbol{v}}$, at angle θ, the vector can be represented as the sum of a horizontal vector $\vec{\boldsymbol{v}}_x$ and a vertical vector $\vec{\boldsymbol{v}}_y$. This is useful because the horizontal vector $\vec{\boldsymbol{v}}_x$ gives us the component (portion) of the vector in the *x*-direction, and the vertical vector $\vec{\boldsymbol{v}}_y$ gives us the component of the vector in the *y*-direction.



Notice that $\vec{\boldsymbol{v}}_x$ remains constant, but $\vec{\boldsymbol{v}}_y$ changes (because of the effects of gravity).

Because perpendicular vectors do not affect each other, we can apply equations to the two directions separately.

As you saw in projectile motion (which you learned about in physics 1), we use the equation $\vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}_o t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$, applying it separately in the *x*- and *y*-directions. This gives us two equations.

In the horizontal (*x*)-direction:

$$\vec{\boldsymbol{d}}_x = \vec{\boldsymbol{v}}_{o,x} t + \cancel{\tfrac{1}{2}\vec{\boldsymbol{a}}_x t^2}^{\,0}$$
$$\vec{\boldsymbol{d}}_x = \vec{\boldsymbol{v}}_x t$$

In the vertical (*y*)-direction:

$$\vec{\boldsymbol{d}}_y = \vec{\boldsymbol{v}}_{o,y} t + \frac{1}{2}\vec{\boldsymbol{a}}_y t^2$$
$$\vec{\boldsymbol{d}}_y = \vec{\boldsymbol{v}}_{o,y} t + \frac{1}{2}\vec{\boldsymbol{g}} t^2$$

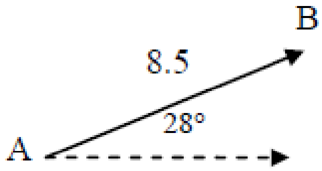
Use this space for summary and/or additional notes:

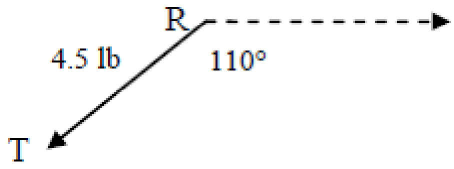
Note that each of the vector quantities ($\vec{d}$, $\vec{v}_o$ and $\vec{a}$) has independent *x*- and *y*-components. For example, $\vec{v}_{o,x}$ (the component of the initial velocity in the *x*-direction) is independent of $\vec{v}_{o,y}$ (the component of the initial velocity in the *x*-direction). This means *we treat them as completely separate variables*, and we can solve for one without affecting the other.

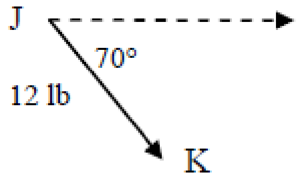
Use this space for summary and/or additional notes:

Homework Problems

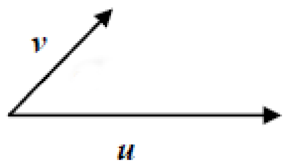
Label the magnitude and direction of each of the following:

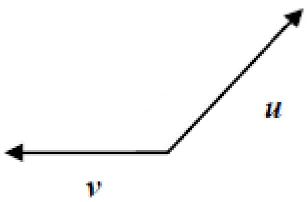
1. 

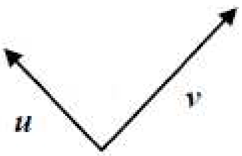
2. 

3. 

Sketch the resultant of each of the following.

4. 

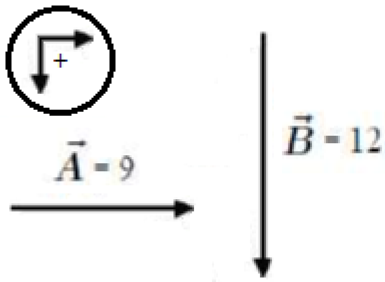
5. 

6. 

Use this space for summary and/or additional notes:

Consider the following vectors $\vec{A} \& \vec{B}$.

Vector $\vec{A}$ has a magnitude of 9 and its direction is the positive horizontal direction (to the right).



7. $\vec{A} - \vec{B}$ ..Sketch the resultant of $\vec{A} + \vec{B}$, and determine its magnitude and direction.

8. Sketch the resultant of $\vec{A} - \vec{B}$ (which is the same as $\vec{A} + (-\vec{B})$, and determine its magnitude and direction.

Use this space for summary and/or additional notes:

Vectors *vs.* Scalars in Physics

Unit: Mathematics

MA Curriculum Frameworks (2016): SP5

AP® Physics 2 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Identify vector *vs.* scalar quantities in physics.

Success Criteria:

- Quantity is correctly identified as a vector or a scalar.

Language Objectives:

- Explain why some quantities have a direction and others do not.

Tier 2 Vocabulary: vector, magnitude, direction

Notes:

In physics, most numbers represent quantities that can be measured or calculated from measurements. Most of the time, there is no concept of a "deficit" of a measured quantity. For example, quantities like mass, energy, and power can only be nonnegative, because in classical mechanics there is no such thing as "anti-mass," "anti-energy," or "anti-power."

However, vector quantities have a direction as well as a magnitude, and direction can be positive or negative.

A rule of thumb that works *most* of the time in a high school physics class is:

Scalar quantities. These are usually positive, with a few notable exceptions (*e.g.,* work and electric charge).

Vector quantities. Vectors have a direction associated with them, which is conveyed by defining a direction to be "positive". Vectors in the positive direction will be expressed as positive numbers, and vectors in the opposite (negative) direction will be expressed as negative numbers.

In some cases, you will need to split a vector in two component vectors, one vector in the x-direction, and a separate vector in the y-direction. In these cases, you will need to choose which direction is positive and which direction is negative for *both* the x- and y-axes. Once you have done this, every vector quantity must be assigned a positive or negative value, according to the directions you have chosen.

Use this space for summary and/or additional notes:

Differences. The difference or change in a variable is indicated by the Greek letter Δ in front of the variable. Any difference can be positive or negative. However, note that a difference can either be a vector, indicating a change relative to the positive direction (*e.g.,* $\vec{\Delta x}$, which indicates a change in position), or scalar, indicating an increase or decrease (*e.g.,* ΔV, which indicates a change in volume).

Example:

Suppose you have a problem that involves throwing a ball straight upwards with a velocity of $15\,\frac{\text{m}}{\text{s}}$. Gravity is slowing the ball down with a downward acceleration of $10\,\frac{\text{m}}{\text{s}^2}$. You want to know how far the ball has traveled in 0.5 s.

Displacement, velocity, and acceleration are all vectors. The motion is happening in the y-direction, so we need to choose whether "up" or "down" is the positive direction. Suppose we choose "up" to be the positive direction. This means:

- When the ball is first thrown, it is moving upwards. This means its velocity is in the *positive* direction, so we would represent the initial velocity as $\vec{v}_o = +15\,\frac{\text{m}}{\text{s}}$.
- Gravity is accelerating the ball downwards, which is the *negative* direction. We would therefore represent the acceleration as $\vec{a} = -10\,\frac{\text{m}}{\text{s}^2}$.
- Time is a scalar quantity, so its value is +0.5 s.

If we had to substitute the numbers into the formula:

$$\vec{d} = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2$$

we would do so as follows:

$$\vec{d} = (+15)(0.5) + (\tfrac{1}{2})(-10)(0.5)^2$$

and we would find out that $\vec{d} = +6.25\,\text{m}$.

The answer is *positive*. Earlier, we defined positive as "up", so the answer tells us that the displacement is upwards from the starting point.

Use this space for summary and/or additional notes:

What if, instead, we had chosen "down" to be the positive direction?

- When the ball is first thrown, it is moving upwards. This means its velocity is now in the *negative* direction, so we would represent the initial velocity as $\vec{\boldsymbol{v}}_o = -15\,\frac{\text{m}}{\text{s}}$.
- Gravity is accelerating the ball downwards, which is the *positive* direction. We would therefore represent the acceleration as $\vec{\boldsymbol{a}} = +10\,\frac{\text{m}}{\text{s}^2}$.
- Time is a scalar quantity, so its value is +0.5 s.

If we had to substitute the numbers into the formula:

$$\vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}_o t + \frac{1}{2}\vec{\boldsymbol{a}} t^2$$

we would do so as follows:

$$\vec{\boldsymbol{d}} = (-15)(0.5) + (\tfrac{1}{2})(10)(0.5)^2$$

and we would find out that $\vec{\boldsymbol{d}} = -6.25\,\text{m}$.

The answer is *negative*. However, remember that we defined "down" to be positive, which means "up" is the negative direction. This means the displacement is *upwards* from the starting point, as before.

In any problem you solve, the choice of which direction is positive *vs.* negative is arbitrary. The only requirement is that *every vector quantity in the problem* needs to be consistent with your choice.

Use this space for summary and/or additional notes:

Vector Multiplication

Unit: Mathematics

MA Curriculum Frameworks (2016): SP5

AP® Physics 2 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Correctly use and interpret the symbols "•" and "×" when multiplying vectors.
- Finding the dot product & cross product of two vectors.

Success Criteria:

- Magnitudes and directions are correct.

Tier 2 Vocabulary: magnitude, direction, dot, cross

Language Objectives:

- Explain how to interpret the symbols "•" and "×" when multiplying vectors.

Tier 2 Vocabulary: vector, product

Notes:

With scalar (ordinary) numbers, there is only one way to multiply them, which you learned in elementary school. Vectors, however, can be multiplied in three different ways.

dot product: multiplication of two vectors that results in a scalar.

$$\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}} = C$$

cross product: multiplication of two vectors that results in a new vector.

$$\vec{\boldsymbol{I}} \times \vec{\boldsymbol{J}} = \vec{\boldsymbol{K}}$$

tensor product: multiplication of two vectors that results in a tensor. (A tensor is an array of vectors that describes the effect of each vector on every other vector in the array. Tensors are beyond the scope of a high school physics course.)

Use this space for summary and/or additional notes:

Multiplying a Vector by a Scalar

Multiplying a vector by a scalar is like multiplying a variable by a number. The magnitude changes, but the direction does not. For example, in physics, displacement equals velocity times time:

$$\vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}t$$

Velocity is a vector; time is a scalar. The magnitude is the velocity times the time, and the direction of the displacement is the same as the direction of the velocity.

If the two vectors have opposite directions, the equation needs a negative sign. For example, the force applied by a spring equals the spring constant (a scalar quantity) times the displacement:

$$\vec{\boldsymbol{F}}_s = -k\vec{\boldsymbol{x}}$$

The negative sign in the equation signifies that the force applied by the spring is in the opposite direction from the displacement.

The Dot (Scalar) Product of Two Vectors

The scalar product of two vectors is called the "dot product". Dot product multiplication of vectors is represented with a dot:

$$\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}}\,^{*}$$

The dot product of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ is:

$$\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}} = AB\cos\theta$$

where A is the magnitude of $\vec{\boldsymbol{A}}$, B is the magnitude of $\vec{\boldsymbol{B}}$, and θ is the angle between the two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$.

For example, in physics, *work* (a scalar quantity) is the dot product of the vectors *force* and *displacement* (distance):

$$W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}} = Fd\cos\theta$$

* pronounced "A dot B"

Use this space for summary and/or additional notes:

The Cross (Vector) Product of Two Vectors

The vector product of two vectors is called the cross product. Cross product multiplication of vectors is represented with a multiplication sign:

$$\vec{A} \times \vec{B}^{*}$$

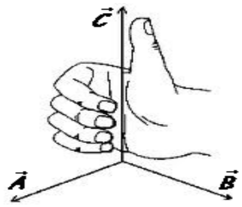
The magnitude of the cross product of vectors $\vec{A}$ and $\vec{B}$ that have an angle of θ between them is given by the formula:

$$\vec{A} \times \vec{B} = AB \sin\theta$$

The direction of the cross product is a little difficult to make sense out of. You can figure it out using the "right hand rule":

$$\vec{A} \times \vec{B} = \vec{C}$$
$$\vec{B} \times \vec{A} = -\vec{C}$$

Position your right hand so that your fingers curl from the first vector to the second. Your thumb points in the direction of the resultant vector.



Note that this means that the resultant vectors for $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ point in *opposite* directions, *i.e.,* the cross product of two vectors is *not* commutative!

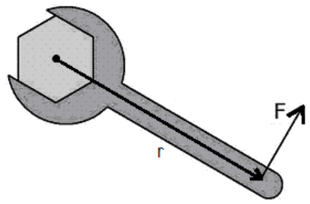
A vector coming out of the page is denoted by a series of ⊙ ⊙ ⊙ ⊙ ⊙ symbols, and a vector going into the page is denoted by a series of ⊗ ⊗ ⊗ ⊗ ⊗ symbols.

Think of these symbols as representing an arrow inside a tube or pipe. The dot represents the tip of the arrow coming toward you, and the "X" represents the fletches (feathers) on the tail of the arrow going away from you.)

* pronounced "A cross B"

Use this space for summary and/or additional notes:

In physics, torque is a vector quantity that is derived by a cross product.



The torque produced by a force $\vec{\boldsymbol{F}}$ acting at a radius $\vec{\boldsymbol{r}}$ is given by the equation:

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} = rF\sin\theta$$

Because the direction of the force is usually perpendicular to the displacement, it is usually true that $\sin\theta = \sin 90° = 1$. This means the magnitude $rF\ \sin\theta = rF(1) = rF$. Using the right-hand rule, we determine that the *direction* of the resultant torque vector is coming out of the page.

(The force generated by the interaction between charges and magnetic fields, a topic covered in AP® Physics 2, is also a cross product.)

Thus, if you are tightening or loosening a nut or bolt that has right-handed (standard) thread, the torque vector will be in the direction that the nut or bolt moves.

Vector Jokes

Now that you understand vectors, here are some bad vector jokes:

Q: What do you get when you cross an elephant with a bunch of grapes?

A:  $\sin\theta$

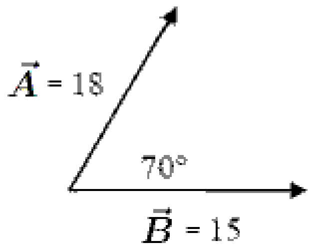
Q: What do you get when you cross an elephant with a mountain climber?

A: You can't do that! A mountain climber is a scalar ("scaler," meaning someone who scales a mountain).

Use this space for summary and/or additional notes:

Homework Problems

For the following vectors $\vec{\boldsymbol{A}} \& \vec{\boldsymbol{B}}$:



1. Determine $\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}}$

2. Determine $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$ (both magnitude and direction)

Use this space for summary and/or additional notes:

Logarithms

Unit: Mathematics

MA Curriculum Frameworks (2016): N/A

Old MA Curriculum Frameworks (2006): N/A

AP® Physics 2 Learning Objectives: N/A

Knowledge/Understanding:

- What logarithms represent and an intuitive understanding of logarithmic quantities.

Skills:

- Use logarithms to solve for a variable in an exponent.

Language Objectives:

- Understand the use of the terms "exponential" and "logarithm" and understand the vernacular use of "log" (otherwise a Tier 1 word) as an abbreviation for "logarithm".

Tier 2 Vocabulary: function

Notes:

The logarithm may well be the least well-understood function encountered in high school mathematics.

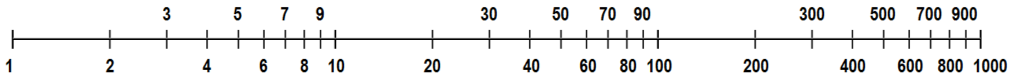
The simplest logarithm to understand is the base-ten logarithm. You can think of the (base-ten) logarithm of a number as the number of zeroes after the number.

x		$\log_{10}(x)$
100 000	10^5	5
10 000	10^4	4
1 000	10^3	3
100	10^2	2
10	10^1	1
1	10^0	0
0.1	10^{-1}	−1
0.01	10^{-2}	−2
0.001	10^{-3}	−3
0.000 1	10^{-4}	−4
0.000 01	10^{-5}	−5

As you can see from the above table, the logarithm of a number turns a set of numbers that vary exponentially (powers of ten) into a set that vary linearly.

Use this space for summary and/or additional notes:

You can get a visual sense of the logarithm function from the logarithmic number line below:



Notice that the *distance* from 1 to 10 is the same as the *distance* from 10 to 100 and from 100 to 1000. In fact, the relative distance to every number on this number line is the logarithm of the number.

x	$\log_{10}(x)$	distance from beginning of number line
10^0	0	0
$10^{0.5} \approx 3.16$	0.5	½ cycle
$10^1 = 10$	1	1 cycle
$10^2 = 100$	2	2 cycles
$10^3 = 1000$	3	3 cycles

By inspection, you can see that the same is true for numbers that are not exact powers of ten. The logarithm function compresses correspondingly more as the numbers get larger.

The most useful mathematical property of logarithms is that they move an exponent into the linear part of the equation:

$$\log_{10}(10^3) = 3\log_{10}(10) = (3)(1) = 3$$

In fact, the logarithm function works the same way for any base, not just 10:

$$\log_2(2^7) = 7\log_2(2) = (7)(1) = 7$$

(In this case, the word "base" means the base of the exponent.) The general equation is:

$$\log_x(a^b) = b\log_x(a)$$

This is a powerful tool in solving for the exponent in an equation. This is, in fact, precisely the purpose of using logarithms in most mathematical equations.

Use this space for summary and/or additional notes:

Sample problem:

Q: Solve $3^x = 15$ for x.

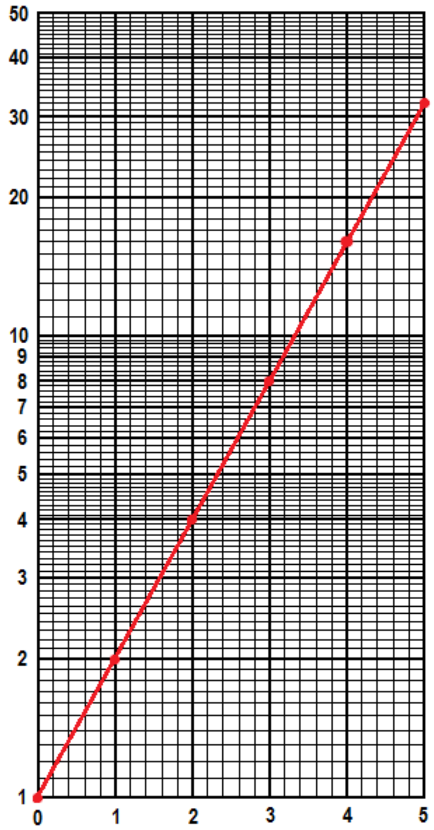
A: Take the logarithm (any base) of both sides. (Note that writing "log" without supplying a base implies that the base is 10.)

$$\log(3^x) = \log(15)$$
$$x\log(3) = \log(15)$$
$$(x)(0.477) = 1.176$$
$$x = \frac{1.176}{0.477} = 2.465$$

This is the correct answer, because $3^{2.465} = 15$

Logarithmic Graphs

A powerful tool that follows from this is using logarithmic graph paper to solve equations. If you plot an exponential function on semilogarithmic ("semi-log") graph paper (meaning graph paper that has a logarithmic scale on one axis but not the other), you get a straight line.



The graph at the right is the function $y = 2^x$. Notice where the following points appear on the graph:

Domain	Range
0	1
1	2
2	4
3	8
4	16
5	32

Notice also that you can use the graph to find intermediate values. For example, at x = 2.6, the graph shows that y = 6.06.

Use this space for summary and/or additional notes:

Natural Logarithms

The natural logarithm comes from calculus—it is the solution to the problem:

$$\int \frac{1}{x}\,dx = \log_e(x)$$

where the base of this logarithm, "*e*," is a constant (sometimes called "Euler's number") that is an irrational number equal to approximately 2.71828 18284 59045…

The natural logarithm is denoted "ln", so we would actually write:

$$\int \frac{1}{x}\,dx = \ln(x)$$

The number "*e*" is often called the exponential function. In an algebra-based physics class, the exponential function appears in some equations whose derivations come from calculus, notably some of the equations relating to resistor-capacitor (RC) circuits.

Finally, just as:

$$\log(10^x) = x \text{ and } 10^{\log(x)} = x$$

it is similarly true that:

$$\ln(e^x) = x \text{ and } e^{\ln(x)} = x$$

Use this space for summary and/or additional notes:

Introduction: Fluids & Pressure

Unit: Fluids & Pressure

Topics covered in this chapter:

In this chapter you will learn about pressure and behaviors of fluids.

- *Pressure* explains pressure as a force spread over an area. Pressure is the property that is central to the topic of fluid mechanics.
- *Hydraulic Pressure and Hydrostatic Pressure* describe how pressure acts in two common situations.
- *Buoyancy* descries the upward pressure exerted by a fluid that causes objects to float.
- *Fluid Motion & Bernoulli's Law* describes the relationship between pressure and fluid motion.

This chapter focuses on real-world applications of fluids and pressure, including more demonstrations than most other topics. One of the challenges in this chapter is relating the equations to the behaviors seen in the demonstrations.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-1. Analyze data to support the claim that Newton's second law of motion is a mathematical model describing change in motion (the acceleration) of objects when acted on by a net force.

HS-PS2-10(MA). Use free-body force diagrams, algebraic expressions, and Newton's laws of motion to predict changes to velocity and acceleration for an object moving in one dimension in various situations.

Use this space for summary and/or additional notes:

AP®

AP® Physics 2 Learning Objectives:

1.E.1.1: The student is able to predict the densities, differences in densities, or changes in densities under different conditions for natural phenomena and design an investigation to verify the prediction. [**SP 4.2, 6.4**]

1.E.1.2: The student is able to select from experimental data the information necessary to determine the density of an object and/or compare densities of several objects. [**SP 4.1, 6.4**]

3.C.4.1: The student is able to make claims about various contact forces between objects based on the microscopic cause of those forces. [**SP 6.1**]

3.C.4.2: The student is able to explain contact forces (tension, friction, normal, buoyant, spring) as arising from interatomic electric forces and that they therefore have certain directions. [**SP 6.2**]

5.B.10.1: The student is able to use Bernoulli's equation to make calculations related to a moving fluid. [**SP 2.2**]

5.B.10.2: The student is able to use Bernoulli's equation and/or the relationship between force and pressure to make calculations related to a moving fluid. [**SP 2.2**]

AP®

5.B.10.3: The student is able to use Bernoulli's equation and the continuity equation to make calculations related to a moving fluid. [**SP 2.2**]

5.B.10.4: The student is able to construct an explanation of Bernoulli's equation in terms of the conservation of energy. [**SP 6.2**]

5.F.1.1: The student is able to make calculations of quantities related to flow of a fluid, using mass conservation principles (the continuity equation). [**SP 2.1, 2.2, 7.2**]

Skills learned & applied in this chapter:

- Before & after problems.

Use this space for summary and/or additional notes:

Pressure

Unit: Fluids & Pressure

MA Curriculum Frameworks (2016): HS-PS2-10(MA), HS-PS2-1

AP® Physics 2 Learning Objectives: 3.4.C.1, 3.4.C.2

Mastery Objective(s): (Students will be able to…)

- Calculate pressure as a force applied over an area.

Success Criteria:

- Pressures are calculated correctly and have correct units.

Language Objectives:

- Understand and correctly use the terms "force", "pressure" and "area" as they apply in physics.
- Explain the difference between how "pressure" is used in the vernacular *vs.* in physics.

Tier 2 Vocabulary: fluid, pressure

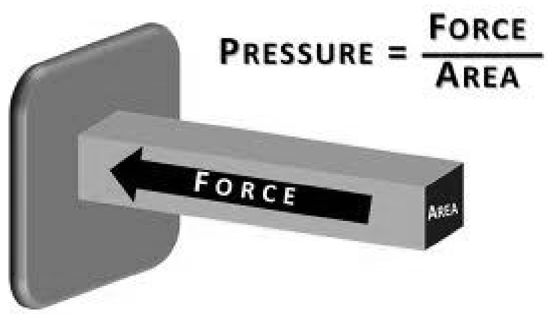
Labs, Activities & Demonstrations:

- Balloon.
- Pinscreen (pin art) toy.
- Balloon & weights on small bed of nails.
- Full-size bed of nails.

Notes:

pressure: the exertion of force upon a surface by an object, fluid, *etc.* that is in contact with it.

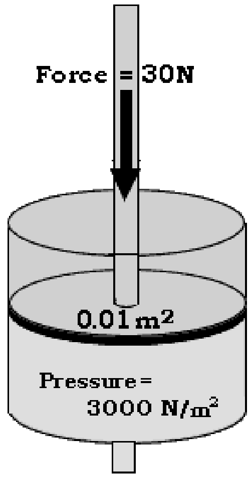
Mathematically, pressure is defined as force divided by area:



$$P = \frac{F}{A}$$

Use this space for summary and/or additional notes:

The S.I. unit for pressure is the pascal (Pa).



$$1\,\text{Pa} \equiv 1\,\frac{\text{N}}{\text{m}^2} \equiv 1\,\frac{\text{kg}}{\text{ms}^2}$$

(Note that Pa is a two-letter symbol.)

Some other common pressure units are:

- bar: 1 bar ≡ 100 000 Pa
- pound per square inch (psi)
- atmosphere (atm): the average atmospheric pressure on Earth at sea level.

  1 atm ≡ 101 325 Pa ≡ 1.01325 bar = 14.696 psi

  In this course, we will use the approximation that 1 atm ≈ 1 bar, meaning that standard atmospheric pressure is 1 bar ≡ 100 000 Pa.

Air pressure can be described relative to a total vacuum (absolute pressure), but is more commonly described relative to atmospheric pressure (gauge pressure):

- absolute pressure: the total pressure on a surface. An absolute pressure of zero means there is zero force on the surface.
- gauge pressure: the difference between the pressure on a surface and atmospheric pressure. A gauge pressure of zero means the same as atmospheric pressure. The pressure in car tires is measured as gauge pressure. For example, a tire pressure of 30 psi (30 pounds per square inch, or $30\,\frac{\text{lb.}}{\text{in.}^2}$) would mean that the air inside the tires is pushing against the air outside the tires with a pressure of 30 psi.

A flat tire would have a gauge pressure of zero and an absolute pressure of about 1 bar.

Sample Problem

Q: What is the pressure caused by a force of 25 N acting on a piston with an area of 0.05 m^2?

A: $P = \dfrac{F}{A} = \dfrac{25\,\text{N}}{0.05\,\text{m}^2} = 500\,\text{Pa}$

Use this space for summary and/or additional notes:

Homework Problems

1. **(M = Must Do)** A person wearing snowshoes does not sink into the snow, whereas the same person without snowshoes sinks into the snow. Explain.

2. **(S = Should Do)** A balloon is inflated to a pressure of 0.2 bar. A 5.0 kg book is balanced on top of the balloon. With what surface area does the balloon contact the book? (*Hint: Remember that* 1 bar = 100 000 Pa.)

   Answer: 0.002 5 m^2

3. **(S = Should Do)** A carton of paper has a mass of 22.7 kg. The area of the bottom is 0.119 m^2. What is the pressure between the carton and the floor?

   Answer: 1 908 Pa

4. **(S = Should Do)** A 1000 kg car rests on four tires, each inflated to 2.2 bar. What surface area does *each* tire have in contact with the ground? (Assume the weight is evenly distributed on each wheel.)

   Answer: 0.011 4 m^2

Use this space for summary and/or additional notes:

5. **(A = Aspire to Do)*** A student with a mass of 75.0 kg is sitting on 4-legged lab stool that has a mass of 3.0 kg. Each leg of the stool is circular and has a diameter of 2.50 cm. Find the pressure under each leg of the stool. (*Hints: (1) Remember to convert* cm^2 *to* m^2 *for the area of the legs of the stool. (2) Remember that the stool has four legs. (3) Note that the problem gives the* diameter *of the legs of the stool, not the radius.*)

   Answer: 397 250 Pa

6. **(M = Must Do)** A student has a mass of 75 kg.

   a. **(M)** The student is lying on the floor of the classroom. The area of the student that is in contact with the floor is 0.6 m^2. What is the pressure between the student and the floor? Express your answer both in pascals and in bar.

   Answer: 1 250 Pa or 0.0125 bar

   b. **(M)** The same student is lying on a single nail, which has a cross-sectional area of $0.1\,\text{mm}^2 = 1\times 10^{-7}\,\text{m}^2$. What is the pressure (in bar) that the student exerts on the head of the nail?

   Answer: 7.5×10^{9} Pa = 75 000 bar

   c. **(M)** The same student is lying on a bed of nails. If the student is in contact with 1 500 nails, what is the pressure (in bar) between the student and each nail?

   Answer: 5×10^{6} Pa = 50 bar

* This is a nuisance problem, not a difficult problem.

Use this space for summary and/or additional notes:

Hydraulic Pressure

Unit: Fluids & Pressure

MA Curriculum Frameworks (2016): HS-PS2-10(MA), HS-PS2-1

AP® Physics 2 Learning Objectives: 3.4.C.1, 3.4.C.2

Mastery Objective(s): (Students will be able to…)

- Calculate the force applied by a piston given the force on another piston and areas of both in a hydraulic system.

Success Criteria:

- Pressures are calculated correctly and have correct units.

Language Objectives:

- Understand and correctly use the term "hydraulic pressure."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to hydraulic pressure.

Tier 2 Vocabulary: fluid, pressure

Labs, Activities & Demonstrations:

- Syringe (squirter)
- Hovercraft

Notes:

Pascal's Principle, which was discovered by the French mathematician Blaise Pascal, states that any pressure applied to a fluid is transmitted uniformly throughout the fluid.

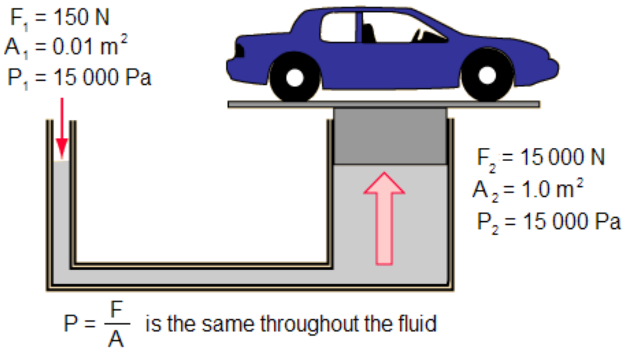
Because $P=\frac{F}{A}$, if the pressure is the same everywhere in the fluid, then $\frac{F}{A}$ must be the same everywhere in the fluid.

Use this space for summary and/or additional notes:

If you have two pistons whose cylinders are connected, the pressure is the same throughout the fluid, which means the force on each piston is proportional to its own area. Thus:

$$P_1 = P_2 \quad \text{which means} \quad \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

This principle is called "hydraulics." If you have a lift that has two pistons, one that is 100 times larger than the other, the larger one can supply 100 times as much force.



This seems like we're getting something for nothing—we're lifting a car by applying only 150 N of force (approximately 35 lbs.). However, conservation of energy tells us that the work done by F_1 must equal the work done by F_2, which means F_1 must act over a considerably larger distance than F_2. In order to lift the car on the right 10 cm (about 4 in.), you would have to press the plunger on the left 10 m.

You could also figure this out by realizing that the volume of fluid transferred on both sides must be the same and multiplying the area by the distance.

This is how hydraulic brakes work in cars. When you step on the brake pedal, the hydraulic pressure is transmitted to the master cylinder and then to the slave cylinders. The master cylinder is much smaller in diameter than the slave cylinders, which means the force applied to the brake pads is considerably greater than the force from your foot.

Sample Problem

Q: In a hydraulic system, a force of 25 N will be applied to a piston with an area of $0.50\ \text{m}^2$. If the force needs to lift a weight of 500. N, what must be the area of the piston supporting the 500. N weight?

A: $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$ $\qquad \dfrac{25}{0.50} = \dfrac{500}{A_2}$

$$25\,A_2 = (500)(0.50)$$
$$25\,A_2 = 250$$
$$A_2 = 10\ \text{m}^2$$

Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** A student who weighs 700. N stands on a hydraulic lift. The lift has a lever, which you push down in order to lift the student. The cross-sectional area of the piston pressing on the fluid under the student is 1 m^2, and the cross-sectional area of the piston pressing on the fluid under the lever is 0.1 m^2. How much force is needed to lift the student?

   Answer: 70 N

2. **(M)** A hovercraft is made from a circle of plywood and a wet/dry vacuum cleaner. The vacuum cleaner motor blows air with a force of 10 N through a hose that has a radius of 1.5 cm (0.015 m). The base of the hovercraft has a radius of 0.6 m. How much weight (in newtons) can the hovercraft lift?

   Answer: 16 000 N (which is approximately 3 600 lbs.)

Use this space for summary and/or additional notes:

Hydrostatic Pressure

Unit: Fluids & Pressure

MA Curriculum Frameworks (2016): HS-PS2-10(MA), HS-PS2-1

AP® Physics 2 Learning Objectives: 1.E.1.1, 1.E.1.2

Mastery Objective(s): (Students will be able to…)

- Calculate the hydrostatic pressure exerted by a column of fluid of a given depth and density.

Success Criteria:

- Pressures are calculated correctly with correct units.

Language Objectives:

- Explain how gravity causes a column of fluid to exert a pressure.

Labs, Activities & Demonstrations:

- Bottle with hole (feel suction, pressure at exit)
- Burette & funnel manometer
- Syphon hose
- Cup of water & index card
- Magdeburg hemispheres
- Shrink-wrap students

Notes:

<u>hydrostatic pressure</u>: the pressure caused by the weight of a column of fluid

The force of gravity pulling down on the particles in a fluid creates pressure. The more fluid there is above a point, the higher the pressure at that point.

The atmospheric pressure that we measure at the surface of the Earth is caused by the air above us, all the way to the highest point in the atmosphere, as shown in the picture at right.

Use this space for summary and/or additional notes:

Assuming the density of the fluid is constant, the pressure in a column of fluid is caused by the weight (force of gravity) acting on an area. Because the force of gravity is *mg* (where $g = 10\,\frac{\text{N}}{\text{kg}}$), this means:

$$P_H = \frac{F_g}{A} = \frac{mg}{A}$$

where:

P_H = hydrostatic pressure

g = strength of gravitational field ($10\,\frac{\text{N}}{\text{kg}}$ on Earth)

A = area of the surface the fluid is pushing on

We can cleverly multiply and divide our equation by volume:

$$P_H = \frac{mg}{A} = \frac{mg \cdot V}{A \cdot V} = \frac{m}{V} \cdot \frac{gV}{A}$$

Then, we need to recognize that (1) density (ρ*) is mass divided by volume, and (2) the volume of a region is the area of its base times the height (h). Thus the equation becomes:

$$P_H = \rho \cdot \frac{gV}{A} = \rho \cdot \frac{g\cancel{A}h}{\cancel{A}}$$

$$P_H = \rho g h$$

Finally, if there is an external pressure, P_o, above the fluid, we have to add it to the hydrostatic pressure from the fluid itself, which gives us the familiar form of the equation:

$$P = P_o + P_H = P_o + \rho g h$$

where:

P_H = hydrostatic pressure

P_o = pressure above the fluid (if relevant)

ρ = density of the fluid (this is the Greek letter "rho")

g = strength of gravity ($10\,\frac{\text{N}}{\text{kg}}$ on Earth)

h = height of the fluid ***above*** the point of interest

Although the depth of the fluid is called the "height," the term is misleading. The pressure is caused by gravity pulling down on the fluid ***above*** it.

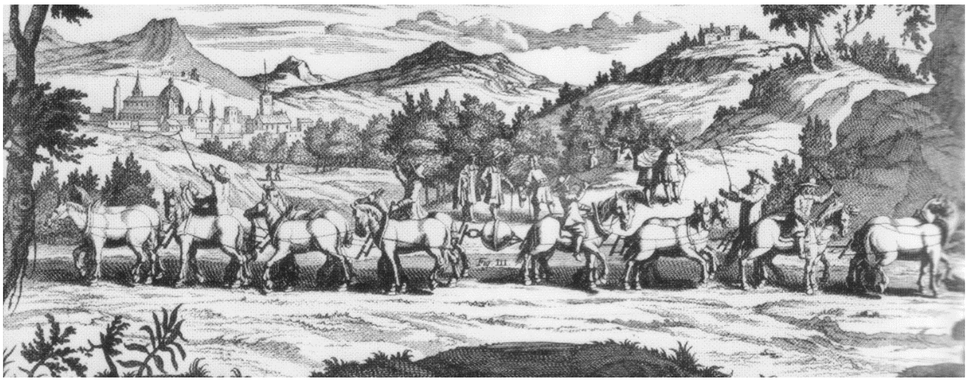
* Note that physicists use the Greek letter ρ ("rho") for density. You need to pay careful attention to the difference between the Greek letter ρ and the Roman letter "p".

Use this space for summary and/or additional notes:

In 1654, Otto von Guericke, a German scientist and the mayor of the town of Magdeburg, invented an apparatus that demonstrates atmospheric pressure.

In 1650, Guericke had invented the vacuum pump. To demonstrate his invention, Guericke built an apparatus consisting of a fitted pair of hemispheres. Guericke pumped (most of) the air out from inside the hemispheres. Because of the pressure from the atmosphere on the outside of the hemispheres, it was difficult if not impossible to pull them apart.

In 1654, Guericke built a large vacuum pump and a large pair of hemispheres. In a famous demonstration, two teams of horses were unable to pull the hemispheres apart.



The hemispheres are called Magdeburg hemispheres, after the town that Guericke was mayor of.

Sample Problem

Q: What is the water pressure in the ocean at a depth of 25 m? The density of sea water is $1025\,\frac{\text{kg}}{\text{m}^3}$.

A: $P_H = \rho g h = (1025)(10)(25) = 256\,250\,\text{Pa} = 2.56\,\text{bar}$

Use this space for summary and/or additional notes:

Homework Problems

For all problems, assume that the density of fresh water is $1000\,\frac{\text{kg}}{\text{m}^3}$.

1. **(S)** A diver dives into a swimming pool and descends to a maximum depth of 3.0 m. What is the pressure on the diver due to the water at this depth? Give your answer in both pascals (Pa) and in bar.

   Answer: 30 000 Pa or 0.3 bar

2. **(M)** A wet/dry vacuum cleaner is capable of creating enough of a pressure difference to lift a column of water to a height of 1.5 m at 20 °C. How much pressure can the vacuum cleaner apply?

   Answer: 15 000 Pa

3. **(S)** A standard water tower is 40 m above the ground. What is the resulting water pressure at ground level? Express your answer in pascals, bar, and pounds per square inch. (1 bar = 14.5 psi)

   Answer: 400 000 Pa or 4 bar or 58 psi

Use this space for summary and/or additional notes:

4. **(M)** A set of Magdeburg hemispheres has a radius of 6 cm (0.06 m). Atmospheric pressure is 1 bar and all of the air inside is pumped out (*i.e.,* the pressure inside is zero).

   a. Calculate the force needed to pull the hemispheres apart. (The formula for the surface area of a sphere is $S = 4\pi r^2$).

   Answer: 1 130 N (which is about 250 lbs.)

   b. Assume that the density of air is $1\frac{\text{kg}}{\text{m}^3}$. If the density of the atmosphere were uniform, how high above the Earth would the top of the atmosphere be?

   Answer: 10 000 m

   c. The actual height of the atmosphere is approximately 10^7 m (10 000 km), which means the atmosphere cannot have a uniform density. Why is it reasonable to assume that water has a uniform density, but not air?

Use this space for summary and/or additional notes:

Buoyancy

Unit: Fluids & Pressure

MA Curriculum Frameworks (2016): HS-PS2-10(MA), HS-PS2-1

AP® Physics 2 Learning Objectives: 1.E.1.1, 1.E.1.2, 3.4.C.1, 3.4.C.2

Mastery Objective(s): (Students will be able to…)

- Solve problems involving the buoyant force on an object.
- Use a free-body diagram to represent the forces on an object surrounded by a fluid.

Success Criteria:

- Problems are set up & solved correctly with the correct units.

Language Objectives:

- Explain why a fluid exerts an upward force on an object surrounded by it.

Tier 2 Vocabulary: float, displace

Labs, Activities & Demonstrations:

- Upside-down beaker with tissue
- Ping-pong ball or balloon under water
- beaker floating in water
  - right-side-up with weights
  - upside-down with trapped air
- Spring scale with mass in & out of water on a balance
- Cartesian diver
- Aluminum foil & weights
- Cardboard & duct tape canoes

Notes:

<u>displace</u>: to push out of the way

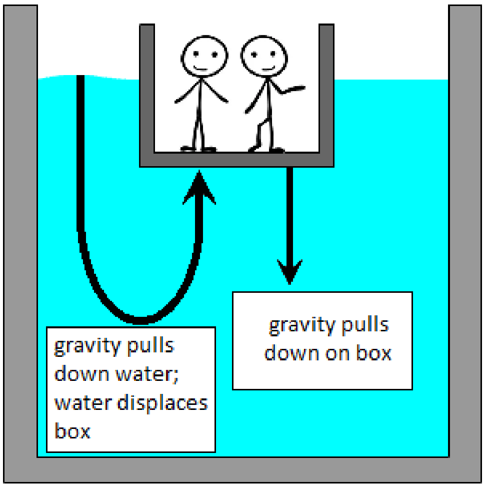
Use this space for summary and/or additional notes:

buoyancy: a net upward force caused by the differences in hydrostatic pressure at different levels within a fluid.

Buoyancy is ultimately caused by gravity:

1. Gravity pulls down on an object.
2. The object displaces water (or whatever fluid it's in).
3. Gravity pulls down on the water.
4. The water attempts to displace the object.

The force of the water attempting to displace the object is the buoyant force $(\vec{F}_B)$.

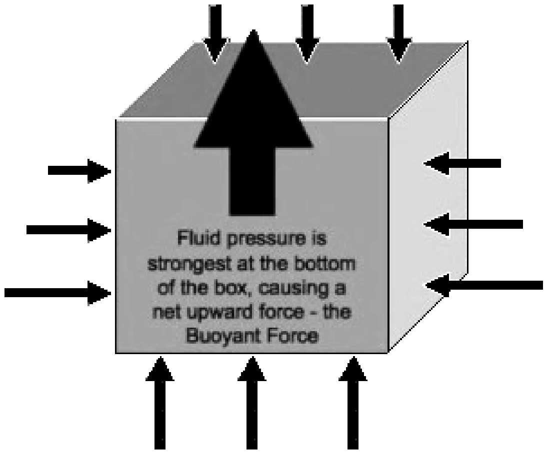


If the object floats, it reaches its equilibrium when the weight of the object and the weight of the water that was displaced (and is trying to displace the object) are equal.

If the object sinks, it is because the object can only displace its own volume. If an equal volume of water would weigh less than the object, the weight of the water is unable to apply enough force to lift the object.

Use this space for summary and/or additional notes:

The reason the object moves upwards is because the hydrostatic pressure is stronger at the bottom of the object than at the top. This slight difference causes a net upward force on the object.



When an object displaces a fluid:

1. The volume of the fluid displaced equals the volume of the submerged part of the object: $V_{fluid\ displaced} = V_{submerged\ part\ of\ object}$
2. The weight of the fluid displaced equals the buoyant force (F_B).
3. The net force on the object, if any, is the difference between its weight and the buoyant force: $F_{net} = F_g - F_B$

The equation for the buoyant force is:

$$F_B = \rho V_d g$$

Where:

F_B = buoyant force (N)

ρ = density of fluid $\left(\frac{\text{kg}}{\text{m}^3}\right)$; fresh water = $1000\,\frac{\text{kg}}{\text{m}^3}$

V_d = volume of fluid displaced (m^3)

g = strength of gravitational field $\left(g = 10\,\frac{\text{N}}{\text{kg}}\right)$

Use this space for summary and/or additional notes:

Maximum Buoyant Force

The maximum buoyant force on an object is conceptually similar to the maximum force of static friction.

Friction	Buoyancy
Static friction is a reaction force that is equal to the force that caused it.	Buoyancy is a reaction force that is equal to the force that caused it (the weight of the object).
When static friction reaches its maximum value, the object starts moving.	When the buoyant force reaches its maximum value (*i.e.,* when the volume of water displaced equals the volume of the object), the object sinks.
When the object is moving, there is still friction, but the force is not strong enough to stop the object from moving.	When an object sinks, there is still buoyancy, but the force is not strong enough to cause the object to float.

Detailed Explanation

If the object floats, there is no net force, which means the weight of the object is equal to the buoyant force. This means:

$$F_g = F_B$$
$$mg = \rho V_d g$$

Cancelling g from both sides gives $m = \rho V_d$, which can be rearranged to give the equation for density:

$$\rho = \frac{m}{V_d}$$

Therefore:

- If the object floats, the mass of the object equals the mass of the fluid displaced.
- The volume of the fluid displaced equals the volume of the object that is submerged.
- The density of the object (including any air inside of it that is below the fluid level) is less than the density of the fluid. (This is why a ship made of steel can float.)

Use this space for summary and/or additional notes:

If the object sinks, the weight of the object is greater than the buoyant force. This means:

$$F_B = \rho V_d g$$
$$F_g = mg$$

Therefore:

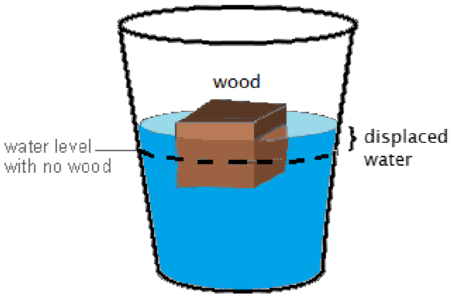
- The weight of the submerged object is $F_{net} = F_g - F_B$

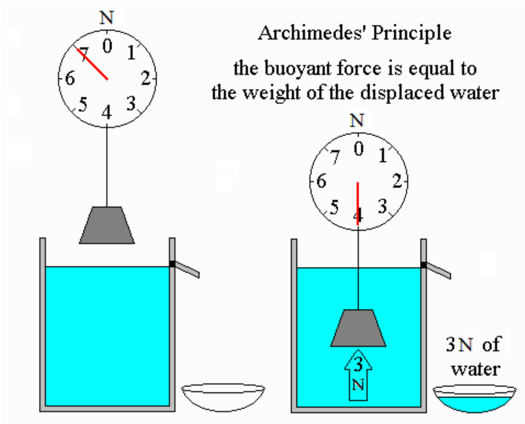
Note that if the object is resting on the bottom of the container, the net force must be zero, which means the normal force and the buoyant force combine to supply the total upward force. *I.e.,* for an object resting on the bottom:

$$F_{net} = 0 = F_g - (F_B + F_N)$$

which means:

$$F_g = F_B + F_N$$





This concept is known as Archimedes' Principle, named for the ancient Greek scientist who discovered it.

The buoyant force can be calculated from the following equation:

$$F_B = m_d g = \rho V_d g$$

where:

F_B	=	buoyant force
m_d	=	mass of fluid displaced by the object
g	=	strength of gravitational field ($10\,\frac{\text{N}}{\text{kg}}$ on Earth)
ρ	=	density of the fluid applying the buoyant force (*e.g.,* water, air)
V_d	=	volume of fluid displaced by the object

Use this space for summary and/or additional notes:

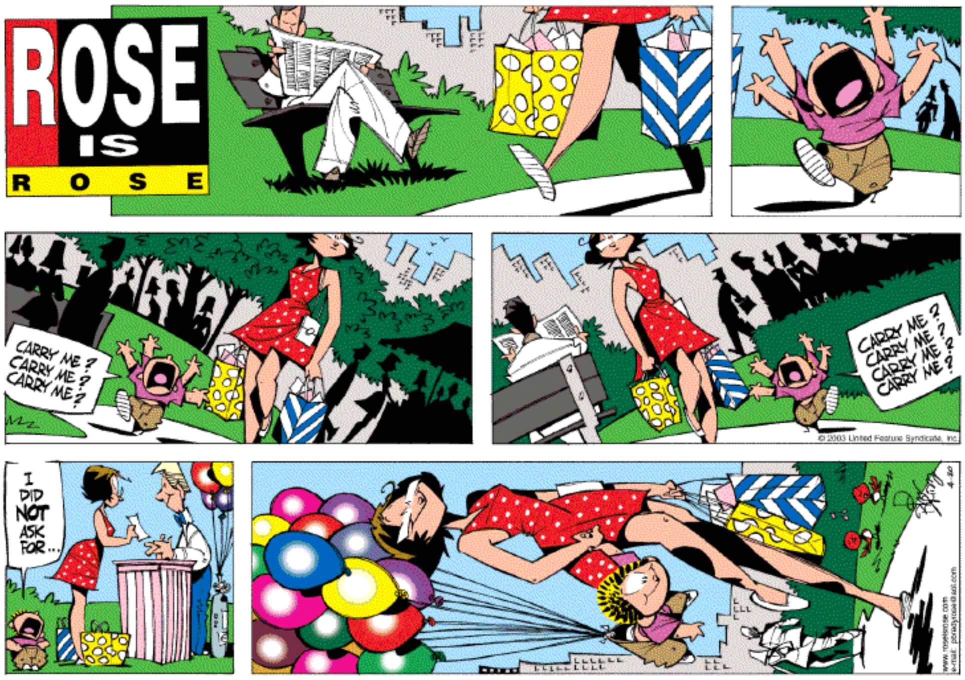
Sample Problems:

Q: A cruise ship displaces 35 000 tonnes of water when it is floating. (1 tonne = 1 000 kg) If sea water has a density of $1025\,\frac{\text{kg}}{\text{m}^3}$, what volume of water does the ship displace? What is the buoyant force on the ship?

A:
$$\rho = \frac{m}{V_d}$$
$$1025 = \frac{(35\,000)(1000)}{V_d}$$
$$V_d = 34\,146\,\text{m}^3$$
$$F_B = \rho V_d = (1025)(34\,146)(10) = 3.5 \times 10^8\ \text{N}$$

Use this space for summary and/or additional notes:

Q: Consider the following cartoon:



Given the following assumptions:

- The balloons are standard 11" balloons, meaning that they have a diameter of 11 inches (28 cm), which equals a radius of 14 cm = 0.14 m.
- The temperature is 20°C. At this temperature, air has a density of $1.200\,\frac{\text{kg}}{\text{m}^3}$, and helium has a density of $0.166\,\frac{\text{kg}}{\text{m}^3}$.
- Pasquale (the child) is probably about four years old. The average mass a four-year-old boy is about 16 kg.
- The mass of an empty balloon plus string is 2.37 g = 0.00237 kg

How many balloons would it actually take to lift Pasquale?

Use this space for summary and/or additional notes:

A: In order to lift Pasquale, $F_B = F_g$.

$$F_g = mg = (16)(10) = 160\,\text{N}$$

$$F_B = \rho_{air} V_d g = (1.2)\,V_d\,(10)$$

Because $F_B = F_g$, this means:

$$160 = 12\,V_d$$
$$V_d = 13.\overline{3}\,\text{m}^3$$

Assuming spherical balloons, the volume of one balloon is:

$$V = \frac{4}{3}\pi r^3 = (\tfrac{4}{3})(3.14)(0.14)^3 = 0.0115\,\text{m}^3$$

Therefore, we need $\dfrac{13.\overline{3}}{0.0115}$ = 1 160 balloons to lift Pasquale.

However, the problem with this answer is that it doesn't account for the mass of the helium, the balloons and the strings.

Each balloon contains $0.0115\,\text{m}^3 \times 0.166\,\frac{\text{kg}}{\text{m}^3} = 0.00191\,\text{kg}$ of helium.

Each empty balloon (including the string) has a mass of 2.37 g = 0.00237 kg.

The total mass of each balloon full of helium is
1.91 g + 2.37 g = 4.28 g = 0.00428 kg.

This means if we have n balloons, the total mass of Pasquale plus the balloons is 16 + 0.00428n kilograms. The total weight (in newtons) of Pasquale plus the balloons is therefore this number times 10, which equals 160 + 0.0428n.

The buoyant force of one balloon is:

$$F_B = \rho_{air} V_d g = (1.2)(0.0115)(10) = 0.138\,\text{N}$$

Therefore, the buoyant force of n balloons is 0.138n newtons.

For Pasquale to be able to float, $F_B = F_g$, which means

$$0.138n = 0.0428n + 160$$
$$0.0952n = 160$$
$$n = \boxed{1\,680 \text{ balloons}}$$

Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** A block is 0.12 m wide, 0.07 m long and 0.09 m tall and has a mass of 0.50 kg. The block is floating in water with a density of $1000\,\frac{\text{kg}}{\text{m}^3}$.

   a. What volume of the block is below the surface of the water?

   Answer: $5\times10^{-4}\ \text{m}^3$

   b. If the entire block were pushed under water, what volume of water would it displace?

   Answer: $7.56\times10^{-4}\ \text{m}^3$

   c. How much *additional* mass could be piled on top of the block before it sinks?

   Answer: 0.256 kg

2. **(S)** The SS United Victory was a cargo ship launched in 1944. The ship had a mass of 15 200 tonnes fully loaded. (1 tonne = 1 000 kg). The density of sea water is $1025\,\frac{\text{kg}}{\text{m}^3}$. What volume of sea water did the SS United Victory displace when fully loaded?

   Answer: 14 829 m^3

Use this space for summary and/or additional notes:

3. **(S)** An empty box is 0.11 m per side. It will slowly be filled with sand that has a density of $3500\,\frac{\text{kg}}{\text{m}^3}$. What volume of sand will cause the box to sink in water? Assume water has a density of $1000\,\frac{\text{kg}}{\text{m}^3}$. Assume the box is neutrally buoyant, which means you may neglect the weight of the box.

   Strategy:

   a. *Find the volume of the box.*
   b. *Find the mass of the water displaced.*
   c. *Find the volume of that same mass of sand.*

   Answer: $3.80\times10^{-4}\ \text{m}^3$

Use this space for summary and/or additional notes:

Fluid Motion & Bernoulli's Law

Unit: Fluids & Pressure

MA Curriculum Frameworks (2016): HS-PS2-10(MA), HS-PS2-1

AP® Physics 2 Learning Objectives: 5.B.10.1, 5.B.10.2, 5.B.10.3, 5.F.1.1

Mastery Objective(s): (Students will be able to…)

- Solve problems involving fluid flow using Bernoulli's Equation.

Success Criteria:

- Problems are set up & solved correctly with the correct units.

Language Objectives:

- Explain why a fluid has less pressure when the flow rate is faster.

Tier 2 Vocabulary: fluid, velocity

Labs, Activities & Demonstrations:

- Blow across paper (unfolded & folded)
- Blow between two empty cans.
- Ping-pong ball and air blower (without & with funnel)
- Venturi tube
- Leaf blower & large ball

Notes:

flow: the net movement of a fluid

velocity of a fluid: the average velocity of a particle of fluid as the fluid flows past a reference point. (unit = $\frac{m}{s}$)

volumetric flow rate: the volume of a fluid that passes through a section of pipe in a given amount of time. (unit = $\frac{m^3}{s}$)

mass flow rate: the mass of fluid that passes through a section of pipe in a given amount of time. (unit = $\frac{kg}{s}$)

Use this space for summary and/or additional notes:

Continuity

If a pipe has only one inlet and one outlet, all of the fluid that flows in must flow out, which means the volumetric flow rate through the pipe $\frac{V}{t}$ must be constant.

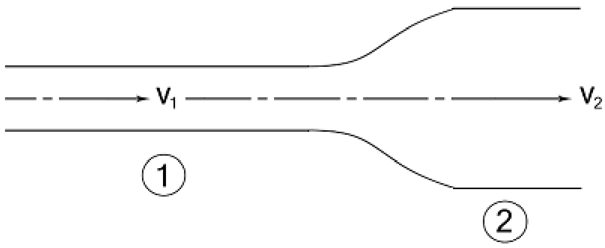
Because volume is area times length, we can write the volumetric flow rate as:

$$\frac{V}{t} = \frac{Ad}{t}$$

Assuming the velocity is constant through a section of the pipe as long as the size and elevation are not changing, we can substitute $v = \frac{d}{t}$, giving:

$$\frac{V}{t} = \frac{Ad}{t} = A \cdot \frac{d}{t} = Av = \text{constant}$$

If the volumetric flow rate remains constant but the diameter of the pipe changes:



In order to squeeze the same volume of fluid through a narrower opening, the fluid needs to flow faster. Because *Av* must be constant, the cross-sectional area times the velocity in one section of the pipe must be the same as the cross-sectional velocity in the other section.

$$Av = \text{constant}$$
$$A_1 v_1 = A_2 v_2$$

This equation is called the *continuity equation*, and it is one of the important tools that you will use to solve these problems.

Note that ***the continuity equation applies only in situations in which the flow rate is constant***, such as inside of a pipe.

Use this space for summary and/or additional notes:

Dynamic Pressure

When a fluid is flowing, the fluid must have kinetic energy, which equals the work that it takes to move that fluid.

Recall the equations for work and kinetic energy:

$$K = \tfrac{1}{2}mv^2$$
$$W = \Delta K = F_{||}d$$

Combining these (the work-energy theorem) gives $\tfrac{1}{2}mv^2 = F_{||}d$.

Solving $P_D = \dfrac{F}{A}$ for force gives $F = P_DA$. Substituting this into the above equation gives:

$$\tfrac{1}{2}mv^2 = F_{||}d = P_DAd$$

Rearranging the above equation to solve for dynamic pressure gives the following. Because volume is area times distance ($V = Ad$), we can then substitute V for Ad:

$$P_D = \frac{\tfrac{1}{2}mv^2}{Ad} = \frac{\tfrac{1}{2}mv^2}{V}$$

Finally, rearranging $\rho = \dfrac{m}{V}$ to solve for mass gives $m = \rho V$. This means our equation becomes:

$$P_D = \frac{\tfrac{1}{2}mv^2}{V} = \frac{\tfrac{1}{2}\rho \cancel{V} v^2}{\cancel{V}} = \tfrac{1}{2}\rho v^2$$
$$P_D = \tfrac{1}{2}\rho v^2$$

Use this space for summary and/or additional notes:

Bernoulli's Principle

Bernoulli's Principle, named for Dutch-Swiss mathematician Daniel Bernoulli states that the pressures in a moving fluid are caused by a combination of:

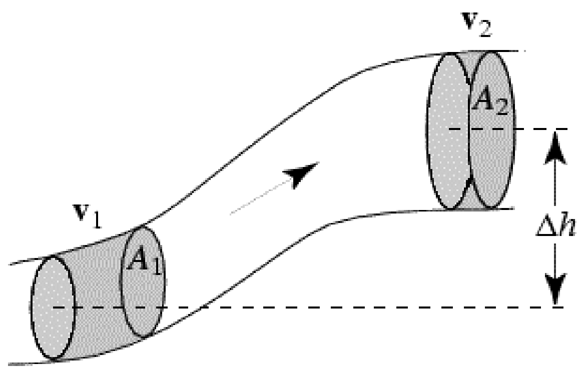
- The hydrostatic pressure: $P_H = \rho g h$
- The dynamic pressure: $P_D = \frac{1}{2}\rho v^2$
- The "external" pressure, which is the pressure that the fluid exerts on its surroundings. (This is the pressure we would measure with a pressure gauge.)

A change in any of these pressures affects the others, which means:

$$P_{ext.} + P_H + P_D = \text{constant}$$
$$P_{ext.} + \rho g h + \tfrac{1}{2}\rho v^2 = \text{constant}$$

The above equation is Bernoulli's equation.

For example, consider the following situation:



- The velocity of the fluid is changing (because the cross-sectional area is changing—remember the continuity equation $A_1 v_1 = A_2 v_2$). This means the dynamic pressure, $P_D = \frac{1}{2}\rho v^2$ is changing.
- The height is changing, which means the hydrostatic pressure, $P_H = \rho g h$ is changing.
- The external pressures will also be different, in order to satisfy Bernoulli's Law.

This means Bernoulli's equation becomes:

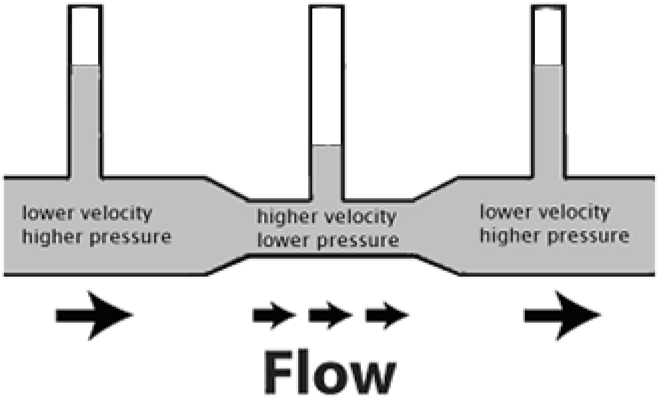
$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$

Use this space for summary and/or additional notes:

Note particularly that Bernoulli's equation tells us that increasing the fluid velocity (v) increases the dynamic pressure. (If v increases, then $P_D = \frac{1}{2}\rho v^2$ increases.)

This means if more of the total pressure is in the form of dynamic pressure, that means the hydrostatic and/or external pressures will be less.

Consider the following example:



This pipe is horizontal, which means h is constant; therefore ρgh is constant. This means that if $\frac{1}{2}\rho v^2$ increases, then pressure (P) must decrease so that

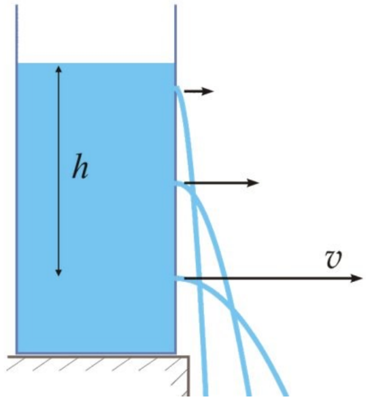
$P_{ext.} + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$.

Although Bernoulli published his principle in 1738, the application to fluids in constricted channels was not published until 1797 by Italian physicist Giovanni Venturi. The above apparatus is named after Venturi and is called a Venturi tube.

Use this space for summary and/or additional notes:

Torricelli's Theorem

A special case of Bernoulli's Principle was discovered almost 100 years earlier, in 1643 by Italian physicist and mathematician Evangelista Torricelli. Torricelli observed that in a container with fluid effusing (flowing out) through a hole, the more fluid there is above the opening, the faster the fluid comes out.



Torricelli found that the velocity of the fluid was the same as the velocity would have been if the fluid were falling straight down, which can be calculated from the change of gravitational potential energy to kinetic energy:

$$\tfrac{1}{2}\cancel{m}v^2 = \cancel{m}gh \;\rightarrow\; v^2 = 2gh \;\rightarrow\; v = \sqrt{2gh}$$

Torricelli's theorem can also be derived from Bernoulli's equation*:

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$

- The external pressures (P_1 and P_2) are both equal—atmospheric pressure—so they cancel.
- The fluid level is going down slowly enough that the velocity of the fluid inside the container (v_1) is essentially zero.
- Once the fluid exits the container, the hydrostatic pressure is zero $(\rho g h_2 = 0)$.

This leaves us with:

$$\cancel{\rho} g h_1 = \tfrac{1}{2}\cancel{\rho} {v_2}^2 \;\rightarrow\; 2gh_1 = {v_2}^2 \;\rightarrow\; \sqrt{2gh_1} = v_2$$

We could do a similar proof from the kinematic equation: $v^2 - v_o^2 = 2ad$

Substituting $a = g, d = h,$ and $v_o = 0$ gives $v^2 = 2gh$ and therefore $v = \sqrt{2gh}$

Note: as described in Hydrostatic Pressure, starting on page 158, hydrostatic pressure is caused by the fluid ***above*** the point of interest, meaning that height is measured *upward*, not downward. In the above situation, the two points of interest for the application of Bernoulli's law are actually:

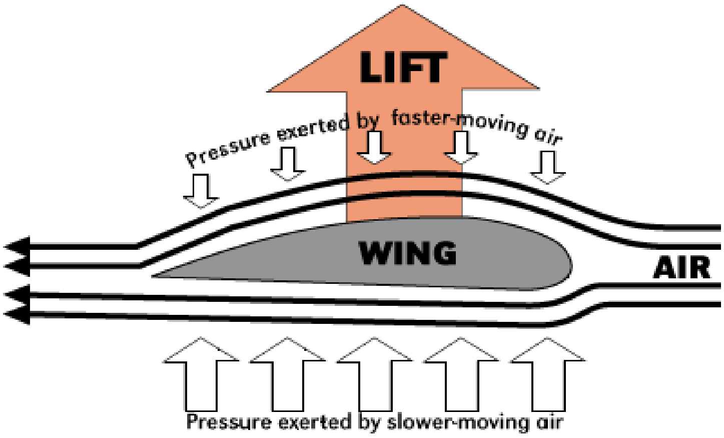
- inside the container next to the opening, where there is fluid above, but essentially no movement of fluid (v = 0, but h ≠ 0)
- outside the opening where there is no fluid above, but the jet of fluid is flowing out of the container (h = 0, but v ≠ 0)

* On the AP® Physics exam, you must start problems from equations that are on the formula sheet. This means you may not use Torricelli's Theorem on the exam unless you first derive it from Bernoulli's Equation.

Use this space for summary and/or additional notes:

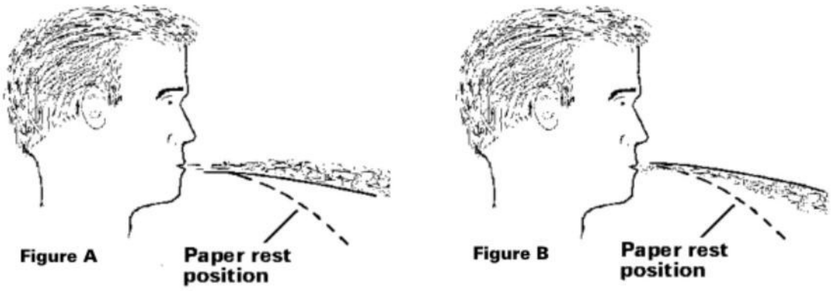
Big Ideas | Details

The decrease in pressure caused by an increase in fluid velocity explains one of the ways in which an airplane wing provides lift:



(Of course, most of an airplane's lift comes from the fact that the wing is inclined with an angle of attack relative to its direction of motion, an application of Newton's third law.)

A common demonstration of Bernoulli's Law is to blow across a piece of paper:



The air moving across the top of the paper causes a decrease in pressure, which causes the paper to lift.

Use this space for summary and/or additional notes:

Sample Problems:

Q: A fluid in a pipe with a *diameter* of 0.40 m is moving with a velocity of $0.30\,\frac{\text{m}}{\text{s}}$. If the fluid moves into a second pipe with half the diameter, what will the new fluid velocity be?

A: The cross-sectional area of the first pipe is:

$A_1 = \pi r^2 = (3.14)(0.20)^2 = 0.126\text{ m}^2$

The cross-sectional area of the second pipe is:

$A_2 = \pi r^2 = (3.14)(0.10)^2 = 0.0314\text{ m}^2$

Using the continuity equation:

$$A_1 v_1 = A_2 v_2$$

$A_1 v_1 = A_2 v_2$ $(0.126)(0.30) = (0.0314) v_2$

$$v_2 = 1.2\,\tfrac{\text{m}}{\text{s}}$$

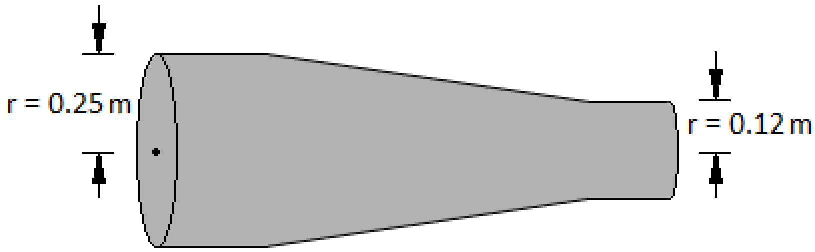
Q: A fluid with a density of $1250\,\frac{\text{kg}}{\text{m}^3}$ has a pressure of 45 000 Pa as it flows at $1.5\,\frac{\text{m}}{\text{s}}$ through a pipe. The pipe rises to a height of 2.5 m, where it connects to a second, smaller pipe. What is the pressure in the smaller pipe if the fluid flows at a rate of $3.4\,\frac{\text{m}}{\text{s}}$ through it?

A:
$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$
$$45\,000 + (1250)(10)(0) + \left(\tfrac{1}{2}\right)(1250)(1.5)^2 = P_2 + (1250)(10)(2.5) + \left(\tfrac{1}{2}\right)(1250)(3.4)^2$$
$$45\,000 + 1406 = P_2 + 31\,250 + 7225$$
$$P_2 = 7931\text{ Pa}$$

Use this space for summary and/or additional notes:

Homework Problems

1. **(S)** A pipe has a radius of 0.25 m at the entrance and a radius of 0.12 m at the exit, as shown in the figure below:



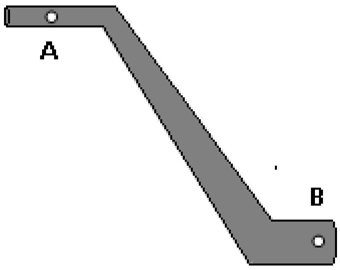
If the fluid in the pipe is flowing at $5.2\,\frac{m}{s}$ at the inlet, then how fast is it flowing at the outlet?

Answer: $22.6\,\frac{m}{s}$

Use this space for summary and/or additional notes:

Big Ideas | Details

2. **(M)** At point A on the pipe to the right, the water's speed is $4.8\,\frac{m}{s}$ and the external pressure (the pressure on the walls of the pipe) is 52.0 kPa. The water drops 14.8 m to point B, where the pipe's cross-sectional area is twice that at point A.



a. Calculate the velocity of the water at point B.

Answer: $2.4\,\frac{m}{s}$

b. Calculate the external pressure (the pressure on the walls of the pipe) at point B.

Answer: 208 600 Pa or 208.6 kPa

Use this space for summary and/or additional notes:

Introduction: Thermal Physics (Heat)

Unit: Thermal Physics (Heat)

Topics covered in this chapter:

This chapter is about heat as a form of energy and the ways in which heat affects objects, including how it is stored and how it is transferred from one object to another.

- *Heat & Temperature* describes the concept of heat as a form of energy and how heat energy is different from temperature.
- *Heat Transfer* describes how to calculate the rate of the transfer of heat energy from one object to another.
- *Specific Heat Capacity & Calorimetry* describes different substances' and objects' abilities to store heat energy.
- *Phase Diagrams* describes how to use a phase diagram to determine the state of matter of a substance at a given temperature and pressure.
- *Phases & Phase Changes* and *Heating Curves* addresses the additional calculations that apply when a substance goes through a phase change (such as melting or boiling).
- *Thermal Expansion* describes the calculation of the change in size of an object caused by heating or cooling.

New challenges specific to this chapter include looking up and working with constants that are different for different substances.

Use this space for summary and/or additional notes:

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-6. Communicate scientific and technical information about why the molecular-level structure is important in the functioning of designed materials.

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.

HS-PS3-2. Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

HS-PS3-4. Plan and conduct an investigation to provide evidence that the transfer of thermal energy when two components of different temperature are combined within a closed system results in a more uniform energy distribution among the components in the system (second law of thermodynamics).

AP® only

AP® Physics 2 Learning Objectives:

1.E.3.1: The student is able to design an experiment and analyze data from it to examine thermal conductivity. [**SP 4.1, 4.2, 5.1**]

4.C.3.1: The student is able to make predictions about the direction of energy transfer due to temperature differences based on interactions at the microscopic level. [**SP 6.4**]

5.A.2.1: The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. [**SP 6.4, 7.2**]

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Thermal Properties**, such as temperature, heat transfer, specific and latent heats, and thermal expansion.
- **Laws of Thermodynamics**, such as first and second laws, internal energy, entropy, and heat engine efficiency.

1. Heat and Temperature
2. The Kinetic Theory of Gases & the Ideal Gas Law
3. The Laws of Thermodynamics
4. Heat Engines

Use this space for summary and/or additional notes:

Skills learned & applied in this chapter:

- Working with material-specific constants from a table.
- Working with more than one instance of the same quantity in a problem.
- Combining equations and graphs.

Use this space for summary and/or additional notes:

Heat & Temperature

Unit: Thermal Physics (Heat)

MA Curriculum Frameworks (2016): HS-PS3-1, HS-PS3-2, HS-PS4-3a

AP® Physics 2 Learning Objectives: 4.C.3.1, 7.A.2.1, 7.A.2.2

Mastery Objective(s): (Students will be able to…)

- Explain heat energy in macroscopic and microscopic terms.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain the difference between heat and temperature.

Tier 2 Vocabulary: heat, temperature

Labs, Activities & Demonstrations:

- Heat a small weight and large weight to slightly different temperatures.
- Fire syringe.
- Steam engine.
- Incandescent light bulb in water.
- Mixing (via molecular motion/convection) of hot *vs.* cold water (with food coloring).

Vocabulary:

heat: energy that can be transferred by moving atoms or molecules via transfer of momentum.

temperature: a measure of the average kinetic energy of the particles (atoms or molecules) of a system.

thermometer: a device that measures temperature, most often via thermal expansion and contraction of a liquid or solid.

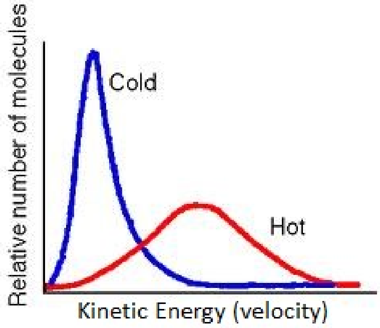
Notes:

Heat is energy that is stored as the translational kinetic energy of the particles that make up an object or substance.

You may remember from chemistry that particles (atoms or molecules) are always moving (even at absolute zero), and that energy can transfer via elastic collisions between the particles of one object or substance and the particles of another. (We will explore these concepts in more detail in the topic *Kinetic-Molecular Theory*, starting on page 233.)

Use this space for summary and/or additional notes:

Note that heat is the energy itself, whereas temperature is a measure of the quality of the heat—the average of the kinetic energies of the individual molecules:



Note that the particles of a substance have a range of kinetic energies, and the temperature is the average. Notice that when a substance is heated, the particles acquire a wider range of kinetic energies, with a higher average.

When objects are placed in contact, heat is transferred from each object to the other via the transfer of momentum that occurs when the individual molecules collide. Molecules that have more energy transfer more energy than they receive. Molecules that have less energy receive more energy than they transfer. This means three things:

1. Individual collisions transfer energy in both directions. The particles of a hot substance transfer energy to the cold substance, but the particles of the cold substance also transfer energy to the hot substance.

2. The net (overall) flow of energy is from objects with a higher temperature (more kinetic energy) to objects with a lower temperature (less kinetic energy). *I.e.,* more energy is transferred from the hot substance to the cold substance than *vice versa*.

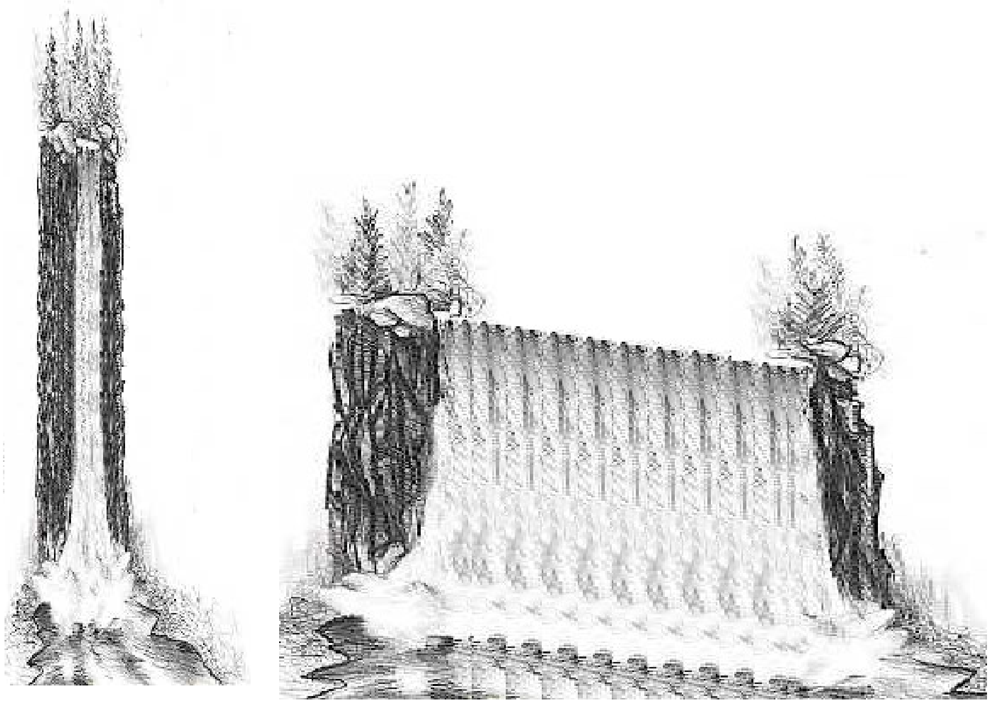
3. If you wait long enough, all of the molecules will have the same temperature (*i.e.,* the same average kinetic energy).

Use this space for summary and/or additional notes:

This means that the ***temperature*** of one object relative to another determines which direction the heat will flow, much like the way the elevation (vertical position) of one location relative to another determines which direction water will flow.

However, the total heat (energy) contained in an object depends on the mass as well as the temperature, in the same way that the total change in energy of the water going over a waterfall depends on the mass of the water as well as the height.

Consider two waterfalls, one of which is twice the height of the second, but the second of which has ten times as much water going over it as the first:



$\Delta U = mg(2h)$ $\Delta U = (10m)gh$

In the above pictures, each drop of water falling from the waterfall on the left has more gravitational potential energy, but more total energy goes over the waterfall on the right.

Similarly, each particle in an object at a higher temperature has more thermal energy than each particle in another object at a lower temperature.

If we built a waterway between the two falls, water could flow from the top of the first waterfall to the top of the second, but not *vice versa*.

Similarly, the net flow of heat is from a smaller object with higher temperature to a larger object with a lower temperature, but not *vice versa*.

Use this space for summary and/or additional notes:

Heat Flow

<u>system</u>: the region or collection of objects under being considered in a problem.

<u>surroundings</u>: everything that is outside of the system.

E.g., if a metal block is heated, we would most likely define the <u>system</u> to be the block, and the <u>surroundings</u> to be everything else.

We generally use the variable Q to represent heat in physics equations.

Heat flow is always represented in relation to the <u>system</u>.

Heat Flow	**Sign of *Q***	**System**	**Surroundings**
from the surroundings into the system	+ (positive)	gains heat (gets warmer)	lose heat (get colder)
from the system out to the surroundings	− (negative)	loses heat (gets colder)	gain heat (get hotter)

A positive value of Q means heat is flowing <u>*into*</u> the system. Because the heat is transferred from the molecules outside the system to the molecules in the system, the energy of the system increases, and the energy of the surroundings decreases.

A negative value of Q means heat is flowing <u>*out of*</u> the system. Because the heat is transferred from the molecules in the system to the molecules outside the system, the energy of the system decreases, and the energy of the surroundings increases.

This can be confusing. Suppose you set a glass of ice water on a table. When you pick up the glass, your hand gets colder because heat is flowing from your hand (which is part of the surroundings) into the system (the glass of ice water). This means the system (the glass of ice water) is gaining heat, and the surroundings (your hand, the table, *etc.*) are losing heat. The value of Q would be positive in this example.

In simple terms, you need to remember that your hand is part of the *surroundings*, not part of the system.

<u>thermal equilibrium</u>: when all of the particles in a system have the same average kinetic energy (temperature). When a system is at thermal equilibrium, no net heat is transferred. (*I.e.,* collisions between particles may still transfer energy, but the average temperature of the particles in the system—what we measure with a thermometer—is not changing.)

Use this space for summary and/or additional notes:

Heat Transfer

Unit: Thermal Physics (Heat)

MA Curriculum Frameworks (2016): HS-PS3-4a

AP® Physics 2 Learning Objectives: 1.E.3.1, 5.B.6.1

Mastery Objective(s): (Students will be able to…)

- Explain heat transfer by conduction, convection and radiation.
- Calculate heat transfer using Fourier's Law of Heat Conduction.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the mechanisms by which heat is transferred.

Tier 2 Vocabulary: conduction, radiation

Labs, Activities & Demonstrations:

- Radiometer & heat lamp.
- Almond & cheese stick.
- Flammable soap bubbles.
- Drop of food coloring in water *vs.* ice water

Notes:

Heat transfer is the flow of heat energy from one object to another. Heat transfer usually occurs through three distinct mechanisms: conduction, radiation, and convection.

conduction: transfer of heat through collisions of particles by objects that are *in direct contact* with each other. Conduction occurs when there is a net transfer of momentum from the molecules of an object with a higher temperature transfer to the molecules of an object with a lower temperature.

thermal conductivity (k): a measure of the amount of heat that a given length of a substance can conduct in a specific amount of time. Thermal conductivity is measured in units of $\frac{\text{J}}{\text{m}\cdot\text{s}\cdot{}^\circ\text{C}}$ or $\frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$.

Use this space for summary and/or additional notes:

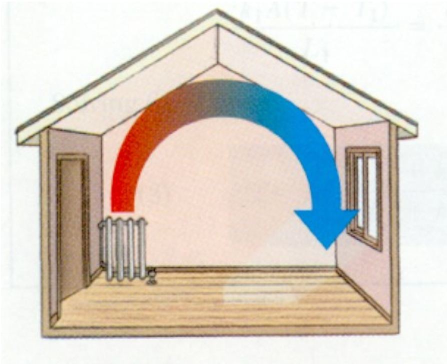
<u>conductor</u>: an object that allows heat to pass through itself easily; an object with high thermal conductivity.

<u>insulator</u>: an object that does not allow heat to pass through itself easily; a poor conductor of heat; an object with low thermal conductivity.

<u>radiation</u>: transfer of heat *<u>through space</u>* via electromagnetic waves (light, microwaves, *etc.*)

<u>convection</u>: transfer of heat *<u>by motion of particles</u>* that have a higher temperature exchanging places with particles that have a lower temperature. Convection usually occurs when air moves around a room.

<u>Natural convection</u> occurs when particles move because of differences in density. In a heated room, because cool air is more dense than warm air, the force of gravity is stronger on the cool air, and it is pulled harder toward the ground than the warm air. The cool air displaces the warm air, pushing it upwards out of the way.



In a room with a radiator, the radiator heats the air, which causes it to expand and be displaced upward by the cool air nearby. When the (less dense) warm air reaches the ceiling, it spreads out, and it continues to cool as it spreads. When the air reaches the opposite wall, it is forced downward toward the floor, across the floor, and back to the radiator.

<u>Forced convection</u> can be achieved by moving heated or cooled air using a fan. Examples of this include ceiling fans and convection ovens. If your radiator does not warm your room enough in winter, you can use a fan to speed up the process of convection. (Make sure the fan is moving the air in the same direction that would happen from natural convection. Otherwise, the fan will be fighting against physics!)

Use this space for summary and/or additional notes:

Calculating Heat Transfer by Conduction

Heat transfer by conduction can be calculated using Fourier's Law of Heat Conduction:

$$P = \frac{Q}{t} = \pm kA\frac{\Delta T}{L}$$

where:

P = power (W)

Q = heat transferred (J)

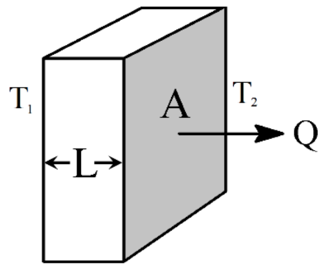
t = time (s)

k = coëfficient of thermal conductivity $\left(\frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}\right)$

A = cross-sectional area (m^2)

ΔT = temperature difference (K or °C)

L = length (m)



The ± sign means that the value can be positive or negative, because the sign for Q is chosen based on whether the heat transfers into (+) or out of (−) the system.

Note that for insulation (the kind you have in the walls and attic of your home), you want the lowest possible thermal conductivity—you don't want the insulation to conduct the heat from the inside of your house to the outside! Because most people think that bigger numbers are better, the industry has created a measure of the effectiveness of insulation called the "R value". It is essentially the reciprocal of $\frac{k}{L}$, which means lower conductivity and more thickness gives better insulation.

Sample Problem:

Q: A piece of brass is 5.0 mm (0.0050 m) thick and has a cross-sectional area of 0.010 m^2. If the temperature on one side of the metal is 65°C and the temperature on the other side is 25°C, how much heat will be conducted through the metal in 30. s? The coefficient of thermal conductivity for brass is $120\,\frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$.

A: $$\frac{Q}{t} = kA\frac{\Delta T}{L}$$

$$\frac{Q}{30} = (120)(0.010)\left(\frac{65-25}{0.0050}\right) = 9600$$

$$Q = 288\,000\text{ J} = 288\text{ kJ}$$

(Note that because the quantities of heat that we usually measure are large, values are often given in kilojoules or megajoules instead of joules.)

Use this space for summary and/or additional notes:

honors (not AP®)

Calculating Heat Transfer by Radiation

Heat transfer by radiation is based on the temperature of a substance and its ability emit heat (emissivity). The equation is:

$$P = \frac{Q}{t} = \varepsilon \sigma A T^4$$

where:

P = power (W)

Q = heat (J)

t = time (s)

ε = emissivity (dimensionless; "black body" ≡ 1)

σ = Stefan-Boltzmann constant $\left(\sigma = 5.67\times10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)$

A = area (m^2)

T = temperature (K)

Note that because the equation contains T (rather than ΔT), the temperature needs to be in Kelvin.

<u>emissivity</u> (ε): a ratio of the amount of heat radiated by a substance to the amount of heat that would be radiated by a perfect "black body" of the same dimensions.

Emissivity is a dimensionless number (meaning that it has no units, because the units cancel), and is specific to the substance.

<u>black body</u>: an object that absorbs all of the heat energy that comes in contact with it (and reflects none of it).

<u>Stefan-Boltzmann constant</u> (σ): the constant that makes the above equation come out in watts. Note that the Stefan-Boltzmann constant is defined from other constants:

$\sigma = \dfrac{2\pi^5 k_B^4}{15h^3c^2}$, where k_B is the Boltzmann constant, h is Planck's constant, and c is the speed of light in a vacuum.

Use this space for summary and/or additional notes:

Homework Problems

You will need to look up coefficients of thermal conductivity in *Table K. Thermal Properties of Selected Materials* on page 615 of your reference tables.

1. **(S)** The surface of a hot plate is made of 12.0 mm (0.012 m) thick aluminum and has an area of 64 cm^2 (which equals 0.0064 m^2). If the heating coils maintain a temperature of 80.°C underneath the surface and the air temperature is 22°C, how much heat can be transferred through the plate in 60. s?

   Answer: 464 000 J* or 464 kJ

2. **(S)** A cast iron frying pan is 5.0 mm thick. If it contains boiling water (100°C), how much heat will be transferred into your hand if you place your hand against the bottom for two seconds?
   (Assume your hand has an area of 0.0040 m^2, and that body temperature is 37°C.)

   Answer: +8 064 J or +8.064 kJ
   (positive because the direction is stated as "<u>*into*</u> your hand")

3. **(M)** A plate of metal has thermal conductivity k and thickness L. One side has a temperature of T_h and the other side has a temperature of T_c, derive an expression for the cross-sectional area A that would be needed in order to transfer a certain amount of heat, Q, through the plate in time t.

   Answer: $A = \dfrac{QL}{kt\,(T_h - T_c)}$

---

* Note: Questions #1 and #3 do not specify the direction of heat transfer, so the answer could be either positive or negative.

Use this space for summary and/or additional notes:

4. **(M)** A glass window in a house has an area of 0.67 m^2 and a thickness of 2.4 mm $(2.4\times10^{-3}\text{ m})$. The temperature inside the house is 21 °C, and the outside temperature is 0 °C.

   a. **(M)** How much heat is lost through the window in 1 hour (3600 s) due to conduction?

      Use $1.0\,\frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$ for the thermal conductivity of the glass.

      Answer: −21 100 000 J = −21 100 kJ
      (negative because heat is *lost* through the window)

Honors (not AP®)

   b. **(M – honors; A – AP®)** How much heat is lost through the window in 1 hour (3600 s) due to radiation? (Assume the temperature of the entire glass is 21 °C for this problem.)

      Hint: Remember to convert the temperature to Kelvin.

      Answer: −940 000 J = −940 kJ
      (negative because heat is *lost* through the window)

   c. **(M – honors; A – AP®)** Which mode of heat transfer (conduction *vs.* radiation) accounts for the greater amount of heat loss?

Use this space for summary and/or additional notes:

honors (not AP®)

Specific Heat Capacity & Calorimetry*

Unit: Thermal Physics (Heat)

MA Curriculum Frameworks (2016): HS-PS3-1, HS-PS4-3a

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Calculate the heat transferred when an object with a known specific heat capacity is heated.
- Perform calculations related to calorimetry.
- Describe what is happening at the molecular level when a system is in thermal equilibrium.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what the specific heat capacity of a substance measures.
- Explain how heat is transferred between one substance and another.

Tier 2 Vocabulary: heat, specific heat capacity, "coffee cup" calorimeter

Labs, Activities & Demonstrations:

- Calorimetry lab.

Notes:

Different objects have different abilities to hold heat. For example, if you enjoy pizza, you may have noticed that the sauce holds much more heat (and burns your mouth much more readily) than the cheese or the crust.

The amount of heat that a given mass of a substance can hold is based on its specific heat capacity.

* Calorimetry is usually taught in chemistry. However, the topic was moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016.

Use this space for summary and/or additional notes:

honors (not AP®)

specific heat capacity (C): a measure of the amount of heat required per gram of a substance to produce a specific temperature change in the substance.

C_p: specific heat capacity, measured at constant pressure. For gases, this means the measurement was taken allowing the gas to expand as it was heated.

C_v: specific heat capacity, measured at constant volume. For gases, this means the measurement was made in a sealed container, allowing the pressure to rise as the gas was heated.

For solids and liquids, $C_p \approx C_v$ because the pressure and volume change very little as they are heated. For gases, $C_p > C_v$ (always). For ideal gases, $C_p - C_v = R$, where R is a constant known as "the gas constant."

When there is a choice, C_p is more commonly used than C_v because it is easier to measure. When dealing with solids and liquids, most physicists just use C for specific heat capacity and don't worry about the distinction.

Calculating Heat from a Temperature Change

The amount of heat gained or lost when an object changes temperature is given by the equation:

$$Q = mC\Delta T$$

where:

Q = heat (J or kJ)

m = mass (g or kg)

C = specific heat capacity ($\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$)

ΔT = temperature change (K or °C)*

Because problems involving heat often involve large amounts of energy, specific heat capacity is often given in kilojoules per kilogram per degree Celsius.

Note that $1\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}} \equiv 1\,\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}} \equiv 1\,\frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$ and $1\,\frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}} \equiv 1\,\frac{\text{kcal}}{\text{kg}\cdot{}^\circ\text{C}} = 4.18\,\frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}} = 4.18\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

You need to be careful with the units. If the mass is given in kilograms (kg), your specific heat capacity will have units of $\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}$ and the heat energy will come out in kilojoules (kJ). If mass is given in grams, you will use units of $\frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$ and the heat energy will come out in joules (J).

* Because 1 K is the same size as 1 °C, the two units are equivalent for ΔT values. Note, however, that T in equations must be in kelvin, because a temperature of 0 in an equation must mean absolute zero.

Use this space for summary and/or additional notes:

honors (not AP®)

Specific Heat Capacities of Some Substances

Substance	Specific Heat Capacity ($\frac{kJ}{kg \cdot K}$)	Substance	Specific Heat Capacity ($\frac{kJ}{kg \cdot K}$)
water at 20 °C	4.181	aluminum	0.897
ethylene glycol (anti-freeze)	2.460	glass	0.84
		iron	0.450
ice at −10 °C	2.080	copper	0.385
steam at 100 °C	2.11	brass	0.380
steam at 130 °C	1.99	silver	0.233
vegetable oil	2.00	lead	0.160
air	1.012	gold	0.129

Calorimetry

calorimetry: the measurement of heat flow

In a calorimetry experiment, heat flow is calculated by measuring the mass and temperature change of an object and applying the specific heat capacity equation.

calorimeter: an insulated container for performing calorimetry experiments.

coffee cup calorimeter: a calorimeter that is only an insulated container—it does not include a thermal mass (such as a mass of water). It is usually made of Styrofoam, and is often nothing more than a Styrofoam coffee cup.

bomb calorimeter: a calorimeter for measuring the heat produced by a chemical reaction. A bomb calorimeter is a double-wall metal container with water between the layers of metal. The heat from the chemical reaction makes the temperature of the water increase. Because the mass and specific heat of the calorimeter (water and metal) are known, the heat produced by the reaction can be calculated from the increase in temperature of the water.

It has a great name, but a bomb calorimeter doesn't involve actually blowing anything up. ☺

Use this space for summary and/or additional notes:

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Solving Coffee Cup Calorimetry Problems

Most coffee cup calorimetry problems involve placing a hot object in contact with a colder one. Many of them involve placing a hot piece of metal into cold water.

To solve the problems, assume that both objects end up at the same temperature.

If we decide that heat gained (going into a substance) by each object that is getting hotter is positive, and heat lost (coming out of a substance) by every substance that is getting colder is negative, then the basic equation is:

$$\text{Heat Lost} + \text{Heat Gained} = \text{Change in Thermal Energy}$$
$$\sum Q_{lost} + \sum Q_{gained} = \Delta Q$$

If the calorimeter is insulated, then no heat is gained or lost by the entire system (which means $\Delta Q = 0$).

If we have two substances (#1 and #2), one of which is getting hotter and the other of which is getting colder, then our equation becomes:

$$\text{Heat Lost} + \text{Heat Gained} = \text{Change in Thermal Energy}$$
$$\sum Q_{lost} + \sum Q_{gained} = \Delta Q = 0$$
$$m_1 C_1 \Delta T_1 + m_2 C_2 \Delta T_2 = 0$$

In this example, ΔT_1 would be negative and ΔT_2 would be positive.

To solve a calorimetry problem, there are six quantities that you need: the two masses, the two specific heat capacities, and the two temperature changes. (You might be given initial and final temperatures for either or both, in which case you'll need to subtract. Remember that if the temperature increases, ΔT is positive, and if the temperature decreases, ΔT is negative.) The problem will usually give you all but one of these and you will need to find the missing one.

If you need to find the final temperature, use $\Delta T = T_f - T_i$ on each side. You will have both T_i numbers, so the only variable left will be T_f. (The algebra is straightforward, but ugly.)

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Sample Problems:

Q: An 0.050 kg block of aluminum is heated and placed in a calorimeter containing 0.100 kg of water at 20. °C. If the final temperature of the water was 30. °C, to what temperature was the aluminum heated?

A: To solve the problem, we need to look up the specific heat capacities for aluminum and water in *Table K. Thermal Properties of Selected Materials* on page 615 of your Physics Reference Tables. The specific heat capacity of aluminum is $0.898\frac{\text{J}}{\text{g}\cdot{}^{\circ}\text{C}}$, and the specific heat capacity for water is $4.181\frac{\text{J}}{\text{g}\cdot{}^{\circ}\text{C}}$.

We also need to realize that we are looking for the initial temperature of the aluminum. ΔT is always ***final – initial***, which means $\Delta T_{\text{Al}} = 30 - T_{i,\text{Al}}$. (Because the aluminum starts out at a higher temperature, this will give us a negative number, which is what we want.)

$$\begin{aligned}
m_{\text{Al}}C_{\text{Al}}\Delta T_{\text{Al}} \quad + \quad m_{\text{w}}C_{\text{w}}\Delta T_{\text{w}} &= 0\\
(0.050)(0.897)(30-T_i)+(0.100)(4.181)(30-20) &= 0\\
0.0449(30-T_i)+ \quad 4.181 &= 0\\
1.3455-0.0449T_i+ \quad 4.181 &= 0\\
5.5265 &= 0.0449T_i\\
T_i = \frac{5.5265}{0.0449} &= 123.2\,{}^{\circ}\text{C}
\end{aligned}$$

Q: An 0.025 kg block of copper at 95°C is dropped into a calorimeter containing 0.075 kg of water at 25°C. What is the final temperature?

A: We solve this problem the same way. The specific heat capacity for copper is $0.385\frac{\text{J}}{\text{g}\cdot{}^{\circ}\text{C}}$, and $\Delta T_{\text{Cu}} = T_f - 95$ and $\Delta T_{\text{w}} = T_f - 25$. This means T_f will appear in two places. The algebra will be even uglier, but it's still a straightforward Algebra 1 problem:

$$\begin{aligned}
m_{\text{Cu}}C_{\text{Cu}}\Delta T_{\text{Cu}} \quad + \quad m_{\text{w}}C_{\text{w}}\Delta T_{\text{w}} &= 0\\
(0.025)(0.385)(T_f-95_i) \;+\; (0.075)(4.181)(T_f-25) &= 0\\
0.009625(T_f-95) \;+\; 0.3138(T_f-25) &= 0\\
0.009625\,T_f-(0.009625)(95)+0.3136\,T_f-(0.3138)(25) &= 0\\
0.009625\,T_f-0.9144+0.3138\,T_f-7.845 &= 0\\
0.3234\,T_f &= 8.759\\
T_f = \frac{8.759}{0.3234} &= 27\,{}^{\circ}\text{C}
\end{aligned}$$

Use this space for summary and/or additional notes:

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Homework Problems

You will need to look up specific heat capacities in *Table K. Thermal Properties of Selected Materials* on page 615 of your Physics Reference Tables.

1. **(S)** 375 kJ of heat is added to a 25.0 kg granite rock. If the temperature increases by 19.0 °C, what is the specific heat capacity of granite?

   Answer: $0.790\,\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}$

2. **(M)** A 0.040 kg block of copper at 95 °C is placed in 0.105 kg of water at an unknown temperature. After equilibrium is reached, the final temperature is 24 °C. What was the initial temperature of the water?

   Answer: 21.5 °C

3. **(S)** A sample of metal with a specific heat capacity of $0.50\,\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}$ is heated to 98 °C and then placed in an 0.055 kg sample of water at 22 °C. When equilibrium is reached, the final temperature is 35 °C. What was the mass of the metal?

   Answer: 0.0948 kg

Use this space for summary and/or additional notes:

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4. **(S)** A 0.280 kg sample of a metal with a specific heat capacity of $0.430\,\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}$ is heated to 97.5 °C then placed in an 0.0452 kg sample of water at 31.2 °C. What is the final temperature of the metal and the water?

   Answer: 57 °C

5. **(M)** A sample of metal with mass *m* is heated to a temperature of T_m and placed into a mass of water *M* with temperature T_w. Once the system reaches equilibrium, the temperature of the water is T_f. Derive an expression for the specific heat capacity of the metal, C_m.

   Answer: $C_m = \dfrac{MC_w(T_f - T_w)}{m(T_m - T_f)}$

6. **(A)** You want to do an experiment to measure the conversion of gravitational potential energy to kinetic energy to heat by dropping 2.0 kg of copper off the roof of LEHS, a height of 14 m. How much will the temperature of the copper increase?
   (*Hint: Remember that potential energy is measured in* J *but specific heat capacity problems usually use* kJ.)

   Answer: 0.36°C

Use this space for summary and/or additional notes:

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Phase Diagrams*

Unit: Thermal Physics (Heat)

MA Curriculum Frameworks (2016): HS-PS1-11(MA)

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Identify the phase of a substance at any combination of temperature and pressure.
- Determine the melting and boiling points of a substance any pressure.

Success Criteria:

- Phases are correctly identified as solid, liquid, gas, supercritical fluid, *etc.,* in accordance with the temperature and pressure indicated on the phase diagram.
- Melting and boiling point temperatures are correctly identified for a substance from its phase diagram for a given pressure.
- The effects of a pressure or temperature change (*e.g.,* substance would melt, sublime, *etc.*) are correctly explained based on the phase diagram.

Language Objectives:

- Explain the regions of a phase diagram and the relationship between each region and the temperature and pressure of the substance..

Tier 2 Vocabulary: phase, curve, fusion, solid, liquid, gas, vapor

Notes:

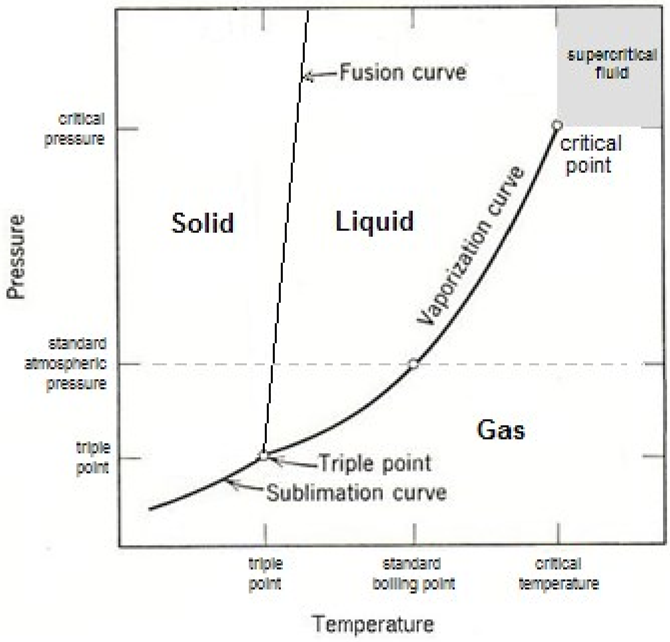
The phase of a substance (solid, liquid, gas) depends on its temperature and pressure.

phase diagram: a graph showing the phase(s) present at different temperatures and pressures.

* Phase diagrams are usually taught in chemistry. However, they relate to the topics of phase changes and heating curves, which were moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016.

Use this space for summary and/or additional notes:

honors (not AP®)



<u>fusion curve</u>: the set of temperatures and pressures at which a substance melts/freezes.

<u>vaporization curve</u>: the set of temperatures & pressures at which a substance vaporizes/condenses.

<u>sublimation curve</u>: the set of temperatures & pressures at which a substance sublimes/deposits.

<u>triple point</u>: the temperature and pressure at which a substance can exist simultaneously as a solid, liquid, and gas.

<u>critical point</u>: the highest temperature at which the substance can exist as a liquid. The critical point is the endpoint of the vaporization curve.

<u>supercritical fluid</u>: a substance whose temperature and pressure are above the critical point. The substance would be expected to be a liquid (due to the pressure), but the molecules have so much energy that the substance behaves more like a gas.

Use this space for summary and/or additional notes:

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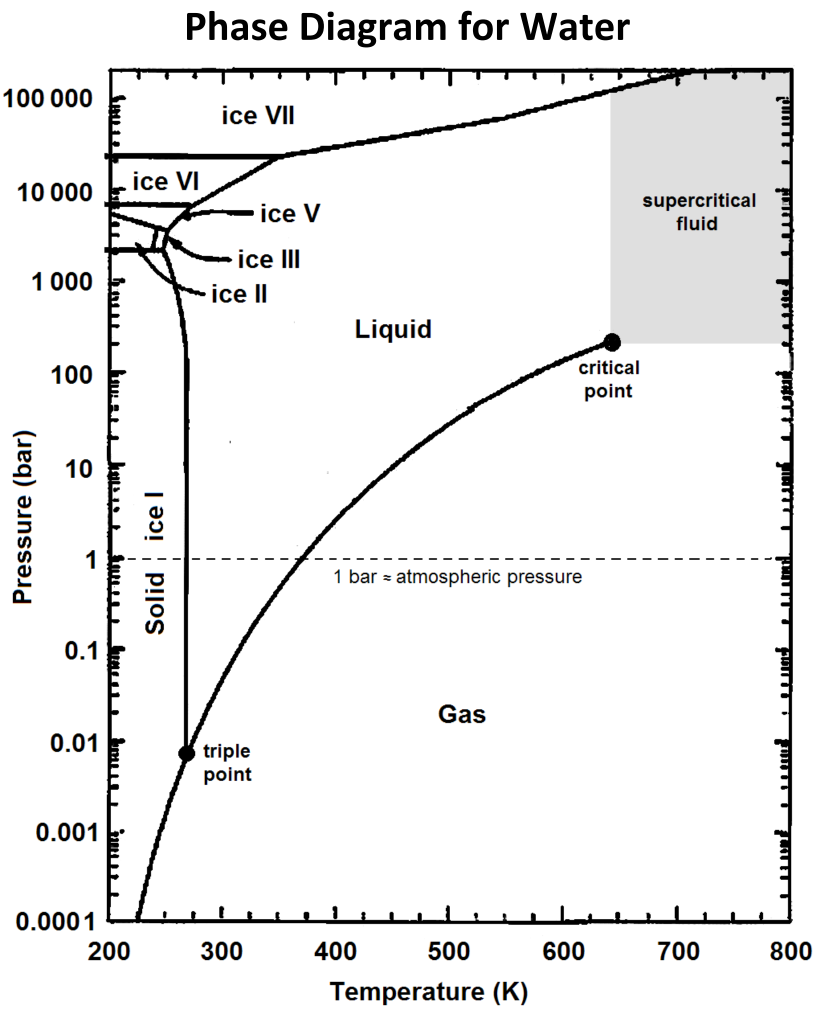

Note that pressure is on a logarithmic scale, and that standard atmospheric pressure is 1 bar ≈ 1 atm.

Note also that the temperature is in kelvin. To convert degrees Celsius to kelvin, add 273. (*e.g.,* 25 °C + 273 = 298 K.)

Notice that the slope of the fusion curve (melting/freezing line) is negative. This is because ice I is less dense than liquid water. At temperatures near the melting point and pressures less than about 2 000 bar, increasing the pressure will cause ice to melt. Water is one of the only known substances that exhibits this behavior.

Use this space for summary and/or additional notes:

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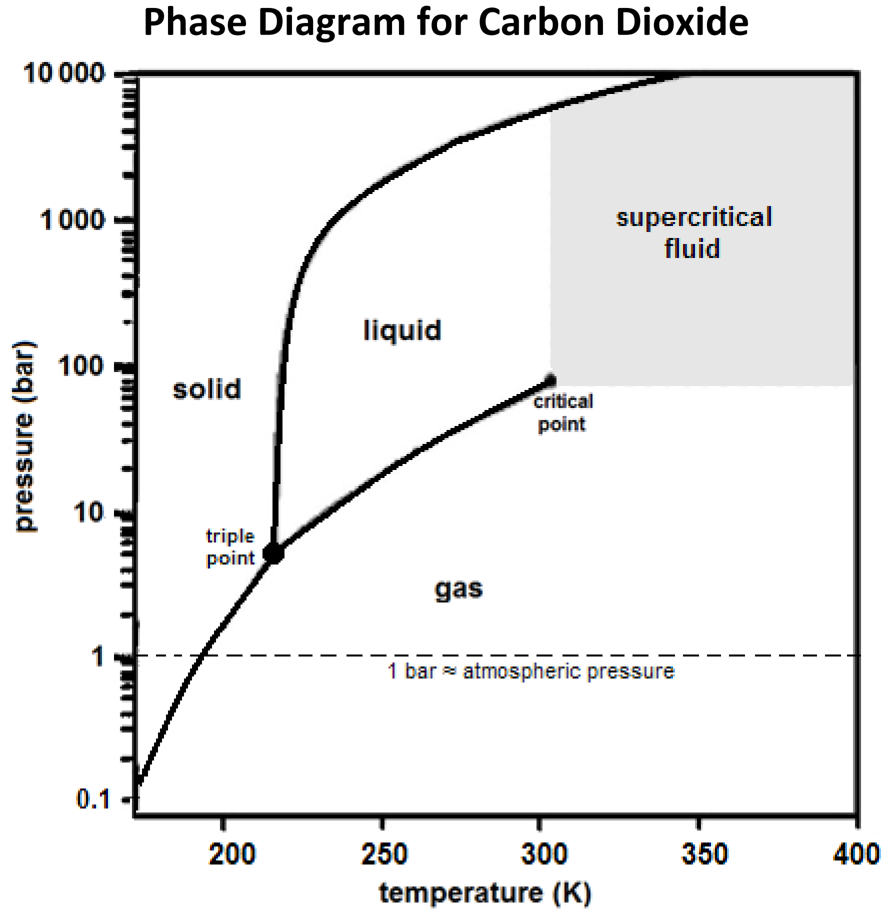


Notice that the pressure of the triple point for CO_2 is about 5 bar, which means CO_2 cannot be a liquid at atmospheric pressure. This is why dry ice (solid CO_2) sublimes directly from a solid to a gas.

Use this space for summary and/or additional notes:

Homework Problems

honors (not AP®)

Answer these questions based on the phase diagrams for water and carbon dioxide on the previous pages.

1. **(M)** Approximately what pressure would be necessary to boil water at a temperature of 350 K?

2. **(M)** What is the minimum pressure necessary for water to exist as a liquid at 350 K?

3. **(S)** At approximately what temperature would water boil if the pressure is 10 bar?

4. **(S)** What is the highest temperature at which carbon dioxide can exist as a liquid?

5. **(M)** At 1.0 bar of pressure, what is the temperature at which carbon dioxide sublimes?

6. **(S)** At room temperature (25 °C ≈ 300 K), what is the minimum pressure at which liquid carbon dioxide can exist?

7. **(M)** Describe the phase transitions and temperatures for water going from 200 K to 400 K at a pressure of 0.1 bar.

8. **(S)** Describe the phase transitions and temperatures for carbon dioxide going 200 K to 300 K at a pressure of 10 bar.

Use this space for summary and/or additional notes:

honors (not AP®)

Phases & Phase Changes*

Unit: Thermal Physics (Heat)

MA Curriculum Frameworks (2016): HS-PS1-3, HS-PS2-8(MA)

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Compare observable states of matter and phase transitions with behavior at the molecular level.

Success Criteria:

- Descriptions include connectedness and motion of molecules.
- Descriptions include comparative descriptions of molecular speed.
- Descriptions relate molecular motion and speed to temperature.

Language Objectives:

- Explain phase changes in terms of changes in molecular behavior.

Tier 2 Vocabulary: phase, solid, liquid, gas, vapor

Labs, Activities & Demonstrations:

- evaporation from boiling water on cloth

Notes:

macroscopic: objects or bulk properties of matter that we can observe directly.

microscopic: objects or properties of matter that are too small to observe directly.

Note that *macroscopic* properties of a substance are often determined by *microscopic* interactions between the individual molecules.†

phase: a term that relates to how rigidly the atoms or molecules in a substance are connected.

* Phase changes are generally taught in chemistry classes. However, because the calorimetry and heating curves topics were moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016, it is useful to review them here.

† In this section, the term "molecules" is used to refer to the particles that make up a substance. In chemistry, a molecule is a group of atoms that are covalently bonded together, and a substance can be made of individual atoms, molecules, crystals, or other types of particles. In these notes, the term "particles" is preferred, but "molecules" is used in this section because it conjures the impression of particles that are attached or bonded together in some way. This gives most students a reasonably correct picture of entities that are firmly attached to each other and cannot be pulled apart by physical means.

Use this space for summary and/or additional notes:

honors (not AP®)

solid: molecules are rigidly connected. A solid has a definite shape and a definite volume.

liquid: molecules are loosely connected; bonds are continuously forming and breaking. A liquid has a definite volume, but not a definite shape.

gas: molecules are not connected. A gas has neither a definite shape nor a definite volume. Gases will expand to fill whatever space they occupy.

plasma: the system has enough heat to remove electrons from atoms, which means the system is comprised of charged particles moving very rapidly.

phase change: when an object or substance changes from one phase to another through gaining or losing heat.

Breaking bonds requires energy. Forming bonds releases energy. This is true for the intermolecular bonds that hold a solid or liquid together as well as for chemical bonds.

As you probably know from experience, you need to add energy to turn a solid to a liquid (melt it), or to turn a liquid to a gas (boil it).

- This is why evaporation causes cooling—because the system (the water) needs to absorb heat from its surroundings in order to make the change from a liquid to a gas (vapor).
- This is also why lids keep drinks hot. The lid is a barrier which significantly reduces the amount of evaporation.
- When you perspire, the water absorbs heat from you in order to evaporate, which cools you off.

It is less obvious that energy is released when a gas condenses or a liquid freezes.

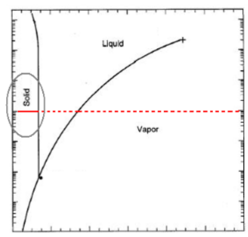
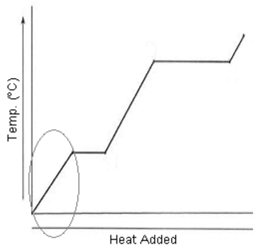
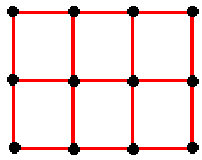
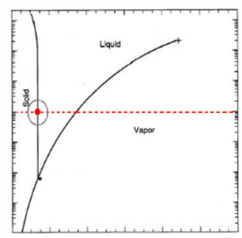
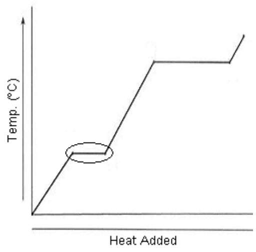
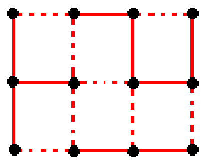
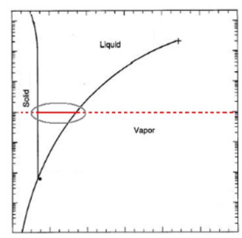
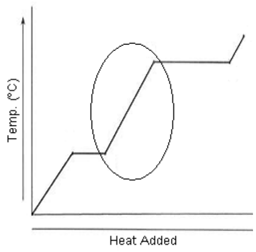
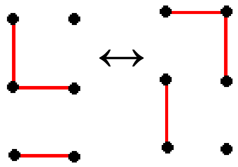
- Ice in your ice tray needs to give off heat in order to freeze. (Your freezer needs to remove that heat in order to make this happen.)
- Burns from steam are much more dangerous than burns from water, because the steam releases a large amount of heat (which is absorbed by your body) as it condenses.

Use this space for summary and/or additional notes:

States of Matter

honors (not AP®)

The following table shows interactions between the molecules and some observable properties for solids, liquids and gases. (The table includes heating curves, which will be discussed in more detail later in the next section, *Heating Curves* starting on page 212. For now, understand that a heating curve shows how the temperature changes as heat is added. Notice in particular that the temperature stays constant during melting and boiling.)

state	phase diagram	heating curve	molecules
solid			rigidly bonded 
	adding energy makes molecules move faster; temperature increases		
melting			some bonds breaking 
	adding energy breaks some of the bonds; temperature remains constant		
liquid			bonds breaking & re-forming rapidly 
	adding energy makes molecules move faster; temperature increases		

Use this space for summary and/or additional notes:

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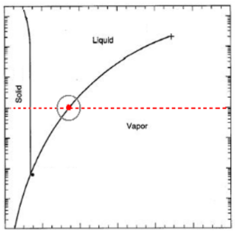
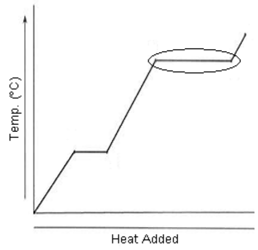
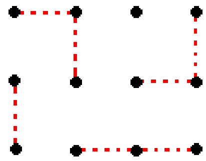
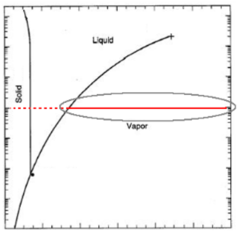
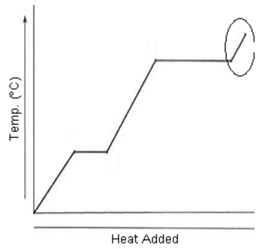
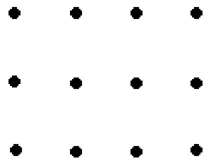
<table>
<tr><th>state</th><th>phase diagram</th><th>heating curve</th><th>molecules</th></tr>
<tr><td>boiling</td><td></td><td></td><td>all bonds breaking
</td></tr>
<tr><td></td><td colspan="3">adding energy breaks all remaining bonds;
temperature remains constant</td></tr>
<tr><td>vapor (gas)</td><td></td><td></td><td>molecules moving freely
</td></tr>
<tr><td></td><td colspan="3">adding energy makes molecules move faster;
temperature increases</td></tr>
</table>

Note that because liquids are continually forming and breaking bonds, if a liquid molecule at the surface breaks its bonds with other liquids, it can "escape" from the attractive forces of the other liquid molecules and become a vapor molecule. This is how evaporation happens at temperatures that are well below the boiling point of the liquid.

Use this space for summary and/or additional notes:

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Heating Curves*

Unit: Thermal Physics (Heat)

MA Curriculum Frameworks (2016): HS-PS3-1, HS-PS3-2, HS-PS3-4a

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Determine the amount of heat required for all of the phase changes that occur over a given temperature range.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what the heat is used for in each step of a heating curve.

Tier 2 Vocabulary: specific heat capacity, curve, phase

Labs, Activities & Demonstrations:

- Evaporation from washcloth.
- Fire & ice (latent heat of paraffin).

Notes:

phase: a term that relates to how rigidly the atoms or molecules in a substance are connected.

solid: molecules are rigidly connected. A solid has a definite shape and volume.

liquid: molecules are loosely connected; bonds are continuously forming and breaking. A liquid has a definite volume, but not a definite shape.

gas: molecules are not connected. A gas has neither a definite shape nor a definite volume. Gases will expand to fill whatever space they occupy.

plasma: the system has enough heat to remove electrons from atoms, which means the system is comprised of particles with rapidly changing charges.

* Heating curves are usually taught in chemistry. However, the topic was moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016.

Use this space for summary and/or additional notes:

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phase change: when an object or substance changes from one phase to another through gaining or losing heat.

Breaking bonds requires energy. Forming bonds releases energy. This is true for the bonds that hold a solid or liquid together as well as for chemical bonds (regardless of what previous teachers may have told you!)

I.e., you need to add energy to turn a solid to a liquid (melt it), or to turn a liquid to a gas (boil it). Energy is released when a gas condenses or a liquid freezes. (*E.g.,* ice in your ice tray needs to give off heat in order to freeze. Your freezer needs to remove that heat in order to make this happen.)

The reason evaporation causes cooling is because the system (the water) needs to absorb heat from its surroundings (*e.g.,* your body) in order to make the change from a liquid to a gas (vapor). When the water absorbs heat from you and evaporates, you have less heat, which means you have cooled off.

Calculating the Heat of Phase Changes

heat of fusion (ΔH_{fus}) (sometimes called "latent heat" or "latent heat of fusion"): the amount of heat required to melt one kilogram of a substance. This is also the heat released when one kilogram of a liquid substance freezes. For example, the heat of fusion of water is $334\,\frac{\text{J}}{\text{g}} \equiv 334\,\frac{\text{kJ}}{\text{kg}}$. The heat required to melt a sample of water is therefore:

$$Q = m\Delta H_{fus} = m(334\,\tfrac{\text{J}}{\text{g}})$$

heat of vaporization (ΔH_{vap}): the amount of heat required to vaporize (boil) one kilogram of a substance. This is also the heat released when one kilogram of a gas condenses. For example, the heat of vaporization of water is $2260\,\frac{\text{J}}{\text{g}} \equiv 2260\,\frac{\text{kJ}}{\text{kg}}$. The heat required to boil a sample of water is therefore:

$$Q = m\Delta H_{vap} = m(2260\,\tfrac{\text{J}}{\text{g}})$$

(Again, pay attention to the units. If your mass is in kg, you will need to use the units $\frac{\text{kJ}}{\text{kg}}$ for your ΔH values; if your mass is in g, you will need to use the units $\frac{\text{J}}{\text{g}}$ for your ΔH values.)

Use this space for summary and/or additional notes:

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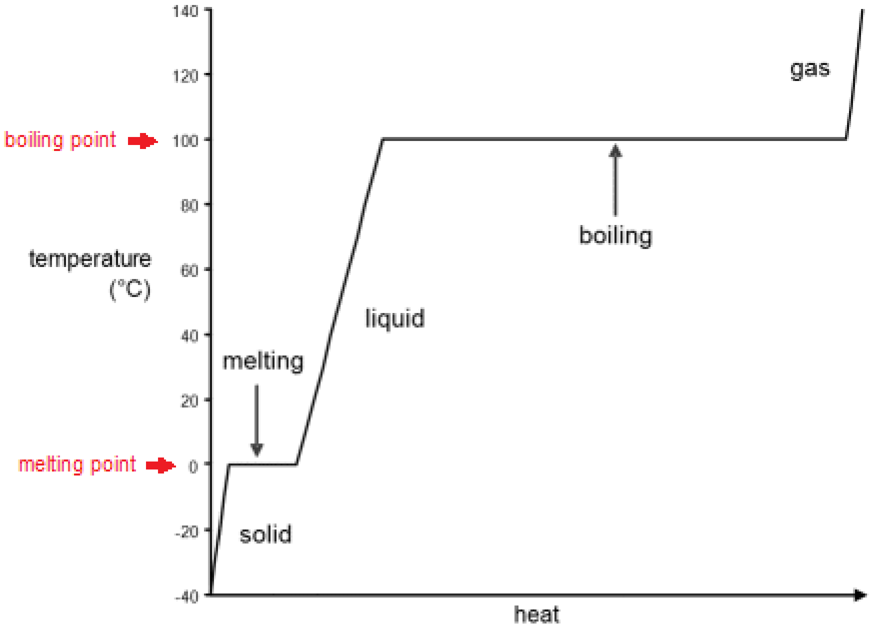
<u>heating curve</u>: a graph of temperature *vs.* heat added. The following is a heating curve for water:



In the "solid" portion of the curve, the sample is solid water (ice). As heat is added, the temperature increases. The specific heat capacity of ice is $2.11\,\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}$, so the heat required is:

$$Q_{solid} = mC\Delta T = m(2.11\,\tfrac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}})(\Delta T)$$

In the "melting" portion of the curve, the sample is a mixture of ice and water. As heat is added, the ice melts, but the temperature remains at 0°C until all of the ice is melted. The heat of fusion of ice is $334\,\frac{\text{kJ}}{\text{kg}}$, so the heat required is:

$$Q_{melt} = m\Delta H_{fus} = m(334\,\tfrac{\text{kJ}}{\text{kg}})$$

In the "liquid" portion of the curve, the sample is liquid water. As heat is added, the temperature increases. The specific heat capacity of liquid water is $4.18\,\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}$, so the heat required is:

$$Q_{liquid} = mC\Delta T = m(4.18\,\tfrac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}})(\Delta T)$$

Use this space for summary and/or additional notes:

honors (not AP®)

In the "boiling" portion of the curve, the sample is a mixture of water and water vapor (steam). As heat is added, the water boils, but the temperature remains at 100°C until all of the water has boiled. The heat of vaporization of water is $2260\,\frac{\text{kJ}}{\text{kg}}$, so the heat required is:

$$Q_{boil} = m\Delta H_{vap} = m(2260\,\tfrac{\text{kJ}}{\text{kg}})$$

In the "gas" portion of the curve, the sample is water vapor (steam). As heat is added, the temperature increases. The specific heat capacity of steam is approximately $2.08\,\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}$ (at 100°C; the specific heat capacity of steam decreases as the temperature increases), so the heat required is:

$$Q_{gas} = mC\Delta T = m(2.08\,\tfrac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}})(\Delta T)$$

Steps for Solving Heating Curve Problems

A heating curve problem is a problem in which a substance is heated across a temperature range that passes through the melting and/or boiling point of the substance, which means the problem includes heating or cooling steps and melting/freezing or boiling/condensing steps.

1. Sketch the heating curve for the substance over the temperature range in question. Be sure to include the melting and boiling steps as well as the heating steps.

2. From your sketch, determine whether the temperature range in the problem passes through the melting and/or boiling point of the substance.

3. Split the problem into:
   a. Heating (or cooling) steps within each temperature range.
   b. Melting or boiling (or freezing or condensing) steps.

4. Find the heat required for each step.
   a. For the heating/cooling steps, use the equation $Q = mC\Delta T$.
   b. For melting/freezing steps, use the equation $Q = m\Delta H_{fus}$.
   c. For boiling/condensing steps, use the equation $Q = m\Delta H_{vap}$.

5. Add the values of Q from each step to find the total.

Use this space for summary and/or additional notes:

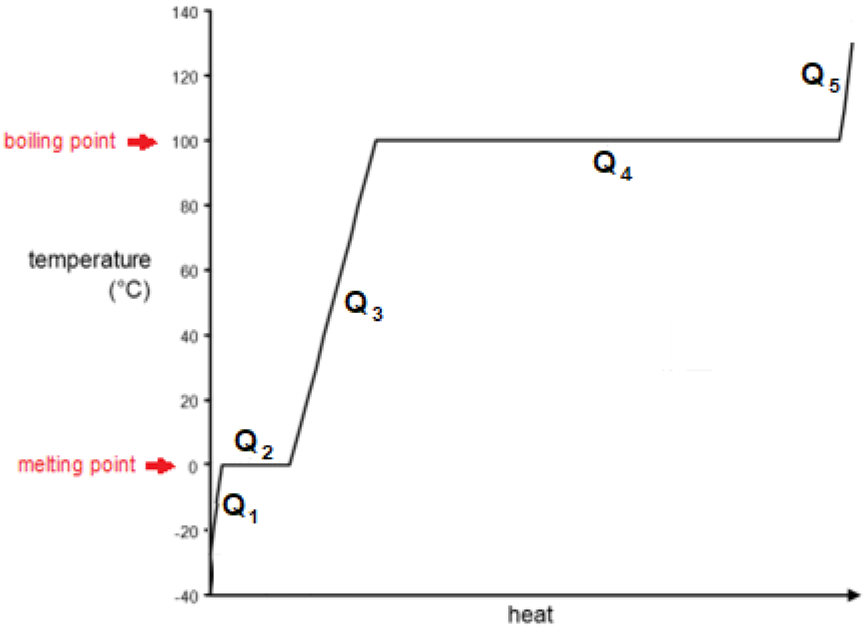
Heating Curves

honors (not AP®)

Q: How much heat would it take to raise the temperature of 15.0 g of H_2O from −25.0 °C to +130.0 °C?

A: The H_2O starts out as ice. We need to:

1. Heat the ice from −25.0 °C to its melting point (0 °C).
2. Melt the ice.
3. Heat the water up to its boiling point (from 0 °C to 100 °C).
4. Boil the water.
5. Heat the steam from 100 °C to 130 °C.
6. Add up the heat for each step to find the total.



1. heat solid: $Q_1 = mC\Delta T = (15)(2.11)(25) = 791.25\text{ J}$
2. melt ice: $Q_2 = m\Delta H_{fus} = (15)(334) = 5\,010\text{ J}$
3. heat liquid: $Q_3 = mC\Delta T = (15)(4.181)(100) = 6\,270\text{ J}$
4. boil water: $Q_4 = m\Delta H_{vap} = (15)(2260) = 33\,900\text{ J}$
5. heat gas: $Q_5 = mC\Delta T = (15)(2.08)(30) = 936\text{ J}$
6. $Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$
   $Q = 791 + 5\,010 + 6\,270 + 33\,900 + 936 = 46\,910\text{ J}$

Use this space for summary and/or additional notes:

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Homework Problems

For the following problems, use data from the following table:

	C (sol.) $\left(\frac{kJ}{kg\cdot°C}\right)$	M.P. (°C)	ΔH_{fus} $\left(\frac{kJ}{kg}\right)$	C (liq) $\left(\frac{kJ}{kg\cdot°C}\right)$	B.P. (°C)	ΔH_{vap} $\left(\frac{kJ}{kg}\right)$	C_p (gas) $\left(\frac{kJ}{kg\cdot°C}\right)$
water	2.11	0	334	4.18	100	2260	2.08*
potassium	0.560	62	61.4	1.070	760	2025	0.671
mercury	0.142	−39	11.3	0.140	357	293	0.104
silver	0.217	962	111	0.318	2212	2360	—

*Note that because of the volume change from heating, the specific heat capacity of gases, C_p, increases with increasing temperature.

1. **(M)** A 0.0250 kg sample of water is heated from −40.0°C to 150. °C.

   a. Sketch the heating curve for the above process. Label the starting temperature, melting point, boiling point, and final temperature on the *y*-axis.

   b. Calculate the heat required for each step of the heating curve, and the total heat required.

   Answer: 80.01 kJ

Use this space for summary and/or additional notes:

honors (not AP®)

2. **(S)** A 0.085 kg sample of mercury is heated from 25°C to 500. °C.

   a. Sketch the heating curve for the above process. Label the starting temperature, melting point, boiling point, and final temperature on the *y*-axis.

   b. Calculate the heat required for each step of the heating curve, and the total heat required.

   Answer: 30.12 kJ

Use this space for summary and/or additional notes:

honors (not AP®)

3. **(S)** A 0.045 kg block of silver at a temperature of 22°C is heated with 20.0 kJ of energy. You want to determine the final temperature and what the physical state (solid, liquid, gas) of the silver at that temperature.

   a. Sketch the heating curve for silver. Label the starting temperature, melting point, and boiling point on the *y*-axis.

   b. Calculate the total heat required by calculating the heat for each step until the entire 20.0 kJ is accounted for.

   Answers: liquid, 1 369°C

Use this space for summary and/or additional notes:

honors (not AP®)

Thermal Expansion

Unit: Thermal Physics (Heat)

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Calculate changes in length & volume for solids, liquids and gases that are undergoing thermal expansion or contraction.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what the heat is used for in each step of a heating curve.

Tier 2 Vocabulary: expand, contract

Labs, Activities & Demonstrations:

- Balloon with string & heat gun.
- Brass ball & ring.
- Bi-metal strip.

Notes:

expand: to become larger

contract: to become smaller

thermal expansion: an increase in the length and/or volume of an object caused by a change in temperature.

When a substance is heated, the particles it is made of move farther and faster. This causes the particles to move farther apart, which causes the substance to expand.

Solids tend to keep their shape when they expand. (Liquids and gases do not have a definite shape to begin with.)

A few materials are known to contract with increasing temperature over specific temperature ranges. One well-known example is liquid water, which contracts as it heats from 0 °C to 4 °C. (Water expands as the temperature increases above 4 °C.)

Use this space for summary and/or additional notes:

honors (not AP®)

Thermal Expansion of Solids and Liquids

Thermal expansion is quantified in solids and liquids by defining a coëfficient of thermal expansion. The changes in length and volume are given by the equation:

Length: $\Delta L = \alpha L_i \Delta T$

Volume: $\Delta V = \beta V_i \Delta T$

where:

ΔL = change in length (m)
L_i = initial length (m)
α = linear coëfficient of thermal expansion (°C^{-1} or K^{-1})

ΔV = change in volume (m^3)
V_i = initial volume (m^3)
β = volumetric coëfficient of thermal expansion (°C^{-1} or K^{-1})

ΔT = temperature change (°C or K)

Values of α and β at 20°C for some solids and liquids:

Substance	α (°C^{-1})	β (°C^{-1})	**Substance**	α (°C^{-1})	β (°C^{-1})
aluminum	2.3×10^{-5}	6.9×10^{-5}	gold	1.4×10^{-5}	4.2×10^{-5}
copper	1.7×10^{-5}	5.1×10^{-5}	iron	1.18×10^{-5}	3.33×10^{-5}
brass	1.9×10^{-5}	5.6×10^{-5}	lead	2.9×10^{-5}	8.7×10^{-5}
diamond	1×10^{-6}	3×10^{-6}	mercury	6.1×10^{-5}	1.82×10^{-4}
ethanol		7.5×10^{-4}	silver	1.8×10^{-5}	5.4×10^{-5}
glass	8.5×10^{-6}	2.55×10^{-6}	water (liq.)	6.9×10^{-5}	2.07×10^{-4}

Use this space for summary and/or additional notes:

honors (not AP®)

expansion joint: a space deliberately placed between two objects to allow room for the objects to expand without coming into contact with each other.

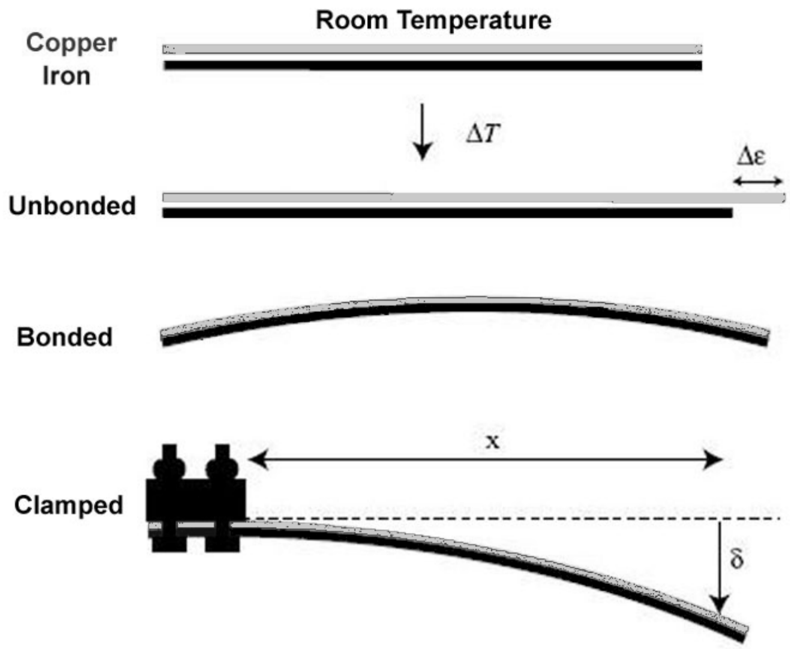
Bridges often have expansion joints in order to leave room for sections of the bridge to expand or contract without damaging the bridge or the roadway.

Railroad rails are sometimes welded together in order to create a smoother ride, which enables high-speed trains to use them. Unfortunately, if expansion joints are not placed at frequent enough intervals, thermal expansion can cause the rails to bend and buckle, resulting in derailments:

Use this space for summary and/or additional notes:

honors (not AP®)

bimetal strip: a strip made from two metals with different coëfficients of thermal expansion that are bonded together. When the strip is heated or cooled, the two metals expand or contract different amounts, which causes the strip to bend. When the strip is returned to room temperature, the metals revert back to their original lengths.



Sample Problems:

Q: Find the change in length of an 0.40 m brass rod that is heated from 25 °C to 980 °C.

A: For brass, $\alpha = 1.9\times10^{-5}\ {}^\circ\mathrm{C}^{-1}$.

$\Delta L = \alpha L_i \Delta T$

$\Delta L = (1.9\times10^{-5})(0.40)(955)$

$\Delta L = 0.0073\ \mathrm{m}$

Use this space for summary and/or additional notes:

honors (not AP®)

Q: A typical mercury thermometer contains about $0.22\ \text{cm}^3$ (about 3.0 g) of mercury. Find the change in volume of the mercury in a thermometer when it is heated from 25 °C to 50. °C.

A: For mercury, $\beta = 1.82\times10^{-4}\ {}^{\circ}\text{C}^{-1}$.

$$\Delta V = \beta V_i \Delta T$$
$$\Delta V = (1.82\times10^{-4})(0.22)(25)$$
$$\Delta V = 0.000\,91\ \text{cm}^3$$

If the distance from the 25 °C to the 50 °C mark is about 3.0 cm, we could use this information to figure out the bore (diameter of the column of mercury) of the thermometer:

$$V = \pi r^2 h$$
$$0.000\,91 = (3.14)r^2(3.0)$$
$$r^2 = \frac{0.000\,91}{(3.14)(3.0)} = 9.66\times10^{-5}$$
$$r = \sqrt{9.66\times10^{-5}} = 0.0098\ \text{cm}$$

The bore is the diameter, which is twice the radius, so the bore of the thermometer is (2)(0.0098) = 0.0197 cm, which is about 0.20 mm.

Use this space for summary and/or additional notes:

Homework Problems

honors (not AP®)

You will need to look up coëfficients of thermal expansion in *Table K. Thermal Properties of Selected Materials* on page 615 of your Physics Reference Tables.

1. **(S)** A brass rod is 27.50 cm long at 25 °C. How long would the rod be if it were heated to 750. °C in a flame?

   Answer: 27.88 cm

2. **(M)** A steel bridge is 625 m long when the temperature is 0 °C.

   a. If the bridge did not have any expansion joints, how much longer would the bridge be on a hot summer day when the temperature is 35 °C?
   (Use the linear coëfficient of expansion for iron.)

   Answer: 0.258 m

   b. Why do bridges need expansion joints?

3. **(M)** A 15.00 cm long bimetal strip is aluminum on one side and copper on the other. If the two metals are the same length at 20.0 °C, how long will each be at 800. °C?

   Answers: aluminum: 15.269 cm; copper: 15.199 cm

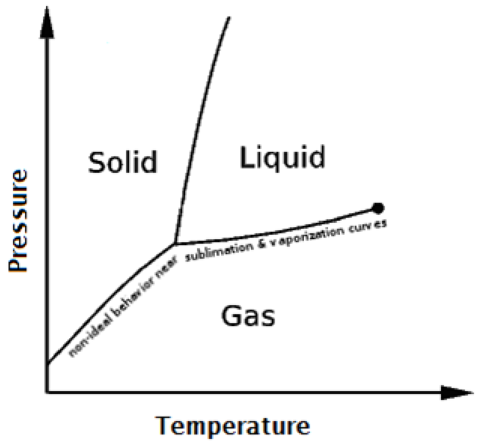
4. **(S)** A glass volumetric flask is filled with water exactly to the 250.00 mL line at 50. °C. What volume will the water occupy after it cools down to 20. °C?

   Answer: 248.45 mL

Use this space for summary and/or additional notes:

honors (not AP®)

Thermal Expansion of Gases



<u>ideal gas</u>: a gas that behaves as if each molecule acts independently, according to kinetic-molecular theory. Most gases behave ideally except at temperatures and pressures near the vaporization curve on a phase diagram. (*I.e.,* gases stop behaving ideally when conditions are close to those that would cause the gas to condense to a liquid or solid.)

For an ideal gas, the change in volume for a change in temperature (provided that the pressure and number of molecules are kept constant) is given by Charles' Law:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where V_1 and T_1 are the initial volume and temperature, and V_2 and T_2 are the final volume and temperature, respectively. Volume can be any volume unit (as long as it is the same on both sides), but temperature *<u>must be</u>* in Kelvin.

Sample Problem:

Q: If a 250 mL container of air is heated from 25 °C to 95 °C, what is the new volume?

A: Temperatures must be in Kelvin, so we need to convert first.

T_1 = 25 °C + 273 = 298 K T_2 = 95 °C + 273 = 368 K

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$
$$\frac{250}{298} = \frac{V_2}{368}$$
$$V_f = 308.7 \approx 310 \text{ mL}$$

Because we used mL for V_1, the value of V_2 is therefore also in mL.

Use this space for summary and/or additional notes:

honors (not AP®)

Homework Problems

1. **(S)** A sample of argon gas was cooled, and its volume went from 380. mL to 250. mL. If its final temperature was −45.0 °C, what was its original temperature?

   Answer: 347 K or 74 °C

2. **(M)** A balloon contains 250. mL of air at 50 °C. If the air in the balloon is cooled to 20.0 °C, what will be the new volume of the air?

   Answer: 226.8mL

Use this space for summary and/or additional notes:

Introduction: Thermodynamics

Unit: Thermodynamics

Topics covered in this chapter:

This chapter is about heat as a form of energy and the ways in which heat affects objects, including how it is stored and how it is transferred from one object to another.

- *Kinetic-Molecular Theory* explains the implications of the theory that gases are made of large numbers of independently-moving particles.
- *Gas Laws* and *The Ideal Gas Law* describe and explain relationships between pressure, volume, temperature and the number of particles in a sample of gas.
- *Energy Conversion* describes conversion between heat and other forms of energy.
- *Thermodynamics* describes the transfer of energy into or out of a sample of gas.
- *Pressure-Volume (PV) Diagrams* and *Heat Engines* describe the relationship between changes in pressure and volume, heat, and work done on or by a gas.
- *Efficiency* describes the relationship between the work obtained from changes to a sample of gas and the maximum amount of energy that is theoretically available.

New challenges specific to this chapter include looking up and working with constants that are different for different substances.

Use this space for summary and/or additional notes:

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

HS-PS2-6. Communicate scientific and technical information about why the molecular-level structure is important in the functioning of designed materials.

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.

HS-PS3-2. Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

HS-PS3-4. Plan and conduct an investigation to provide evidence that the transfer of thermal energy when two components of different temperature are combined within a closed system results in a more uniform energy distribution among the components in the system (second law of thermodynamics).

AP® only

AP® Physics 2 Learning Objectives:

4.C.3.1: The student is able to make predictions about the direction of energy transfer due to temperature differences based on interactions at the microscopic level. [**SP 6.4**]

5.A.2.1: The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. [**SP 6.4, 7.2**]

5.B.4.1: The student is able to describe and make predictions about the internal energy of systems. [**SP 6.4, 7.2**]

5.B.4.2: The student is able to calculate changes in kinetic energy and potential energy of a system, using information from representations of that system. [**SP 1.4, 2.1, 2.2**]

5.B.5.4: The student is able to make claims about the interaction between a system and its environment in which the environment exerts a force on the system, thus doing work on the system and changing the energy of the system (kinetic energy plus potential energy). [**SP 6.4, 7.2**]

5.B.5.5: The student is able to predict and calculate the energy transfer to (*i.e.,* the work done on) an object or system from information about a force exerted on the object or system through a distance. [**SP 2.2, 6.4**]

Use this space for summary and/or additional notes:

AP® only

5.B.5.6: The student is able to design an experiment and analyze graphical data in which interpretations of the area under a pressure-volume curve are needed to determine the work done on or by the object or system. [**SP 4.2, 5.1**]

5.B.6.1: The student is able to describe the models that represent processes by which energy can be transferred between a system and its environment because of differences in temperature: conduction, convection, and radiation. [**SP 1.2**]

5.B.7.1: The student is able to predict qualitative changes in the internal energy of a thermodynamic system involving transfer of energy due to heat or work done and justify those predictions in terms of conservation of energy principles. [**SP 6.4, 7.2**]

5.B.7.2: The student is able to create a plot of pressure versus volume for a thermodynamic process from given data. [**SP 1.1**]

5.B.7.3: The student is able to use a plot of pressure versus volume for a thermodynamic process to make calculations of internal energy changes, heat, or work, based upon conservation of energy principles (*i.e.,* the first law of thermodynamics). [**SP 1.1**, **1.4**, **2.2**]

7.A.1.1: The student is able to make claims about how the pressure of an ideal gas is connected to the force exerted by molecules on the walls of the container, and how changes in pressure affect the thermal equilibrium of the system. [**SP 6.4**, **7.2**]

7.A.1.2: Treating a gas molecule as an object (*i.e.,* ignoring its internal structure), the student is able to analyze qualitatively the collisions with a container wall and determine the cause of pressure, and at thermal equilibrium, to quantitatively calculate the pressure, force, or area for a thermodynamic problem given two of the variables. [**SP 1.4**, **2.2**]

7.A.2.1: The student is able to qualitatively connect the average of all kinetic energies of molecules in a system to the temperature of the system. [**SP 7.1**]

7.A.2.2: The student is able to connect the statistical distribution of microscopic kinetic energies of molecules to the macroscopic temperature of the system and to relate this to thermodynamic processes. [**SP 7.1**]

7.A.3.1: The student is able to extrapolate from pressure and temperature or volume and temperature data to make the prediction that there is a temperature at which the pressure or volume extrapolates to zero. [**SP 6.4**, **7.2**]

Use this space for summary and/or additional notes:

AP® only

7.A.3.2: The student is able to design a plan for collecting data to determine the relationships between pressure, volume, and temperature, and amount of an ideal gas, and to refine a scientific question concerning a proposed incorrect relationship between the variables. [**SP 3.2**, **4.2**]

7.A.3.3: The student is able to analyze graphical representations of macroscopic variables for an ideal gas to determine the relationships between these variables and to ultimately determine the ideal gas law $PV = nRT$. [**SP 5.1**]

7.B.1.1: The student is able to extrapolate from pressure and temperature or volume and temperature data to make the prediction that there is a temperature at which the pressure or volume extrapolates to zero. [**SP 6.4**, **7.2**]

7.B.2.1: The student is able to connect qualitatively the second law of thermodynamics in terms of the state function called entropy and how it (entropy) behaves in reversible and irreversible processes. [**SP 7.1**]

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Laws of Thermodynamics**, such as first and second laws, internal energy, entropy, and heat engine efficiency.

1. Heat and Temperature
2. The Kinetic Theory of Gases & the Ideal Gas Law
3. The Laws of Thermodynamics
4. Heat Engines

Skills learned & applied in this chapter:

- Working with material-specific constants from a table.
- Working with more than one instance of the same quantity in a problem.
- Combining equations and graphs.

Use this space for summary and/or additional notes:

Kinetic-Molecular Theory

Unit: Thermodynamics

MA Curriculum Frameworks (2016): HS-PS2-8(MA)

AP® Physics 2 Learning Objectives: 7.A.1.2

Mastery Objective(s): (Students will be able to…)

- Explain how each aspect of Kinetic-Molecular Theory applies to gases.

Success Criteria:

- Descriptions account for behavior at the molecular level.
- Descriptions account for measurable properties, *e.g.,* temperature, pressure, volume, *etc.*

Language Objectives:

- Explain how gas molecules behave and how their behavior relates to properties we can measure.

Tier 2 Vocabulary: kinetic, gas, ideal, real

Notes:

In chemistry you learned about matter, including its composition, structure, and changes that it can undergo. In physics, we are interested in matter to the extent that it can be used to bring objects or energy in contact with each other and transfer forces, energy or momentum from one object or collection of objects to another. This chapter is about gases and using properties of gases to convert between mechanical and thermal energy.

Properties of Different States of Matter

State	Description	Uses
solid	Particles rigidly bonded. Bonds difficult to break. (Definite shape & definite volume)	Construction materials where structure is important. Conduction of heat and/or electricity. Storage of heat as thermal mass.
liquid	Particles loosely bonded and have limited movement. Bonds continuously breaking & reforming. (Definite volume, but indefinite shape.)	Chemical reactions & heat transfer where continual mixing of materials is needed. Storage of heat as thermal mass.
gas	Particles not bonded and able to move freely. (Indefinite shape & volume.)	Heat and materials transfer in large spaces. ***Conversion of energy between heat and mechanical work.***

Use this space for summary and/or additional notes:

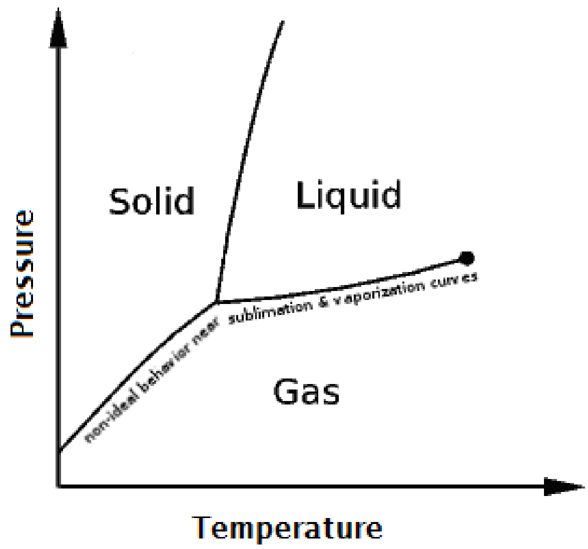
Kinetic-Molecular Theory (KMT)

Kinetic-Molecular Theory (KMT) is a theory, developed by James C. Maxwell and Ludwig Boltzmann, that predicts the behavior of gases by modeling them as moving molecules. (“kinetic” = “moving”.) The theory states that:

- Gases are made of very large numbers of molecules*
- Molecules are constantly moving (obeying Newton’s laws of motion), and their speeds are constant
- Molecules are very far apart compared with their diameter
- Molecules collide with each other and walls of container in elastic collisions
- Molecules behaving according to KMT are not reacting† or exerting any other forces (attractive or repulsive) on each other.

ideal gas: a gas whose molecules behave according to KMT. Most gases are ideal under *some* conditions (but not all). In general, gases behave ideally when they are not close to the solid or liquid regions of the phase diagram for the substance.

real gas: a gas whose molecules do ***not*** behave according to KMT. This can occur with all gases, most commonly at temperatures and pressures that are close to the solid or liquid regions of the phase diagram for the substance.



* In this chapter we will use the terms “particle” and “molecule” interchangeably, with apologies to chemists.

† Of course, reactions can occur, but chemical reactions are part of collision theory, which is separate from KMT.

Use this space for summary and/or additional notes:

Measurable Properties of Gases

All gases have the following properties that can be measured:

Property	Variable	S.I. Unit	Description
amount	N	—	amount of gas (particles)
	n	mole (mol)	amount of gas (moles) (1 mol = 6.02×10^{23} particles)
volume	V	cubic meter (m^3)	space that the gas takes up
temperature	T	kelvin (K)	ability to transfer heat through collisions with other molecules (average kinetic energy of the particles)
pressure	P	pascal (Pa)	average force on the walls of the container due to collisions between the molecules and the walls

Notes about calculations:

- Moles are based on the definition that $1\,\text{mole} = 6.022\,140\,76 \times 10^{23}\ \text{particles}$.
  1 mole was originally the number of carbon atoms in exactly 12 grams of carbon-12, such that the molar mass of a substance is the same number of grams as the average atomic mass of one atom in atomic mass units. This definition persisted, despite the fact that the base mass unit of the MKS system is the kilogram.
- Temperature must be absolute, which means you *must* use Kelvin. A temperature of 0 in a gas laws calculation can only mean absolute zero.
- Pressures must be absolute. (For example, you can't use a tire gauge because it measures "gauge pressure," which is the difference between atmospheric pressure and the pressure inside the tire.) A pressure of 0 in a gas laws calculation can only mean that there are no molecules colliding with the walls.

Other Common Units

- **Volume** can be measured in liters (L) or milliliters (mL).
  1 m^3 = 1 000 L and 1 L = 1 000 mL
- **Pressure** can be measured in many different units.
  1 atm = 101 325 Pa = 14.696 psi = 760 mm Hg = 29.92 in. Hg
  1 bar = 100 000 Pa = 100 kPa ≈ 1 atm

Use this space for summary and/or additional notes:

Gas Laws

Unit: Thermodynamics

MA Curriculum Frameworks (2016): HS-PS2-8(MA)

AP® Physics 2 Learning Objectives: 3.4.C.1, 3.4.C.2, 4.C.3.1, 7.A.3.3, 7.B.1.1

Mastery Objective(s): (Students will be able to…)

- Qualitatively describe the relationship between any two of the quantities: *number of particles*, *temperature*, *pressure*, and *volume* in terms of Kinetic Molecular Theory (KMT).
- Quantitatively determine the *number of particles*, *temperature*, *pressure*, or *volume* in a before & after problem in which one or more of these quantities is changing.

Success Criteria:

- Descriptions relate behavior at the molecular level to behavior at the macroscopic level.
- Solutions have the correct quantities substituted for the correct variables.
- Chosen value of the gas constant has the same units as the other quantities in the problem.
- Algebra and rounding to appropriate number of significant figures is correct.

Language Objectives:

- Identify each quantity based on its units and assign the correct variable to it.
- Understand and correctly use the terms "pressure," "volume," and "temperature," and "ideal gas."
- Explain the placement of each quantity in the ideal gas law.

Tier 2 Vocabulary: ideal, law

Labs, Activities & Demonstrations:

- Vacuum pump (pressure & volume) with:
  - balloon (air vs. water)
  - shaving cream
- Absolute zero apparatus (pressure & temperature)
- Balloon with tape (temperature & volume)
- Can crush (pressure, volume & temperature)

Use this space for summary and/or additional notes:

Notes:

<u>ideal gas</u>: a gas that behaves as if each molecule acts independently, according to kinetic-molecular theory. Specifically, this means the molecules are far apart, and move freely in straight lines at constant speeds. When the molecules collide, the collisions are perfectly elastic, which means they bounce off each other with no energy or momentum lost. (See the section on *Kinetic-Molecular Theory*, starting on page 233.)

Avogadro's Principle

In 1811, Italian physicist Amedeo Avogadro (whose full name was Lorenzo Romano Amedeo Carlo Avogadro di Quaregna e di Cerreto) published the principle that equal volumes of an ideal gas at the same temperature and pressure must contain equal numbers of particles.

Demonstration	Outcome	What the molecules are doing	Conclusion
put more (moles of) air into a balloon $n\uparrow$	the volume of the balloon got larger $V\uparrow$	crowding each other → pushing each other farther away	n and V are directly proportional. $\frac{V}{n} = \text{constant}$

If the pressure and temperature are constant, then for an ideal gas:

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} \quad \text{and} \quad \frac{V_1}{n_1} = \frac{V_2}{n_2}\,^*$$

(particles) (moles)

Although Avogadro's principle was originally stated in terms of particles (N) of gas, it is almost always more convenient to work with moles (n).

(Note that by convention, gas laws use subscripts "1" and "2" instead of "o" for initial no subscript for final.)

* Avogadro's principle is usually stated $\frac{n_1}{V_1} = \frac{n_2}{V_2}$. I have inverted it in these notes so that the quantities in the numerator and denominator are the same as the quantities in the numerator and denominator of the combined gas law.

Use this space for summary and/or additional notes:

Boyle's Law

In 1662, British physicist and chemist Robert Boyle published his findings that the pressure and volume of a gas were inversely proportional.

Demonstration	Outcome	What the molecules are doing	Conclusion
decrease pressure by putting a balloon in a vacuum chamber $P\downarrow$	the volume of the air inside the balloon increased $V\uparrow$	expanding the space = more surface area → less force per unit area (less pressure)	P and V are inversely proportional. PV = constant

Therefore, if the temperature and the number of particles of gas are constant, then for an ideal gas:

$$P_1V_1 = P_2V_2$$

Charles' Law

In the 1780s, French physicist Jacques Charles discovered that the volume and temperature of a gas were directly proportional.

Demonstration	Outcome	What the molecules are doing	Conclusion
place masking tape around balloon and heat with hot air gun $T\uparrow$	the volume of the air got larger and expanded the balloon except where the tape pinched it $V\uparrow$	moving more slowly → pushing each other less far away	V and T are directly proportional. $\frac{V}{T} = \text{constant}$

If pressure and the number of particles are constant, then for an ideal gas:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Use this space for summary and/or additional notes:

Gay-Lussac's Law

In 1702, French physicist Guillaume Amontons discovered that there is a relationship between the pressure and temperature of a gas. However, precise thermometers were not invented until after Amontons' discovery so it wasn't until 1808, more than a century later, that French chemist Joseph Louis Gay-Lussac confirmed this law mathematically. The pressure law is most often attributed to Gay-Lussac, though some texts refer to it as Amontons' Law.

Demonstration	Outcome	What the molecules are doing	Conclusion
increase temperature by heating a metal sphere full of air $T\uparrow$	the pressure of the air increased $P\uparrow$	moving faster → colliding with more force	P and T are directly proportional. $\frac{P}{T} = \text{constant}$

If volume and the number of particles are constant, then for an ideal gas:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

The Combined Gas Law

We can combine each of the above principles. When we do this (keeping P and V in the numerator and n (or N) and T in the denominator for consistency), we get following relationship for an ideal gas:

$$\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2} = \text{constant} \qquad \frac{P_1 V_1}{N_1 T_1} = \frac{P_2 V_2}{N_2 T_2} = \text{constant}$$

using moles using particles

Note, however, that in most situations where we want to calculate properties of a gas, the number of moles or particles remains constant. This means $n_1 = n_2$ or $N_1 = N_2$, and we can cancel it from the equation. This gives:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

The above equation is called the "combined gas law", which is used to solve most "before/after" problems involving ideal gases.

Use this space for summary and/or additional notes:

When using the combined gas law, any quantity that is not changing may be cancelled out of the equation. (If a quantity is not mentioned in the problem, you can assume that it is constant and may be cancelled.)

For example, suppose a problem doesn't mention anything about temperature. That means T is constant and you can cancel it. When you cancel T from both sides of the combined gas law, you get:

$$\frac{P_1 V_1}{\cancel{T_1}} = \frac{P_2 V_2}{\cancel{T_2}} \text{ which simplifies to } P_1 V_1 = P_2 V_2 \text{ (Boyle's Law)}$$

Solving Problems Using the Combined Gas Law

You can use this method to solve any "before/after" gas law problem:

1. Determine which variables you have
2. Determine which values are *initial* (#1) *vs.* *final* (#2).
3. Start with the combined gas law and cancel any variables that are explicitly not changing or omitted (assumed not to be changing).
4. Substitute your numbers into the resulting equation and solve. (Make sure all initial and final quantities have the same units, and don't forget that temperatures *must* be in Kelvin!)

Note: because quantities appear on both sides of the equation, it is not necessary to use S.I. units when solving problems using the combined gas law. It is, however, important to ***use the same units for the same quantity on both sides of the equation***.

Use this space for summary and/or additional notes:

Sample problem:

Q: A gas has a temperature of 25 °C and a pressure of 1.5 bar. If the gas is heated to 35 °C, what will the new pressure be?

A: 1. Find which variables we have.

We have two temperatures (25 °C and 35 °C), and two pressures (1.5 bar and the new pressure that we're looking for).

2. Find the action being done on the gas ("heated"). Anything that was true about the gas *before* the action is time "1", and anything that is true about the gas *after* the action is time "2".

Time 1 ("before"):	**Time 2 ("after")**:
P_1 = 1.5 bar	$P_2 = P_2$
T_1 = 25 °C + 273 = 298 K	T_2 = 35 °C + 273 = 308 K

3. Set up the formula. We can cancel volume (V), because the problem doesn't mention it:

$$\frac{P_1 \cancel{V_1}}{T_1} = \frac{P_2 \cancel{V_2}}{T_2} \text{ which gives us } \frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ (Gay-Lussac's Law)}$$

4. Plug in our values and solve:

$$\frac{1.5\,\text{bar}}{298\,\text{K}} = \frac{P_2}{308\,\text{K}} \quad \rightarrow \quad \boxed{P_2 = 1.55\text{ bar}}$$

Use this space for summary and/or additional notes:

Homework Problems

Solve these problems using one of the gas laws in this section. Remember to convert temperatures to Kelvin!

1. **(M)** A sample of oxygen gas occupies a volume of 250. mL at a pressure of 740. torr. What volume will it occupy at 800. torr?

   Answer: 231.25 mL

2. **(M)** A sample of O_2 is at a temperature of 40.0 °C and occupies a volume of 2.30 L. To what temperature should it be raised to occupy a volume of 6.50 L?

   Answer: 612 °C

3. **(S)** H_2 gas was cooled from 150. °C to 50. °C. Its new pressure is 750 torr. What was its original pressure?

   Answer: 980 torr

4. **(S)** A 2.00 L container of N_2 had a pressure of 3.20 atm. What volume would be necessary to decrease the pressure to 98.0 kPa?

   (*Hint: notice that the pressures are in different units. You will need to convert one of them so that both pressures are in either* atm *or* kPa.)

   Answer: 6.62 L

Use this space for summary and/or additional notes:

5. **(S)** A sample of air has a volume of 60.0 mL at S.T.P. What volume will the sample have at 55.0 °C and 745 torr?

   Answer: 73.5 mL

6. **(M)** N_2 gas is enclosed in a tightly stoppered 500. mL flask at 20.0 °C and 1 atm. The flask, which is rated for a maximum pressure of 3.00 atm, is heated to 680. °C. Will the flask explode?

   Answer: P_2 = 3.25 atm. Yes, the flask will explode.

7. A scuba diver's 10. L air tank is filled to a pressure of 210 bar at a dockside temperature of 32.0 °C. When the diver is breathing the air underwater, the water temperature is 8.0 °C, and the pressure is 2.1 bar.

   a. **(M)** What volume of air does the diver use?

      Answer: 921 L

   b. **(S)** If the diver uses air at the rate of 8.0 L/min, how long will the diver's air last?

      Answer: 115 min

Use this space for summary and/or additional notes:

Ideal Gas Law

Unit: Thermodynamics

MA Curriculum Frameworks (2016): HS-PS2-8(MA)

AP® Physics 2 Learning Objectives: 3.4.C.1, 3.4.C.2, 4.C.3.1, 7.A.3.3, 7.B.1.1

Mastery Objective(s): (Students will be able to…)

- Describe the relationship between any two variables in the ideal gas law.
- Apply the ideal gas law to problems involving a sample of gas.

Success Criteria:

- Solutions have the correct quantities substituted for the correct variables.
- Chosen value of the gas constant has the same units as the other quantities in the problem.
- Algebra and rounding to appropriate number of significant figures is correct.

Language Objectives:

- Identify each quantity based on its units and assign the correct variable to it.
- Explain the placement of each quantity in the ideal gas law.

Tier 2 Vocabulary: ideal, law

Notes:

ideal gas: a gas that behaves according to Kinetic-Molecular Theory (KMT).

When we developed the combined gas law, before we cancelled the number of moles or particles, we had the equations:

$$\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2} = \frac{PV}{nT} = R \text{ (constant)} \qquad \frac{P_1 V_1}{N_1 T_1} = \frac{P_2 V_2}{N_2 T_2} = \frac{PV}{NT} = k_B \text{ (constant)}$$

using moles — using particles

where n is the number of moles of gas, and N is the number of gas particles. One mole is 6.02×10^{23} particles, which means $N = (6.02 \times 10^{23})n$

Because P, V, n and T are all of the quantities needed to specify the conditions of an ideal gas, this expression must be true for *any ideal gas* under *any conditions*. If V is in m^3, P is in Pa, n is in moles, and T is in Kelvin, then:

$$R = 8.31 \, \tfrac{\text{J}}{\text{mol·K}} \quad \text{and} \quad k_B = 1.38 \times 10^{-23} \, \tfrac{\text{J}}{\text{K}}$$

R is called "the gas constant," and k_B is Boltzmann's constant.

We can rearrange $\frac{PV}{nT} = R$ and $\frac{PV}{NT} = k_B$ to get the ideal gas law in its familiar form:

$$PV = nRT \quad \text{and} \quad PV = Nk_B T$$

Use this space for summary and/or additional notes:

Other Values of R

The purpose of the gas constant R is to convert the quantity nT from units of mol·K to units of pressure × volume. This constant can have different values, depending on the units that it needs to cancel:

$$R = 8.31\,\frac{\text{J}}{\text{mol·K}} \equiv 8.31\,\frac{\text{m}^3\text{·Pa}}{\text{mol·K}} \equiv 8.31\,\frac{\text{L·kPa}}{\text{mol·K}} \equiv 8.31\times10^{-3}\,\frac{\text{kJ}}{\text{mol·K}}$$

$$R = 0.0821\,\frac{\text{L·atm}}{\text{mol·K}} \qquad R = 62.4\,\frac{\text{L·torr}}{\text{mol·K}} \qquad R = 1.987\,\frac{\text{cal}}{\text{mol·K}} \equiv 1.987\,\frac{\text{BTU}}{\text{lb-mol·}^\circ\text{R}}$$

Use of non-S.I. units, such as atm or torr, is more common in chemistry. In this course, we will use the S.I. units of m^3 for volume and Pa for pressure. The unit Pa·m^3 is equivalent to a joule.

Solving Problems Using the Ideal Gas Law

If a gas behaves according to the ideal gas law, simply substitute the values for pressure, volume, number of moles (or particles), and temperature into the equation. Be sure your units are correct (especially that temperature is in Kelvin), and that you use the correct constant, depending on whether you know the number of particles or the number of moles of the gas.

Sample Problem:

A 3.50 (n) mol sample of an ideal gas has a pressure of 120 000 (P) Pa and a temperature of 35 °C. What is its volume?
$T \rightarrow K$ $\quad$ V

Answer:

Note that because pressure is given in pascals (Pa), we need to use the value of the gas constant that also uses Pa: $R = 8.31\,\frac{\text{m}^3\cdot\text{Pa}}{\text{mol}\cdot\text{K}}$

P = 120 000 Pa

V = V

n = 3.50 mol

$R = 8.31\,\frac{\text{m}^3\cdot\text{Pa}}{\text{mol}\cdot\text{K}}$

T = 35 °C + 273 = 308 K

Then we substitute these numbers into the ideal gas law and solve:

$$PV = nRT$$

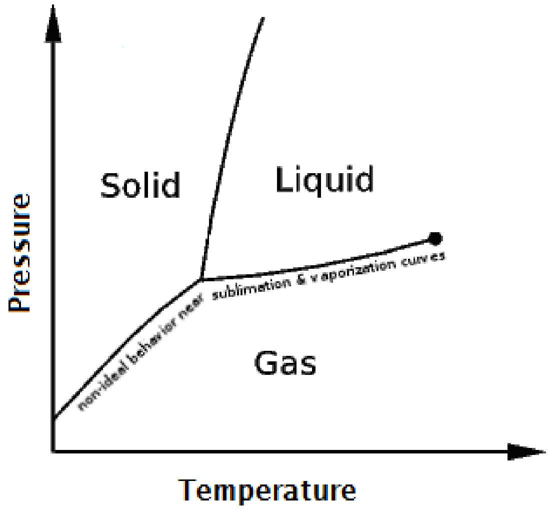
$$V = \frac{nRT}{P} = \frac{(3.50)(8.31)(308)}{120\,000} = 0.0747\,\text{m}^3$$

Use this space for summary and/or additional notes:

honors (not AP®)

Real Gases

As stated previously, when the temperature and pressure of a gas are close to the solid or liquid regions of the phase diagram for the substance, gases start to exhibit non-ideal behaviors. Recall the following definition of a real gas:



real gas: a gas whose molecules do ***not*** behave according to kinetic-molecular theory (KMT). This occurs most commonly at temperatures and pressures that are close to the solid or liquid regions of the phase diagram for the substance.

In the late 19th century, the Dutch physicist Johannes van der Waals published a correction to the ideal gas law that can be applied to real gases.

The van der Waals Equation applies correction factors to the pressure and volume terms in the equation:

$$\left(P + a\frac{n^2}{V^2}\right)(V - nb) = nRT$$

in this equation, the constants a and b are properties specific to a gas and must be looked up or determined experimentally.

The corrected pressure term ($P + a\frac{n^2}{V^2}$ instead of P) is because molecules attract each other slightly at low pressures, but repel each other when they are forced close together. This repulsion acts like additional pressure.

The corrected volume term ($V - nb$ instead of V) term is because the ideal gas law assumes that the molecules are far enough apart that we do not need to consider the volumes of the molecules themselves as part of the volume of their container. As the molecules are brought closer together, we have to subtract the space taken up by n moles of molecules from the available volume.

You will not need to solve problems using the van der Waals equation in this course.

Use this space for summary and/or additional notes:

Homework Problems

Use the ideal gas law to solve the following problems. Be sure to choose the appropriate value for the gas constant and to convert temperatures to Kelvin.

1. **(M)** A sample of 1.00 moles of oxygen at 50.0 °C and 98.6 kPa occupies what volume?

   Answer: 27.2 L

2. **(S)** If a steel cylinder with a volume of 1.50 L contains 10.0 moles of oxygen, under what pressure is the oxygen if the temperature is 27.0 °C?

   Answer: 164 atm = 125 000 torr = 16 600 kPa

3. **(S)** In a gas thermometer, the pressure of 0.500 L of helium is 113.30 kPa at a temperature of −137 °C. How many moles of gas are in the sample?

   Answer: 0.050 mol

4. **(M)** A sample of 4.25 mol of hydrogen at 20.0 °C occupies a volume of 25.0 L. Under what pressure is this sample?

   Answer: 4.09 atm = 3 108 torr = 414 kPa

Use this space for summary and/or additional notes:

Energy Conversion

Unit: Thermodynamics

MA Curriculum Frameworks (2016): HS-PS3-1

AP® Physics 2 Learning Objectives: 5.B.4.2, 5.B.5.4, 5.B.5.5

Mastery Objective(s): (Students will be able to…)

- Describe the conversion of energy between heat and other forms.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Describe and explain an example of conversion of heat into mechanical work.

Tier 2 Vocabulary: heat, energy

Labs, Activities & Demonstrations:

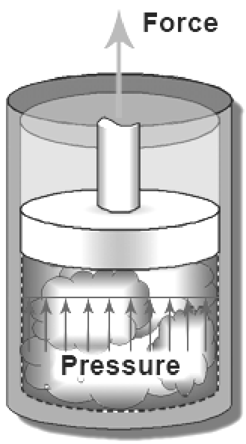
- steam engine
- fire syringe
- metal spheres & paper

Notes:

The law of conservation of energy states that total energy is always conserved, but that energy can be converted from one form to another.

We have already seen this in mechanics with the conversion between gravitational potential energy and kinetic energy.

Heat is energy. Like other forms of energy, it can do work. For example, in a steam engine, heat is used to boil water in a sealed container. As more water boils, there is more gas in the boiler, which makes the pressure increase. If the gas can only expand by pushing against something (like a piston), the force from the pressure can do work by moving the piston and whatever it's connected to.

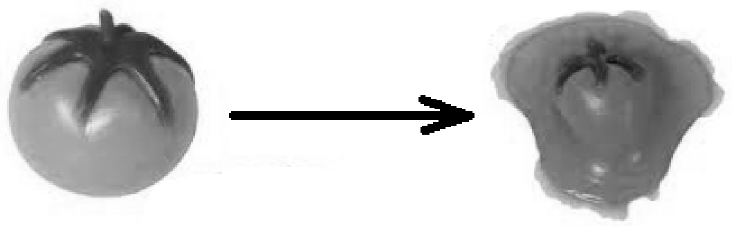


In mechanics, recall that collisions can be elastic or inelastic. In an elastic collision, kinetic energy is conserved; in an inelastic collision, some of the kinetic energy is converted to other forms, mostly heat.

Use this space for summary and/or additional notes:

We can use the law of conservation of energy to estimate the amount of energy converted to heat in a completely inelastic collision.

Consider a 0.150 kg tomato hitting the wall at a velocity of $20.0\,\frac{\text{m}}{\text{s}}$.



After the collision, the velocity of the tomato and the wall are both zero. This means the kinetic energy of the tomato after the collision is zero. Because energy must be conserved, this means all of the kinetic energy from the tomato must have been converted to heat.

$$E_k = \tfrac{1}{2}mv^2$$
$$E_k = (\tfrac{1}{2})(0.150)(20.0)^2 = 30.0\text{ J}$$

Now consider the same tomato with a mass of 0.150 kg and a velocity of $20.0\,\frac{\text{m}}{\text{s}}$ hitting a 1.00 kg block of wood that is initially at rest. This is still an inelastic collision, but now the wood is free to move, which means it has kinetic energy after the collision.

To solve this problem, we need to use conservation of momentum to find the velocity of the tomato + wood after the collision, and then use the velocity before and after to calculate the change in kinetic energy.

Before the collision:

$$\vec{\boldsymbol{p}} = m_t\vec{\boldsymbol{v}}_t + \cancelto{0}{m_w\vec{\boldsymbol{v}}_w}$$
$$\vec{\boldsymbol{p}} = (0.150)(+20.0) + 0 = +3.00\,\text{N}\cdot\text{s}$$

$$K = \tfrac{1}{2}m_t v_t^2 + \cancelto{0}{\tfrac{1}{2}m_w v_w^2}$$
$$K = (\tfrac{1}{2})(0.150)(20.0)^2 + 0 = 30.0\text{ J}$$

After the collision:

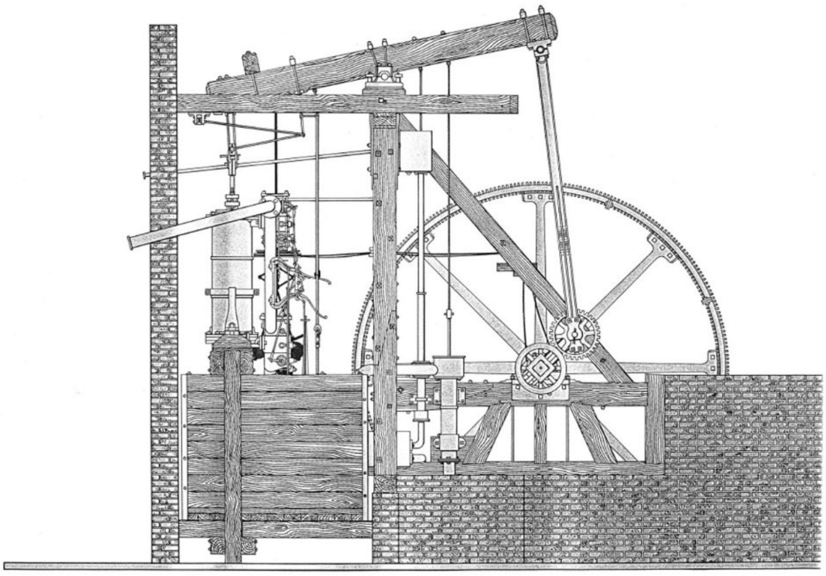
$$\vec{\boldsymbol{p}} = (m_t + m_w)\,\vec{\boldsymbol{v}}$$
$$+3.00 = (0.150 + 1.00)\,\vec{\boldsymbol{v}} = 1.15\vec{\boldsymbol{v}}$$
$$\vec{\boldsymbol{v}} = +2.61\,\tfrac{\text{m}}{\text{s}}$$

$$K = \tfrac{1}{2}mv^2$$
$$K = (\tfrac{1}{2})(1.15)(2.61)^2 = 3.91\text{ J}$$

This means there is 30.0 – 3.91 = 26.1 J of kinetic energy that is "missing" after the collision. This "missing" energy is mostly converted to heat. If you could measure the temperature of the tomato and the wood extremely accurately before and after the collision, you would find that both would be slightly warmer as a result of the "missing" 26.1 J of energy.

Use this space for summary and/or additional notes:

The first instance of a machine using heat to do work was in 1698, when Thomas Savery patented a steam-driven water pump. In 1769, Scottish engineer James Watt and investor John Roebuck patented a steam engine that could be used for a variety of purposes, including running sawmills, cotton mills, and anything else that required a large amount of force. Watt built his first prototype steam engine in 1788.



James Watt's prototype steam engine, 1788

The invention of the steam engine was a significant factor in the spread of the the industrial revolution, and all of the societal changes that went with it.

Thermodynamics is the study of heat energy and its conversion to other forms of energy. In chemistry, thermodynamics is heat energy that drives chemical reactions. In physics, thermodynamics is heat energy that is converted to mechanical work (which you may recall from physics 1, is a force applied over a distance).

Use this space for summary and/or additional notes:

Thermodynamics

Unit: Thermodynamics

MA Curriculum Frameworks (2016): HS-PS2-6

AP® Physics 2 Learning Objectives: 5.B.4.1, 5.B.5.4, 5.B.7.1, 7.B.2.1

Mastery Objective(s): (Students will be able to…)

- Calculate kinetic energy, internal energy and work done by the particles of a gas.

Success Criteria:

- Solutions have the correct quantities substituted for the correct variables.
- Algebra and rounding to appropriate number of significant figures is correct.

Language Objectives:

- Describe the different types of energy (kinetic, internal, work) and explain what they measure.

Tier 2 Vocabulary: internal, energy, work

Labs, Activities & Demonstrations:

- heat exchange dice game
- dice distribution game
- entropy (microstates) percentile dice game

Notes:

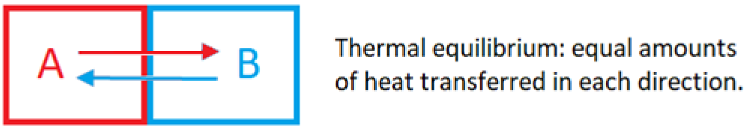
<u>thermodynamics</u>: the study of heat-related (thermal) energy changes (dynamics)

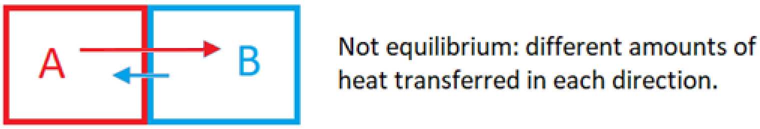
Thermodynamics is an application of the law of conservation of energy. In Physics 1, we studied changes between gravitational potential energy and kinetic energy. Thermodynamic changes involve the same principle; the details and the equations, however, are quite different.

As was the case with gas laws, the topic of thermodynamics is studied by both chemists and physicists. Chemists tend to be more concerned with the heat produced and consumed by chemical changes and reactions. Physicists tend to be more concerned with the conversion between thermal energy (regardless of how it is produced) and other forms of energy, particularly mechanical.

Use this space for summary and/or additional notes:

thermal equilibrium: if two systems "A" and "B" are in thermal equilibrium, heat is transferred from A to B at the same rate as heat is transferred from B to A.





temperature: a measure of the average kinetic energy of the particles in a substance. (K)

$$K_{ave.} = \tfrac{1}{2} m v_{ave.}^2 = \tfrac{3}{2} k_B T$$

where:

$K_{ave.} =$ average kinetic energy (J)
$m =$ mass of a particle (kg)
$M =$ molar mass (mass of one mole of particles) ($\frac{\text{kg}}{\text{mol}}$)*
$v_{ave.} =$ average velocity of a particle ($\frac{\text{m}}{\text{s}}$)
$k_B =$ Boltzmann's constant = $1.38 \times 10^{-23}\ \frac{\text{J}}{\text{K}}$
$T =$ temperature (K)

The factor of $\frac{3}{2}$ is because there is an equal probability of a collision between particles in the *x*-, *y*-, and *z*-directions, and $(3)(\frac{1}{2}) = \frac{3}{2}$.

root mean square velocity (v_{rms}): the geometric mean (average) velocity of a particle. ($\frac{\text{m}}{\text{s}}$) The rms velocity is derived by solving $\frac{1}{2} m v_{ave.}^2 = \frac{3}{2} k_B T$ for the average velocity:

$$v_{rms} = \sqrt{v_{ave.}^2} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{M}}$$

* Chemists express molar mass in $\frac{\text{g}}{\text{mol}}$ rather than $\frac{\text{kg}}{\text{mol}}$. Because we use the MKS system in physics, we need to express molar mass in $\frac{\text{kg}}{\text{mol}}$.

Use this space for summary and/or additional notes:

internal energy (U[*]): the total thermal energy of a system due to the kinetic energy of its particles.

If the kinetic energy of a single particle is $K_{ave.} = \frac{3}{2}k_B T$, then the total kinetic energy in a system that has N particles would be:

$$U = NK_{ave.} = \frac{3}{2}Nk_B T$$

Because it is generally unwieldy to perform calculations for systems with large numbers of particles, it is more convenient to use moles. Substituting nR for Nk_B gives the equation for the internal energy of a system that has n moles of particles:

$$U = \frac{3}{2}nRT$$

Similarly, a *change* in internal energy (ΔU) is related to the corresponding *change* in temperature (ΔT):

$$\Delta U = \frac{3}{2}nR\Delta T \quad (= \frac{3}{2}Nk_B\Delta T)$$

heat (Q): thermal energy transferred into or out of a system. (J)

work (W): mechanical energy (such as the application of a force over a distance) transferred into or out of a system. (J)

The work that a gas can do comes from its ability to move an object by applying a force on it as it expands. If the pressure is constant:

$$W = Fd = F\Delta x$$

$$P = \frac{F}{A} \quad \rightarrow \quad F = PA$$

$$\therefore \; W = (PA)\Delta x$$

$$\Delta V = A\Delta x$$

$$\therefore \; W = P\Delta V$$

If a gas does work by expanding, the energy is transferred from the gas (the system) to the object that the gas is pushing against (the surroundings). This means that when the volume increases (ΔV is positive), energy is leaving the system (W is negative). Conversely, if work is done to compress a gas, energy is entering the system in order to compress the gas (W is positive), and the volume decreases (ΔV is negative). This means that W and $P\Delta V$ must have opposite signs, which gives the equation:

$$W = -P\Delta V$$

assuming that pressure is constant.

[*] Chemistry textbooks often use the variable E instead of U.

Use this space for summary and/or additional notes:

If pressure is not constant, then $W = -\Delta(PV)$, which means you would need to calculate PV at each point, taking the limit as the distance between the data points shrinks to zero, and add them up. In calculus, this is the integral: $W = -\int P\,\mathrm{d}V$, where P is a function of V.

In an algebra-based course, we will limit ourselves to problems where the pressure is constant, or where the pressure change is linear and you can use the average pressure, giving:

$$W = -P_{ave.}\Delta V$$

<u>entropy</u> (S): "unusable" thermal energy in a system. Energy in the form of entropy is unavailable because it has "escaped" or "spread out". (Entropy will be discussed further in the Second Law of Thermodynamics.)

Laws of Thermodynamics

The laws of thermodynamics describe the behavior of systems with respect to changes in heat energy.

For historical reasons, the laws are numbered from 0–3* instead of 1–4, because the 0 law was added after the others, and the laws are often referred to by their number.

0. If a system is at thermal equilibrium, every component of the system has the same temperature. ("You have to play the game.")
1. Heat always flows from a region of higher internal energy to a region of lower internal energy. Because internal energy is directly proportional to temperature, this is equivalent to saying that heat flows from a region of higher temperature to a region of lower temperature. This means you can't get more heat out of a system than you put in. ("You can't win.")
2. In almost every change, some energy is irretrievably lost to the surroundings. Entropy is a measure of this "lost" energy. The entropy of the universe is always increasing, which means on any practical scale, you will always get out less energy than you put in. ("You can't break even.")
3. Conservation of energy always applies. In any closed system, the total energy (internal energy + entropy + work) remains constant. If energy was "lost," it turned into an increase in entropy. ("You can't get out of the game.")

* There is one type of person in the world: those who start counting from zero and those who start counting from one.

Use this space for summary and/or additional notes:

Zeroth Law (or Zero Law)

The zeroth law says that if you have multiple systems in thermal equilibrium (the heat transferred from "A" to "B" is equal to the heat transferred from "B" to "A"), then the systems must have the same temperature. The consequences of this are:

- If we have three (or more) systems "A," "B," and "C," and A is in thermal equilibrium with B, and B is in thermal equilibrium with C, this means that A, B, and C must all have the same temperature, and A is therefore in thermal equilibrium with C. (This is akin to the transitive property of equality in mathematics.)
- If an object with a higher temperature (a "hotter" object) is in contact with an object with a lower temperature (a "colder" object), heat will flow from the object with higher temperature to the object with lower temperature until the temperatures are the same (the objects are in thermal equilibrium).

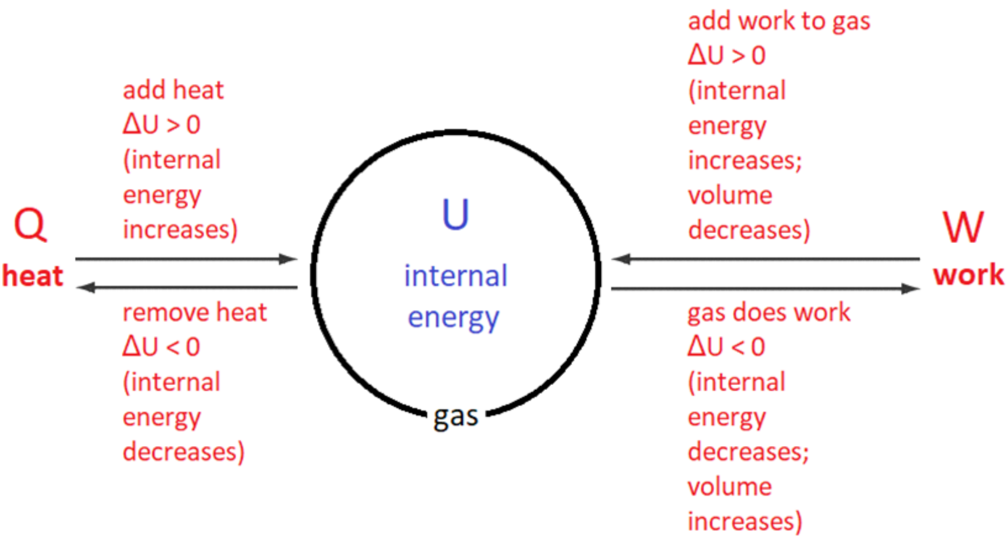
Use this space for summary and/or additional notes:

First Law

Consider an isolated system—*i.e.,* a system where heat and other forms of energy cannot enter nor leave the system. This system is doing no work, but it has internal energy.

Internal energy is similar to potential energy—it is a property of a system that is not doing work currently, but has the potential to do work in the future.

According to the First Law, the internal energy of a system increases $(\Delta U > 0)$ if energy is added in the form of heat $(Q > 0)$ or work $(W > 0)$. The internal energy decreases if energy is removed from the system, by removing heat $(Q < 0)$ or by using the internal energy of the system (the gas) to do work on the surroundings ($W < 0$ because work is going out of the system).



In equation form, the First Law looks like this:

$$\Delta U = Q + W^{*}$$

The First Law is simply the law of conservation of energy—the change in internal energy comes from the heat added to or removed from the system combined with the work done on or by the system.

Combining the First Law with the definition of internal energy gives:

$$\Delta U = \tfrac{3}{2} nR\Delta T = Q + W$$

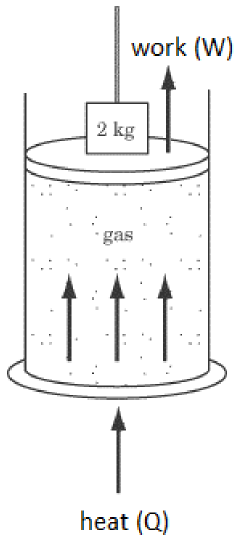
* Some textbooks define work exclusively as work done ***by*** the system ***on*** the surroundings (*i.e.,* energy ***leaving*** the system). Using this definition would reverse the sign of W in the equation, giving:

$$\Delta U = Q - W$$

Use this space for summary and/or additional notes:

Sample Problem:

Q: A cylinder containing 1.8 mol of an ideal gas with a temperature of 275 K has a piston with a weight on top. The combined mass of the piston plus the weight is 2.0 kg, and the cross-sectional area of the piston is 0.01 m^2. The volume of the gas in the cylinder is 0.033 m^3.



Heat is added, and the volume of the gas increases to 0.040 m^3. How much heat was added to the gas?

A: When the gas is heated, the following occur:

1. In order to increase the volume, the gas has to expand, which means the temperature needs to increase. We know how much the volume increased, and the pressure remains constant (the piston pushes the same amount throughout the process). We can use $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$ to find the final temperature of the gas.

$$\frac{\cancel{P_1}V_1}{T_1}=\frac{\cancel{P_2}V_2}{T_2} \rightarrow \frac{0.033}{275}=\frac{0.040}{T_2} \rightarrow T_2 = 333\,\text{K}$$

2. The increase in temperature means the internal energy of the gas increased. We can use $\Delta U=\frac{3}{2}nR\Delta T$ to find out how much the internal energy of the gas increased (ΔU). The equation $\Delta U = Q+W$ tells us that some of the heat energy needed to be used for the increase in internal energy, and the rest of it was used to do the work of raising the piston.

$$\Delta U=\tfrac{3}{2}nR\Delta T \quad \rightarrow \quad \Delta U=(\tfrac{3}{2})(1.8)(8.31)(333-275)=1301\,\text{J}$$

3. We can calculate the work used to raise the piston from the work equation from physics 1: $W=mg\Delta h$ (Note that $\Delta V=A\Delta h$.)

$$W=-mg\Delta h=-mg\frac{\Delta V}{A}=-(2)(10)\frac{0.007}{0.01}=-14\,\text{J}$$

The work is negative because the energy is going out of the system (remember that the system is the gas) and into the surroundings.

4. Once we have ΔU and W, we can find Q by applying the first law:

$$\Delta U = Q+W$$
$$1301 = Q+(-14)$$
$$1315\,\text{J}=Q$$

Use this space for summary and/or additional notes:

Alternatively, we could calculate the work using $W=-P\Delta V$, but we would need to use gauge pressure rather than absolute pressure. (See the explanation below and *gauge pressure* on page 152.)

The first two steps are the same as above.

1. This step is the same as step #1 above—we need to find the temperature change necessary to produce the change in volume.

2. This step is the same as step #2 above—we need to calculate the change in internal energy of the gas caused by the change in temperature.

3. Instead of calculating the work using equations from physics 1, we can use $W=-P\Delta V$. However, pressure needs to be the amount of pressure that is doing the work, which is the difference in pressure between the inside of the piston and the outside of the piston. (This would be the gauge pressure inside the cylinder.) We can calculate this using the pressure equation from the fluids unit (see Pressure, starting on page 151):

$$P=\frac{F}{A}=\frac{(2)(10)}{0.01}=2\,000\text{ Pa}$$

   This is the pressure at which the gas needs to do work.

   Once we have the pressure, the work is given by:

$$\begin{aligned}&W=-P\Delta V\\&\Delta V=0.040-0.033=0.007\text{ m}^3\\&W=-(2000)(0.007)=-14\text{ J}\end{aligned}$$

4. Once we have ΔU and *W*, we can find Q by applying the first law:

$$\begin{aligned}\Delta U&=Q+W\\1301&=Q+(-14)\\1315\text{ J}&=Q\end{aligned}$$

Use this space for summary and/or additional notes:

Second Law

The Second law tells us that heat energy cannot flow from a colder system to a hotter one unless work is done on the system. This is why your coffee gets cold and your ice cream melts.

One consequence of this law is that no machine can work at 100% efficiency; all machines generate some heat, and some of that heat is always lost to the surroundings.

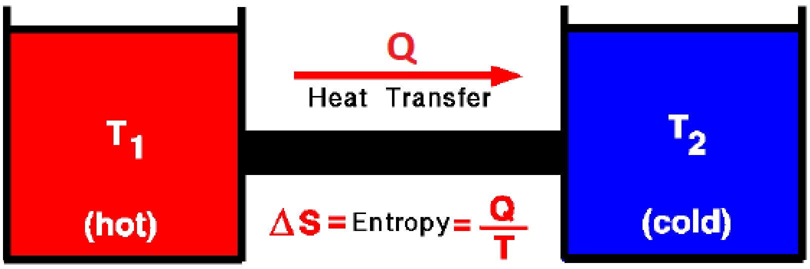
Entropy

Because energy must be conserved, we have to account for energy that still exists, but has "gotten lost" ("escaped" or "spread out") and is no longer usable by the system. Energy that has spread out and cannot be recovered is called entropy.

For example, when an egg falls to the floor and breaks, gravitational potential energy is converted to a combination of internal energy (the measurable increase in the temperature of the egg), and entropy (heat energy that is radiated to the environment and "lost"). Over time, the internal energy in the egg is also radiated to the environment and "lost" as the egg cools off. Ultimately, all of the gravitational potential energy ends up converted to entropy, which is the heat energy that is dissipated and cannot be recovered.

Entropy is sometimes called "disorder" or "randomness", but in the thermodynamic sense this is not correct. The entropy of your room is a *thermodynamic* property of the heat energy in your room, not a commentary on the amount of dirty laundry on the floor!

Whenever heat is transferred from an object with a higher temperature to an object with a lower temperature, the heat "spreads out" as it warms the colder object. The amount of energy that goes into this "spread" is called entropy.



Use this space for summary and/or additional notes:

If the heat is transferred in a way that is completely reversible (which is impossible and would take an infinite amount of time), then you would be able to recover the energy that was converted to entropy when transferring it back to the hotter object. We call this fictitious heat "reversible heat," denoted Q_{rev}.

$$\Delta S = \frac{Q_{rev}}{T}$$

Real energy transfers that take place in finite amounts of time can never recover all of the energy that was turned into entropy. This means the actual increase in entropy is always more than the amount that would occur in a reversible process:

$$\Delta S = \frac{Q_{rev}}{T} \geq \frac{Q}{T}$$

In other words, the energy that is lost to entropy by transfer of an amount of heat Q would be exactly $\frac{Q}{T}$ for a completely reversible process (*i.e.,* $Q = Q_{rev}$), and more than that for any real process.

Because actual heat transfer in a finite amount of time cannot be completely reversible, some heat is lost to the surroundings and the actual entropy change is always greater than the actual heat change at a given temperature. The concept of a reversible process is an idealization that represents the maximum amount of work that could theoretically be extracted from the process.

A consequence of the Second Law is that *the entropy of the universe is always increasing*.

In physics, there is a hierarchy of thinking. Conservation of energy, conservation of momentum, and the Second Law are at the top of the hierarchy. Just as special relativity tells us that time, distance and mass all need to be changeable in order to maintain conservation of energy and momentum, the Second Law explains why time cannot move backwards—to do so would require a decrease in the entropy of the universe.

Third Law

The Third Law tells us that in an isolated system, the total energy of the system must be constant. (An isolated system is a system for which it is not possible to exchange energy with the surroundings.) This makes intuitive sense; because energy must be conserved, if no energy can be added or taken away, then the total energy cannot change.

Use this space for summary and/or additional notes:

Thermodynamic Quantities and Equations

Because energy is complex and exists in so many forms, there are many thermodynamic quantities that can be calculated in order to quantify the energy of different portions of a system. The following is a list of some of the more familiar ones:

Selected Thermodynamic Quantities

Variable	Name	Description
Q	heat	Thermal energy (heat) transferred into or out of a system due to a difference in temperature.
W	work	Mechanical energy transferred into or out of a system through the action of a force applied over a distance. $W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}} = -P\Delta V$
U*	internal energy	Total thermal (non-chemical) energy contained within the particles of a system because of their kinetic energy. $U = \frac{3}{2}nRT \qquad \Delta U = Q + W = \frac{3}{2}nR\Delta T$
S	entropy	Energy that is "lost" (inaccessible) because it has spread to the surroundings or has spread to separate microstates and cannot be utilized by the particles of the system.
A	Helmholtz free energy	Useful work that could theoretically be obtained from a system. $A = U - TS \qquad \Delta A = \Delta U - T\Delta S$
H	enthalpy	Heat energy available in a chemical reaction. $H = U + PV \qquad \Delta H = \Delta U + P\Delta V = \Delta U - W$
G	Gibbs free energy	Total energy available in a chemical reaction. $G = H - TS \qquad \Delta G = \Delta H - T\Delta S$

Because this is a physics course, we will leave enthalpy and Gibbs free energy to the chemists.

* Some chemistry textbooks use the variable *E* instead of *U*.

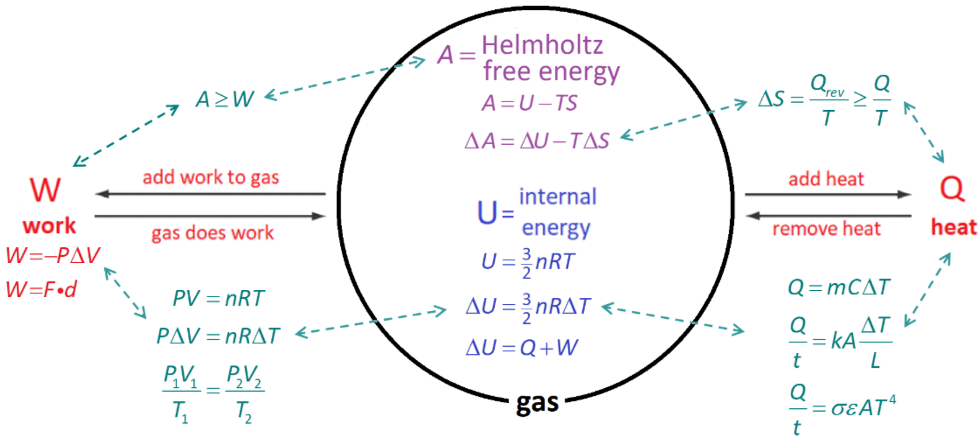
Use this space for summary and/or additional notes:

Thermodynamics Equations Used in This Course

Most of the thermodynamics problems encountered in this course are applications of the following equations:

Equation	Quantities that are Changing
$U=\frac{3}{2}nRT$ $\Delta U=\frac{3}{2}nR\Delta T$	internal energy *vs.* temperature
$\Delta U=Q+W$	internal energy *vs.* heat & work
$PV=nRT$ $P\Delta V=nR\Delta T$	pressure & volume *vs.* temperature
$W=-P\Delta V$ *	work *vs.* volume

What makes thermodynamics challenging is that there are many relationships between the quantities in these equations, as shown in the following thermodynamics equation map:



It is often necessary to combine equations. For example:

$$\Delta U=\frac{3}{2}nR\Delta T=Q+W$$

$$-W=P\Delta V=nR\Delta T$$

(Note that we moved the negative sign from $W=-P\Delta V$ to the other side of the equation.)

* In an algebra-based course, we need to restrict ourselves to problems in which the pressure remains constant during volume changes. In a calculus-based course, this equation would be $W=-\int P\,dV$.

Use this space for summary and/or additional notes:

The problems that you will encounter will involve a change in a measurable state variable (pressure, volume and/or temperature). To solve these problems, you will need to:

1. Determine what the change involves:
   - heat transfer (Q)
   - work (W) resulting from a change in volume (ΔV).
   - a change in internal energy (ΔU) resulting from a change in temperature (ΔT).

   (There can be more than one of these happening at the same time.)

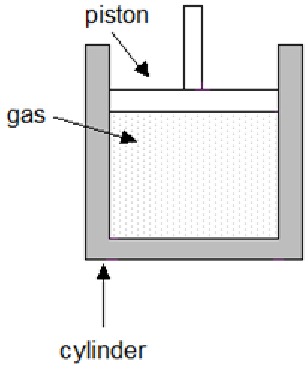
2. If necessary, determine initial and/or final values of these state variables in relation to other variables using equations such as:
   - $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$
   - $PV = nRT$

3. Apply algebraic combinations of these equations to find each of the necessary quantities to answer the question.

Use this space for summary and/or additional notes:

Homework Problems

Each of the problems below consists of a gas in a cylinder, as shown in the diagram to the right.



The gas in the cylinder has an initial volume of 0.01 m^3 and an initial temperature of 300 K. The piston has an area of 0.2 m^2, moves freely (with negligible friction), and has a weight of 250 N. Atmospheric pressure outside the cylinder is 100 000 Pa.

You may assume that the cylinder is perfectly insulated, which means the amount of heat that escapes from the cylinder in each situation is negligible.

1. **(M)** What is the initial pressure inside the cylinder? (*This is a fluids problem.*)

   Answer: 101 250 Pa

2. **(M)** How many moles of gas are in the cylinder?

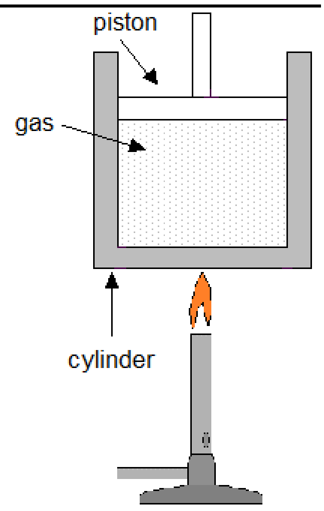
   Answer: 0.406 mol

3. **(M)** What is the internal energy of the gas in the cylinder?

   Answer: 1 519 J

Use this space for summary and/or additional notes:

4. **(M)** A Bunsen burner is placed below the cylinder and is turned on. Heat is added to the gas (with the pressure remaining constant) until the volume increases to 0.012 m^3.



a. What is the new temperature of the gas?

Answer: 360 K

b. What is the new internal energy of the gas?

Answer: 1822.5 J

c. How much work was done to raise the piston? (If you use $W = P\Delta V$, remember to use gauge pressure by subtracting the atmospheric pressure that is pushing down on the piston.)

Answer: 2.5 J

d. How much total heat was added to the gas?

Answer: 306.25 J

Use this space for summary and/or additional notes:

5. **(M)** Suppose instead that the piston from problem #4 above was fixed in position and was not allowed to move, so the volume remains constant at 0.01 m^3 while the 306.25 J of heat from question 4d was added.

   a. What is the new temperature of the gas?

      Answer: 360.5 K

   b. What is the new pressure of the gas?

      Answer: 121 667 Pa

   c. If the piston is then released and allowed to move freely, what will the new pressure be inside the cylinder?

      Answer: 101 250 Pa (the same as in problem #1 above)

Use this space for summary and/or additional notes:

Pressure-Volume (PV) Diagrams

Unit: Thermodynamics

MA Curriculum Frameworks (2016): HS-PS2-6

AP® Physics 2 Learning Objectives: 5.B.5.6, 5.B.7.2, 5.B.7.3, 7.A.3.2, 7.A.3.3

Mastery Objective(s): (Students will be able to…)

- Determine changes in heat, work, internal energy and entropy from a pressure-volume (PV) diagram.

Success Criteria:

- Correct equation is chosen.
- Solutions have the correct quantities substituted for the correct variables.
- Sign is correct for work (positive *vs.* negative).
- Algebra is correct.

Language Objectives:

- Explain what is physically happening to a gas for each section of a PV diagram.

Tier 2 Vocabulary: internal, energy, heat, work

Notes:

P-V diagram: a graph that shows changes in pressure *vs.* changes in volume.

Recall that:

$$W = -\int P\,dV$$

On a graph, the integral is the area "under the curve" (meaning the area between the curve and the *x*-axis).

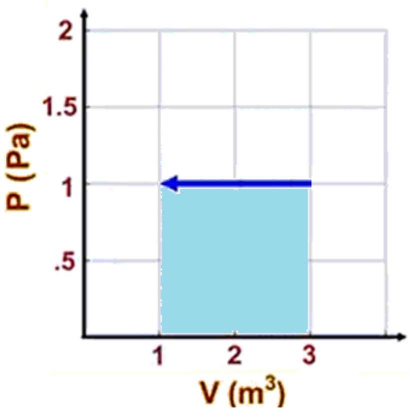
Therefore, if we plot a graph of pressure *vs.* volume with pressure on the *y*-axis and volume on the *x*-axis, the integral would therefore be represented by the area between the curve (pressure) and the *x*-axis.

This means that ***the work done by a thermodynamic change equals the area under a P-V graph***.*

* While the above explanation requires calculus, as stated earlier we will limit ourselves to areas that can be calculated using simple geometry equations. Note that some of these will result in situations that would not be realistically achievable in the "real world".

Use this space for summary and/or additional notes:

In the following example, suppose that a gas is compressed from 3 m^3 to 1 m^3 at a pressure of 1 Pa. (A pressure of 1 Pa is much smaller than you would encounter in any real problem; these numbers were chosen to keep the math simple.)



The pressure is P = 1 Pa, and the change in volume is $\Delta V = -2\ m^3$. Because pressure is constant, we can use $W = -P\Delta V = -(1)(-2) = +2\text{ J}$.

$P\Delta V$ is the area under the graph. Because it is a rectangular region, the area is the base of the rectangle times the height. The base is 2 m^3 and the height is 1 Pa, which gives an area of 2 J.

Note that the arrow showing the change points to the *left*, which indicates that the volume is *decreasing*. Because work must be put *into* the gas in order to compress it, this means that the work done ***on*** the gas will be positive.* This is where the negative sign comes from. $W = -P\Delta V$ means that:

- if work is done on the gas (work is positive), the gas is compressed and the change in volume is therefore negative.
- If work is done by the gas on the surroundings (work is negative), the gas expands and the change in volume is therefore positive.

We will look at the effects of changes in pressure *vs.* volume in four types of pressure-volume changes:

- isochoric (constant volume)
- isothermal (constant temperature)
- adiabatic (no heat loss)
- isobaric (constant pressure).

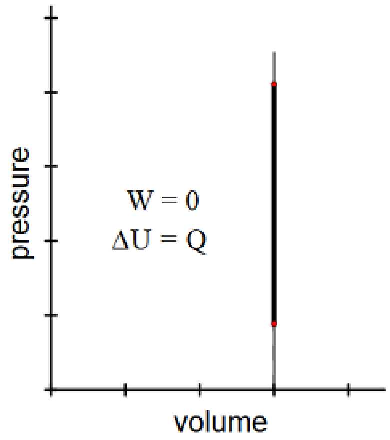
* Unless explicitly stated otherwise, positive work means work done ***on*** the gas, meaning that energy is added to the gas and the internal energy of the gas increases.

Use this space for summary and/or additional notes:

Isochoric

From Greek "iso" (same) and "khoros" (volume). An isochoric change is one in which volume remains constant, but pressure and temperature may vary.

An example is any rigid, closed container, such as a thermometer.



$$\frac{P_1 \cancel{V_1}}{T_1} = \frac{P_2 \cancel{V_2}}{T_2} \quad \rightarrow \quad \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Because the volume is not changing, there is no way for the gas to displace anything. (Recall from Physics 1 that $W = \vec{F} \bullet \vec{d}$.) If there is no displacement, there is no work, which means $W = 0$.

$$\Delta U = Q + \cancelto{0}{W} = \frac{3}{2} nR\Delta T$$

$$\Delta U = Q = \frac{3}{2} nR\Delta T$$

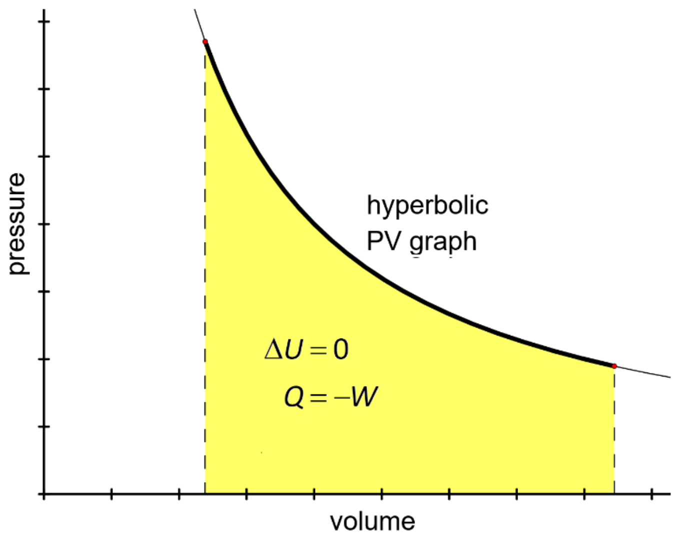
Another way to think of a constant volume change is that if you add heat to a rigid container of gas, none of the energy can be converted to work, so all of it must be converted to an increase in internal energy (*i.e.,* an increase in temperature).

Use this space for summary and/or additional notes:

Isothermal

Constant temperature. From Greek "iso" (same) and "thermotita" (heat). An isothermal change is one in which temperature remains constant, but pressure and volume may vary.

An example is any "slow" process, such as breathing out through a wide open mouth.



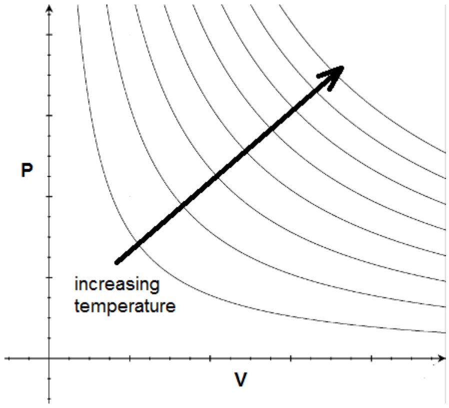
$$\frac{P_1 V_1}{\cancel{T_1}} = \frac{P_2 V_2}{\cancel{T_2}} \quad \rightarrow \quad P_1 V_1 = P_2 V_2$$

Because $\Delta T = 0$ (definition of isothermal), this means

$$\Delta U = \frac{3}{2} nR \cancelto{0}{\Delta T} = 0$$

Further, because:

$$\Delta U = Q + W = 0$$
$$Q = -W$$

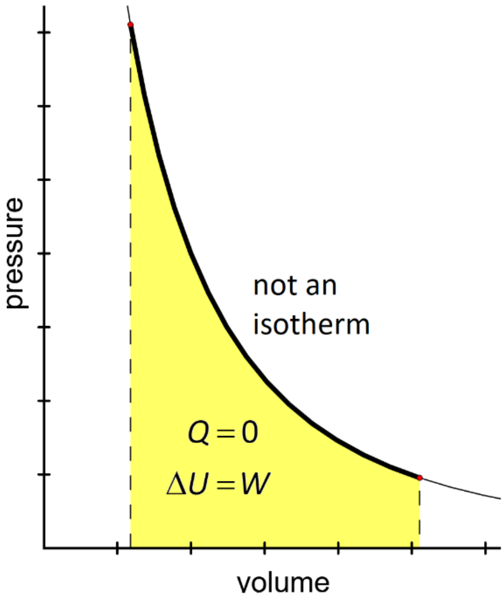


The P-V curve for an isothermal process is called an <u>isotherm</u>. Note that isotherms are hyperbolas, *i.e.,* they are solutions to the equation PV = constant. You may recall that allowing pressure and volume to change while keeping temperature constant is represented by Boyle's Law: $P_1 V_1 = P_2 V_2$

As temperature increases, the isotherm moves farther away from the origin.

Use this space for summary and/or additional notes:

Adiabatic



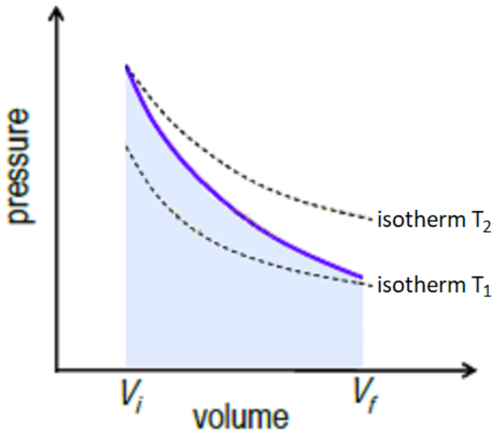
An adiabatic process is one in which there is no heat exchange with the environment. From Greek "a" (not) + "dia" (through) + "batos" (passable).

An example is any "fast" process, such as forcing air out through pursed lips or a bicycle tire pump.

Because the definition of an adiabatic process is one for which Q = 0, this means:

$$Q=0$$
$$\Delta U=\cancel{Q}^{\,0}+W=\tfrac{3}{2}nR\Delta T$$
$$\Delta U=W=\tfrac{3}{2}nR\Delta T$$

Note that adiabatic expansion (sudden increase in volume without time for heat transfer) results in a decrease in temperature, and adiabatic compression (sudden decrease in volume without time for heat transfer) results in an increase in temperature.

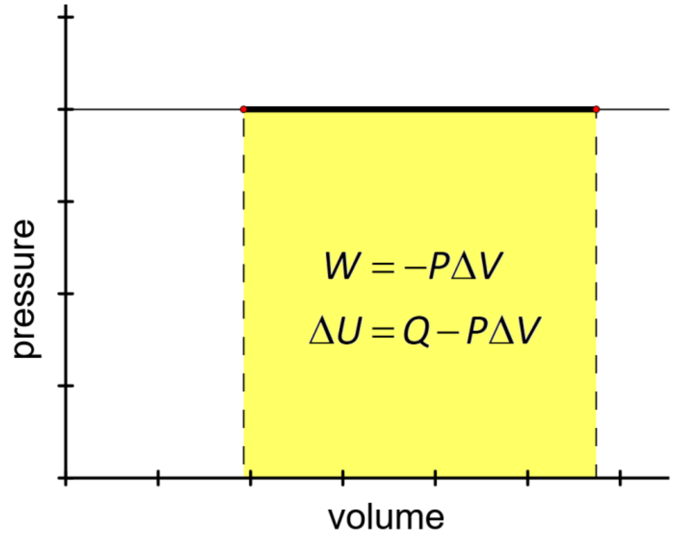


Use this space for summary and/or additional notes:

Isobaric

From Greek "iso" (same) and "baros" (weight). An isobaric change is one in which pressure remains constant, but volume and temperature may vary.

Some examples include a weighted piston, a flexible container in earth's atmosphere, or a hot air balloon.



$$\frac{\cancel{P_1} V_1}{T_1} = \frac{\cancel{P_2} V_2}{T_2} \quad \rightarrow \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Isobaric changes involve changes in both Q and W, because to change the volume of a gas while keeping pressure constant, you need to add or remove heat, but the resulting change in volume means that work is being done.

$$W = -\Delta(PV) = -P\Delta V$$
$$\Delta U = Q + W$$
$$\Delta U = Q + (-P\Delta V)$$
$$\Delta U = Q - P\Delta V$$

Adding $P\Delta V$ to both sides gives $Q = \Delta U + P\Delta V$.

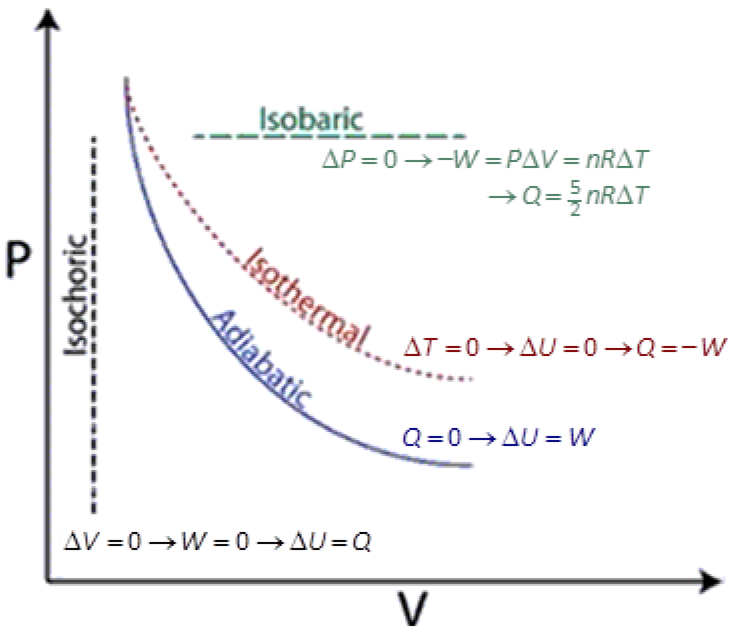
Now, because $\Delta U = \frac{3}{2} nR\Delta T$ and $P\Delta V = nR\Delta T = \frac{2}{2} nR\Delta T$, that means:

$$Q = \Delta U + P\Delta V$$
$$Q = \frac{3}{2} nR\Delta T + \frac{2}{2} nR\Delta T$$
$$Q = \frac{5}{2} nR\Delta T$$

This makes sense, because some of the heat is used to do the work of expanding the gas $(P\Delta V = nR\Delta T)$, and some of the heat is used to increase the temperature. $(\Delta U = \frac{3}{2} nR\Delta T)$.

Use this space for summary and/or additional notes:

If we wanted to compare all four processes on the same PV diagram, they would look like this:



Positive *vs.* Negative Work

In thermodynamics problems, whether work is represented by a positive or negative number depends on how the problem is stated.

Work done ***on*** the gas: a positive number means work is coming from the surroundings into the gas.

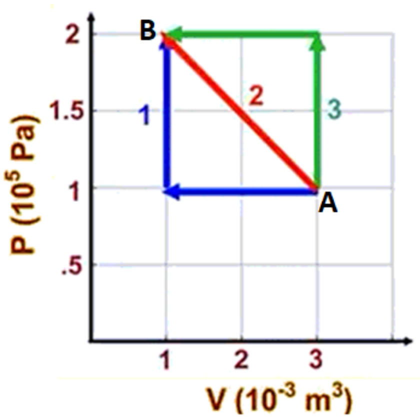
Work done ***by*** the gas: a positive number means work is going from the gas out to the surroundings.

If the problem does not specify otherwise, the convention is to use a positive number to indicate work done on (*i.e.,* going into) the gas.

Use this space for summary and/or additional notes:

Sample Problem

Q: Calculate the work done as the pressure and volume of a gas are taken from point A to point B along each of paths 1, 2, and 3.



A: Process #1 is first isobaric (constant pressure), then isochoric (constant volume).

For the isobaric part of the process:

$$W = -P\Delta V$$
$$W = -(1\times10^{5})(1\times10^{-3} - 3\times10^{-3})$$
$$W = -(1\times10^{5})(-2\times10^{-3})$$
$$W = 2\times10^{2} = 200\,\text{J}$$

For the isochoric process, there is no change in volume, which means the gas does no work (because it cannot push against anything). Therefore $W = 0$.

The total work for process #1 is therefore 200 J.

Notice that the work is equal to the area under the PV graph, which is a rectangular area with a base of 2×10^{-3} m^3 and a height of 1×10^5 Pa.

$$W = (1\times10^{5})(2\times10^{-3}) = 200\text{ J}$$

Note that because the arrow points to the left, this means the *volume is decreasing*. That means *work is being done on the gas*, which means the *work is represented by a positive number*. (We have to make this determination any time we use the graph to calculate the work.)

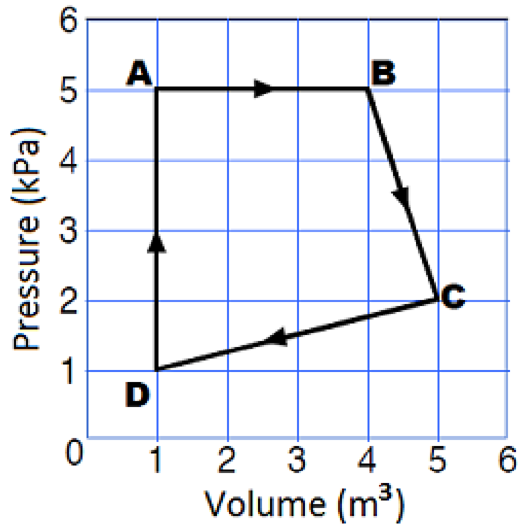
For process #2, the area is the 200 J square that we calculated for process #1 plus the area of the triangle above it, which is $\frac{1}{2}bh = \frac{1}{2}(2\times10^{-3})(1\times10^{5}) = 100\text{ J}$. Therefore, 200 J + 100 J = 300 J.

For process #3, the area under the curve is $W = (2\times10^{5})(2\times10^{-3}) = 400\text{ J}$.

Use this space for summary and/or additional notes:

Homework Problems*

Problems #1–8 refer to the following PV diagram, in which 2 moles of gas undergo the pressure and volume changes represented by the path from point A to B to C to D and back to A.



1. **(M)** Which thermodynamic process takes place along the path from point A to point B?

2. **(M)** Which thermodynamic process takes place along the path from point D to point A?

3. **(M)** How much work is done ***by the gas*** as it undergoes a change along the curve from point B → C? (Remember to use a positive number for work done on the gas by the surroundings, and a negative number for work done by the gas on the surroundings.)

   Answer: +3500 J

* These problems are from a worksheet by Tony Wayne.

Use this space for summary and/or additional notes:

4. **(S)** How much work is done ***on** the gas* as it undergoes a change along the curve from point C → D?

   Answer: +6000 J

5. **(S)** How much net work is done ***by** the gas* on the surroundings as it undergoes a change along the curve from point A → B → C → D → A?

   Answer: +12 500 J

6. **(S)** What is the temperature of the 2 moles of gas at point A?

   Answer: 300.8 K

7. **(M)** What is the change in internal energy of the gas during the process from point D → A?

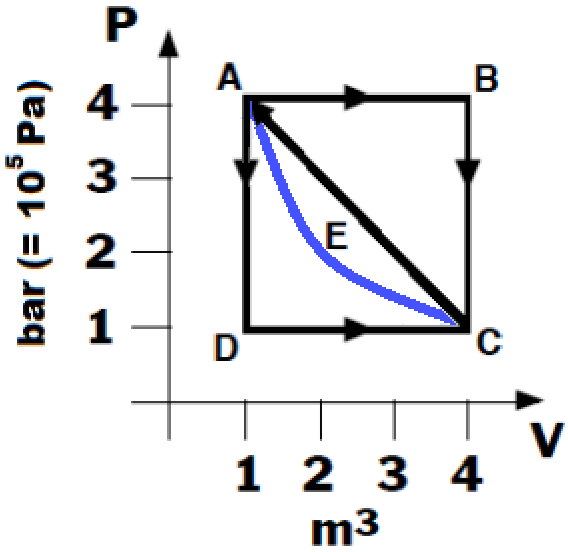
   Answer: 6000 J

8. **(M)** How much work is done on or by the gas during the process from point D → A?

   Answer: zero

Use this space for summary and/or additional notes:

Problems #9–13 refer to the following diagram:



9. **(S)** For which process(es) is $Q = \frac{5}{2}nR\Delta T$? Show calculations to justify your answer.

   Answer: A → B and D → C

10. **(M)** For which process(es) is no work done? Explain.

    Answer: A → D and B → C

Use this space for summary and/or additional notes:

11. **(M)** Which thermodynamic process takes place along path E?

12. **(M)** Calculate the heat exchanged in process A → B? Is heat added or released? Explain.

Answer: $3\times10^{6}\text{ J}$; heat is added because the temperature increases.

13. **(M)** Does path A → D → C → E → A require more or less work than path A → D → C → A? Explain.

14. **(S)** Calculate the work done ***by the gas*** in processes A → B → C → A and A → D → C → A.

Answer: A → B → C → A: 450 000 J
A → D → C → A: −450 000 J

Use this space for summary and/or additional notes:

Heat Engines

Unit: Thermodynamics

MA Curriculum Frameworks (2016): HS-PS2-6

AP® Physics 2 Learning Objectives: 5.B.5.4, 5.B.5.5, 5.B.7.3, 7.B.2.1

Mastery Objective(s): (Students will be able to…)

- Calculate the energy produced or used by a heat engine.

Success Criteria:

- Correct equation is chosen.
- Solutions have the correct quantities substituted for the correct variables.
- Sign is correct for work (positive *vs.* negative).
- Algebra is correct.

Language Objectives:

- Explain what is happening to a gas through each of the steps of a heat engine cycle.

Tier 2 Vocabulary: internal, energy, heat, engine, work

Labs, Activities & Demonstrations:

- Stirling engine

Notes:

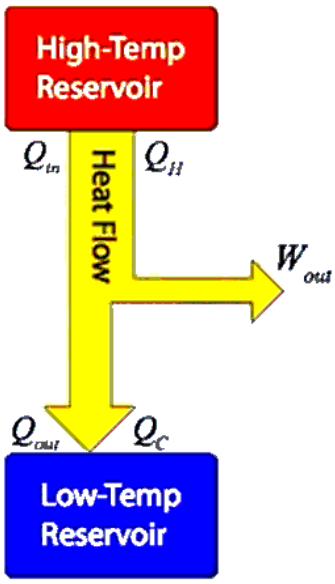
heat engine: a device that turns heat energy into mechanical work.

A heat engine operates by taking heat from a hot place (heat source), converting some of that heat into work, and dumping the rest of the heat into a cooler reservoir (heat sink).

A large number of the machines we use—most notably cars—employ heat engines.

Use this space for summary and/or additional notes:

The basic principle of heat engine is the first law of thermodynamics (heat flows from a region of higher temperature to a region of lower temperature). Because heat is a form of energy, some of that energy can be harnessed to do work.

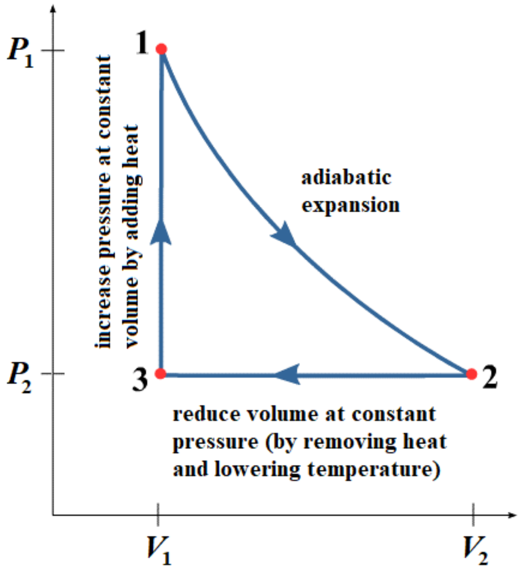


The law of conservation of energy tells us that all of the energy that we put into the heat engine must go somewhere. Therefore, the work done plus the heat that comes out must equal the heat we put in.

This means:

$$Q_{in} = Q_{out} + W_{out}$$

The above picture is easier to understand in the context of a PV diagram.



1. Starting from point 1 (V_1, P_1), the gas is expanded adiabatically (without losing heat) to point 2 (V_2, P_2). The area under the graph from point 1–2 represents the work that is done by the gas (W_{out}) as it expands and cools.

2. Ultimately, we need to get the gas back to point 1 so we can begin the cycle over again, but in a way that costs less work than we got out. There are many ways to accomplish this. In this example, the next step is to reduce the volume isobarically, by removing heat (Q_{out}). (This is what the low temperature reservoir is for.) This results in a decrease in temperature as well as a decrease in volume, which gets us to point 3 (V_1, P_2).

3. Now we increase the pressure to get from point 3 (V_1, P_2) to point 1 (V_1, P_1) by adding heat to the gas without letting the volume change. We do this by bringing the gas back to the high temperature reservoir so it can absorb the heat (Q_{in}).

Use this space for summary and/or additional notes:

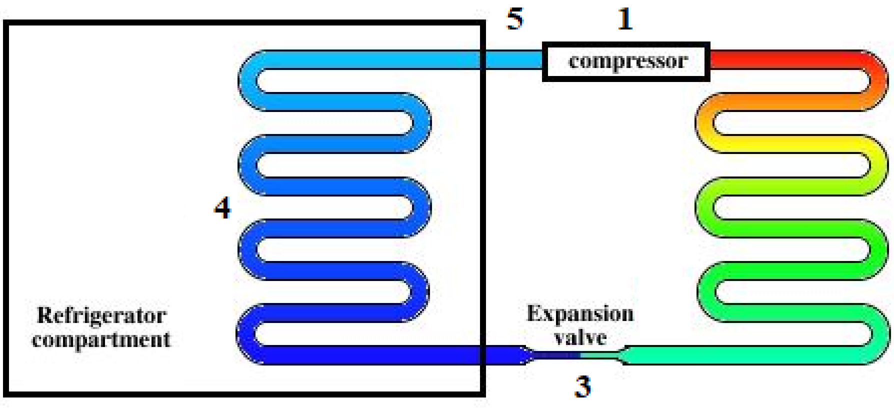
Heat Pumps

A heat pump is a device, similar to a heat engine, that "pumps" heat from one place to another. A refrigerator and an air conditioner are examples of heat pumps. A refrigerator uses a fluid ("refrigerant") to transfer (or "pump") heat from the inside of the refrigerator to the outside of the refrigerator (and into your kitchen).

This is why you can't cool off the kitchen by leaving the refrigerator door open—even if you had a 100 % efficient refrigerator (most refrigerators are actually only 20–40 % efficient), all of the heat that you pumped out of the refrigerator is still in the kitchen!

A refrigerator works by the following process.

1. Work is put in to compress a refrigerant (gas). In most cases, the gas is compressed until it turns into a liquid, which means additional energy is stored in the phase change. This increases the temperature of the refrigerant to about 70 °C.
2. The refrigerant (now a liquid) passes through cooling coils on the back of the refrigerator. The liquid is cooled through convection by the air in the kitchen to about 25 °C
3. The refrigerant (still a liquid) is pumped to the inside of the refrigerator and allowed to expand to a gas adiabatically. Work comes out of the gas, and the temperature drops to about −20 °C.
4. Heat is transferred via convection from the contents (the food) to the refrigerant.
5. The refrigerant (still a gas) is pumped out of the refrigerator, which brings us back to step 1.

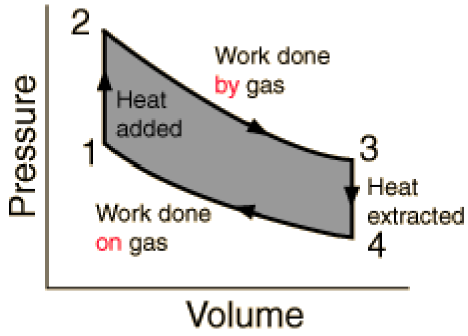


A heater can operate under the same principle, by putting the cooling coils inside the room and having the expansion (which cools the refrigerant) occur outside. Individual rooms in homes are sometimes heated and cooled by reversible heat pumps called "mini-splits".

Use this space for summary and/or additional notes:

Heat Engines and PV Diagrams

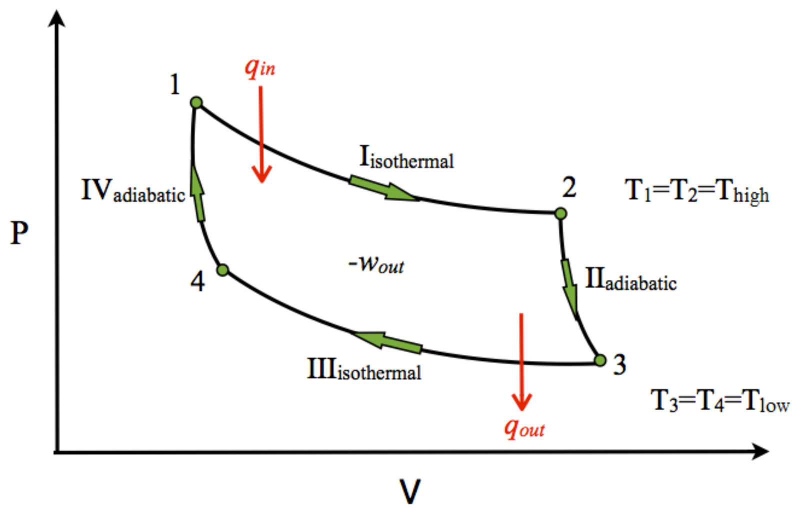
On a PV diagram, a heat engine is any closed loop or cycle:



Recall that on a PV diagram, a curve that moves from left to right represents work done *by* the gas on the surroundings. (Work is leaving the system, so $\Delta W < 0$.) A curve that moves from right to left represents work done *on* the gas by the surroundings. (Work is entering the system so $\Delta W > 0$.)

A heat engine is a clockwise cycle, which means more work is done going to the right than to the left, which means there is a net flow of work out of the system (*i.e.,* the heat is being used to do work). A refrigerator is a counterclockwise cycle, in which more work is put in and more heat is taken out.

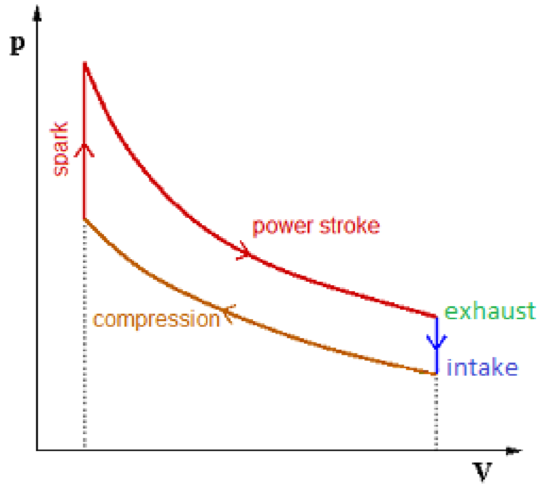
The Carnot cycle, named after the French physicist Nicolas Carnot, is the most efficient type of heat engine. The Carnot cycle, which uses only adiabatic (no heat loss) and isothermal processes, is the basis for heat pumps (including refrigerators and air conditioners) :



Use this space for summary and/or additional notes:

The internal combustion engine in a car is also a type of heat engine. The engine is called a "four-stroke" engine, because the piston makes four strokes (a back or forth motion) in one complete cycle. The four strokes are:

1. The piston moves down (intake), sucking a mixture of gasoline and air into the cylinder.
2. The piston moves up (compression), compressing the gases in the cylinder.
3. The spark plug creates a spark, which combusts the gases. This increases the temperature in the cylinder to approximately 250°C.
4. The gas expands (power stroke), which is the work that the engine provides to make the car go.
5. The piston raises again, forcing the exhaust gases out of the cylinder (exhaust).



Note that, at the end of the cycle, the gas is hotter than its original temperature. The hot gas from the cylinder is dumped out the exhaust pipe, and fresh (cool) gas and fuel is added. This is why the blue intake arrow on the right moves downward—the intake is at a lower temperature (lower isotherm).

The energy to move the piston for the intake and exhaust strokes is provided by the power strokes of the other pistons.

This cycle—constant temperature compression, constant volume heating (spark), constant temperature expansion (power), and constant volume gas exchange (exhaust) is called the Otto cycle, named after Nikolaus August Otto, who used this type of heat engine to build the first commercially successful internal combustion engine.

Use this space for summary and/or additional notes:

Efficiency

Unit: Thermodynamics

MA Curriculum Frameworks (2016): HS-PS2-6

AP® Physics 2 Learning Objectives: 5.B.5.4, 5.B.5.5, 5.B.7.3, 7.B.2.1

Mastery Objective(s): (Students will be able to…)

- Calculate the efficiency of a thermodynamic process.

Success Criteria:

- Correct equation is chosen.
- Solutions have the correct quantities substituted for the correct variables.
- Sign is correct for work (positive *vs.* negative).
- Algebra is correct.

Language Objectives:

- Explain how the efficiency of a process relates to the energy it uses and the work it produces.

Tier 2 Vocabulary: energy, heat, work

Notes:

efficiency (η): the ratio of the energy consumed by a device or process to the energy output by the device or process.

Assume that a heat engine starts with a certain temperature, which means a certain internal energy (U). The engine takes heat from a heat source at the incoming temperature T_{in}, does work (W), and exhausts heat at the higher temperature T_{out}. Assuming the internal energy of the machine itself stays constant, this means $\Delta U = 0$. Therefore, from the First law:

$$\Delta U = 0 = \Delta Q - \Delta W$$
$$0 = Q_{in} - Q_{out} - \Delta W$$
$$\Delta W = Q_{in} - Q_{out}$$

A 100% efficient heat engine would turn all of the heat into work, and would exhaust no heat ($Q_{out} = 0$, which would mean $\Delta W = Q_{in}$). Of course, real engines cannot do this, so we define efficiency, *e*, as the ratio of work out to heat in, *i.e.*:

$$e = \frac{\Delta W}{\Delta Q_{in}} = \frac{\Delta Q_{in} - \Delta Q_{out}}{\Delta Q_{in}} = \frac{\Delta Q_{in}}{\Delta Q_{in}} - \frac{\Delta Q_{out}}{\Delta Q_{in}} = 1 - \frac{\Delta Q_{out}}{\Delta Q_{in}}$$

Because the engine is doing work, $\Delta W > 0$, which means $0 \le e \le 1$. Furthermore, a consequence of the Second law is that some energy is always lost to the surroundings (entropy), which means $Q_{out} > 0$ and therefore $0 \le e < 1$.

Use this space for summary and/or additional notes:

Sample Problem

Q: 80. J of heat is injected into a heat engine, causing it to do work. The engine then exhausts 20. J of heat into a cool reservoir. What is the efficiency of the engine?

A: $Q_{in} = 80\,\text{J}$ and $Q_{out} = 20\,\text{J}$. Therefore:

$$e = 1 - \frac{\Delta Q_{out}}{\Delta Q_{in}} = 1 - \frac{20}{80} = 1 - 0.25 = 0.75$$

Because efficiency is usually expressed as a percentage, we would say that the engine is 75% efficient.

The following table gives energy conversion efficiencies for common devices and processes. In all of these cases, the "lost" energy is converted to heat that is given off to the surroundings.

Energy Conversion Efficiency

Device/Process	Typical Efficiency
gas generator	up to 40%
coal/gas-fired power plant	45%
combined cycle power plant	60%
hydroelectric power plant	up to 90%
wind turbine	up to 59%
solar cell	6–40%; usually 15%
hydrogen fuel cell	up to 85%
internal combustion engine	25%
electric motor, small (10–200 W)	50–90%
electric motor, large (> 200 W)	70–99%
photosynthesis in plants	up to 6%
human muscle	14–27%
refrigerator	20%
refrigerator, energy-saving	40–50%
light bulb, incandescent	0.7–5%
light bulb, fluorescent	8–16%
light bulb, LED	4–15%
electric heater	100%
firearm	30%

Use this space for summary and/or additional notes:

You may notice that an electric heater is 100% efficient, because all of its energy is converted to heat. However, this does not mean that electric heat is necessarily the best choice for your home, because the power plant that generated the electricity is probably only 45% efficient.

Heating Efficiency

Heating efficiency is calculated in a similar way. The difference is that the energy produced by the heater *is* Q_{out}, which means:

$$\eta = \frac{Q_{out}}{Q_{in}} = \frac{\text{usable heat out}}{\text{total energy in}}$$

"Usable heat out" means heat that is not lost to the environment. For example, if the boiler or furnace in your house is 70% efficient, that means 70% of the energy from the gas or oil that it burned was used to heat the steam, hot water or hot air that was used to heat your house. The other 30% of the energy heated the air in the boiler or furnace, and that heat was lost to the surroundings when the hot air went up the chimney.

Older boilers and furnaces (pre-1990s) were typically 70% efficient. Newer boilers and furnaces are around 80% efficient, and high-efficiency boilers and furnaces that use heat exchangers to collect the heat from the exhaust air before it goes up the chimney can be 90−97% efficient.

Efficiency of a Heat Pump

Carnot's theorem states that the maximum possible efficiency of a heat pump is related to the ratio of the temperature of the heat transfer fluid (liquid or gas) when it enters the heat pump to the temperature when it exits:

$$\eta \le 1 - \frac{T_{in}}{T_{out}}$$

Note that the Carnot equation is really the same as the efficiency equation in the previous section. Recall that for heating or cooling a substance:

$$Q = mC\Delta T = mC(T_{out} - T_{in})$$

The refrigerant is the same substance, which means mC is the same for the input as for the output, and it drops out of the equation.

Use this space for summary and/or additional notes:

It is a little counter-intuitive that a higher temperature difference means the heat pump is more efficient, but you should think about this in terms of heat transfer. (See the section on Heat Transfer starting on page 190.) Recall from Fourier's Law of Conduction that a higher temperature difference means a higher rate of heat transfer from one side to the other. In other words, the more heat you pump into the refrigerant, the higher its temperature will be when it leaves the system, and therefore the more efficiently the pump is moving heat. Conversely, if $T_{out} = T_{in}$, then the heat pump cannot transfer any heat, and the efficiency is zero.

Sample Problem

Q: Refrigerant enters a heat pump at 20. °C (293 K) and exits at 300. °C (573 K). What is the Carnot efficiency of this heat pump?

A: Carnot's equation states that:

$$\eta = 1 - \frac{T_{in}}{T_{out}} = 1 - \frac{293}{573}$$
$$\eta = 1 - 0.51 = 0.49$$

I.e., this heat pump is 49% efficient.

Use this space for summary and/or additional notes:

Introduction: Electric Force, Field & Potential

Unit: Electric Force, Field & Potential

Topics covered in this chapter:

This chapter discusses static electric charges, how they behave, and how they relate to each other.

- *Electric Charge* and *Coulomb's Law* describe the behavior of individual charged particles and their effects on each other.
- *Electric Fields* describes the behavior of an electric force field on charged particles.
- *Electric Field Vectors* and *Equipotential Lines & Maps* describe ways of representing electric fields in two dimensions.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-4. Use mathematical representations of Newton's Law of Gravitation and Coulomb's Law to describe and predict the gravitational and electrostatic forces between objects.

HS-PS3-1. Use algebraic expressions and the principle of energy conservation to calculate the change in energy of one component of a system when the change in energy of the other component(s) of the system, as well as the total energy of the system including any energy entering or leaving the system, is known. Identify any transformations from one form of energy to another, including thermal, kinetic, gravitational, magnetic, or electrical energy, in the system.

HS-PS3-2. Develop and use a model to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles and objects or energy stored in fields.

Use this space for summary and/or additional notes:

HS-PS3-5. Develop and use a model of magnetic or electric fields to illustrate the forces and changes in energy between two magnetically or electrically charged objects changing relative position in a magnetic or electric field, respectively.

AP® only

AP® Physics 2 Learning Objectives:

1.B.1.1: The student is able to make claims about natural phenomena based on conservation of electric charge. [**SP 6.4**]

1.B.1.2: The student is able to make predictions, using the conservation of electric charge, about the sign and relative quantity of net charge of objects or systems after various charging processes, including conservation of charge in simple circuits. [**SP 6.4**, **7.2**]

1.B.2.2: The student is able to make a qualitative prediction about the distribution of positive and negative electric charges within neutral systems as they undergo various processes. [**SP 6.4**, **7.2**]

1.B.2.3: The student is able to challenge claims that polarization of electric charge or separation of charge must result in a net charge on the object. [**SP 6.1**]

1.B.3.1: The student is able to challenge the claim that an electric charge smaller than the elementary charge has been isolated. [**SP 1.5**, **6.1**, **7.2**]

2.C.1.1: The student is able to predict the direction and the magnitude of the force exerted on an object with an electric charge *q* placed in an electric field using the mathematical model of the relation between an electric force and an electric field: ; a vector relation.[**SP 6.4**, **7.2**] E⍰ EF ⍰ ⍰ q ⍰

2.C.1.2: The student is able to calculate any one of the variables — electric force, electric charge, and electric field — at a point given the values and sign or direction of the other two quantities.[**SP 2.2**]

2.C.2.1: The student is able to qualitatively and semi-quantitatively apply the vector relationship between the electric field and the net electric charge creating that field. [**SP 2.2**, **6.4**]

2.C.3.1: The student is able to explain the inverse square dependence of the electric field surrounding a spherically symmetric electrically charged object. [**SP 6.2**]

2.C.4.1: The student is able to distinguish the characteristics that differ between monopole fields (gravitational field of spherical mass and electrical field due to single point charge) and dipole fields (electric dipole field and magnetic field) and make claims about the spatial behavior of the fields using qualitative or semi-quantitative arguments based on vector addition of fields due to each point source, including identifying the locations and signs of sources from a vector diagram of the field. [**SP 2.2**, **6.4**, **7.2**]

Use this space for summary and/or additional notes:

AP® only

2.C.4.2: The student is able to apply mathematical routines to determine the magnitude and direction of the electric field at specified points in the vicinity of a small set (2–4) of point charges, and express the results in terms of magnitude and direction of the field in a visual representation by drawing field vectors of appropriate length and direction at the specified points. [**SP 1.4**, **2.2**]

2.C.5.1: The student is able to create representations of the magnitude and direction of the electric field at various distances (small compared to plate size) from two electrically charged plates of equal magnitude and opposite signs, and is able to recognize that the assumption of uniform field is not appropriate near edges of plates. [**SP 1.1**, **2.2**]

2.C.5.2: The student is able to calculate the magnitude and determine the direction of the electric field between two electrically charged parallel plates, given the charge of each plate, or the electric potential difference and plate separation. [**SP 2.2**]

2.C.5.3: The student is able to represent the motion of an electrically charged particle in the uniform field between two oppositely charged plates and express the connection of this motion to projectile motion of an object with mass in the Earth's gravitational field. [**SP 1.1**, **2.2**, **7.1**]

2.E.1.1: The student is able to construct or interpret visual representations of the isolines of equal gravitational potential energy per unit mass and refer to each line as a gravitational equipotential. [**SP 1.4**, **6.4**, **7.2**]

2.E.2.1: The student is able to determine the structure of isolines of electric potential by constructing them in a given electric field. [**SP 6.4**, **7.2**]

2.E.2.2: The student is able to predict the structure of isolines of electric potential by constructing them in a given electric field and make connections between these isolines and those found in a gravitational field. [**SP 6.4**, **7.2**]

2.E.2.3: The student is able to qualitatively use the concept of isolines to construct isolines of electric potential in an electric field and determine the effect of that field on electrically charged objects. [**SP 1.4**]

2.E.3.1: The student is able to apply mathematical routines to calculate the average value of the magnitude of the electric field in a region from a description of the electric potential in that region using the displacement along the line on which the difference in potential is evaluated. [**SP 2.2**]

2.E.3.2: The student is able to apply the concept of the isoline representation of electric potential for a given electric charge distribution to predict the average value of the electric field in the region. [**SP 1.4**, **6.4**]

3.A.2.1: The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. [**SP 1.1**]

Use this space for summary and/or additional notes:

AP® only

3.A.3.2: The student is able to challenge a claim that an object can exert a force on itself. [**SP 6.1**]

3.A.3.3: The student is able to describe a force as an interaction between two objects and identify both objects for any force. [**SP 1.4**]

3.A.3.4: The student is able to make claims about the force on an object due to the presence of other objects with the same property: mass, electric charge. [**SP 6.1**, **6.4**]

3.A.4.1: The student is able to construct explanations of physical situations involving the interaction of bodies using Newton's third law and the representation of action-reaction pairs of forces. [**SP 1.4**, **6.2**]

3.A.4.2: The student is able to use Newton's third law to make claims and predictions about the action-reaction pairs of forces when two objects interact. [**SP 6.4**, **7.2**]

3.A.4.3: The student is able to analyze situations involving interactions among several objects by using free-body diagrams that include the application of Newton's third law to identify forces. [**SP 1.4**]

3.B.1.3: The student is able to re-express a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. [**SP 1.5**, **2.2**]

3.B.1.4: The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations. [**SP 6.**4, **7.2**]

3.B.2.1: The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. [**SP 1.1**, **1.4**, **2.2**]

3.C.2.1: The student is able to use Coulomb's law qualitatively and quantitatively to make predictions about the interaction between two electric point charges. [**SP 2.2**, **6.4**]

3.C.2.2: The student is able to connect the concepts of gravitational force and electric force to compare similarities and differences between the forces. [**SP 7.2**]

3.C.2.3: The student is able to use mathematics to describe the electric force that results from the interaction of several separated point charges (generally 2–4 point charges, though more are permitted in situations of high symmetry). [**SP 2.2**]

3.G.1.2: The student is able to connect the strength of the gravitational force between two objects to the spatial scale of the situation and the masses of the objects involved and compare that strength to other types of forces. [**SP 7.1**]

Use this space for summary and/or additional notes:

AP® only

3.G.2.1: The student is able to connect the strength of electromagnetic forces with the spatial scale of the situation, the magnitude of the electric charges, and the motion of the electrically charged objects involved. [**SP 7.1**]

4.E.3.1: The student is able to make predictions about the redistribution of charge during charging by friction, conduction, and induction. [**SP 6.4**]

4.E.3.2: The student is able to make predictions about the redistribution of charge caused by the electric field due to other systems, resulting in charged or polarized objects. [**SP 6.4**, **7.2**]

4.E.3.3: The student is able to construct a representation of the distribution of fixed and mobile charge in insulators and conductors. [**SP 1.1**, **1.4**, **6.4**]

4.E.3.4: The student is able to construct a representation of the distribution of fixed and mobile charge in insulators and conductors that predicts charge distribution in processes involving induction or conduction. [**SP 1.1**, **1.4**, **6.4**]

4.E.3.5: The student is able to plan and/or analyze the results of experiments in which electric charge rearrangement occurs by electrostatic induction, or is able to refine a scientific question relating to such an experiment by identifying anomalies in a data set or procedure. [**SP 3.2**, **4.1**, **4.2**, **5.1**, **5.3**]

5.A.2.1: The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. [**SP 6.4**, **7.2**]

5.B.2.1: The student is able to calculate the expected behavior of a system using the object model (*i.e.,* by ignoring changes in internal structure) to analyze a situation. Then, when the model fails, the student can justify the use of conservation of energy principles to calculate the change in internal energy due to changes in internal structure because the object is actually a system. [**SP 1.4**, **2.1**]

5.C.2.1: The student is able to predict electric charges on objects within a system by application of the principle of charge conservation within a system. [**SP 6.4**]

5.C.2.2: The student is able to design a plan to collect data on the electrical charging of objects and electric charge induction on neutral objects and qualitatively analyze that data. [**SP 4.2**, **5.1**]

5.C.2.3: The student is able to justify the selection of data relevant to an investigation of the electrical charging of objects and electric charge induction on neutral objects. [**SP 4.1**]

Use this space for summary and/or additional notes:

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Electric Fields, Forces, and Potentials**, such as Coulomb's law, induced charge, field and potential of groups of point charges, and charged particles in electric fields

1. Electric Charge
2. Electric Force
3. Electric Field
4. Electric Potential
5. Conductors and Insulators

Skills learned & applied in this chapter:

- Working with isolines.

Use this space for summary and/or additional notes:

Electric Charge

Unit: Electric Force, Field & Potential

MA Curriculum Frameworks (2016): HS-PS3-5

AP® Physics 2 Learning Objectives: 1.B.1.1, 1.B.1.2, 1.B.2.2, 1.B.2.3, 1.B.3.1, 2.C.3.1, 2.C.4.1, 2.C.5.3, 3.A.3.2, 3.A.3.3, 3.A.3.4, 3.A.4.1, 3.A.4.2, 3.C.2.2, 4.E.3.1, 4.E.3.4, 4.E.3.5, 5.C.2.1, 5.C.2.2, 5.C.2.3

Mastery Objective(s): (Students will be able to…)

- Describe properties of positive and negative electric charges.
- Describe properties of conductors and insulators.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why the mass of the pendulum does not affect its period.

Tier 2 Vocabulary: charge

Labs, Activities & Demonstrations:

- charged balloon making hairs repel, attracting water molecules.
- charged balloon sticking to wall (draw on one side of balloon to show that charges do not move)
- charged balloon pulling meter stick
- build & demonstrate electroscope
- Wimshurst machine
- Van de Graaff generator

Notes:

charge:

1. A physical property of matter which causes it to experience a force when near other electrically charged matter. (Sometimes called "electric charge".) Measured in coulombs (C).
2. A single microscopic object (such as a proton or electron) that carries an electric charge. (Sometimes called a "point charge.) Denoted by the variable *q*.

Use this space for summary and/or additional notes:

3. The total amount of electric charge on a macroscopic object (caused by an accumulation of microscopic charged objects). Denoted by the variable *Q*.

4. (verb) To cause an object to acquire an electric charge.

<u>positive charge</u>: the charge of a proton. Originally defined as the charge left on a piece of glass when rubbed with silk. The glass becomes positively charged because the silk pulls electrons off the glass.

<u>negative charge</u>: the charge of an electron. Originally defined as the charge left on a piece of amber (or rubber) when rubbed with fur (or wool). The amber becomes negatively charged because the amber pulls the electrons off the fur.

<u>static electricity</u>: stationary electric charge, such as the charge left on silk or amber in the above definitions.

<u>elementary charge</u>: the magnitude (amount) of charge on one proton or one electron. One elementary charge equals 1.60×10^{-19} C. Because ordinary matter is made of protons and electrons, the amount of charge carried by any object must be an integer multiple of the elementary charge.

Note however that quarks, which protons and neutrons are made of, carry fractional charges; up-type quarks carry a charge of +⅔ of an elementary charge, and down-type quarks carry a charge of −⅓ of an elementary charge. A proton is made of two up quarks and one down quark and carries a charge of +1 elementary charge. A neutron is made of one up quark and two down quarks and carries no charge.

Use this space for summary and/or additional notes:

electric current (sometimes called electricity): the movement of electrons through a medium (substance) from one location to another. Note, however, that electric current is defined as the direction a *positively* charged particle would move. Thus electric current "flows" in the opposite direction from the actual electrons.

WE WERE GOING TO USE THE TIME MACHINE TO PREVENT THE ROBOT APOCALYPSE, BUT THE GUY WHO BUILT IT WAS AN ELECTRICAL ENGINEER.

Some Devices that Produce, Use or Store Charge

capacitor: a device that stores electric charge.

battery: a device that uses chemical reactions to produce an electric current.

generator: a device that converts mechanical energy (motion) into an electric current.

motor: a device that converts an electric current into mechanical energy.

Conductors *vs.* Insulators

conductor: a material that allows charges to move freely through it. Examples of conductors include metals and liquids with positive and negative ions dissolved in them (such as salt water). When charges are transferred to a conductor, the charges distribute themselves evenly throughout the substance.

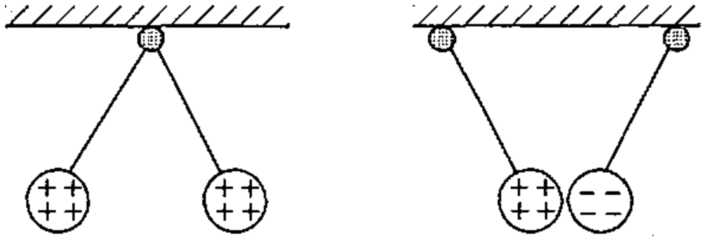
insulator: a material that does not allow charges to move freely through it. Examples of insulators include nonmetals and most pure chemical compounds (such as glass or plastic). When charges are transferred to an insulator, they cannot move, and remain where they are placed.

Use this space for summary and/or additional notes:

Behavior of Charged Particles

- **Like charges repel.** A pair of the same type of charge (two positive charges or two negative charges) exert a force that pushes the charges away from each other.

- **Opposite charges attract.** A pair of opposite types of charge (a positive charge and a negative charge) exert a force that pulls the charges toward each other.



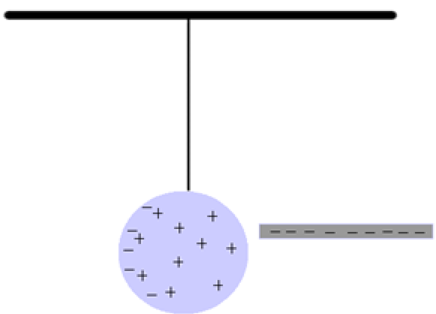
- **Charge is conserved.** Electric charges cannot be created or destroyed, but can be transferred from one location or medium to another. (This is analogous to the laws of conservation of mass and energy.)

Note that if you were to place a charge (either positive or negative) on a solid metal sphere, the charges would repel, and the result would be that the charges would be spread equally over the *outside* surface, but not inside the sphere.

Charging by Induction

induction: when an electrical charge on one object causes a charge in a second object.

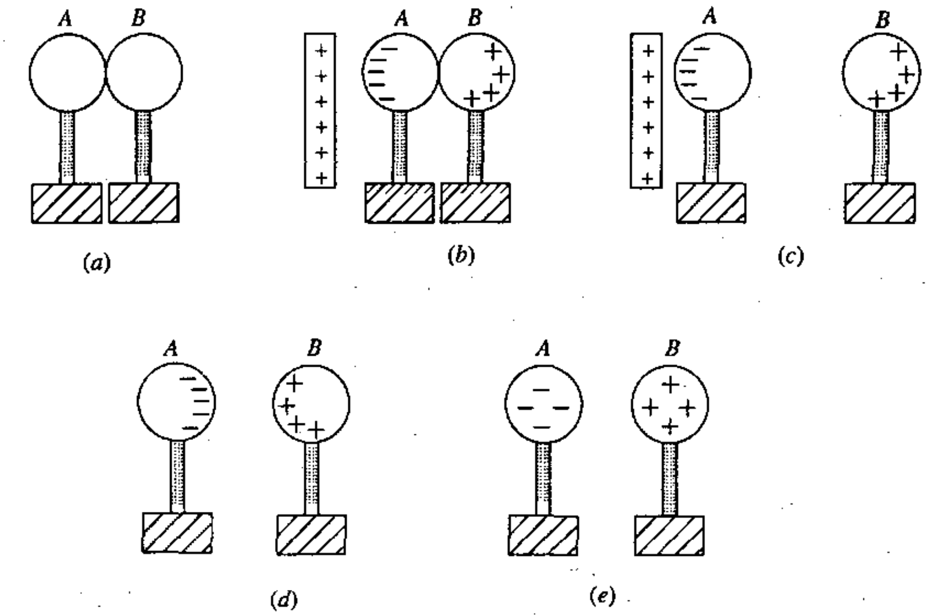
When a charged rod is brought near a neutral object, the charge on the rod attracts opposite charges and repels like charges that are near it. The diagram to the right shows a negatively-charged rod repelling negative charges.



If the negatively-charged rod were touched to the sphere, some of the charges from the rod would be transferred to the sphere at the point of contact. This would cause the sphere to have an overall negative charge.

Use this space for summary and/or additional notes:

A procedure for inducing charges in a pair of metal spheres is shown below:



(a) Metal spheres *A* and *B* are brought into contact.

(b) A positively charged object is placed near (but not in contact with) sphere *A*. This induces a negative charge in sphere *A*, which in turn induces a positive charge in sphere *B*.

(c) Sphere *B* (which is now positively charged) is moved away.

(d) The positively charged object is removed.

(e) The charges distribute themselves throughout the metal spheres.

Use this space for summary and/or additional notes:

honors (not AP®)

Charge Density

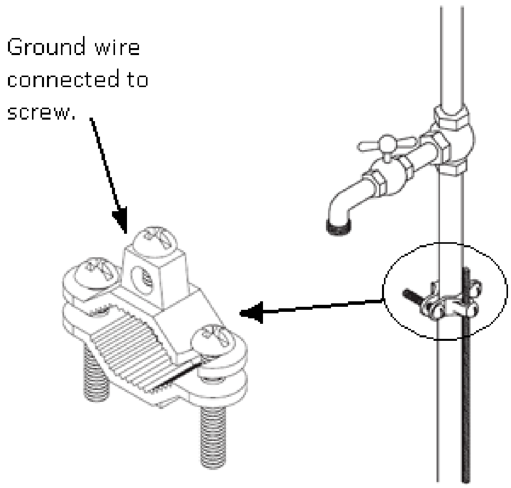
The amount of electric charge on a surface is called the charge density. As with density (in the mass/volume sense), the variable used is usually the Greek letter rho with a subscript *q* indicating charge (ρ_q). Charge density can be expressed in terms of length, area, or volume, which means, the units for charge density can be $\frac{\text{C}}{\text{m}}$, $\frac{\text{C}}{\text{m}^2}$, or $\frac{\text{C}}{\text{m}^3}$.

Grounding

For the purposes of our use of electric charges, the ground (Earth) is effectively an endless supply of both positive and negative charges. Under normal circumstances, if a charged object is touched to the ground, electrons will move to neutralize the charge, either by flowing from the object to the ground or from the ground to the object.

Grounding a charged object or circuit means neutralizing the electrical charge on an object or portion of the circuit. *The charge of any object that is connected to ground is zero, by definition.*

In buildings, the metal pipes that bring water into the building are often used to ground the electrical circuits. The metal pipe is a good conductor of electricity, and carries the unwanted charge out of the building and into the ground outside.



Use this space for summary and/or additional notes:

Coulomb's Law

Unit: Electric Force, Field & Potential

MA Curriculum Frameworks (2016): HS-PS2-4

AP® Physics 2 Learning Objectives: 3.A.3.4, 3.C.2.1, 3.C.2.2, 3.C.2.3, 3.G.1.2, 3.G.2.1

Mastery Objective(s): (Students will be able to…)

- Solve problems using Coulomb's Law
- Quantitatively predict the effects on the electrostatic force when one of the variables (amount of electric charge or distance) in Coulomb's Law is changed.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how force and distance both affect the amount of force between two charged objects.

Tier 2 Vocabulary: charge

Labs, Activities & Demonstrations:

- Charged balloon or Styrofoam sticking to wall.
- Charged balloon pushing meter stick.
- Van de Graaff generator with negative electrode attached to inertia balance pan.

Notes:

Electric charge is measured in Coulombs (abbreviation "C"). One Coulomb is the amount of electric charge transferred by a current of 1 ampere for a duration of 1 second.

+1 C is the charge of 6.2415×10^{18} protons.
−1 C is the charge of 6.2415×10^{18} electrons.

A single proton or electron therefore has a charge of $\pm 1.6022 \times 10^{-19}$ C. This amount of charge is called the elementary charge, because it is the charge of one elementary particle.

Use this space for summary and/or additional notes:

An object can only have an integer multiple of this amount of charge, because it is impossible* to have a charge that is a fraction of a proton or electron.

Because charged particles attract or repel each other, that attraction or repulsion must be a force, which can be measured and quantified. The force is directly proportional to the strengths of the charges, and inversely proportional to the square of the distance. The formula is:

$$F_e = \frac{kq_1q_2}{r^2}$$

where:

F_e = electrostatic force of repulsion between electric charges. A positive value of F_e denotes that the charges are repelling (pushing away from) each other; a negative value of F_e denotes that the charges are attracting (pulling towards) each other.

k = electrostatic constant = $9.0\times10^9\ \frac{\text{N·m}^2}{\text{C}^2}$.

q_1 , q_2 = the charges on objects #1 and #2 respectively

r = distance (radius, because it goes outward in every direction) between the centers of the two charges

This formula is Coulomb's Law, named for its discoverer, the French physicist Charles-Augustin de Coulomb.

Sample problems:

Q: Find the force of electrostatic attraction between the proton and electron in a hydrogen atom if the radius of the atom is 37.1 pm

A: The charge of a single proton is 1.60×10^{-19} C, and the charge of a single electron is -1.60×10^{-19} C.

$$37.1\,\text{pm} = 3.71\times10^{-11}\text{m}$$

$$F_e = \frac{kq_1q_2}{r^2} = \frac{(8.99\times10^9)(1.60\times10^{-19})(-1.60\times10^{-19})}{(3.71\times10^{-11})^2} = -1.67\times10^{-7}\,\text{N}$$

The value of the force is negative, which signifies that the force is attractive. However, rather than memorize whether a positive or negative indicates attraction or repulsion, it's easier to reason that the charges are opposite, so the objects attract. *Never memorize what you can understand!*

* This is true for macroscopic objects. Certain quarks, which are the particles that protons and neutrons are made of, have charges of $\frac{1}{3}$ or $\frac{2}{3}$ of an elementary charge.

Use this space for summary and/or additional notes:

Q: Two charged particles, each with charge $+q$ (which means $q_1 = q_2 = q$) are separated by distance d. If the amount of charge on one of the particles is halved and the distance is doubled, what will be the effect on the force between them?

A: To solve this problem, we first set up Coulomb's Law:

$$F_e = \frac{kq_1q_2}{r^2}$$

Now, we replace one of the charges with half of itself—let's say q_1 will become $(0.5\ q_1)$. Similarly, we replace the distance r with $(2r)$. This gives:

$$F_e = \frac{k(0.5q_1)q_2}{(2r)^2}$$

Simplifying and rearranging this expression gives:

$$F_e = \frac{0.5kq_1q_2}{4r^2} = \frac{0.5}{4} \cdot \frac{kq_1q_2}{r^2} = \frac{1}{8} \cdot \frac{kq_1q_2}{r^2}$$

Therefore, the new F_e will be $\frac{1}{8}$ of the old F_e.

An easier way to solve this problem is to do a "before and after" calculation. Set the value of every quantity in the "before" equation to 1:

$$F_e = \frac{kq_1q_2}{r^2} \quad \rightarrow \quad \frac{1 \cdot 1 \cdot 1}{1^2} = 1$$

For the "after" equation, replace quantities that change with their multipliers:

$$F_e = \frac{kq_1q_2}{r^2} \quad \rightarrow \quad \frac{1 \cdot 1 \cdot 0.5}{2^2} = \frac{0.5}{4} = \frac{1}{8}$$

The "before" value for F_e was 1, and the "after" value was $\frac{1}{8}$, which means the new force will be $\frac{1}{8}$ of the original force.

Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** What is the magnitude of the electric force between two objects, each with a charge of $+2.00\times10^{-6}$ C, which are separated by a distance of 1.50 m? Is the force attractive or repulsive?

   Answer: 0.016 N, repulsive

2. **(M)** An object with a charge of $+q_1$ is separated from a second object with an unknown charge by a distance *r*. If the objects attract each other with a force *F*, what is the charge on the second object?
   (*If you are not sure how to do this problem, do #3 below and use the steps to guide your algebra.*)

   Answer: $q_2 = -\frac{Fr^2}{kq_1}$

3. **(S)** An object with a charge of $+1.50\times10^{-2}$ C is separated from a second object with an unknown charge by a distance of 0.500 m. If the objects attract each other with a force of 1.35×10^{6} N, what is the charge on the second object?
   (*You must start with the equations in your* Physics Reference Tables. *You may only use the answer to question #2 above as a starting point if you have already solved that problem.*)

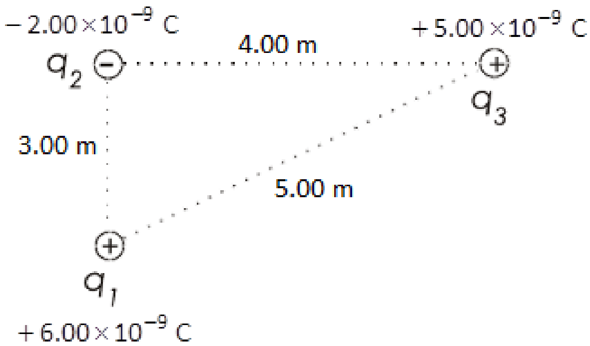
   Answer: -2.50×10^{-3} C

Use this space for summary and/or additional notes:

4. **(M)** The distance between an alpha particle (+2 elementary charges) and an electron (−1 elementary charge) is 2.00×10^{-25} m. If that distance is tripled, what will be the effect on the force between the charges?

Answer: The new F_e will be $\frac{1}{9}$ of the old F_e.

5. **(A)** Three elementary charges, particle q_1 with a charge of $+6.00\times10^{-9}$ C, particle q_2 with a charge of -2.00×10^{-9} C, and particle q_3 with a charge of $+5.00\times10^{-9}$ C, are arranged as shown in the diagram below.



What is the net force (magnitude and direction) on particle q_3?
(*Hint: this is a forces-at-an-angle problem like you saw in Physics 1.*)

Answer: 7.16×10^{-9} N at an angle of 65.2° above the *x*-axis.

Use this space for summary and/or additional notes:

Electric Fields

Unit: Electric Force, Field & Potential

MA Curriculum Frameworks (2016): HS-PS3-1, HS-PS3-2, HS-PS3-5

AP® Physics 2 Learning Objectives: 2.C.1.1, 2.C.1.2, 2.C.2.1, 2.C.3.1, 2.C.4.1, 2.C.4.2, 2.C.5.1, 2.C.5.2, 2.C.5.3, 3.A.3.4

Mastery Objective(s): (Students will be able to…)

- Sketch electric field lines and vectors around charged particles or objects.
- Solve problems involving the forces on a charge due to an electric field.

Success Criteria:

- Sketches show arrows pointing from positive charges to negative charges.
- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how the electric force on a charged particle changes as you get closer to or farther away from another charged object.

Tier 2 Vocabulary: charge, field

Labs, Activities & Demonstrations:

- students holding copper pipe in one hand and zinc-coated steel pipe in other—measure with voltmeter. (Can chain students together.)

Notes:

force field: a region in which an object experiences a force because of some intrinsic property of the object that enables the force to act on it. Force fields are vectors, which means they have both a magnitude and a direction.

electric field ($\vec{E}$) : an electrically charged region (force field) that exerts a force on any charged particle within the region.

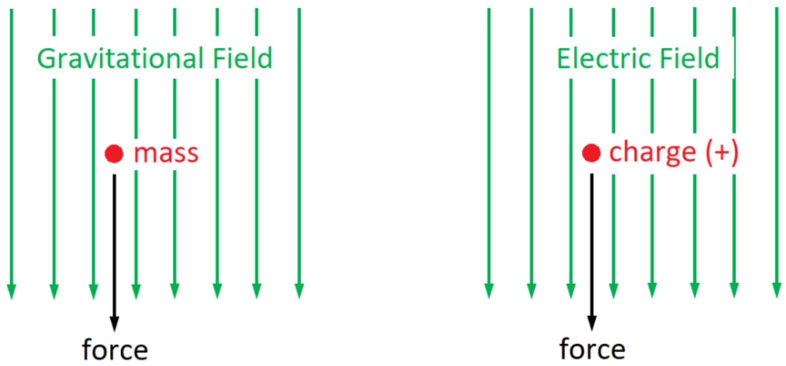
An electric field applies a force to an object based on its electrical charge. $\vec{F}_e = q\vec{E}$, where $\vec{E}$ represents the magnitude and direction of the electric field.

Use this space for summary and/or additional notes:

Because gravity is a familiar concept, it is useful to use gravitational fields as a way to explain force fields, and thus electric fields.

Recall that a gravitational field applies a force to an object based on its mass. $\vec{F}_g = m\vec{g}$, where $\vec{g}$ represents the magnitude and direction of the gravitational field.

Just as a gravitational field applies a force to an object that has mass, an electric field applies a force to an object that has charge:



A key difference between the two situations is that there are two kinds of charges—positive and negative—whereas there is only one kind of mass.

The force on an object with mass is always in the direction of the gravitational field. However, the direction of the force on an object with charge depends on whether the charge is positive or negative. *The force on an object with **positive charge** is in the **same direction** as the electric field; the force on an object with **negative charge** is always in the **opposite direction** from the electric field.*

For any force field, the amount of force is the amount of the quantity that the field acts on times the strength of the field:

$$\vec{F}_g = m \vec{g}$$

force	amount of quantity that the field acts on	strength of field

$$\vec{F}_e = q \vec{E}$$

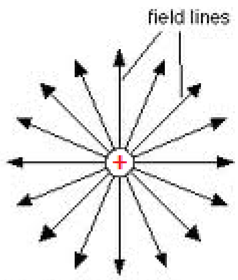
Use this space for summary and/or additional notes:

field lines: lines with arrows that show the direction of an electric field. In the above diagrams, the arrows are the field lines.

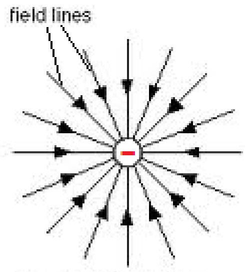
Field lines are lines that show the directions of force on an object. As described above, for an electric field, the object is assumed to be a positively-charged particle. This means that *the direction of the electric field is from positive to negative.* This means that field lines go outward in all directions from a positively-charged particle, and inward from all directions toward a negatively-charged particle.

This means that a positively-charged particle (such as a proton) would move in the direction of the arrows, and a negatively charged particle (such as an electron) would move in the opposite direction.

The simplest electric field is the region around a single charged particle:

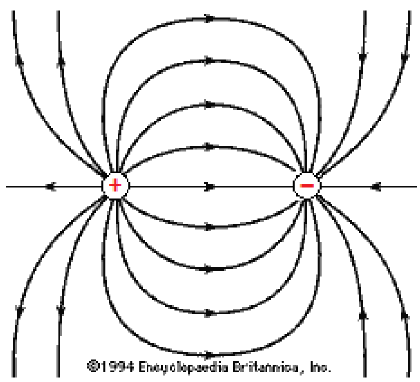


The electric field from an isolated positive charge



The electric field from an isolated negative charge

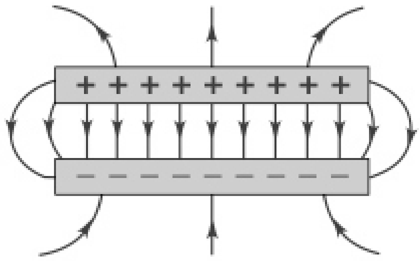
If a positive and a negative charge are near each other, the field lines go from the positive charge toward the negative charge:



Use this space for summary and/or additional notes:

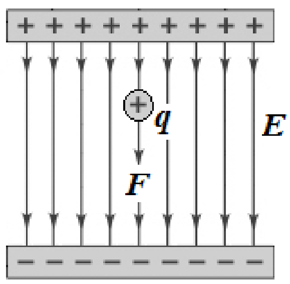
(Note that even though this is a two-dimensional drawing, the field itself is three-dimensional. Some field lines come out of the paper from the positive charge and go into the paper toward the negative charge, and some go behind the paper from the positive charge and come back into the paper from behind toward the negative charge.)

In the case of two charged plates (flat surfaces), the field lines would look like the following:



Electric Field Strength

We can measure the strength of an electric field by placing a particle with a positive charge (q) in the field and measuring the force ($\vec{F}$) on the particle.



Coulomb's Law tells us that the force on the charge is due to the charges from the electric field:

$$F_e = \frac{kq_1q_2}{r^2}$$

If the plates have equal charge densities, the repulsive force from the like-charged plate decreases as the particle moves away from it, but the attractive force from the oppositely-charged plate increases by the same amount as the particle moves toward it.

This means that if the positive and negative charges on the two surfaces that make the electric field have equal charge densities, *the force is the same everywhere in between the two surfaces*. The force on the particle is related only to the strength of the electric field and the charge of the particle.

This results in the equation that defines the electric field ($\vec{E}$) as the force between the electric field and our particle, divided by the charge of our particle:

$$\vec{E} = \frac{\vec{F}}{q} \quad \text{or} \quad \vec{F} = q\vec{E}$$

Use this space for summary and/or additional notes:

Work Done on a Charge by an Electric Field

Recall that work is the dot product of force and displacement:

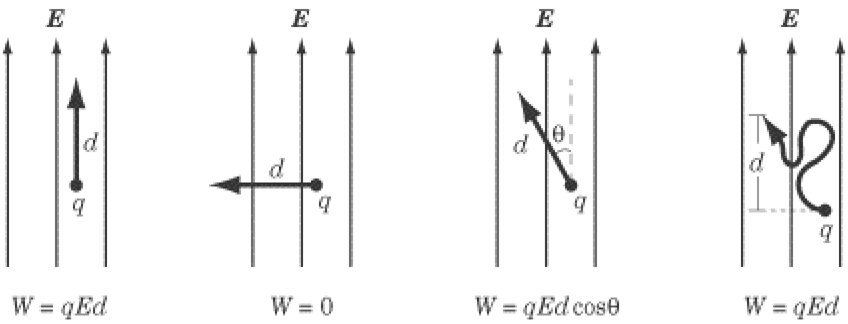
$$W = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{d}} = Fd\cos\theta$$

Because $W = \Delta U$, the potential energy of an electric field is the work that it is able to do. This means:

$$U_e = \vec{\boldsymbol{F}}_e \cdot \vec{\boldsymbol{d}} = \frac{kq_1q_2}{r^2} \cdot r = \frac{kq_1q_2}{r}$$

Because $\vec{\boldsymbol{F}} = q\vec{\boldsymbol{E}}$, we can substitute:

$$W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}} = q\,\vec{\boldsymbol{E}} \bullet \vec{\boldsymbol{d}} = qEd\cos\theta$$



Use this space for summary and/or additional notes:

Electric Potential

Recall from the work-energy theorem that work equals a change in energy. Because an electric field can do work on a charged particle, an electric field must therefore apply energy to the particle.

<u>electric potential</u> (*V*): the electric potential energy of a charged particle in an electric field.

Electric potential is measured in volts (V).

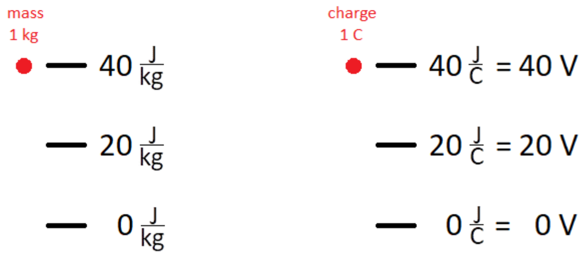
$$1\text{ V} \equiv 1\,\tfrac{\text{N}\cdot\text{m}}{\text{C}} \equiv 1\,\tfrac{\text{J}}{\text{C}}$$

Electric potential is analogous to gravitational potential energy. In a gravitational field, a particle has gravitational potential energy because gravity can make it move. In an electric field, a particle has electric potential (energy) because the electric field can make it move.

For example, if we had a 1 kg mass and we placed it at a height of 4 m above the ground, its gravitational potential would be $U_g = mgh = (1)(10)(4) = 40\text{ J}$.

Similarly, if we put an object with a charge of 1 C at a location that has an electric potential of 40 V, that object would have 40 J of potential energy due to the electric field.

Gravitational Potential Energy | Electric Potential



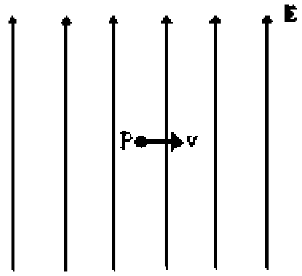
$$\frac{U_g}{m} = \vec{\boldsymbol{g}} \bullet \vec{\boldsymbol{h}} \qquad\qquad V = \frac{W}{q} = \vec{\boldsymbol{E}} \bullet \vec{\boldsymbol{d}}$$

gravitational potential energy per unit of mass | **electric potential** (already per unit of charge)

Use this space for summary and/or additional notes:

Sample Problem:

Q: A proton has a velocity of $1\times 10^{5}\ \frac{\text{m}}{\text{s}}$ when it is at point *P* in a uniform electric field that has an intensity of $1\times 10^{4}\ \frac{\text{N}}{\text{C}}$. Calculate the force (magnitude and direction) on the proton and sketch its path.

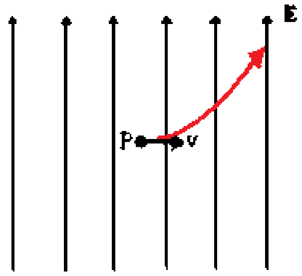


A: The force on the proton is given by:

$$\vec{F}_e = q\vec{E} = (1.6\times 10^{-19})(1\times 10^{4}) = 1.6\times 10^{-15}\ \text{N}$$

The direction of the force is the same direction as the electric field, which in this problem is upwards.

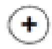
An upward force causes acceleration upwards. Because the proton starts with a velocity only to the right, upward acceleration means that its velocity will have a continuously increasing vertical component.



Use this space for summary and/or additional notes:

Homework Problems

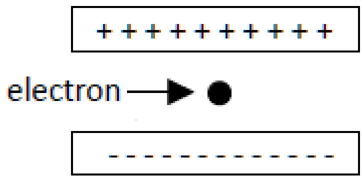
1. **(M)** Sketch the electric field in all directions around each of the following charged particles. (Assume that each particle has the same amount of charge.)

   a. 

   b. 

   c. 

2. **(M)** An electron is placed exactly halfway between two charged parallel plates, as shown in the diagram at the right. The electric field strength between the plates is $4.8\times10^{-11}\ \frac{\text{N}}{\text{C}}$.



   a. Sketch field lines to represent the electric field between the plates.

   b. Which direction does the electron move?

   c. As the electron moves, does the force acting on it increase, decrease, or remain the same?

   d. What is the net force on the electron?

   Answer: -7.68×10^{-30} N
   (Negative means the opposite direction of the electric field.)

Use this space for summary and/or additional notes:

Electric Field Vectors

Unit: Electric Force, Field & Potential

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 2.C.4.2, 2.E.1.1, 2.E.2.1, 2.E.2.2, 2.E.2.3, 2.E.3.1, 2.E.3.2

Mastery Objective(s): (Students will be able to…)

- Sketch & interpret electric field vector diagrams.

Success Criteria:

- Sketches show arrows pointing from positive charges toward negative charges.
- Electric field vectors show longer arrows where charges are larger and shorter arrows where charges are smaller.

Language Objectives:

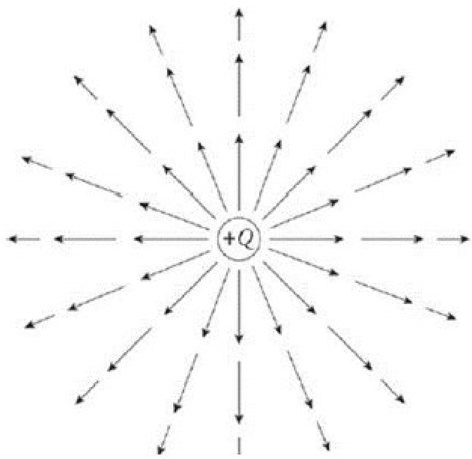
- Explain how the electric force on a charged particle changes as you get closer to or farther away from another charged object.

Tier 2 Vocabulary: charge, field

Notes:

electric field vector: an arrow representing the strength and direction of an electric field at a point represented on a map.

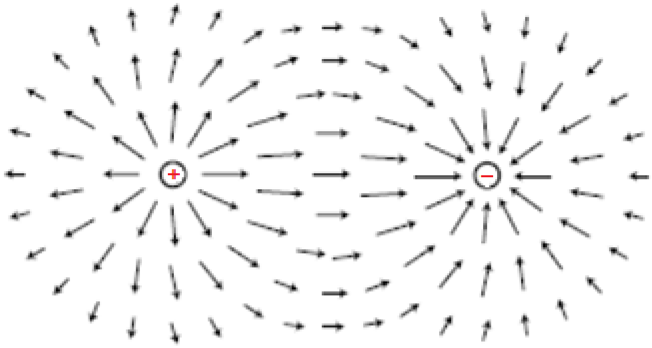
A map of an electric field can be drawn using field vectors instead of field lines. Electric field vectors are preferred, because in addition to showing the direction of the electric field at a given location, they also show the relative strength. For example, this diagram shows the electric field around a positive charge. Notice that:



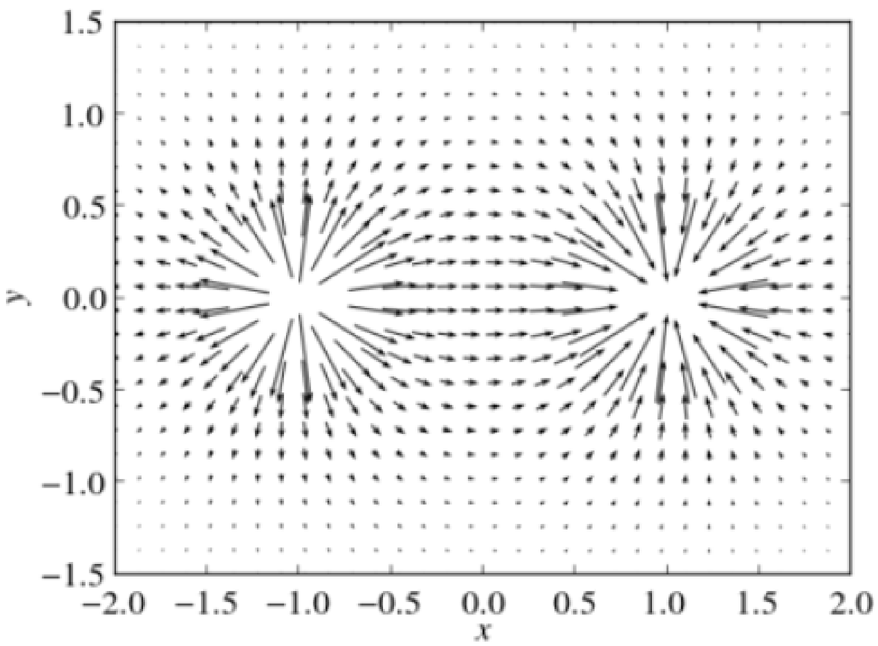
- The vectors point in the direction of the electric field (from positive to negative).
- The vectors are longer where the electric field is stronger and shorter where the electric field is weaker.

Use this space for summary and/or additional notes:

The electric field vectors around a pair of point charges, one positive and one negative, would look like the following:



If the point charges were not shown, you could use a field vector diagram to determine their locations:

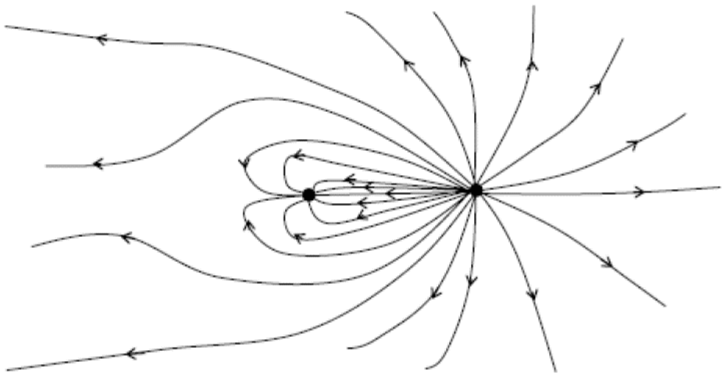


In the above example, there must be a positive point charge at coördinates (−1.0, 0) and a negative point charge at coördinates (+1.0, 0)

Use this space for summary and/or additional notes:

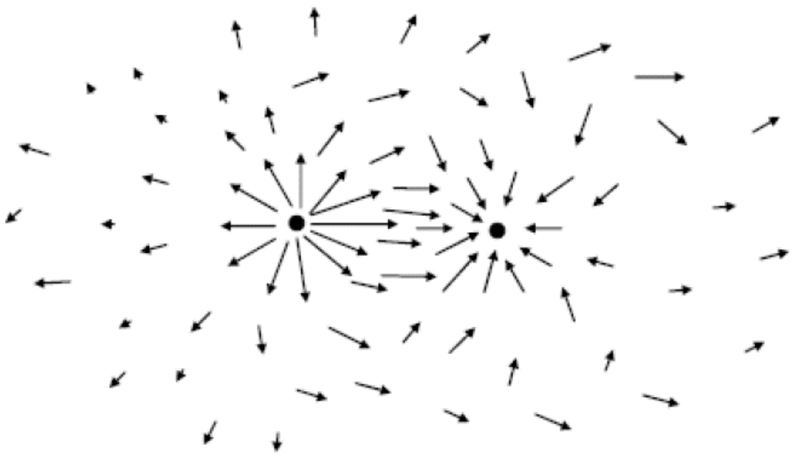
Homework Problems

1. **(M)** In the following electric field diagram: (Note that this is not an electric field vector diagram.)



   a. Label the point charges (the black dots) with the sign of their respective charges (positive or negative).
   b. Which of the two charges is stronger? Explain how you can tell.

2. **(M)** Consider the following electric field vector diagram:



   a. Label the point charges (the black dots) with the sign of their respective charges (positive or negative).
   b. Which of the two charges is stronger? Explain how you can tell.

Use this space for summary and/or additional notes:

Equipotential Lines & Maps

Unit: Electric Force, Field & Potential

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 2.C.4.2, 2.E.1.1, 2.E.2.1, 2.E.2.2, 2.E.2.3, 2.E.3.1, 2.E.3.2

Mastery Objective(s): (Students will be able to…)

- Sketch & interpret equipotential ("isoline") maps.

Success Criteria:

- Isolines are perpendicular to electric field lines.
- Isolines connect regions of equal electric potential.

Language Objectives:

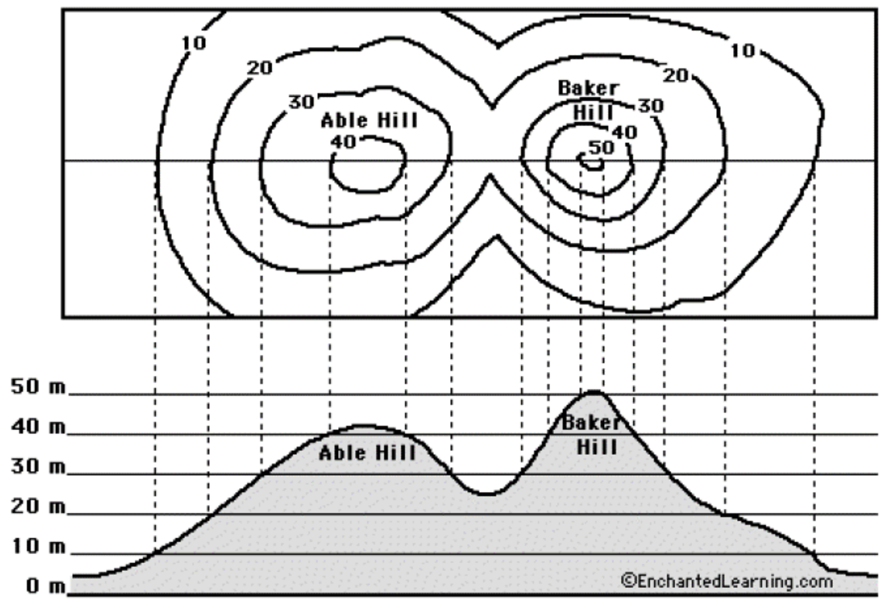
- Explain how isolines are like geographical contour maps.

Tier 2 Vocabulary: isoline, field, map

Notes:

<u>isoline or equipotential line</u>: a line on a map that connects regions of equal electric potential.

Isolines are the equivalent of elevation lines on a contour map. Below is a contour map (top) and side view of the same landscape (bottom):

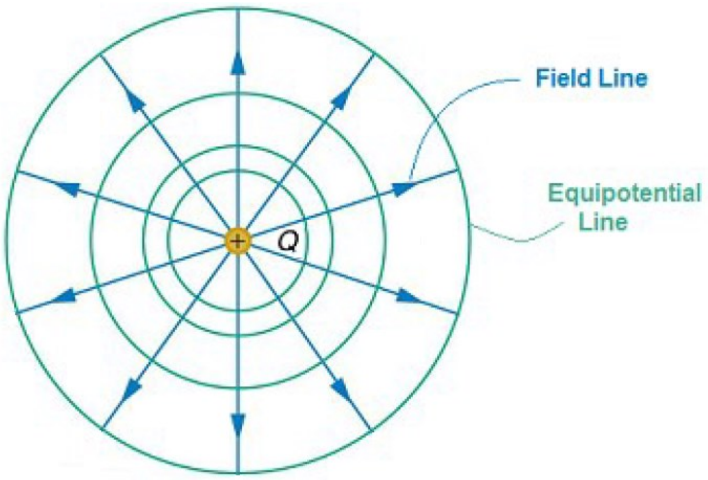


The contour map (top) is a view from above. Each contour line connects points that have the same elevation.

Use this space for summary and/or additional notes:

Similar to contour lines, equipotential lines connect regions of the same electric potential.

For example, the electric field direction is away from a positive point charge. The electric field strength decreases as you get farther away from the point charge. The equipotential lines (isolines) are therefore circles around the point charge; circles closer to the charge have higher electric potential, and circles farther away have lower electric potential.

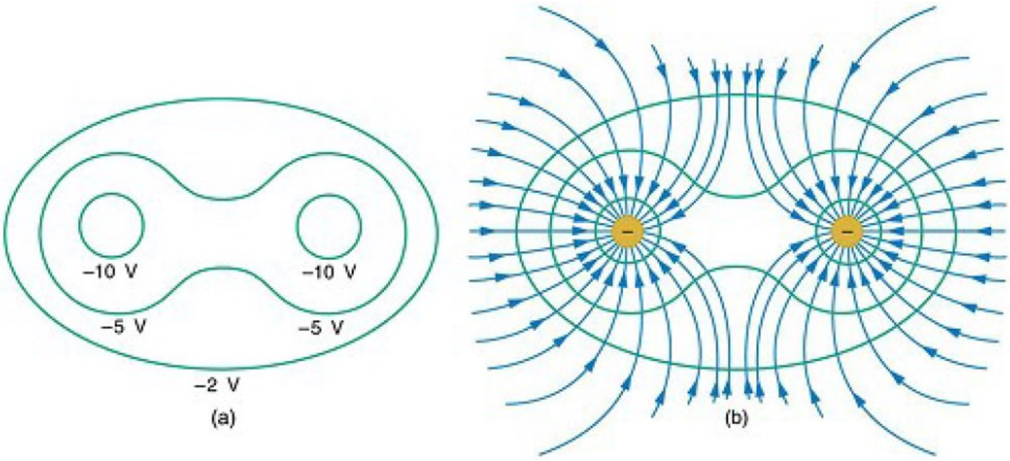


Notice that the equipotential lines are perpendicular to the field lines. As you travel along a field line, the electric potential becomes continuously less positive or more negative. The equipotential lines are the mileposts that show what the electric potential is at that point.

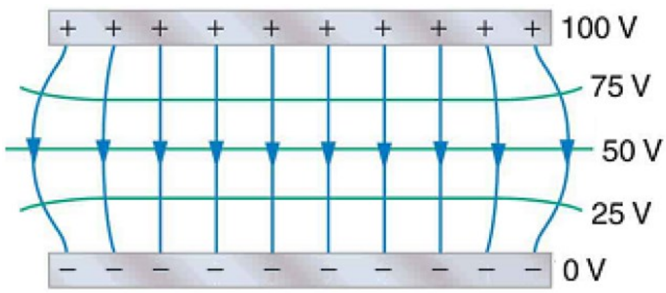
Use this space for summary and/or additional notes:

If you were given only the equipotential lines, you could determine what the resulting field lines looked like. For example, in the illustration below, if you were given the isolines in diagram (a), you would infer that the regions inside each of the smaller isolines must be negative point charges, and that the electric field becomes more and more negative as you approach those points.

Because the electric field lines go from positive to negative, the field lines must therefore point into the points, resulting in diagram (b). Notice again that the field lines are always perpendicular to the isolines.



The electric field lines and isolines for the region between two parallel plates would look like the following:

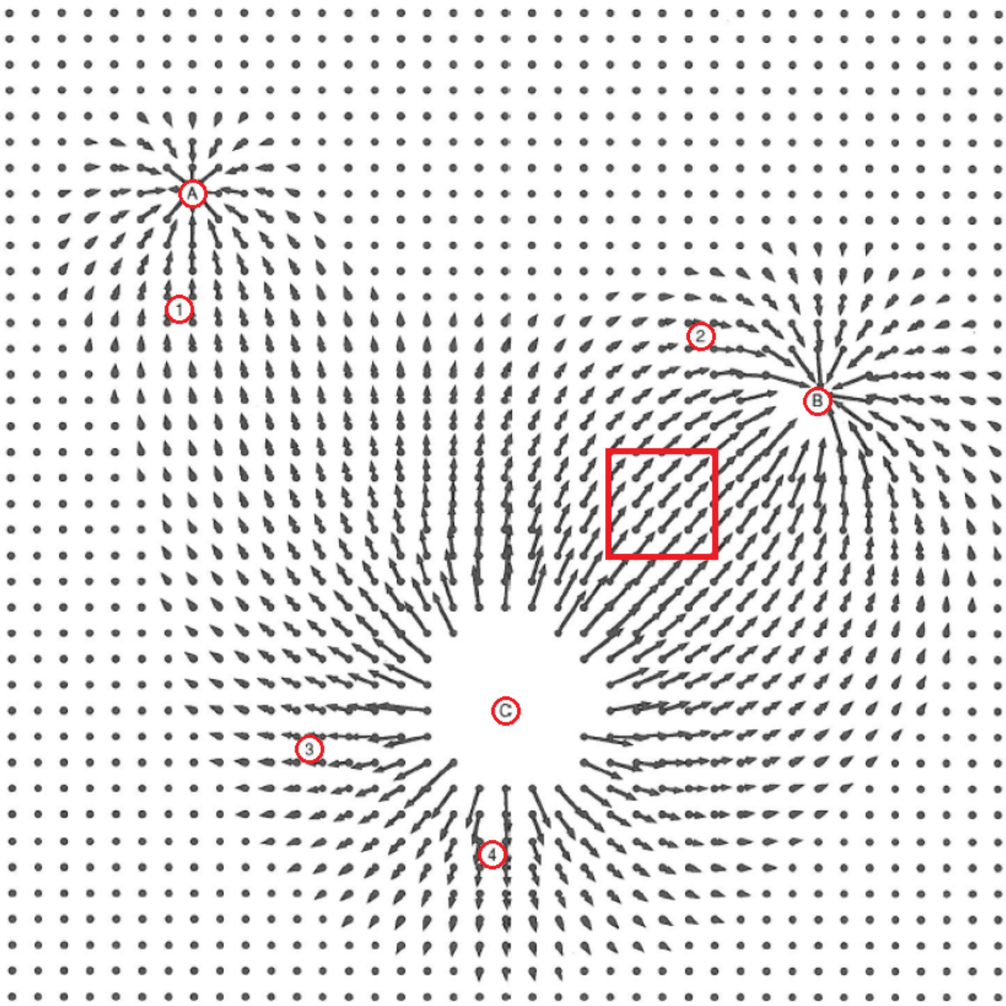


Use this space for summary and/or additional notes:

Homework Problems

Consider the following electric field vector map. The length and direction of each vector arrow represents the magnitude and direction of the electric field in that location. Empty space (such as the space around point C) represents regions where the electric field is extremely strong.

There are three point charges, labeled A, B, and C. There are four numbered locations, 1, 2, 3, and 4, and a region of interest surrounded by a square box between points B and C.



Use this space for summary and/or additional notes:

1. **(M)** On the diagram on the previous page, indicate the sign (positive or negative) of charges A, B, and C.

2. **(M)** Rank the charges from strongest to weakest (regardless of sign).

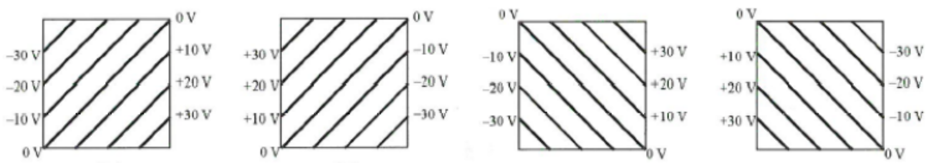
3. **(M)** Based on the lengths and directions of the field vectors, draw equipotential lines (isolines) connecting regions of the same electric potential on the diagram on the previous page.

4. **(M)** Indicate the direction of the force that would act on a proton placed at point 1 and point 2.

5. **(M)** Indicate the direction of the force that would act on an electron placed at point 3 and point 4.

6. **(M)** At which numbered point is the electric field strongest in magnitude?

7. **(M)** Circle the diagram that could represent isolines in the boxed region between charges B and C.



Explain your rationale.

Use this space for summary and/or additional notes:

Introduction: DC Circuits

Unit: DC Circuits

Topics covered in this chapter:

This chapter discusses DC Circuits, particularly those containing batteries, resistors and/or capacitors., how they behave, and how they relate to each other.

- *Electric Current & Ohm's Law* describes equations and calculations involving the flow of charged particles (electric current).
- *Electrical Components* shows pictures of and circuit diagram symbols for common electrical components.
- *EMF & Internal Resistance of a Battery* explains the difference between the voltage supplied by the chemical cells in a battery and the voltage that the battery is actually able to supply in a circuit.
- *Circuits*, *Series Circuits*, *Parallel Circuits*, and *Mixed Series & Parallel Circuits* describe arrangements of circuits that contain batteries and resistors (or other components that have resistance) and the equations that relate to them.
- *Measuring Voltage, Current & Resistance* describes how to correctly measure those quantities for components in a circuit.
- *Capacitance* and *Capacitors in Series & Parallel Circuits* describes capacitors and how they behave in circuits.

Use this space for summary and/or additional notes:

- *DC Resistor-Capacitor (RC) Circuits* describes calculations for time-varying circuits that contain a resistor and a capacitor.

One of the new challenges encountered in this chapter is interpreting and simplifying circuit diagrams, in which different equations may apply to different parts of the circuit.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-4. Use mathematical representations of Newton's Law of Gravitation and Coulomb's Law to describe and predict the gravitational and electrostatic forces between objects.

HS-PS3-1. Use algebraic expressions and the principle of energy conservation to calculate the change in energy of one component of a system when the change in energy of the other component(s) of the system, as well as the total energy of the system including any energy entering or leaving the system, is known. Identify any transformations from one form of energy to another, including thermal, kinetic, gravitational, magnetic, or electrical energy, in the system.

HS-PS3-2. Develop and use a model to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles and objects or energy stored in fields.

HS-PS3-5. Develop and use a model of magnetic or electric fields to illustrate the forces and changes in energy between two magnetically or electrically charged objects changing relative position in a magnetic or electric field, respectively.

Use this space for summary and/or additional notes:

AP® only
AP® only

AP® Physics 2 Learning Objectives:

1.E.2.1: The student is able to choose and justify the selection of data needed to determine resistivity for a given material. [**SP 4.1**]

4.E.4.1: The student is able to make predictions about the properties of resistors and/or capacitors when placed in a simple circuit, based on the geometry of the circuit element and supported by scientific theories and mathematical relationships. [**SP 2.2, 6.4**]

4.E.4.2: The student is able to design a plan for the collection of data to determine the effect of changing the geometry and/or materials on the resistance or capacitance of a circuit element and relate results to the basic properties of resistors and capacitors. [**SP 4.1, 4.2**]

4.E.4.3: The student is able to analyze data to determine the effect of changing the geometry and/or materials on the resistance or capacitance of a circuit element and relate results to the basic properties of resistors and capacitors. [**SP 5.1**]

4.E.5.1: The student is able to make and justify a quantitative prediction of the effect of a change in values or arrangements of one or two circuit elements on the currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel. [**SP 2.2**, **6.4**]

4.E.5.2: The student is able to make and justify a qualitative prediction of the effect of a change in values or arrangements of one or two circuit elements on currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel. [**SP 6.1**, **6.4**]

4.E.5.3: The student is able to plan data collection strategies and perform data analysis to examine the values of currents and potential differences in an electric circuit that is modified by changing or rearranging circuit elements, including sources of emf, resistors, and capacitors. [**SP 2.2**, **4.2**, **5.1**]

5.B.9.4: The student is able to analyze experimental data including an analysis of experimental uncertainty that will demonstrate the validity of Kirchhoff's loop rule. [**SP 5.1**]

5.B.9.5: The student is able to use conservation of energy principles (Kirchhoff's loop rule) to describe and make predictions regarding electrical potential difference, charge, and current in steady-state circuits composed of various combinations of resistors and capacitors. [**SP 6.4**]

5.B.9.6: The student is able to mathematically express the changes in electric potential energy of a loop in a multi-loop electrical circuit and justify this expression using the principle of the conservation of energy. [**SP 2.1**, **2.2**]

Use this space for summary and/or additional notes:

AP® only

5.B.9.7: The student is able to refine and analyze a scientific question for an experiment using Kirchhoff's Loop rule for circuits that includes determination of internal resistance of the battery and analysis of a non-ohmic resistor. [**SP 4.1**, **4.2**, **5.1**, **5.3**]

5.B.9.8: The student is able to translate between graphical and symbolic representations of experimental data describing relationships among power, current, and potential difference across a resistor. [**SP 1.5**]

5.C.3.4: The student is able to predict or explain current values in series and parallel arrangements of resistors and other branching circuits using Kirchhoff's junction rule and relate the rule to the law of charge conservation. [**SP 6.4**, **7.2**]

5.C.3.5: The student is able to determine missing values and direction of electric current in branches of a circuit with resistors and ***no*** capacitors from values and directions of current in other branches of the circuit through appropriate selection of nodes and application of the junction rule. [**SP 1.4**, **2.2**]

5.C.3.6: The student is able to determine missing values and direction of electric current in branches of a circuit with ***both*** resistors ***and*** capacitors from values and directions of current in other branches of the circuit through appropriate selection of nodes and application of the junction rule. [**SP 1.4**, **2.2**]

5.C.3.7: The student is able to determine missing values, direction of electric current, charge of capacitors at steady state, and potential differences within a circuit with resistors and capacitors from values and directions of current in other branches of the circuit. [**SP 1.4**, **2.2**]

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Circuit Elements and DC Circuits**, such as resistors, light bulbs, series and parallel networks, Ohm's law, and Joule's law.
- **Capacitance**, such as parallel-plate capacitors and time-varying behavior in charging/ discharging.

1. Voltage
2. Current
3. Resistance
4. Energy & Power
5. Circuits
6. Capacitors

Skills learned & applied in this chapter:

- Working with material-specific constants from a table.
- Identifying electric circuit components.
- Simplifying circuit diagrams.

Use this space for summary and/or additional notes:

Electric Current & Ohm's Law

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives: 1.E.2.1, 5.B.9.8

Mastery Objective(s): (Students will be able to…)

- Solve problems involving relationships between voltage, current, resistance and power.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

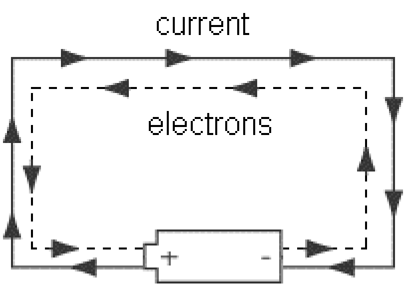
- Describe the relationships between voltage, current, resistance, and power.

Tier 2 Vocabulary: current, resistance, power

Labs, Activities & Demonstrations:

- modeling resistivity with straws
- make a light bulb out of a pencil "lead" (graphite)

Notes:



electric current (I): the flow of charged particles through a conductor, caused by a difference in electric potential. The direction of the electric current is defined as the direction that a positively-charged particle would move. Note, however, that the particles that are actually moving are electrons, which are negatively charged.

This means that electric current "travels" in the *opposite* direction from the electrons. We will use conventional current (pretending that positive particles are flowing through the circuit) throughout this course.

Electric current ($\vec{I}$) is a vector quantity and is measured in amperes (A), often abbreviated as "amps". One ampere is one coulomb per second.

$$I = \frac{\Delta Q}{t}$$

Use this space for summary and/or additional notes:

Note that when electric current is flowing, charged particles move from where they are along the circuit. For example, when a light bulb is illuminated, the electrons that do the work for the first few minutes are already in the filament.

voltage (potential difference) (ΔV)*: the difference in electric potential energy between two locations, per unit of charge.

$$\Delta V = \frac{W}{q}$$

Potential difference is the work (W) done on a charge per unit of charge (q). Potential difference (ΔV) is a scalar quantity (in DC circuits) and is measured in volts (V), which are equal to joules per coulomb.

The total voltage in a circuit is usually determined by the power supply that is used for the circuit (usually a battery in DC circuits).

resistance (R): the amount of electromotive force (electric potential) needed to force a given amount of current through an object in a DC circuit.

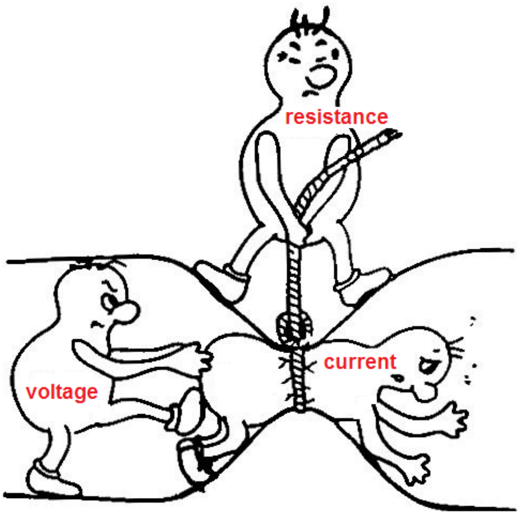
$$R = \frac{\Delta V}{I}$$

Resistance (R) is a scalar quantity and is measured in ohms (Ω). One ohm is one volt per ampere.

This relationship is Ohm's Law, named for the German physicist Georg Ohm. Ohm's Law is more commonly written:

$$I = \frac{\Delta V}{R} \quad \text{or} \quad \Delta V = IR$$

Simply put, Ohm's Law states that an object has an ability to resist electric current flowing through it. The more resistance an object has, the more voltage you need to force electric current through it. Or, for a given voltage, the more resistance an object has, the less current will flow through it.



Resistance is an intrinsic property of a substance. In this course, we will limit problems that involve calculations to ohmic resistors, which means their resistance does not change with temperature.

* Note that most physics texts (and most physicists and electricians) use V for both electric potential and voltage, and students have to rely on context to tell the difference. In these notes, to make the distinction clear (and to be consistent with the AP® Physics 2 exam), we will use V for electric potential, and ΔV for voltage (potential difference).

Use this space for summary and/or additional notes:

Choosing the voltage and the arrangement of objects in the circuit (which determines the resistance) is what determines how much current will flow.

Electrical engineers use resistors in circuits to reduce the amount of current that flows through the components.

honors (not AP®)

<u>impedance</u> (Z): the opposition that a circuit presents to a current when a voltage is applied. In a DC circuit, impedance and resistance are equivalent. In an AC circuit, the oscillating voltage creates changing electric and magnetic fields, which themselves resist the changes caused by the alternating current. This means the opposition to current is constantly changing at the same frequency as the oscillation of the current.

Mathematically, impedance is represented as a complex number, in which the real part is resistance and the imaginary part is reactance, a quantity that takes into account the effects of the oscillating electric and magnetic fields.

<u>resistivity (ρ)</u>: the innate ability of a substance to offer electrical resistance. The resistance of an object is therefore a function of the resistivity of the substance (ρ), and of the length (L) and cross-sectional area (A) of the object. In MKS units, resistivity is measured in ohm-meters (Ω·m).

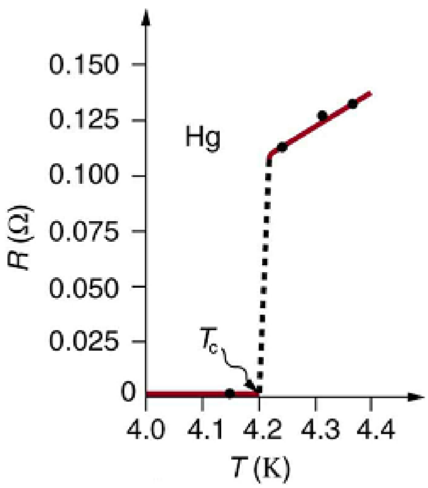
$$R = \frac{\rho L}{A}$$

Resistivity changes with temperature. For small temperature differences (less than 100°C), resistivity is given by:

$$\rho = \rho_o(1 + \alpha \Delta T)$$

where ρ_o is the resistivity at some reference temperature and α is the temperature coëfficient of resistivity for that substance. For conductors, α is positive (which means their resistivity increases with temperature). For metals at room temperature, resistivity typically varies from +0.003 to +0.006 K^{-1}.

Some materials become superconductors (essentially zero resistance) at very low temperatures. The temperature below which a material becomes a superconductor is called the critical temperature (T_c). For example, the critical temperature for mercury is 4.2 K, as shown in the graph to the right.



Use this space for summary and/or additional notes:

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<u>conductivity</u> (σ): the innate ability of a substance to conduct electricity. Conductivity (σ) is the inverse of resistivity, and is measured in siemens (S). Siemens used to be called mhos (symbol ℧). (Note that "mho" is "ohm" spelled backwards.)

$$\sigma = \frac{1}{\rho}$$

<u>ohmic resistor</u>: a resistor whose resistance is the same regardless of voltage and current. The filament of an incandescent light bulb is an example of a non-ohmic resistor, because the current heats up the filament, which increases its resistance. (This is necessary in order for the filament to also produce light.)

<u>capacitance</u> (C): the ability of an object to hold an electric charge. Capacitance (C) is a scalar quantity and is measured in farads (F). One farad equals one coulomb per volt.

$$C = \frac{Q}{\Delta V}$$

<u>power</u> (P): as discussed in the mechanics section of this course, power (P) is the work done per unit of time and is measured in watts (W).

In electric circuits:

$$P = \frac{W}{t} = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

<u>work</u> (W): recall from mechanics that work (W) equals power times time, and is measured in either newton-meters (N·m) or joules (J):

$$W = Pt = I\Delta V t = I^2 R t = \frac{(\Delta V)^2 t}{R} = Vq$$

Electrical work or energy is often measured in kilowatt-hours (kW·h).

$$1\,\text{kW}\cdot\text{h} \equiv 3.6\times 10^6\ \text{J} \equiv 3.6\,\text{MJ}$$

Summary of Terms, Units and Variables

Term	Variable	Unit	Term	Variable	Unit
point charge	q	coulomb (C)	resistance	R	ohm (Ω)
charge	Q	coulomb (C)	capacitance	C	farad (F)
current	I	ampere (A)	power	P	watt (W)
voltage	ΔV	volt (V)	work	W	joule (J)

Use this space for summary and/or additional notes:

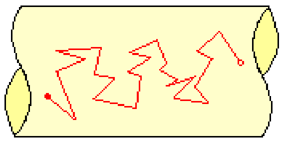
honors (not AP®)

Alternating Current *vs.* Direct Current

Electric current can move in two ways.

direct current: electric current flows through the circuit, starting at the positive terminal of the battery or power supply, and ending at the negative terminal. Batteries supply direct current. A typical AAA, AA, C, or D battery supplies 1.5 volts DC.

However, the net flow of charged particles through a wire is very slow. Electrons continually collide with one another in all directions as they drift slowly through the circuit. Electrons in a DC circuit have a net velocity of about one meter per hour.



alternating current: electric current flows back and forth in one direction and then the other, like a sine wave. The current alternates at a particular frequency. In the U.S., household current is 110 volts AC with a frequency of 60 Hz.

Alternating current requires higher voltages in order to operate devices, but has the advantage that the voltage drop is much less over a length of wire than with direct current.

Sample Problems:

Q: A simple electrical device uses 1.5 A of current when plugged into a 110 V household electrical outlet. How much current would the same device draw if it were plugged into a 12 V outlet in a car?

A: Resistance is a property of a specific object. Because we are not told otherwise, we assume the device is ohmic and the resistance is the same regardless of the current.

Therefore, our strategy is to use the information about the device plugged into a household outlet to determine the device's resistance, then use the resistance to determine how much current it draws in the car.

In the household outlet:

$$R = \frac{\Delta V}{I} = \frac{110}{1.5} = 73.\overline{3}\,\Omega$$

In the car:

$$I = \frac{\Delta V}{R} = \frac{12}{73.\overline{3}} = 0.163\text{ A}$$

Use this space for summary and/or additional notes:

Q: A laptop computer uses 10 W of power. The laptop's power supply adjusts the current so that the power is the same regardless of the voltage supplied. How much current would the computer draw from a 110 V household outlet? How much current would the same laptop computer need to draw from a 12 V car outlet?

A: The strategy for this problem is the same as the previous one.

Household outlet:

$$P = I\Delta V$$
$$I = \frac{P}{\Delta V} = \frac{10}{110} = 0.091\text{ A}$$

Car outlet:

$$I = \frac{P}{\Delta V} = \frac{10}{12} = 0.8\overline{3}\text{ A}$$

Q: A 100 Ω resistor is 0.70 mm in diameter and 6.0 mm long. If you wanted to make a 470 Ω resistor out of the same material (with the same diameter), what would the length need to be? If, instead, you wanted to make a resistor the same length, what would the new diameter need to be?

A: In both cases, $R = \frac{\rho L}{A}$.

For a resistor of the same diameter (same cross-sectional area), ρ and A are the same, which means:

$$\frac{R'}{R} = \frac{L'}{L}$$
$$L' = \frac{R'L}{R} = \frac{(470)(6.0)}{100} = 28.2\text{ mm}$$

For a resistor of the same length, ρ and L are the same, which means:

$$\frac{R'}{R} = \frac{A}{A'} = \frac{\pi r^2}{\pi (r')^2} = \frac{\pi (d/2)^2}{\pi (d'/2)^2} = \frac{d^2}{(d')^2}$$
$$d' = \sqrt{\frac{Rd^2}{R'}} = d\sqrt{\frac{R}{R'}} = 0.70\sqrt{\frac{100}{470}} = 0.70\sqrt{0.213} = 0.323\text{ mm}$$

Use this space for summary and/or additional notes:

Homework Problems

1. **(S)** An MP3 player uses a standard 1.5 V battery. How much resistance is in the circuit if it uses a current of 0.010 A?

   Answer: 150 Ω

2. **(M)** How much current flows through a hair dryer plugged into a 110 V circuit if it has a resistance of 25 Ω?

   Answer: 4.4 A

3. **(S)** A battery pushes 1.2 A of charge through the headlights in a car, which has a resistance of 10 Ω. What is the potential difference across the headlights?

   Answer: 12 V

4. **(M)** A circuit used for electroplating copper applies a current of 3.0 A for 16 hours. How much charge is transferred?

   Answer: 172 800 C

5. **(S)** What is the power when a voltage of 120 V drives a 2.0 A current through a device?

   Answer: 240W

Use this space for summary and/or additional notes:

6. **(S)** What is the resistance of a 40. W light bulb connected to a 120 V circuit?

   Answer: 360 Ω

7. **(M)** If a component in an electric circuit dissipates 6.0 W of power when it draws a current of 3.0 A, what is the resistance of the component?

   Answer: 0.67 Ω

8. **(S)** A 0.7 mm diameter by 60 mm long pencil "lead" is made of graphite, which has a resistivity of approximately $1.0\times10^{-4}\ \Omega\cdot\text{m}$. What is its resistance?

   Hints:

   - *You will need to convert mm to m.*
   - *You will need to convert the diameter to a radius before using $A=\pi r^2$ to find the area.*

   Answer: 15.6 Ω

9. **(M)** A cylindrical object has radius *r* and length *L* and is made from a substance with resistivity *ρ*. A potential difference of Δ*V* is applied to the object. Derive an expression for the current that flows through it.

   Hint: this is a two-step problem.

   Answer: $I=\dfrac{(\Delta V)A}{\rho L}$

Use this space for summary and/or additional notes:

10. **(S)** Some children are afraid of the dark and ask their parents to leave the hall light on all night. Suppose the hall light in a child's house has two 75. W incandescent light bulbs (150 W total), the voltage is 120 V, and the light is left on for 8.0 hours.

    a. How much current flows through the light fixture?

    Answer: 1.25 A

    b. How many kilowatt-hours of energy would be used in one night?

    Answer: 1.2 kW·h

    c. If the power company charges 22 ¢ per kilowatt-hour, how much does it cost to leave the light on overnight?

    Answer: 26.4 ¢

    d. If the two incandescent bulbs are replaced by LED bulbs that use 12.2 W each (24.4 W total) how much would it cost to leave the light on overnight?

    Answer: 4.3 ¢

Use this space for summary and/or additional notes:

Electrical Components

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Identify electrical components using the components themselves and/or the symbols used in circuit diagrams.
- Describe the purpose of various electrical components and how they are used in circuits.

Success Criteria:

- Descriptions correctly identify the component.
- Purpose and use of component is correct.

Language Objectives:

- Explain the components in an actual circuit or a circuit diagram, and describe what each one does.

Tier 2 Vocabulary: component, resistor, fuse

Labs, Activities & Demonstrations:

- Show & tell with actual components.
- How a fuse works.

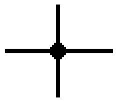
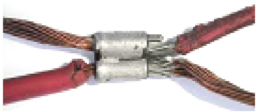
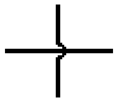
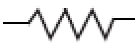
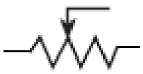
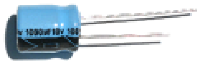
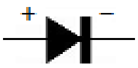
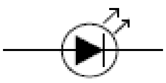
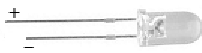
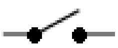
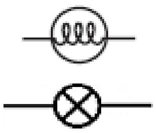
Notes:

electrical component: an object that performs a specific task in an electric circuit. A circuit is a collection of components connected together so that the tasks performed by the individual components combine in some useful way.

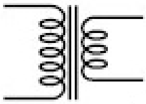
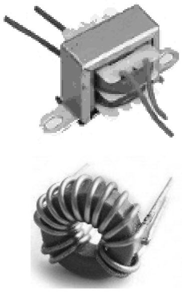
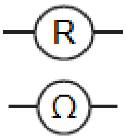
circuit diagram: a picture that represents a circuit, with different symbols representing the different components.

Use this space for summary and/or additional notes:

The following table describes some of the common components of electrical circuits, what they do, and the symbols that are used to represent them in circuit diagrams.

Component	Symbol	Picture	Description
wire			Carries current in a circuit.
junction			Connection between two or more wires.
unconnected wires			Wires pass by each other but are not connected.
battery			Supplies current at a fixed voltage.
resistor			Resists flow of current.
potentiometer (rheostat, dimmer)			Provides variable (adjustable) resistance.
capacitor			Stores charge.
diode			Allows current to flow in only one direction (from + to −).
light-emitting diode (LED)			Diode that gives off light when current flows through it.
switch			Opens / closes circuit.
incandescent lamp (light)			Provides light (and resistance).

Use this space for summary and/or additional notes:

Component	Symbol	Picture	Description
inductor (transformer)			Increases or decreases voltage in an AC circuit.
voltmeter			Measures voltage (volts).
ammeter			Measures current (amperes).
ohmmeter			Measures resistance (ohms).
fuse			Opens circuit if too much current flows through it.
ground		 (clamps to water pipe)	Neutralizes charge.

Use this space for summary and/or additional notes:

honors (not AP®)

Voltaic Cells (Batteries)*

Unit: DC Circuits

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain how a battery works.
- Identify the components of a battery and their function.

Success Criteria:

- Descriptions are accurate and components are identified correctly

Language Objectives:

- Explain how a battery works. (Domains: speaking, writing)
- Identify the components of a battery and their function (Domains: speaking, writing)

Tier 2 Vocabulary: battery, current

Labs, Activities & Demonstrations:

- building a voltaic cell

Notes:

voltaic cell: (also called a galvanic cell) a chemical apparatus that uses an electrochemical reaction to produce electricity. (A battery is a type of galvanic cell.)

electrochemistry: using chemical oxidation & reduction (redox) reactions to produce electricity or vice-versa. In an electrochemical reaction, oxidation and reduction reactions occur in separate containers, and electrons travel from one container to the other. In physics, the chemical energy from the combination of the two reactions is the potential difference (voltage) that moves those electrons through an electric circuit.

electrolytic cell: a cell similar to a galvanic cell, except that the reaction is nonspontaneous, and electricity is used to add the energy needed to make the reaction occur. (Electrolysis of water is an example.)

electrode: a solid metal strip where either oxidation or reduction occurs. The metal strips also conduct the electrons into or out of the electric circuit.

* Voltaic cells are generally taught in AP® Chemistry as part of the topic of electrochemistry. The topic is presented here in order to explain where the electric potential in a battery comes from.

Use this space for summary and/or additional notes:

honors (not AP®)

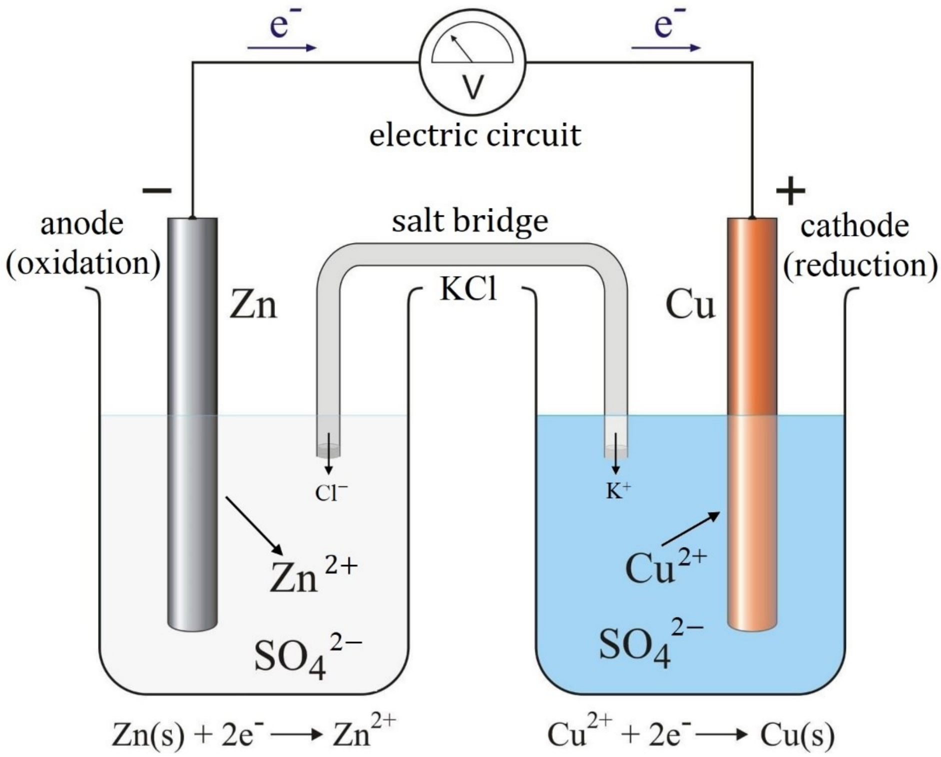
anode: the negatively (−) charged electrode. At the anode:

- Oxidation occurs. (Atoms from the anode are oxidized into positive ions.)
- These positive ions are released into the solution. (*I.e.,* the anode loses mass.)
- The electrons produced by oxidation are forced through the wire toward the cathode via the electric circuit.

cathode: the positively (+) charged electrode. At the cathode:

- Reduction occurs. (Ions from the solution are reduced to neutral metal atoms.)
- These metal atoms are deposited onto the cathode. (*I.e.,* the cathode gains mass.)
- The electrons needed for reduction are brought in through the wire from the anode via the electric circuit.

salt bridge: a salt solution that is connected to both half-cells. The salt bridge provides ions for the two half-cells in order to keep the charges balanced. (If the charges are not allowed to balance, opposite charges would build up in both cells and the reaction would stop.) The salt solution must be made of ions that do not take part in the reactions at the cathode or anode. (KNO_3 is commonly used.)



Use this space for summary and/or additional notes:

honors (not AP®)

standard voltage ($E°$): the voltage (electric potential) of an electrochemical reaction under "standard conditions".

- "Standard conditions" means temperature is 25 °C, all ion concentrations are $1\,\frac{\text{mol}}{\text{L}}$, and all gas pressures are 1 atm.*
- The actual voltage of the cell, V, depends on the temperature, ion concentrations and gas pressures. At standard conditions, $V = E^o$.
- $E°$ values for reduction reactions are published in tables of Standard Reduction Potentials.
- $E°$ for an oxidation reaction is the negative of the $E°$ for the reverse (reduction) reaction. *(I.e.,* if you reverse the reaction, change the sign of $E°$.)
- The standard voltage of a cell is the sum of the standard voltages for the oxidation and reduction half-cells:
  - $E^o = E^o_{reduction} + E^o_{oxidation}$
- If $E^o > 0$, then the reaction happens spontaneously. This is what happens when a battery is used to power a circuit.
- If $E^o < 0$, the reaction does not occur spontaneously, and energy is required to force the reaction to occur. This is what happens while a battery is charging.

* You can tell that "standard conditions" were defined by chemists. If they were defined by physicists, standard pressure would be 1 bar rather than 1 atm.

Use this space for summary and/or additional notes:

EMF & Internal Resistance of a Battery

Unit: DC Circuits

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Solve problems involving relationships between voltage, current, resistance and power.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Describe the relationships between voltage, current, resistance, and power.

Tier 2 Vocabulary: current, resistance, power

Labs, Activities & Demonstrations:

- batteries and resistors (< 10 Ω) to measure emf in a low-resistance circuit

Notes:

An ideal battery always supplies current at the voltage of the electrochemical cells inside of it. In a real battery, the voltage is a little less when the battery is "under load" (supplying current to a circuit) than when it is tested with no load. This difference is caused by real-world limitations of the chemical and physical processes that occur inside of the battery.

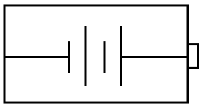
electromotive force (emf): the potential difference (voltage) supplied by a battery without load. This is the stated voltage of the battery. The term "electromotive force" literally means "electron-moving force". EMF is often represented by the variable ε.

voltage: the observed potential difference between two points in a circuit. The voltage of a battery usually means the voltage under load.

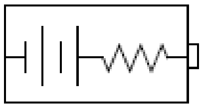
Use this space for summary and/or additional notes:

honors (not AP®)

To account for internal resistance, we model a battery as if it were a power supply with the ideal voltage, plus a resistor that is physically inside of the battery.



ideal



model

Note that this is a model; the actual situation is more complex, because in addition to the resistivity of the battery's component materials, the difference between the internal voltage and the supplied voltage also depends on factors such as electrolyte conductivity, ion mobility, and electrode surface area.

The following table shows the nominal voltage and internal resistance of common Duracell (coppertop) dry cell batteries of different sizes. These numbers are given by the manufacturer for a new battery at room temperature (25°C):

Size	**AAA**	**AA**	**C**	**D**	**9V**
V_{NL} **(V)**	1.5	1.5	1.5	1.5	9
R_{int} **(mΩ)**	250	120	150	137	1 700

The internal resistance can be used to calculate the maximum current that a battery could theoretically supply. If you were to connect a wire from the positive terminal of a battery to the negative terminal, the only resistance in the circuit should be the battery's internal resistance.

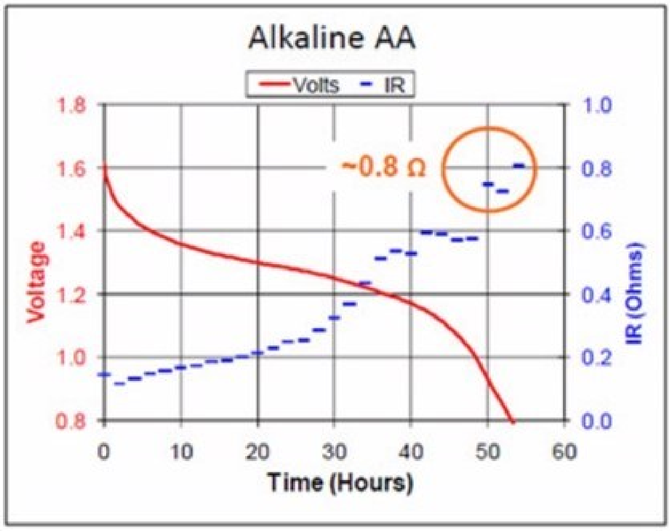
The theoretical maximum current that the battery can supply is therefore the current that would be supplied when the only resistance is the battery's internal resistance, and can be calculated from Ohm's Law:

$$I_{\max} = \frac{\Delta V}{R_{\text{int}}}$$

Use this space for summary and/or additional notes:

honors (not AP®)

Note also that the factors that affect a battery's internal resistance change as the battery ages. The following graph shows the changes in voltage and internal resistance of an alkaline AA battery as it supplied a current of 50 mA.



Internal resistance can be calculated by measuring the voltage with no "load" on the battery (*i.e.,* the voltmeter is connected directly to the battery with nothing else in the circuit) and the voltage with "load" (*i.e.,* the battery is connected to a circuit with measurable resistance):

$$R_{\text{int}} = \left(\frac{\Delta V_{NL}}{\Delta V_{FL}} - 1 \right) R_L$$

where:

R_{int} = internal resistance of battery

ΔV_{FL} = voltage measured with full load (resistor with resistance R_L in circuit)

ΔV_{NL} = voltage measured with no load (voltmeter connected directly to battery)

R_L = resistance of the load (resistor) that is used to experimentally determine the internal resistance of the battery

Use this space for summary and/or additional notes:

Circuits

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives: 4.E.4.1

Mastery Objective(s): (Students will be able to…)

- Identify electrical circuits or sections of circuits as series or parallel.

Success Criteria:

- Descriptions correctly identify the component.
- Descriptions correctly describe which type of circuit (series or parallel) the component is in.

Language Objectives:

- Identify which components are in series *vs.* parallel in a mixed circuit.

Tier 2 Vocabulary: series, parallel

Labs, Activities & Demonstrations:

- Example circuit with light bulbs & switches.
- Fuse demo using a single strand from a multi-strand wire.

Notes:

circuit: an arrangement of electrical components that allows electric current to pass through them so that the tasks performed by the individual components combine in some useful way.

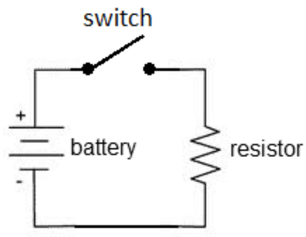
closed circuit: a circuit that has a complete path for current to flow from the positive terminal of the battery or power supply through the components and back to the negative terminal.

open circuit: a circuit that has a gap such that current cannot flow from the positive terminal to the negative terminal.

short circuit: a circuit in which the positive terminal is connected directly to the negative terminal with no load (resistance) in between.

Use this space for summary and/or additional notes:

A diagram of a simple electric circuit might look like the diagram to the right.



When the switch is closed, the electric current flows from the positive terminal of the battery through the switch, through the resistor, and back to the negative terminal of the battery.

An electric circuit needs a power supply (often a battery) that provides current at a specific voltage (electric potential difference), and one or more components that use the energy provided.

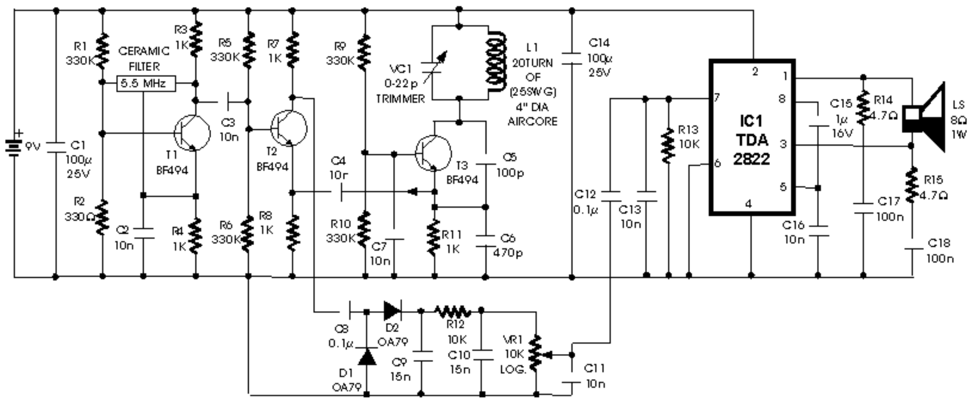
The battery or power supply continues to supply current, provided that:

1. There is a path for the current to flow from the positive terminal to the negative terminal, and

2. The total resistance of the circuit is small enough to allow the current to flow.

If the circuit is broken, current cannot flow and the chemical reactions inside the battery stop.

As circuits become more complex, the diagrams reflect this increasing complexity. The following is a circuit diagram for a metal detector:



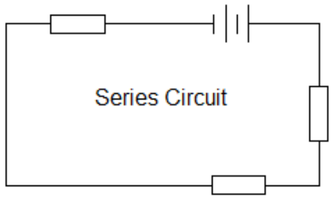
Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance contributed by each component of a circuit.

Use this space for summary and/or additional notes:

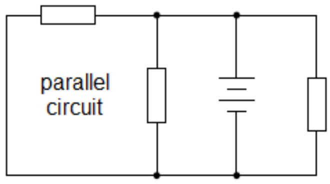
Series vs. Parallel Circuits

If a circuit has multiple components, they can be arranged in series or parallel.

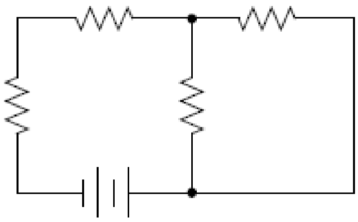
series: Components in series lie along the same path, one after the other.



parallel: Components in parallel lie in separate paths.



Note that complex circuits may have some components that are in series with each other and other components that are in parallel.

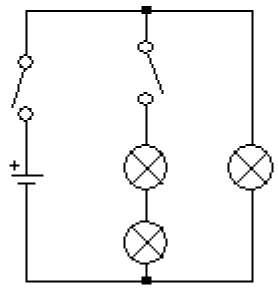


Use this space for summary and/or additional notes:

Sample Problem:

Q: A circuit consists of a battery, two switches, and three light bulbs. Two of the bulbs are in series with each other, and the third bulb is in parallel with the others. One of the switches turns off the two light bulbs that are in series with each other, and the other switch turns off the entire circuit. Draw a schematic diagram of the circuit, using the correct symbol for each component.

A:

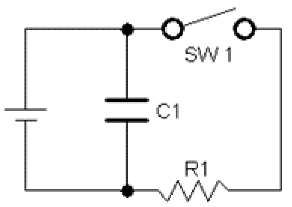


Note that no sensible person would intentionally wire a circuit this way. It would make much more sense to have the second switch on the branch with the one light bulb, so you could turn off either branch separately or both branches by opening both switches. This is an example of a strange circuit that a physics teacher would use to make sure you really can follow exactly what the question is asking!

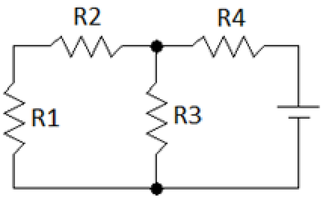
Use this space for summary and/or additional notes:

Homework Problems

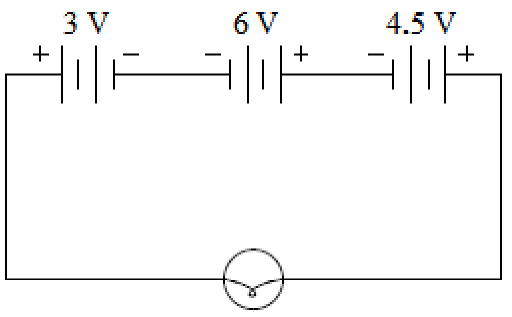
1. **(M)** The circuit shown to the right contains a battery, switch (SW1), capacitor (C1), and resistor (R1). Which of components C1 and SW1 are in series with R1? Which are in parallel with R1?



2. **(M)** The circuit shown to the right contains a battery and four resistors (R1, R2, R3, and R4). Which resistors are in series with R1? Which are in parallel with R1?



3. **(M)** The following bizarre circuit contains three batteries and a light bulb. What is the potential difference across the light bulb?
(*Hint: remember to check the +/− orientation of the batteries.*)



Answer: 7.5 V

Use this space for summary and/or additional notes:

Kirchhoff's Rules

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives: 5.B.9.4, 5.B.9.5, 5.B.9.6, 5.B.9.7, 5.B.9.8, 5.C.3.4, 5.C.3.5, 5.C.3.6, 5.C.3.7

Mastery Objective(s): (Students will be able to…)

- Apply Kirchhoff's junction and loop rules to determine voltages and currents in circuits.

Success Criteria:

- Loop rule correctly applied (electric potential differences add to zero).
- Junction rule correctly applied (total current into a junction equals total current out).

Language Objectives:

- Explain why electric potential has to add to zero around a loop and why current into a junction has to add up to current out.

Tier 2 Vocabulary: loop, junction

Labs, Activities & Demonstrations:

- model a circuit by walking up & down stairs

Notes:

In 1845, the German physicist Gustav Kirchhoff came up with two simple rules that describe the behavior of current in complex circuits. Those rules are:

Kirchhoff's junction rule: the total current coming into a junction must equal the total current coming out of the junction.

The junction rule is based on the concept that electric charge cannot be created or destroyed. Current is simply the flow of electric charge, so any charges that come into a junction must also come out of it.

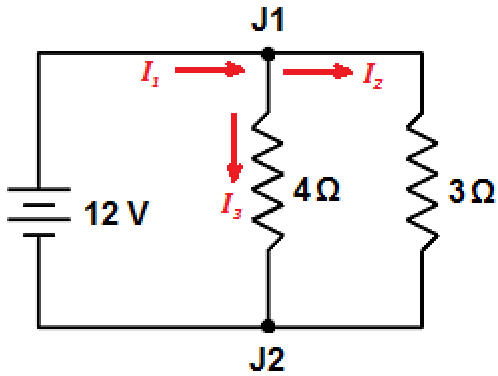
Kirchhoff's loop rule: the sum of the voltages around a closed loop must add up to zero.

The loop rule is based on the concept that voltage is the difference in electric potential between one location in the circuit and another. If you come back to the same point in the circuit, the difference in electric potential between where you started and where you ended (the same place) must be zero. Therefore, any increases and decreases in voltage around the loop must cancel.

Use this space for summary and/or additional notes:

Junction Rule Example:

As an example of the junction rule, consider the following circuit:



The junction rule tells us that the current flowing into junction J1 must equal the current flowing out. If we assume current I_1 flows into the junction, and currents I_2 and I_3 flow out of it, then $I_1 = I_2 + I_3$.

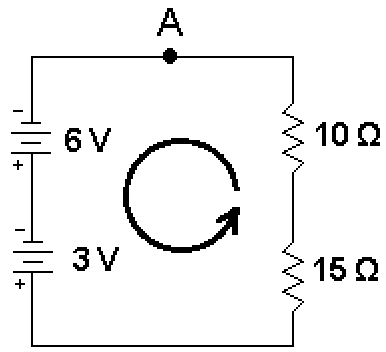
We know that the voltage across both resistors is 12 V. From Ohm's Law we can determine that the current through the 3 Ω resistor is $I_2 = 4\text{ A}$, and the current through the 4 Ω resistor is $I_3 = 3\text{ A}$. The junction rule tells us that the total current must therefore be:

$$I_1 = I_2 + I_3 = 4\text{A} + 3\text{A} = 7\text{A}.$$

Use this space for summary and/or additional notes:

Loop Rule Example:

For the loop rule, consider the circuit to the right:



If we start at point A and move counterclockwise around the loop (in the direction of the arrow), the voltage should be zero when we get back to point A.

For this example, we are moving around the circuit in the same direction that the current flows, because that makes the most intuitive sense. However, it wouldn't matter if we moved clockwise instead—just as with vector quantities, we choose a positive direction and assign each quantity to a positive or negative number accordingly, and the math tells us what is actually happening.

Starting from point A, we first move through the 6 V battery. We are moving from the negative pole to the positive pole of the battery, so the voltage increases by +6 V. When we move through the second battery, the voltage increases by +3 V.

Next, we move through the 15 Ω resistor. When we move through a resistor in the positive direction (of current flow), the voltage drops, so we assign the resistor a voltage of $-15I$ (based on $V = IR$, where I is the current through the resistor). Similarly, the voltage across the 10 Ω resistor is $-10I$. Applying the loop rule gives:

$$6+3+(-15I)+(-10I)=0$$
$$9-25I=0$$
$$9=25I$$
$$I=\frac{9}{25}=0.36\text{ A}$$

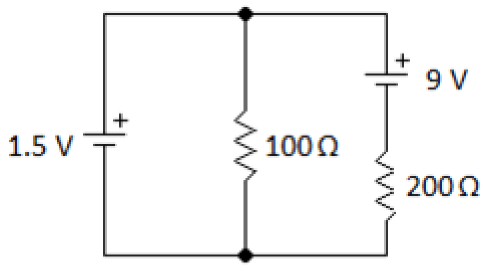
Now that we know the total current, we can use it to find the voltage drop (potential difference) across the two resistors.

$$\Delta V_{10\,\Omega}=IR=(0.36)(10)=3.6\text{ V} \qquad \Delta V_{15\,\Omega}=IR=(0.36)(15)=5.4\text{ V}$$

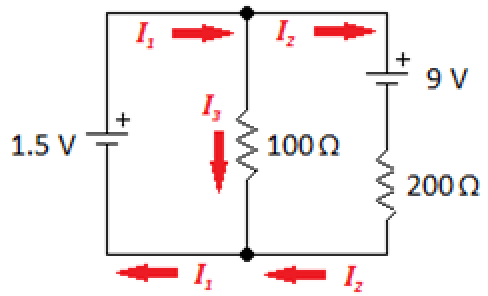
Use this space for summary and/or additional notes:

Sample Problem using Kirchhoff's Rules:

Find the voltage and current across each resistor in the following circuit:

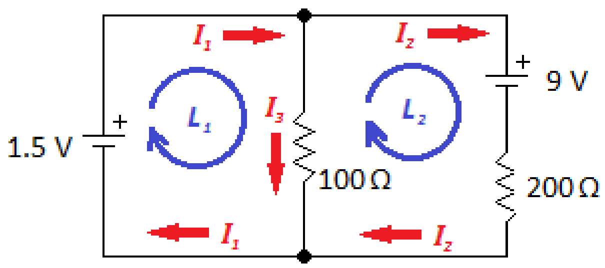


Applying the junction rule, we choose arbitrary directions for current:



$$I_1 = I_2 + I_3$$

By closer inspection, we can see that the direction for I_2 is probably going to be wrong. This means we expect that I_2 will come out to a negative number.



Now that we have defined the directions for current using the junction rule, we apply the loop rule. Again, we choose the direction of each loop arbitrarily, without worrying about which direction we have chosen for current. The loop direction is simply the order in which we inspect each element. The numbers will be determined by the direction we chose for current.

Use this space for summary and/or additional notes:

As we inspect our way around the loop, there are two rules for determining the voltage across each component:

1. Voltage across a battery is *positive* if the loop direction is *from negative to positive* (the "forward" direction).
   Voltage across a battery is *negative* if the loop direction is f*rom positive to negative* (the "backward" direction).

2. Voltage across a resistor is *negative* if the loop direction is *with the current* (the resistor is "using up" voltage).
   Voltage across a resistor is *positive* if the loop direction is *against the current* (we are traveling from a place where the electric potential is lower to a place where it is higher).

Now we inspect our way around each loop, writing the equations for the voltages:

L1: $+1.5-100I_3=0 \rightarrow I_3=0.015\text{ A}$

L2: $-9-200I_2+100I_3=0$

$-9-200I_2+1.5=0$

$-7.5=200I_2$

$I_2=-0.0375\text{ A}$

$I_1=I_2+I_3=-0.0375+0.015=-0.0225\text{ A}$

I_3 came out to a positive number, meaning that the current is flowing in the direction that we chose initially. However, I_1 and I_2 both came out negative, meaning that the current in those two segments of the circuit is actually flowing in the opposite direction from the arbitrary direction that we chose at the beginning of the problem.

Now that we know the current and resistance, we can find the voltage drop across each resistor using Ohm's Law.

$100\,\Omega$: $V=I_3R=(0.015)(100)=1.5\text{ V}$

$200\,\Omega$: $V=I_2R=(-0.0375)(200)=-7.5\text{ V}$

Again, the negative sign shows that the voltage drop (from positive to negative) is in the opposite direction from what we originally chose.

Use this space for summary and/or additional notes:

Series Circuits (Resistance Only)

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives: 4.E.4.1, 4.E.4.2, 4.E.4.3, 4.E.5.1, 4.E.5.2, 4.E.5.3

Mastery Objective(s): (Students will be able to…)

- Calculate voltage, current, resistance and power in series circuits.

Success Criteria:

- Correct relationships are applied for voltage, current, resistance and power in series circuits.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the relationships for voltages, current, resistance and power in series circuits.

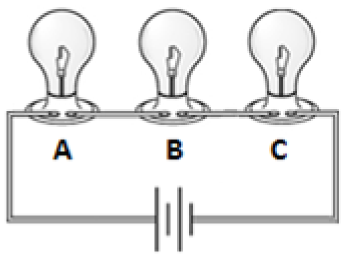
Tier 2 Vocabulary: series, circuit

Labs, Activities & Demonstrations:

- Circuit with light bulbs wired in series.

Notes:

series: Components in series lie along the same path, one after the other.



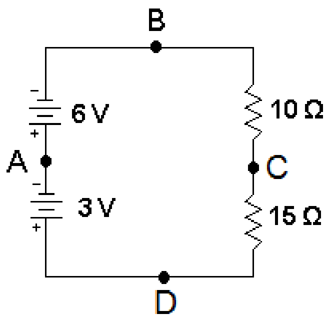
In a series circuit, all of the current flows through every component, one after another. If the current is interrupted anywhere in the circuit, no current will flow. For example, in the following series circuit, if any of the light bulbs A, B, or C is removed, no current can flow and none of the light bulbs will be illuminated.

Use this space for summary and/or additional notes:

Because some of the electric potential energy (voltage) is "used up" by each bulb in the circuit, each additional bulb means the voltage is divided among more bulbs and is therefore less for each bulb. This is why light bulbs get dimmer as you add more bulbs in series.

Christmas tree lights used to be wired in series. This caused a lot of frustration, because if one bulb burned out, the entire string went out, and it could take several tries to find which bulb was burned out.

The diagram to the right shows two batteries and two resistors in series.



Current

Because there is only one path, all of the current flows through every component. This means the current is the same through every component in the circuit:

$$I_{total} = I_1 = I_2 = I_3 = \ldots$$

Voltage

In a series circuit, if there are multiple voltage sources (*e.g.,* batteries), the voltages add:

$$\Delta V_{total} = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots$$

In the above circuit, there are two batteries, one that supplies 6 V and one that supplies 3 V. The voltage from A to B is +6 V, the voltage from A to D is −3 V (note that A to D means measuring from negative to positive), and the voltage from D to B is (+3 V) + (+6 V) = +9 V.

Resistance

If there are multiple resistors, each one contributes to the total resistance and the resistances add:

$$R_{total} = R_1 + R_2 + R_3 + \ldots$$

In the above circuit, the resistance between points B and D is 10Ω + 15Ω = 25Ω.

Power

In all circuits (series and parallel), any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \ldots$$

Use this space for summary and/or additional notes:

Calculations

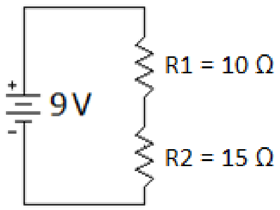
You can calculate the voltage, current, resistance, and power of each component separately, any subset of the circuit, or entire circuit, using the equations:

$$\Delta V = IR \qquad P = I\Delta V = I^2R = \frac{(\Delta V)^2}{R}$$

"Solving" the circuit for these quantities is much like solving a Sudoku puzzle. You systematically decide which variables (for each component and/or the entire circuit) you have enough information to solve for. Each result enables you to determine more and more of the, until you have found all of the quantities you need.

Sample Problem:

Suppose we are given the following series circuit:



and we are asked to fill in the following table:

	R_1	R_2	Total
Voltage (ΔV)			9 V
Current (I)			
Resistance (R)	10 Ω	15 Ω	
Power (P)			

First, we recognize that resistances in series add, which gives us:

	R_1	R_2	Total
Voltage (ΔV)			9 V
Current (I)			
Resistance (R)	10 Ω	15 Ω	***25 Ω***
Power (P)			

Now, we know two variables in the "Total" column, so we use $\Delta V = IR$ to find the current.

$$\Delta V = IR$$
$$9 = (I)(25)$$
$$I = \frac{9}{25} = 0.36\text{ A}$$

Use this space for summary and/or additional notes:

Because this is a series circuit, the total current is also the current through R_1 and R_2.

	R_1	R_2	Total
Voltage (ΔV)			9 V
Current (I)	***0.36 A***	***0.36 A***	***0.36 A***
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)			

As soon as we know the current, we can find the voltage across R_1 and R_2, again using $\Delta V = IR$.

	R_1	R_2	Total
Voltage (ΔV)	***3.6 V***	***5.4 V***	9 V
Current (I)	0.36 A	0.36 A	0.36 A
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)			

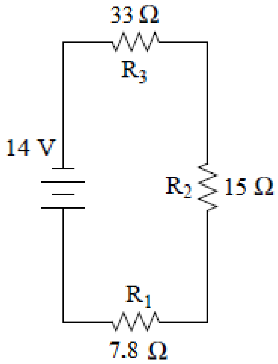
Finally, we can fill in the power, using $P = I\,\Delta V$:

	R_1	R_2	Total
Voltage (ΔV)	3.6 V	5.4 V	9 V
Current (I)	0.36 A	0.36 A	0.36 A
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)	***1.30 W***	***1.94 W***	***3.24 W***

Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** Fill in the table for the following circuit:



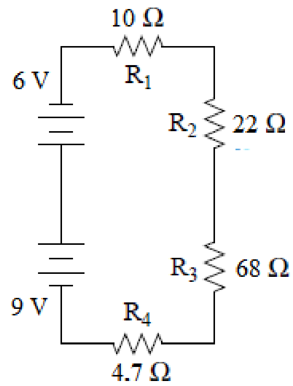
	R_1	R_2	R_3	Total
Voltage (ΔV)				14 V
Current (I)				
Resist. (R)	7.8 Ω	15 Ω	33 Ω	
Power (P)				

(The space below is intentionally left blank for calculations.)

Use this space for summary and/or additional notes:

2. **(M)** Fill in the table for the following circuit.

   (*Hint: Notice that the batteries are oriented in opposite directions.*)



	R_1	R_2	R_3	R_4	Total
Voltage (ΔV)					
Current (I)					
Resist. (R)	10 Ω	22 Ω	68 Ω	4.7 Ω	
Power (P)					

(The space below is intentionally left blank for calculations.)

Use this space for summary and/or additional notes:

Parallel Circuits (Resistance Only)

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives: 4.E.4.1, 4.E.4.2, 4.E.4.3, 4.E.5.1, 4.E.5.2, 4.E.5.3

Mastery Objective(s): (Students will be able to…)

- Calculate voltage, current, resistance and power in parallel circuits.

Success Criteria:

- Correct relationships are applied for voltage, current, resistance and power in parallel circuits.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

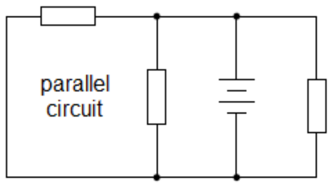
Language Objectives:

- Explain the relationships for voltages, current, resistance and power in parallel circuits.

Tier 2 Vocabulary: parallel

Labs, Activities & Demonstrations:

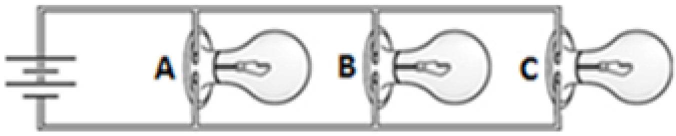
- Circuit with light bulbs wired in parallel.parallel: Components in parallel lie in separate paths.



In a parallel circuit, the current divides at each junction, with some of the current flowing through each path. If the current is interrupted in one path, current can still flow through the other paths.

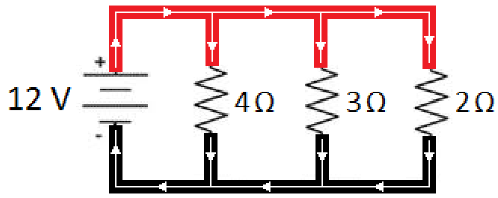
Use this space for summary and/or additional notes:

For example, in the parallel circuit below, if any of light bulbs A, B, or C is removed, current still flows through the remaining bulbs.



Because the voltage across each branch is equal to the total voltage, all of the bulbs will light up with full brightness, regardless of how many bulbs are in the circuit. (However, each separate light bulb draws the same amount of current as if it were the only thing in the circuit, so ***the total current in the circuit increases with each new branch***. This is why you trip a circuit breaker or blow a fuse if you have too many high-power components plugged into the same circuit.)

The following circuit shows a battery and three resistors in parallel:



Current

The current divides at each junction (as indicated by the arrows). This means the current through each path must add up to the total current:

$$I_{total} = I_1 + I_2 + I_3 + \ldots$$

Voltage

In a parallel circuit, the potential difference (voltage) across the battery is always the same (12 V in the above example). Therefore, the potential difference between *any point* on the top wire and *any point* on the bottom wire must be the same. This means the voltage is the same across each path:

$$V_{total} = V_1 = V_2 = V_3 = \ldots$$

Resistance

If there are multiple resistors, the effective resistance of each path becomes less as there are more paths for the current to flow through. The total resistance is given by the formula:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Use this space for summary and/or additional notes:

Some students find it confusing that the combined resistance of a group of resistors in series is always less than any single resistor by itself.

Power

Just as with series circuits, in a parallel circuit, any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \ldots$$

Electric current is analogous to water in a pipe:

- The current corresponds to the flow rate.
- The voltage corresponds to the pressure between one side and the other.
- The resistance would correspond to how small the pipe is (i.e., how hard it is to push water through the pipes). A smaller pipe has more resistance; a larger pipe will let water flow through more easily than a smaller pipe.

The voltage (pressure) drop is the same between one side and the other because less water flows through the smaller pipes and more water flows through the larger ones until the pressure is completely balanced. The same is true for electrons in a parallel circuit.

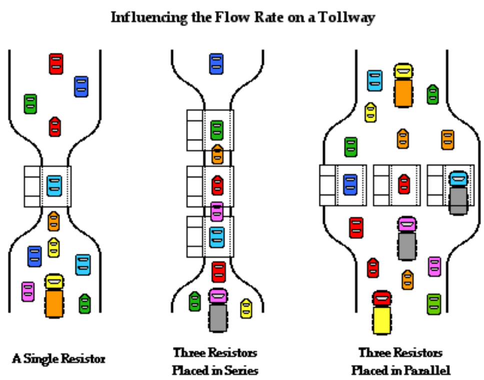
The water will flow through the *set* of pipes more easily than it would through any one pipe by itself. The same is true for resistors. As you add more resistors, you add more pathways for the current, which means less total resistance.

Another common analogy is to compare resistors with toll booths on a highway.

One toll booth slows cars down while the drivers pay the toll.

Multiple toll booths in a row would slows traffic down more. This is analogous to resistors in series.

Multiple toll booths next to each other (in parallel) make traffic flow faster because there are more paths for the cars to follow. Each toll booth further reduces the resistance to the flow of traffic.



Use this space for summary and/or additional notes:

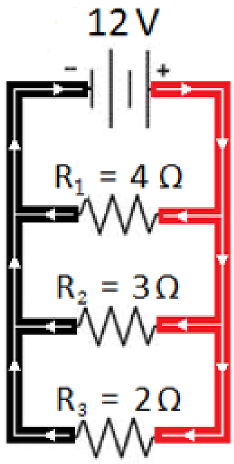
Calculations

Just as with series circuits, you can calculate the voltage, current, resistance, and power of each component and the entire circuit using the equations:

$$\Delta V = IR \qquad P = I\Delta V = I^2R = \frac{(\Delta V)^2}{R}$$

Sample Problem

Suppose we are given the following parallel circuit:



and we are asked to fill in the following table:

	R_1	R_2	R_3	Total
Voltage (ΔV)				12 V
Current (I)				
Resistance (R)	4 Ω	3 Ω	2 Ω	
Power (P)				

Because this is a parallel circuit, the total voltage equals the voltage across all three branches, so we can fill in 12 V for each resistor.

The next thing we can do is use $\Delta V = IR$ to find the current through each resistor:

	R_1	R_2	R_3	Total
Voltage (ΔV)	***12 V***	***12 V***	***12 V***	***12 V***
Current (I)	***3 A***	***4 A***	***6 A***	***13 A***
Resistance (R)	4 Ω	3 Ω	2 Ω	
Power (P)				

In a parallel circuit, the current adds, so the total current is 3 + 4 + 6 = 13 A.

Now, we have two ways of finding the total resistance. We can use $\Delta V = IR$ with the total voltage and current, or we can use the formula for resistances in parallel:

$$\Delta V = IR$$
$$12 = 13R$$
$$R = \frac{12}{13} = 0.923\,\Omega$$

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$\frac{1}{R_{total}} = \frac{1}{4} + \frac{1}{3} + \frac{1}{2} = \frac{3}{12} + \frac{4}{12} + \frac{6}{12} = \frac{13}{12}$$
$$R_{total} = \frac{12}{13} = 0.923\,\Omega$$

Use this space for summary and/or additional notes:

Now we have:

	R_1	R_2	R_3	Total
Voltage (ΔV)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	***0.923 Ω***
Power (P)				

As we did with series circuits, we can calculate the power, using $P = I\,\Delta V$:

	R_1	R_2	R_3	Total
Voltage (V)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	0.923 Ω
Power (P)	***36 W***	***48 W***	***72 W***	***156 W***

Batteries in Parallel

One question that has not been answered yet is what happens when batteries are connected in parallel.

If the batteries have the same voltage, the potential difference (voltage) remains the same, but the total current is the combined current from the two batteries.

However, if the batteries have different voltages there is a problem, because each battery attempts to maintain a constant potential difference (voltage) between its terminals. This results in the higher voltage battery overcharging the lower voltage battery.

Remember that physically, batteries are electrochemical cells—small solid-state chemical reactors with redox reactions taking place in each cell. If one battery overcharges the other, material is deposited on the cathode (positive terminal) until the cathode becomes physically too large for its compartment, at which point the battery bursts and the chemicals leak out.

Use this space for summary and/or additional notes:

Light Bulbs

Electric light bulbs, which were invented by Thomas Edison in 1880, use electrical energy to produce light. For about 100 years, most light bulbs were incandescent bulbs, pass electricity through a tungsten filament until it glows white-hot.

Newer light bulbs, such as fluorescent bulbs or light-emitting diode (LED) bulbs, produce similar amounts of light but much less heat, making them much more energy efficient.

The S.I. unit for light intensity is the lumen (lm).

Intensity (lm)	450	800	1100	1600
Power (W)*	40	60	75	100
Resistance (Ω)†	360	240	192	144

The amount of light that a light bulb produces is proportional to the amount of power it consumes. For over 100 years, incandescent bulbs were sold according to their power rating (in watts), and people developed an understanding of how much light a typical incandescent bulb would produce in a 120 V household circuit. Note that the power consumed by a light bulb is a function of both the current and the voltage: $P = I\Delta V$.

- In a parallel circuit (such as you would find in your house), the voltage is constant. The resistance of a component determines how much current it draws. A component with lower resistance (*e.g.,* a light bulb with a higher "wattage" rating) draws more current, and therefore uses more power. This means a bulb with a higher "wattage" rating will be <u>*brighter*</u> in a parallel circuit.
- In a series circuit, the current through each bulb is constant (because there is only one path). The voltage across the entire circuit is fixed, but the voltage across each component splits according to the component's resistance. A component with less resistance (*e.g.,* a light bulb with a higher "wattage" rating) "uses up" less of the total voltage and therefore uses less power. This means a bulb with a higher "wattage" rating will be <u>*dimmer*</u> in a series circuit.

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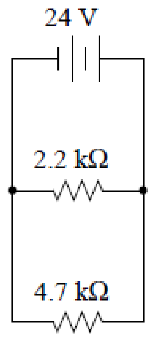
* For an incandescent bulb, assuming a 120 V household circuit.

† For an incandescent bulb. Note that light bulbs are not ohmic resistors, meaning their resistance changes as the current changes. These values are for bulbs in a 120 V household circuit.

Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** Fill in the table for the following circuit:

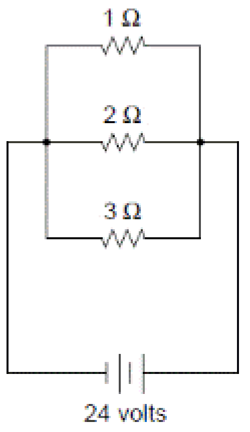


	R_1	R_2	Total
Voltage (ΔV)			24 V
Current (I)			
Resist. (R)	2 200 Ω	4 700 Ω	
Power (P)			

(The space below is intentionally left blank for calculations.)

Use this space for summary and/or additional notes:

2. **(S)** Fill in the table for the following circuit:

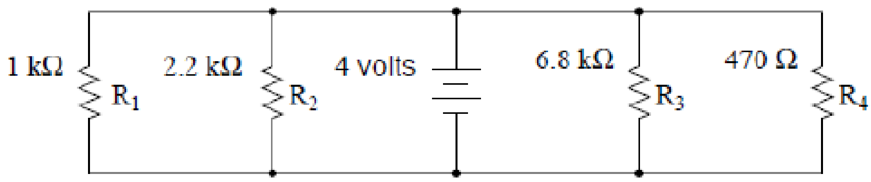


	R_1	R_2	R_3	Total
Voltage (ΔV)				24 V
Current (I)				
Resist. (R)	1 Ω	2 Ω	3 Ω	
Power (P)				

(The space below is intentionally left blank for calculations.)

Use this space for summary and/or additional notes:

3. **(M)** Fill in the table for the following circuit:



	R_1	R_2	R_3	R_4	Total
Voltage (ΔV)					4 V
Current (I)					
Resistance (R)	1 000 Ω	2 200 Ω	6 800 Ω	470 Ω	
Power (P)					

(The space below is intentionally left blank for calculations.)

Use this space for summary and/or additional notes:

Mixed Series & Parallel Circuits (Resistance Only)

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives: 4.E.4.1, 4.E.4.2, 4.E.4.3, 4.E.5.1, 4.E.5.2, 4.E.5.3

Mastery Objective(s): (Students will be able to…)

- Calculate voltage, current, resistance and power in mixed series & parallel circuits.

Success Criteria:

- Correct relationships are applied for voltage, current, resistance and power in mixed series & parallel circuits.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the relationships for voltages, current, resistance and power in mixed series & parallel circuits.

Tier 2 Vocabulary: series, parallel

Labs, Activities & Demonstrations:

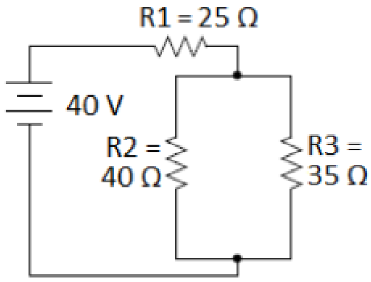
- light bulb mystery circuits

Notes:

If a circuit has mixed series and parallel sections, you can determine the various voltages, currents and resistances by applying Kirchhoff's Rules and/or by "simplifying the circuit." Simplifying the circuit, in this case, means replacing groups of resistors that are in series or parallel with a single resistor of equivalent resistance.

Use this space for summary and/or additional notes:

For example, suppose we need to solve the following mixed series & parallel circuit for voltage, current, resistance and power for each resistor:



Because the circuit has series and parallel sections, we cannot simply use the series and parallel rules across the entire table.

	R_1	R_2	R_3	Total
Voltage (ΔV)				40 V
Current (I)				
Resistance (R)	25 Ω	40 Ω	35 Ω	
Power (P)				

We can use Ohm's Law $(\Delta V = IR)$ and the power equation $(P = I\Delta V)$ on each individual resistor and the totals for the circuit (columns), but we need two pieces of information for each resistor in order to do this.

Our strategy will be:

1. Simplify the resistor network until all resistances are combined into one equivalent resistor to find the total resistance.
2. Use $\Delta V = IR$ to find the total current.
3. Work backwards through your simplification, using the equations for series and parallel circuits in the appropriate sections of the circuit until you have all of the information.

Step 1: If we follow the current through the circuit, we see that it goes through resistor R1 first. Then it splits into two parallel pathways. One path goes through R2 and R3, and the other goes through R4 and R5.

There is no universal shorthand for representing series and parallel components, so let's define the symbols "—" to show resistors in series, and "||" to show resistors in parallel. The above network of resistors could be represented as:

R1 — (R2 || R3)

Now, we simplify the network just like a math problem—start with the innermost parentheses and work your way out.

Use this space for summary and/or additional notes:

Step 2: Combine the parallel 40 Ω and 35 Ω resistors into a single equivalent resistance:

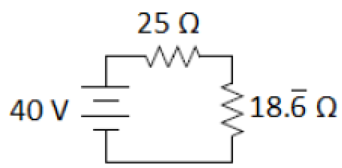
$$25\,\Omega - (40\,\Omega \parallel 35\,\Omega) \longrightarrow 25\,\Omega - (R_{eq.,||})$$

$$\frac{1}{R_{total}} = \frac{1}{40} + \frac{1}{35}$$

$$\frac{1}{R_{total}} = 0.0250 + 0.0286 = 0.0536$$

$$R_{total} = \frac{1}{0.0536} = 18.\overline{6}\,\Omega$$

Now our circuit is equivalent to:



Step 3: Add the two resistances in series to get the total combined resistance of the circuit:

$$25\,\Omega - 18.\overline{6}\,\Omega \longrightarrow R_{total}$$

$$18.\overline{6} + 25 = 43.\overline{6}\,\Omega$$

This gives:

	R_1	R_2	R_3	Total
Voltage (ΔV)				40 V
Current (I)				
Resistance (R)	25 Ω	40 Ω	35 Ω	***$43.\overline{6}$ Ω***
Power (P)				

Use this space for summary and/or additional notes:

Step 4: Now that we know the total voltage and resistance, we can use Ohm's Law to find the total current:

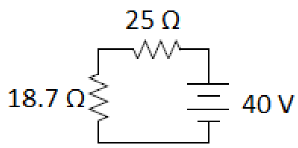
$$\Delta V = IR$$
$$40 = I(43.\overline{6})$$
$$I = \frac{40}{43.\overline{6}} = 0.916\text{ A}$$

While we're at it, let's use $P = I\Delta V = (0.916)(40) = 36.6\text{ W}$ to find the total power.

	R_1	R_2	R_3	Total
Voltage (ΔV)				40 V
Current (I)				***0.916 A***
Resistance (R)	25 Ω	40 Ω	35 Ω	$43.\overline{6}\,\Omega$
Power (P)				***36.6 W***

Now we work backwards.

The next-to-last simplification step was:



The 25 Ω resistor is R1. All of the current goes through it, so the current through R1 must be 0.916 A. Using Ohm's Law, this means the voltage drop across R1 must be:

$$\Delta V = IR = (0.916)(25) = 22.9\text{ V}$$

and the power must be:

$$P = I\Delta V = (0.916)(22.9) = 21.0\text{ W}$$

This means that the voltage across the parallel portion of the circuit (R2 ‖ R3) must be 40 – 22.9 = 17.1 V. Therefore, the voltage is 17.1 V across *both* parallel branches (because voltage is the same across parallel branches).

	R_1	R_2	R_3	Total
Voltage (ΔV)	***22.9 V***	***17.1 V***	***17.1 V***	40 V
Current (I)	***0.916 A***			0.916 A
Resistance (R)	25 Ω	40 Ω	35 Ω	$43.\overline{6}\,\Omega$
Power (P)	***21.0 W***			36.6 W

Use this space for summary and/or additional notes:

We can use this and Ohm's Law to find the current through one branch:

$$\Delta V_{40\,\Omega} = \Delta V_{35\,\Omega} = 40 - \Delta V_1 = 40 - 22.9 = 17.1 V$$

$$\Delta V_{40\,\Omega} = I_{40\,\Omega} R_{40\,\Omega}$$

$$I_{40\,\Omega} = \frac{\Delta V_{40\,\Omega}}{R_{40\,\Omega}} = \frac{17.1}{40} = 0.428\ A$$

We can use Kirchhoff's Junction Rule to find the current through the other branch:

$$I_{total} = I_{40\,\Omega} + I_{35\,\Omega}$$
$$0.916 = 0.428 + I_{35\,\Omega}$$
$$I_{35\,\Omega} = 0.488\ \text{A}$$

This gives us:

	R_1	R_2	R_3	Total
Voltage (ΔV)	22.9 V	17.1 V	17.1 V	40 V
Current (I)	0.916 A	***0.428 A***	***0.488 A***	0.916 A
Resistance (R)	25 Ω	40 Ω	35 Ω	$43.\overline{6}\,\Omega$
Power (P)	21.0 W			36.6 W

Finally, because we now have current and resistance for each of the resistors R_2 and R_3, we can use $P = I\Delta V$ to find the power:

$$P_2 = I_2 \Delta V_2 = (0.428)(17.1) = 7.32\ \text{W}$$

$$P_3 = I_3 \Delta V_3 = (0.488)(17.1) = 8.34\ \text{W}$$

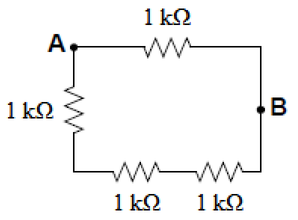
	R_1	R_2	R_3	Total
Voltage (ΔV)	22.9 V	17.1 V	17.1 V	40 V
Current (I)	0.916 A	0.428 A	0.488 A	0.916 A
Resistance (R)	25 Ω	40 Ω	35 Ω	$43.\overline{6}\,\Omega$
Power (P)	21.0 W	***7.32 W***	***8.34 W***	36.6 W

Alternately, because the total power is the sum of the power of each component, once we had the power in all but one resistor, we could have subtracted from the total to find the last one.

Use this space for summary and/or additional notes:

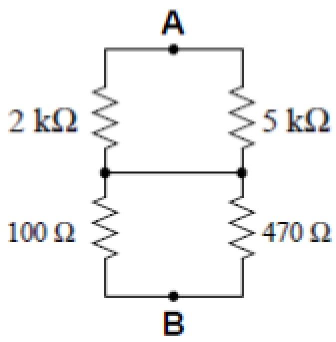
Homework Problems

1. **(M)** What is the equivalent resistance between points **A** and **B**?



Answer: 750 Ω

2. **(M)** What is the equivalent resistance between points **A** and **B**?



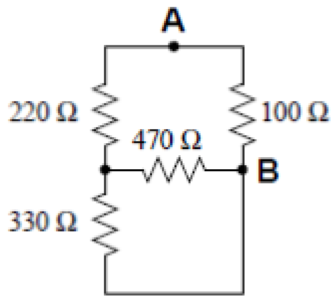
Hints:

- *Convert the resistances from* kΩ *to* Ω*.*
- *Redrawing the circuit to separate the top and bottom halves may make it easier to understand what is going on.*

Answer: 1511 Ω or 1.511 kΩ

Use this space for summary and/or additional notes:

3. **(M)** What is the equivalent resistance between points **A** and **B**?

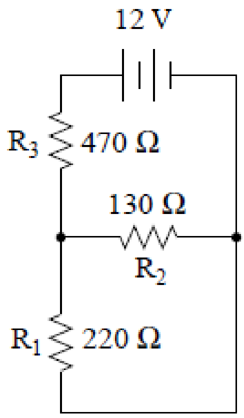


(The space below is intentionally left blank for calculations.)

Answer: 80.5 Ω

Use this space for summary and/or additional notes:

4. **(M)** Fill in the table for the circuit below:



	R_1	R_2	R_3	Total
Voltage (ΔV)				12 V
Current (I)				
Resistance (R)	220 Ω	130 Ω	470 Ω	
Power (P)				

(The space below is intentionally left blank for calculations.)

Use this space for summary and/or additional notes:

Measuring Voltage, Current & Resistance

Unit: DC Circuits

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 4.E.5.3

Mastery Objective(s): (Students will be able to…)

- Accurately measure voltage and current in a DC circuit.

Success Criteria:

- Multimeter wires are plugged in to the correct jacks and dial is set to the correct quantity.
- Measurements are taken at appropriate points in the circuit. (Voltage is measured in parallel and current is measured in series.)

Language Objectives:

- Explain how to set up the multimeter correctly.
- Explain where to take the measurements and why.

Tier 2 Vocabulary: meter

Labs, Activities & Demonstrations:

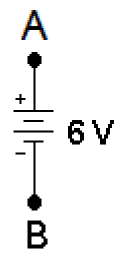
- Show & tell with digital multi-meter.
- Measurement of voltages and currents in a live DC circuit.

Notes:

Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance in each component of a circuit. In order to analyze actual circuits, it is necessary to be able to measure these quantities.

Measuring Voltage

Suppose we want to measure the electric potential (voltage) across the terminals of a 6 V battery. The diagram would look like this:



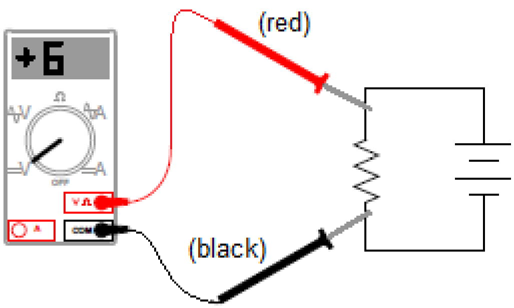
The voltage between points A and B is either +6 V or −6 V, depending on the direction. The voltage from A to B (positive to negative) is +6 V, and the voltage from B to A (negative to positive) is −6 V.

Use this space for summary and/or additional notes:

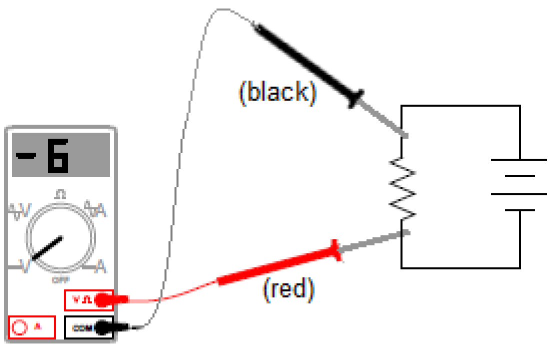
To measure voltage:

1. The circuit needs to be powered up with current flowing through it.
2. Make sure the red lead is plugged into the V Ω socket (for measuring volts or ohms).
3. Make sure that the voltmeter is set for volts (DC or AC, as appropriate).
4. Touch the two leads *in parallel* with the two points you want to measure the voltage across. (Remember that voltage is the same across all branches of a parallel circuit. You want the voltmeter to be one of the branches, and the circuit to be the other branch with the same voltage.)

On a voltmeter (a meter that measures volts or voltage), positive voltage means the current is going from the red (+) lead to the black (−) lead. In the following circuit, if you put the red (+) lead on the end of a resistor that is closer to the positive terminal of the battery, and the black (−) lead on the end that is closer to the negative terminal, the voltage reading will be positive. In this circuit, the voltmeter reads +6 V:



However, if you reverse the leads so that the black (−) lead is closer to the positive terminal and the red (+) lead is closer to the negative terminal, the voltage reading will be negative. In this circuit, the voltmeter reads −6 V:



The reading of −6 V indicates that the current is actually flowing in the opposite direction from the way the voltmeter is measuring—from the black (−) lead to the red (+) lead.

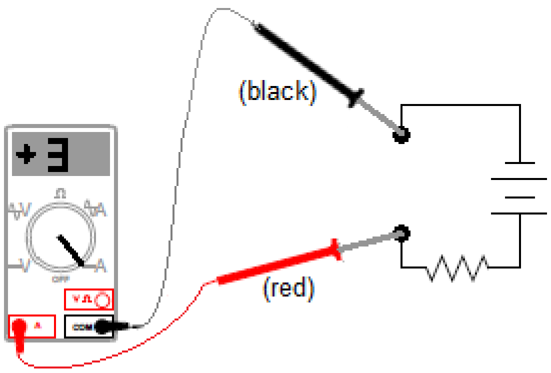
Use this space for summary and/or additional notes:

Measuring Current

To measure current:

1. The circuit needs to be open between the two points where you want to measure the current.
2. Make sure the red lead is plugged into appropriate socket (10 A if the current is expected to be 0.5 A or greater; 1 A or mA/μA if the current is expected to be less than 0.5 A).
3. Make sure the ammeter is set for amperes (A), milliamperes (mA) or microamperes (μA) AC or DC, depending on what you expect the current in the circuit to be.
4. Touch one lead to each of the two contact points, so that the ammeter is *in series* with the rest of the circuit. (Remember that current is the same through all components in a series circuit. You want to measure all of the current, so you want all of the current to flow through the meter.)

On an ammeter (a meter that measures current), the current is measured assuming that it is flowing from the red (+) lead to the black (−) lead. In the following circuit, if you put the red (+) lead on the side that is connected to the positive terminal and the black (−) lead on the end that is connected to the negative terminal, the current reading would be positive. In this circuit, the current is +3 A:

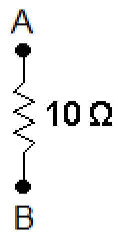


As with the voltage example above, if you switched the leads, the reading would be −3 A instead of +3 A.

Use this space for summary and/or additional notes:

Measuring Resistance

Resistance does not have a direction. If you placed an ohmmeter (a meter that measures resistance) across points A and B, it would read 10 Ω regardless of which lead is on which point.



An ohmmeter supplies a voltage across the component and measures the current. Because the voltage supplied is constant, the Ohm's Law calculation is built into the meter and the readout displays the resistance.

To measure resistance:

1. The circuit needs to be open. Because the meter is applying a voltage and measuring current, you do not want other voltages or currents in the circuit.
2. Make sure the red lead is plugged into the V Ω socket (for measuring volts or ohms).
3. Make sure that the voltmeter is set for Ω.
4. Touch one lead to each end of the resistor.

It is sometimes easier and/or more reliable to measure the voltage and current and calculate resistance using Ohm's Law ($V = IR$).

Use this space for summary and/or additional notes:

Capacitance

Unit: DC Circuits

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 4.E.4.1, 4.E.4.2, 4.E.4.3, 4.E.5.1, 4.E.5.2, 4.E.5.3

Mastery Objective(s): (Students will be able to…)

- Solve problems involving relationships between capacitance, charge and voltage.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Describe what a capacitor does.

Tier 2 Vocabulary: charge, capacitance

Labs, Activities & Demonstrations:

- build a capacitor

Notes:

capacitor: an electrical component that stores electrical charge but does not allow current to flow through.

When a voltage is applied to the circuit, one side of the capacitor will acquire a positive charge, and the other side will acquire an equal negative charge. This process is called *charging the capacitor*.

When a charged capacitor is placed in a circuit (perhaps it was charged previously, and then the voltage source is switched off), charge flows out of the capacitor into the circuit. This process is called *discharging the capacitor*.

No current actually flows through the capacitor, but as it charges, the positive charges that accumulate on one side of the capacitor repel positive charges from the other side into the rest of the circuit. This means that ***an uncharged capacitor acts like a wire*** when it first begins to charge.

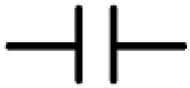
Use this space for summary and/or additional notes:

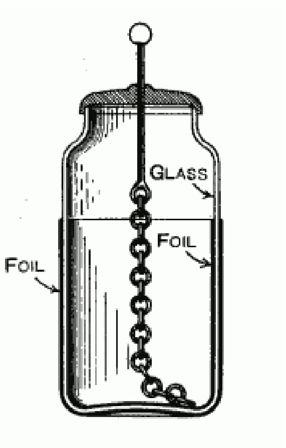
Once the capacitor is fully charged, the amount of potential difference in the circuit is unable to add any more charge, and no more charges flow. This means that ***a fully-charged capacitor*** *in a circuit that has a power supply (e.g., a battery)* ***acts like an open switch or a broken wire***.

If you disconnect the battery and reconnect the capacitor to a circuit that allows the capacitor to discharge, charges will flow out of the capacitor and through the circuit. This means that *a fully-charged capacitor in a circuit without a separate power supply acts like a battery* when it first begins to discharge.

Toys from joke shops that shock people use simple battery-and-capacitor circuits. The battery charges the capacitor gradually over time until a significant amount of charge has built up. When the person grabs the object, the person completes a circuit that discharges the capacitor, resulting in a sudden, unpleasant electric shock.

The simplest capacitor (conceptually) is a pair of parallel metal plates separated by a fixed distance. The symbol for a capacitor is a representation of the two parallel plates.





The first capacitors were made independently in 1745 by the German cleric Ewald Georg von Kleist and by the Dutch scientist Pieter van Musschenbroek. Both von Kleist and van Musschenbroek lined a glass jar with metal foil on the outside and filled the jar with water. (Recall that water with ions dissolved in it conducts electricity.) Both scientists charged the device with electricity and received a severe shock when they accidentally discharged the jars through themselves.

This type of capacitor is named after is called a Leyden jar, after the city of Leiden (Leyden) where van Musschenbroek lived.

Modern Leyden jars are lined on the inside and outside with conductive metal foil. As a potential difference is applied between the inside and outside of the jar, charge builds up between them. The glass, which acts as an insulator (a substance that does not conduct electricity), keeps the two pieces of foil separated and does not allow the charge to flow through.

Because the thickness of the jar is more or less constant, the Leyden jar behaves like a parallel plate capacitor.

Use this space for summary and/or additional notes:

Shortly after the invention of the Leyden jar, Daniel Gralath discovered that he could connect several jars in parallel to increase the total possible stored charge. Benjamin Franklin compared this idea with a “battery” of cannon. (The original meaning of the term “battery” was a collection of cannon for the purpose of battering the enemy.) The term is now used to describe a similar arrangement of electrochemical cells.

Franklin’s most famous experiment was to capture the charge from a lightning strike in Leyden jars, proving that lightning is an electric discharge.

capacitance: a measure of the ability of a capacitor to store charge. Capacitance is measured in farads (F), named after the English physicist Michael Faraday.

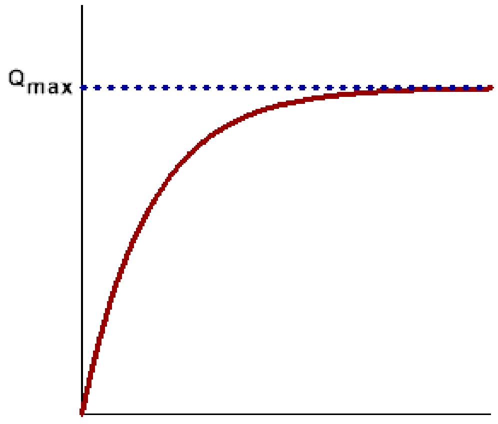
Capacitance is the ratio of the charge stored by a capacitor to the voltage applied:

$$C = \frac{Q}{\Delta V} \quad \text{which is often represented as} \quad Q = C\Delta V$$

Thus one farad is one coulomb per volt. Note, however, that one farad is a ridiculously large amount of capacitance. The capacitors in most electrical circuits are in the millifarad (mF) to picofarad (pF) range.

Capacitance is the theoretical limit of the charge that a capacitor could store at a given potential difference (voltage) if the charge were allowed to build up over an infinite amount of time.

As a capacitor is charged, the positive side increasingly repels additional positive charges coming from the voltage source, and the negative side increasingly repels additional negative charges. This means that the capacitor charges rapidly at first, but the amount of charge stored decreases exponentially as the charge builds up.



Note that Q_{max} is sometimes labeled Q_o. Be careful—in this case, the subscript 0 does ***not*** necessarily mean at time = 0.

Use this space for summary and/or additional notes:

Energy Stored in a Capacitor

Recall that energy is the ability to do work, and that $W = \Delta U$. Because $W = qV$, if we keep the voltage constant and add charge to the capacitor:

$$W = \Delta U = \Delta V \Delta q$$

Applying calculus* gives:

$$dU = \Delta V\, dq \quad \text{and therefore} \quad U = \int_0^Q \Delta V\, dq = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C}$$

Because $Q = C\Delta V$, we can substitute $C\Delta V$ for Q, giving the equation for the stored (potential) energy in a capacitor:

$$U_C = \frac{Q^2}{2C} = \tfrac{1}{2} QV = \tfrac{1}{2} CV^2$$

* Because this is not a calculus-based course, you are not responsible for understanding this derivation. However, you do need to be able to use the resulting equations.

Use this space for summary and/or additional notes:

honors (not AP®)

Parallel-Plate Capacitors and Dielectrics

The capacitance of a parallel plate capacitor is given by the following equation:

$$C = \kappa \varepsilon_o \frac{A}{d}$$

where:

$C =$ capacitance
$\kappa =$ relative permittivity (dielectric constant), vacuum $\equiv 1$
$\varepsilon_o =$ electrical permittivity of free space $= 8.85 \times 10^{-12} \ \frac{\text{F}}{\text{m}}$
$A =$ cross-sectional area
$d =$ distance between the plates of the capacitor

When a capacitor is fully charged, the distance between the plates can be so small that a spark could jump from one plate to the other, shorting out and discharging the capacitor. In order to prevent this from happening, the space between the plates is often filled with a chemical (often a solid material or an oil) called a dielectric.

A dielectric is an electrical insulator (charges do not move, which reduces the possibility of the capacitor shorting out), but has a relatively high value of electric permittivity (ability to support an electric field).

Dielectrics in capacitors serve the following purposes:

- Keep the conducting plates from coming in contact, allowing for smaller plate separations and therefore higher capacitances.
- Increase the effective capacitance by reducing the electric field strength, which allows the capacitor to hold same charge at a lower voltage.
- Reduce the possibility of the capacitor shorting out by sparking (more formally known as dielectric breakdown) during operation at high voltage.

Note that a higher value of κ and lower value of d both enable the capacitor to have a higher capacitance.

Commonly used solid dielectrics include porcelain, glass or plastic. Common liquid dielectrics include mineral oil or castor oil. Common gaseous dielectrics include air, nitrogen and sulfur hexafluoride.

Use this space for summary and/or additional notes:

Capacitors in Series and Parallel Circuits

Unit: DC Circuits

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 4.E.4.1, 4.E.4.2, 4.E.4.3, 4.E.5.1, 4.E.5.2, 4.E.5.3

Mastery Objective(s): (Students will be able to…)

- Calculate voltage, capacitance, charge and potential energy in series and parallel circuits.

Success Criteria:

- Correct relationships are applied for each quantity
- Variables are correctly identified and substituted correctly into the correct equations and algebra is correct.

Language Objectives:

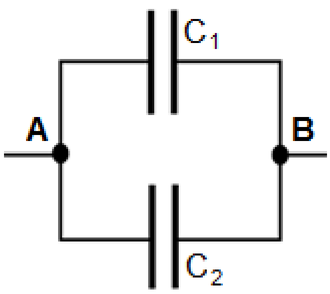
- Explain how capacitors behave similarly to and different from resistors in series and parallel circuits.

Tier 2 Vocabulary: charge, capacitance

Notes:

Capacitors in Parallel

When capacitors are connected in parallel:



The voltage between point **A** and point **B** must be $\Delta V = V_A - V_B$, regardless of the path.

The charge on capacitor C_1 must be $Q_1 = C_1 \Delta V$, and the charge on capacitor C_2 must be $Q_2 = C_2 \Delta V$.

The total charge must be:

$$Q_1 + Q_2 = C_1 \Delta V + C_2 \Delta V = (C_1 + C_2)\Delta V$$

Therefore:

$$C_{eq} = \frac{Q_{total}}{\Delta V} = \frac{Q_1 + Q_2}{\Delta V} = C_1 + C_2$$

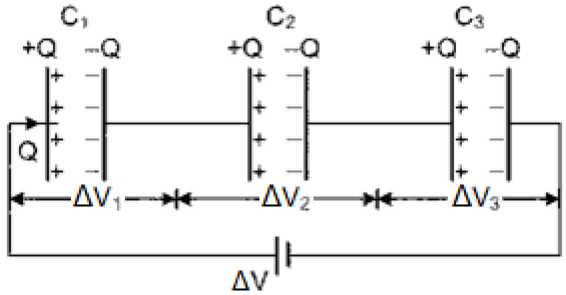
Generalizing this relationship, when capacitors are arranged in parallel, the total capacitance is the sum of the capacitances of the individual capacitors:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

Use this space for summary and/or additional notes:

Capacitors in Series

In a series circuit, the voltage from one end to the other is divided among the components.



Note that the segment of the circuit that goes from the right side of C_1 to the left side of C_2 is isolated from the rest of the circuit. Current does not flow through a capacitor, which means charges cannot enter or leave this segment. Because charge is conserved (electrical charges cannot be created or destroyed), this means the negative charge on C_1 (which is $-Q_1$) must equal the positive charge on C_2 (which is $+Q_2$).

By applying this same argument across each of the capacitors, *all of the charges across capacitors in series must be equal to each other and equal to the total charge in that branch of the circuit*. (Note that this is true *regardless* of whether C_1, C_2 and C_3 have the same capacitance.)

Therefore: $Q = Q_1 = Q_2 = Q_3$

Because the components are in series, we also know that $\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$

Because $\Delta V = \frac{Q}{C}$:

$$\Delta V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

Because $Q = Q_1 = Q_2 = Q_3$:

$$\Delta V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{\Delta V}{Q} = \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Generalizing this relationship, when capacitors are arranged in series, the total capacitance is the sum of the capacitances of the individual capacitors:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

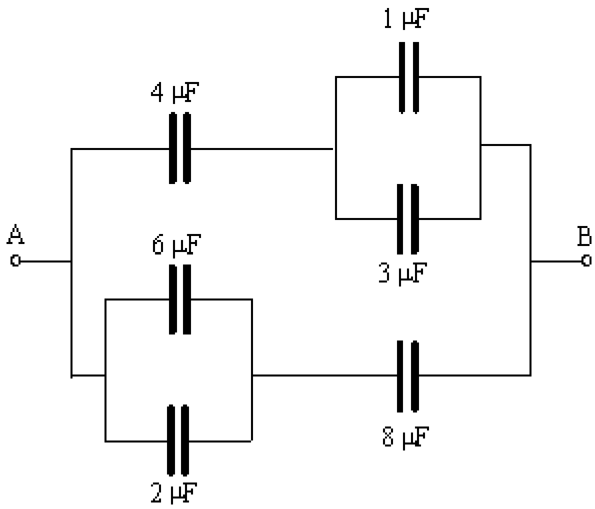
Use this space for summary and/or additional notes:

Mixed Series and Parallel Circuits

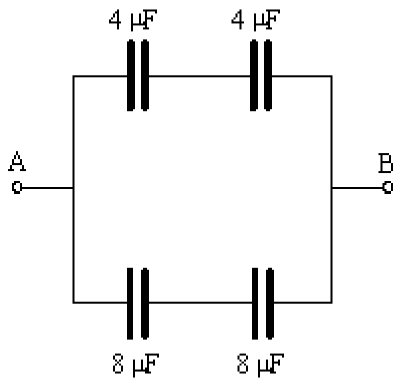
As with resistor networks, mixed circuits involving capacitors in series and in parallel can be simplified by replacing each set of capacitors with an equivalent capacitance, starting with the innermost set of capacitors and working outwards (much like simplifying an equation by starting with the innermost parentheses and working outwards).

Sample Problem:

Simplify the following circuit:



First we would add the capacitances in parallel. On top, $3\,\mu\text{F} + 1\,\mu\text{F} = 4\,\mu\text{F}$. On the bottom, $6\,\mu\text{F} + 2\,\mu\text{F} = 8\,\mu\text{F}$. This gives the following equivalent circuit:



Use this space for summary and/or additional notes:

Next, we combine the capacitors in series on top:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

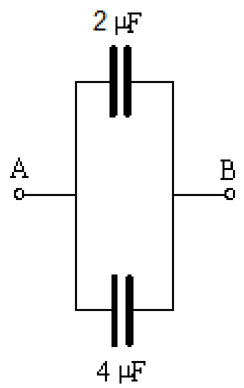
$$\frac{1}{C_{eq}} = \frac{1}{2}; \quad \therefore C_{eq} = 2\ \mu\text{F}$$

and the capacitors in series on the bottom:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$$

$$\frac{1}{C_{eq}} = \frac{1}{4}; \quad \therefore C_{eq} = 4\ \mu\text{F}$$

This gives the following equivalent circuit:



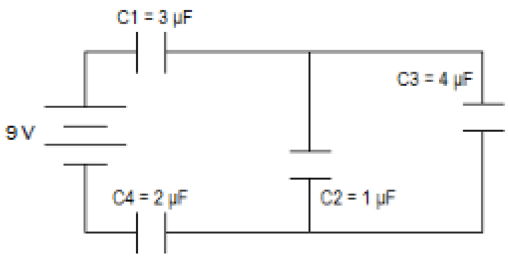
Finally, we combine the last two capacitances in parallel, which gives:

$$C_{eq} = C_1 + C_2 = 2 + 4 = 6\ \mu\text{F}$$

Use this space for summary and/or additional notes:

Sample Problem

Q: Fill in the table for the following circuit:



Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV					9
Capacitance (μF)	C	3	1	4	2	
Charge (μC)						
Energy (μJ)						

A: Let's start by combining the two capacitors in parallel (C2 & C3) to make an equivalent capacitor.

$$C_* = C_2 + C_3 = 1\,\mu\text{F} + 4\,\mu\text{F} = 5\,\mu\text{F}$$

Now we have three capacitors in series: C_1, C_*, and C_4:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_*} + \frac{1}{C_4} = \frac{1}{3} + \frac{1}{5} + \frac{1}{2} = 0.\overline{3} + 0.2 + 0.5 = 1.0\overline{3}$$

$$C_{total} = \frac{1}{1.0\overline{3}} = 0.9677\,\mu\text{F}$$

Now we can calculate Q for the total circuit from $Q = C\Delta V$:

$$Q = C\Delta V = (9)(0.9677) = 8.710\,\mu\text{C}$$

Now we have:

Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV					9
Capacitance (μF)	C	3	1	4	2	**0.9677**
Charge (μC)	Q					**8.710**
Energy (μJ)	U					

Use this space for summary and/or additional notes:

Next, because the charge is equal across all capacitors in series, we know that $Q_1 = Q_* = Q_4 = Q_{total}$, which gives:

Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV					9
Capacitance (μF)	C	3	1	4	2	0.9677
Charge (μC)	Q	**8.710**			**8.710**	8.710
Energy (μJ)	U					

Now we can calculate V_1 and V_4 from $Q = C\Delta V$.

We can also calculate the energy from $U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV	**2.903**			**4.355**	9
Capacitance (μF)	C	3	1	4	2	0.9677
Charge (μC)	Q	8.710			8.710	8.710
Energy (μJ)	U	**12.64**			**18.96**	**39.19**

We know that voltages in series add, so $\Delta V_{total} = \Delta V_1 + \Delta V_* + \Delta V_4$, which means $9 = 2.903 + \Delta V_* + 4.355$, which gives $\Delta V_* = 1.742\text{ V}$.

Because C_2 and C_3 (and therefore ΔV_2 and ΔV_3) are in parallel, $\Delta V_* = \Delta V_2 = \Delta V_3 = 1.742\text{ V}$:

Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV	2.903	**1.742**	**1.742**	4.355	9
Capacitance (μF)	C	3	1	4	2	0.9677
Charge (μC)	Q	8.710			8.710	8.710
Energy (μJ)	U	12.64			18.96	39.19

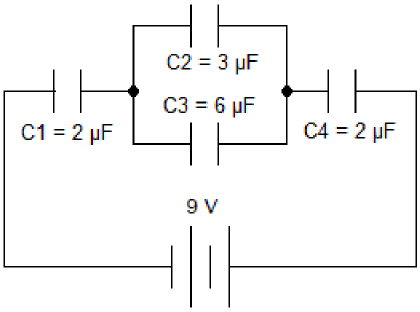
Finally, we can calculate Q from $Q = C\Delta V$ and U from $U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$:

Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV	2.903	1.742	1.742	4.355	9
Capacitance (μF)	C	3	1	4	2	0.9677
Charge (μC)	Q	8.710	**1.742**	**6.968**	8.710	8.710
Energy (μJ)	U	12.64	**1.52**	**6.07**	18.96	39.19

Use this space for summary and/or additional notes:

Homework Problems

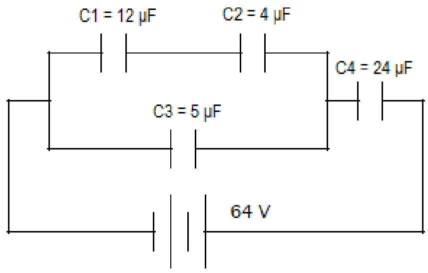
1. **(S)** Fill in the table for the following circuit:



Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV					9
Capacitance (μF)	C	2	3	6	2	
Charge (μC)	Q					
Energy (μJ)	U					

Use this space for summary and/or additional notes:

2. **(M)** Fill in the table for the following circuit:



Quantity (unit)	**Var.**	**C1**	**C2**	**C3**	**C4**	**Total**
Voltage (V)	ΔV					64
Capacitance (μF)	C	12	4	5	24	
Charge (μC)	Q					
Energy (μJ)	U					

Use this space for summary and/or additional notes:

DC Resistor-Capacitor (RC) Circuits

Unit: DC Circuits

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 4.E.4.1, 4.E.4.2, 4.E.4.3, 4.E.5.1, 4.E.5.2, 4.E.5.3

Mastery Objective(s): (Students will be able to…)

- Solve problems involving time-varying circuits with charging and discharging capacitors.

Success Criteria:

- Correct relationships are applied for each quantity
- Variables are correctly identified and substituted correctly into the correct equations and algebra is correct.

Language Objectives:

- Explain why a discharged capacitor behaves like a wire, and why a fully-charged capacitor behaves like an open switch.

Tier 2 Vocabulary: charge, capacitance, resistance

Labs, Activities & Demonstrations:

- RC circuit lab

Notes:

<u>RC circuit</u>: a circuit involving combinations of resistors and capacitors.

In an RC circuit, the amount and direction of current change with time as the capacitor charges or discharges. The amount of time it takes for the capacitor to charge or discharge is determined by the combination of the capacitance and resistance in the circuit. This makes RC circuits useful for intermittent (*i.e.,* with a built-in delay) back-and-forth switching. Some common uses of RC circuits include:

- clocks
- windshield wipers
- pacemakers
- synthesizers

When we studied resistor-only circuits, the circuits were steady-state, *i.e.,* voltage and current remained constant. RC circuits are time-variant, *i.e.,* the voltage, current, and charge stored in the capacitor(s) are all changing with time.

Use this space for summary and/or additional notes:

Charging a Capacitor

Recall that a capacitor is an electrical component that stores charge. No current actually flows through the capacitor. Recall also that capacitance (*C*) is a capacitor's ability to be charged by a given electric potential difference (voltage). Therefore, the maximum charge that a capacitor can hold is:

$$Q_{max} = C\Delta V$$

In the previous section, the charge that we calculated was actually this maximum charge Q_{max}, which is the amount of charge that the capacitor would hold if it had been charged for "a long time" such that it was fully charged.

However, recall also that:

- In a capacitor with zero charge, every charge placed on one side causes an equivalent charge on the opposite side. This means:

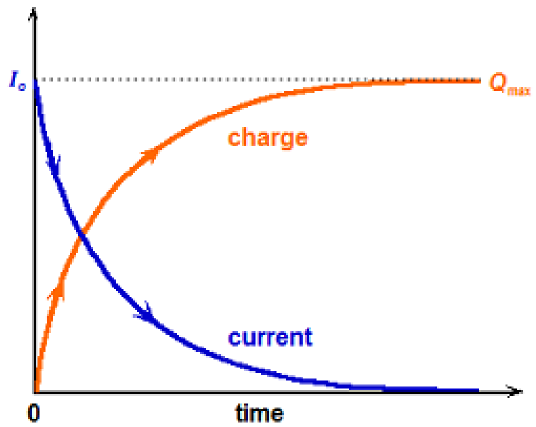
  With respect to current, a capacitor with zero charge initially behaves like a wire.

- When a capacitor is fully charged, no additional charge can be added (unless the voltage is increased). This means:

  With respect to current, a fully-charged capacitor behaves like an open circuit.

This means that the behavior of the capacitor changes as the charges build up inside of it.

When a capacitor that initially has zero charge is connected to a voltage source, the current that flows through the circuit decreases exponentially, and the charge stored in the capacitor asymptotically approaches Q_{max}, the maximum charge that can be stored in that capacitor for the voltage applied.



(Note that the graphs are not to scale; the y-axis scale and units are necessarily different for charge and current.)

Use this space for summary and/or additional notes:

The equations for the charge in a capacitor and the current that flows "through" it as a function of time while a capacitor is charging are:

$$I = I_o e^{-t/RC} = \frac{\Delta V}{R} e^{-t/RC}$$

$$Q = Q_{max}(1 - e^{-t/RC}) = C\Delta V(1 - e^{-t/RC})$$

where:

I = current (A)
I_o = initial current (just after switch was closed) (A)
ΔV = voltage (V)
Q = charge (C)
Q_{max} = (theoretical) maximum charge stored by capacitor at the circuit's voltage (C)
e = base of exponential function = 2.71828…
t = time since switch was closed (s)
R = resistance (Ω)
C = capacitance (F)

We can rearrange the above equations to give:

$$\frac{I}{I_o} = 1 - \frac{Q}{Q_{max}} = e^{-t/RC}$$

Use this space for summary and/or additional notes:

The RC term in the exponent is known as the time constant (τ) for the circuit. Larger values of RC mean the circuit takes longer to charge the capacitor. The following table shows the rate of decrease in current in the charging circuit and the rate of increase in charge on the capacitor as a function of time:

t	$\frac{I}{I_o}=\frac{\Delta V}{\Delta V_o}=e^{-t/RC}$	$\frac{Q}{Q_{\max}}=1-e^{-t/RC}$
0	1	0
¼ RC	0.78	0.22
½ RC	0.61	0.39
0.69 RC	0.5	0.5
RC	0.37	0.63
2 RC	0.14	0.86
4 RC	0.02	0.98
10 RC	4.5×10^{-5}	≈ 1

Note that the half-life of the charging (and discharging) process is approximately 0.69 RC.

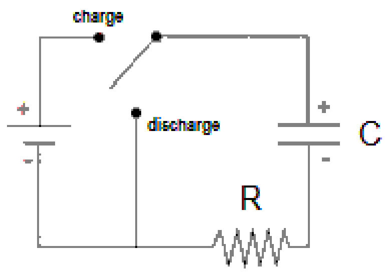
Note also that while Q_{max} depends on the voltage applied, the rate of charging and discharging depend only on the resistance and capacitance in the circuit.

Use this space for summary and/or additional notes:

Discharging a Capacitor

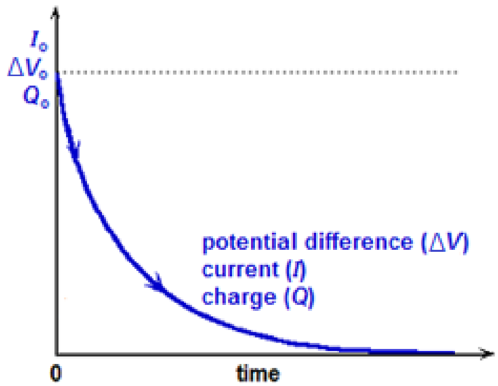
Just as a capacitor charges gradually, it also discharges gradually.

Imagine we have a circuit like the one at the right.



When the switch is in the "charge" position, the battery charges the capacitor. When the switch is flipped to the "discharge" position, the battery is switched out and the circuit contains only the capacitor and the resistor.

When this happens, the capacitor discharges (loses its charge). The capacitor acts as a temporary voltage source, and current temporarily flows out of the capacitor through the resistor.



(Note again that the graphs are not to scale; the y-axis scale and units are necessarily different for current, voltage and charge.)

The driving force for this temporary current is the repulsion from the stored charges in the capacitor. As charge leaves the capacitor there is less repulsion, which causes the voltage and current to decrease exponentially along with the charge.

The equations for discharging a capacitor are therefore identical in form to the equations for charging it:

$$\Delta V = \Delta V_o e^{-t/RC} \qquad Q = Q_o e^{-t/RC} \qquad I = I_o e^{-t/RC}$$

or, dividing each by its value at time zero:

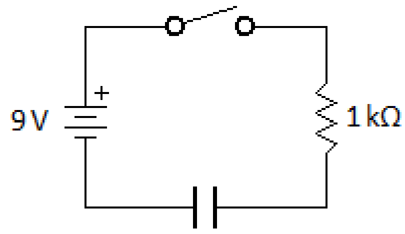
$$\frac{\Delta V}{\Delta V_o} = \frac{Q}{Q_o} = \frac{I}{I_o} = e^{-t/RC}$$

Again the time constant, *RC*, is the relative amount of time it takes for the charge remaining in the capacitor and the voltage and current in the circuit to decay. (Refer to the table on page 396.)

Use this space for summary and/or additional notes:

Sample Problem

Q: A circuit has a 9 V battery, an open switch, a 1 kΩ resistor, and a capacitor in series. The capacitor has no residual charge.



When the switch is closed, the charge in the capacitor climbs to 86 % of its maximum value in 50 ms. What is the capacitance of the capacitor?

A: The charge increases at the rate of:

$$Q(t) = Q_{max}(1 - e^{-t/RC})$$

We are given that $\frac{Q}{Q_{max}} = 0.86$, $t = 0.05\,\text{s}$, and $R = 1000\,\Omega$.

$$\frac{Q}{Q_{max}} = 1 - e^{-t/RC}$$

$$0.86 = 1 - e^{-0.05/1000\,C}$$

$$-0.14 = -e^{-0.05/1000\,C}$$

$$\ln(0.14) = \ln(e^{-0.05/1000\,C}) = \frac{-0.05}{1000\,C}$$

$$-1.97 = \frac{-0.05}{1000\,C}$$

$$1970\,C = 0.05$$

$$C = \frac{0.05}{1970} = 2.5 \times 10^{-5}\,\text{F} = 25\,\mu\text{F}$$

Note that we could have solved this problem by using the table on page 396 to see that the capacitor is 86 % charged $\left(\frac{Q}{Q_{max}} = 0.86\right)$ when $t = 2\,RC$.

Use this space for summary and/or additional notes:

Homework Problems

1. **(S)** A series RC circuit consists of a 9 V battery, a 3 Ω resistor, a 6 μF capacitor and a switch. How long would it take after the switch is closed for the capacitor to reach 63 % of its maximum potential difference?

   Answer: 18 μs

2. **(M)** A circuit contains a 9 V battery, an open switch, a 1 kΩ resistor, and a capacitor, all in series. The capacitor initially has no charge. When the switch is closed, the charge on the capacitor climbs to 86 % of its maximum value in 50 ms. What is the capacitance of the capacitor?

   Answer: 25 μF

Use this space for summary and/or additional notes:

3. **(S)** A heart defibrillator has a capacitance of 25 µF and is charged to a potential difference of 350 V. The electrodes of the defibrillator are attached to the chest of a patient who has suffered a heart attack. The initial current that flows out of the capacitor is 10 mA.

    a. How much time does it take for the current to fall to 0.5 mA?

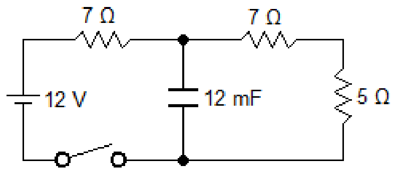
    Answer: 2.62 s

    b. How much charge is left on the defibrillator plates after 1.2 s?

    Answer: 2.22 mC

Use this space for summary and/or additional notes:

4. **(M)** In the following RC circuit:



the switch (S) has been closed for a long time.

a. When the switch is opened, how much time does it takes for charge on the capacitor to drop to 13.5% of its original value?

Answer: 0.288 s

b. What is the maximum current through the 5 resistor the instant the switch is opened?

Answer: 0.63 A

Use this space for summary and/or additional notes:

Introduction: Magnetism & Electromagnetism

Unit: Magnetism & Electromagnetism

Topics covered in this chapter:

This chapter discusses electricity and magnetism, how they behave, and how they relate to each other.

- *Magnetism* describes properties of magnets and what causes objects to be magnetic.
- *Magnetic Fields & Magnetic Flux* explains magnetic fields and magnetic flux and how it is calculated.
- *Electromagnetism* describes the relationship between electric fields and magnetic fields, and how changes in one induce changes in the other.
- *Devices that Use Electromagnetism* lists devices that combine electricity and magnetism and explains how they work.

One of the challenges encountered in this chapter is understanding which set of equations applies to a given situation.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-5. Provide evidence that an electric current can produce a magnetic field and that a changing magnetic field can produce an electric current.

HS-PS3-5. Develop and use a model of magnetic or electric fields to illustrate the forces and changes in energy between two magnetically or electrically charged objects changing relative position in a magnetic or electric field, respectively.

Use this space for summary and/or additional notes:

AP® only

AP® Physics 2 Learning Objectives:

2.C.4.1: The student is able to distinguish the characteristics that differ between monopole fields (gravitational field of spherical mass and electrical field due to single point charge) and dipole fields (electric dipole field and magnetic field) and make claims about the spatial behavior of the fields using qualitative or semi-quantitative arguments based on vector addition of fields due to each point source, including identifying the locations and signs of sources from a vector diagram of the field. [**SP 2.2, 6.4, 7.2**]

2.D.1.1: The student is able to apply mathematical routines to express the force exerted on a moving charged object by a magnetic field. [**SP 2.2**]

2.D.2.1: The student is able to create a verbal or visual representation of a magnetic field around a long straight wire or a pair of parallel wires. [**SP 1.1**]

2.D.3.1: The student is able to describe the orientation of a magnetic dipole placed in a magnetic field in general and the particular cases of a compass in the magnetic field of the Earth and iron filings surrounding a bar magnet. [**SP 1.2**]

2.D.4.1: The student is able to use the representation of magnetic domains to qualitatively analyze the magnetic behavior of a bar magnet composed of ferromagnetic material. [**SP 1.4**]

3.A.2.1: The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. [**SP 1.1**]

3.A.3.2: The student is able to challenge a claim that an object can exert a force on itself. [**SP 6.1**]

AP® only

3.A.3.3: The student is able to describe a force as an interaction between two objects and identify both objects for any force. [**SP 1.4**]

3.A.4.1: The student is able to construct explanations of physical situations involving the interaction of bodies using Newton's third law and the representation of action-reaction pairs of forces. [**SP 1.4**, **6.2**]

3.A.4.2: The student is able to use Newton's third law to make claims and predictions about the action-reaction pairs of forces when two objects interact. [**SP 6.4**, **7.2**]

3.A.4.3: The student is able to analyze situations involving interactions among several objects by using free-body diagrams that include the application of Newton's third law to identify forces. [**SP 1.4**]

3.C.3.1: The student is able to use right-hand rules to analyze a situation involving a current-carrying conductor and a moving electrically charged object to determine the direction of the magnetic force exerted on the charged object due to the magnetic field created by the current-carrying conductor. [**SP 1.4**]

Use this space for summary and/or additional notes:

AP® only

3.C.3.2: The student is able to plan a data collection strategy appropriate to an investigation of the direction of the force on a moving electrically charged object caused by a current in a wire in the context of a specific set of equipment and instruments and analyze the resulting data to arrive at a conclusion. [**SP 4.2**, **5.1**]

4.E.1.1: The student is able to use representations and models to qualitatively describe the magnetic properties of some materials that can be affected by magnetic properties of other objects in the system. [**SP 1.1**, **1.4**, **2.2**]

4.E.2.1: The student is able to construct an explanation of the function of a simple electromagnetic device in which an induced emf is produced by a changing magnetic flux through an area defined by a current loop (i.e., a simple microphone or generator) or of the effect on behavior of a device in which an induced emf is produced by a constant magnetic field through a changing area. [**SP 6.4**]

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Magnetism**, such as permanent magnets, fields caused by currents, particles in magnetic fields, Faraday's law, and Lenz's law.

1. Permanent Magnets
2. Magnetic Force on Charges
3. Magnetic Force on Current-Carrying Wires
4. The Magnetic Field Due to a current
5. Motional EMF
6. Faraday's Law

Skills learned & applied in this chapter:

- Working with material-specific constants from a table.

Use this space for summary and/or additional notes:

Magnetism

Unit: Magnetism & Electromagnetism

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 2.4.C.1, 2.D.2.1, 2.D.3.1, 2.D.4.1, 4.E.1.1

Mastery Objective(s): (Students will be able to…)

- List and explain properties of magnets.

Success Criteria:

- Explanations account for observed behavior.

Language Objectives:

- Explain why we call the ends of a magnet "north" and "south".

Tier 2 Vocabulary: magnet

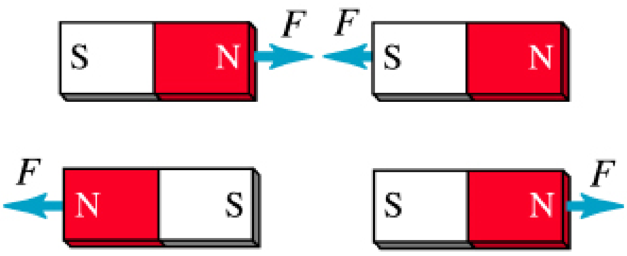
Labs, Activities & Demonstrations:

- neodymium magnets
- ring magnets repelling each other on a dowel
- magnets attracting each other across a gap

Notes:

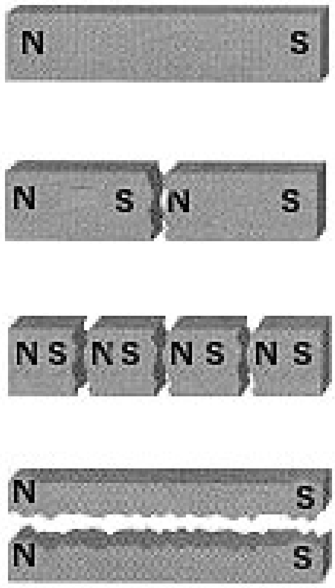
magnet: a material with electrons that can align in a manner that attracts or repels other magnets.

A magnet has two ends or "poles", called "north" and "south". If a magnet is allowed to spin freely, the end that points toward the north on Earth is called the north end of the magnet. The end that points toward the south on Earth is called the south end of the magnet. (The Earth's magnetic poles are near, but not in exactly the same place as its geographic poles.)All magnets have a north and south pole. As with charges, opposite poles attract, and like poles repel.



Use this space for summary and/or additional notes:

If you were to cut a magnet in half, each piece would be a magnet with its own north and south pole:



Electrons and Magnetism

Honors (not AP®)

Magnetism is caused by unpaired electrons in atoms. Electrons within atoms reside in energy regions called "orbitals". Each orbital can hold up to two electrons.

If two electrons share an orbital, they have opposite spins. (Note that the electrons are not actually spinning. "Spin" is the term for the intrinsic property of certain subatomic particles that is believed to be responsible for magnetism.) This means that if one electron aligns itself with a magnetic field, the other electron in the same orbital becomes aligned to oppose the magnetic field, and there is no net force.

However, if an orbital has only one electron, that electron is free to align with the magnetic field, which causes an attractive force between the magnet and the magnetic material. For example, as you may have learned in chemistry, the electron configuration for iron is:

unpaired electrons

1s	2s	2p			3s	3p			4s	3d				
↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑	↑	↑	↑

The inner electrons are paired up, but four of the electrons in the 3d sublevel are unpaired, and are free to align with an external magnetic field.

Use this space for summary and/or additional notes:

Honors (not AP®)

Magnetic measurements and calculations involve fields that act over 3-dimensional space and change continuously with position. This means that most calculations relating to magnetic fields need to be represented using multivariable calculus, which is beyond the scope of this course.

magnetic permeability (magnetic permittivity): the ability of a material to support the formation of a magnetic field. Magnetic permeability is represented by the variable μ. The magnetic permeability of empty space is $\mu_0 = 4\pi \times 10^{-7} \frac{\text{N}}{\text{A}^2}$.

diamagnetic: a material whose electrons cannot align with a magnetic field. Diamagnetic materials have very low magnetic permeabilities.

paramagnetic: a material that has electrons that can align with a magnetic field. Paramagnetic materials have relatively high magnetic permeabilities.

ferromagnetic: a material that can form crystals with permanently-aligned electrons, resulting in a permanent magnet. Ferromagnetic materials can have very high magnetic permeabilities. Some naturally-occurring materials that exhibit ferromagnetism include iron, cobalt, nickel, gadolinium, dysprosium, and magnetite (Fe_3O_4).

magnetic susceptibility: a measure of the degree of magnetization of a material when it is placed into a magnetic field.

Use this space for summary and/or additional notes:

Magnetic Fields & Magnetic Flux

Unit: Magnetism & Electromagnetism

MA Curriculum Frameworks (2016): HS-PS3-5

AP® Physics 2 Learning Objectives: 2.C.4.1, 2.D.3.1, 2.D.4.1, 4.E.2.1

Mastery Objective(s): (Students will be able to…)

- Describe and draw magnetic fields.
- Calculate magnetic flux.

Success Criteria:

- Magnetic field lines connect north and south poles of the magnet.
- Arrows on field lines point from north to south.

Language Objectives:

- Explain how a compass works.

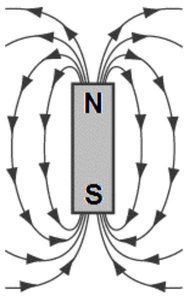
Tier 2 Vocabulary: field, north pole, south pole

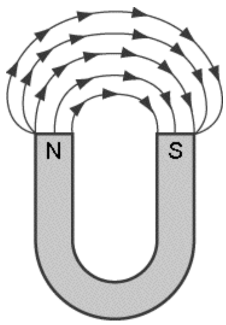
Labs, Activities & Demonstrations:

- magnetic field demonstrator plate
- placing various objects into the gap between two magnets
- ferrofluid
- representation of flux as dots on a balloon

Notes:

magnetic field ($\vec{B}$) : a force field (region in which a force acts on objects that have a certain property) in which magnetic attraction and repulsion are occurring. Similar to an electric field, we represent a magnetic field by drawing field lines. Magnetic field lines point from the north pole of a magnet toward the south pole, and they show the direction that the north end of a compass or magnet would be deflected if it was placed in the field:





Use this space for summary and/or additional notes:

The strength of a magnetic field is measured in teslas (T), named after the physicist Nikola Tesla. One tesla is the magnetic field strength necessary to produce one newton of force when a particle that has a charge of one coulomb is moved through the magnetic field at a velocity of one meter per second. Because of the relationship between magnetism, forces, and electricity, one tesla can be expressed as many different combinations of units:

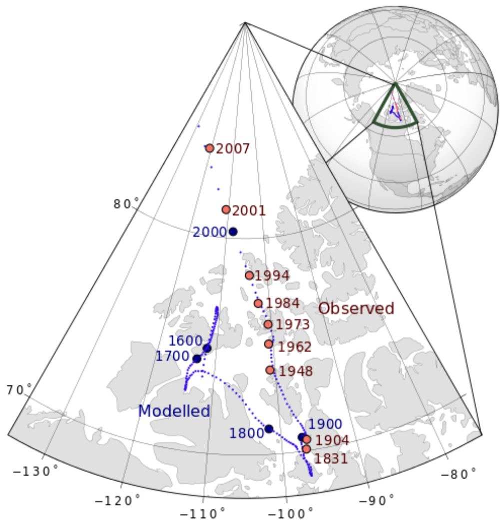
$$1\text{ T} \equiv 1\frac{\text{V}\cdot\text{s}}{\text{m}^2} \equiv 1\frac{\text{N}}{\text{A}\cdot\text{m}} \equiv 1\frac{\text{J}}{\text{A}\cdot\text{m}^2} \equiv 1\frac{\text{kg}}{\text{C}\cdot\text{s}} \equiv 1\frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}} \equiv 1\frac{\text{kg}}{\text{A}\cdot\text{s}^2}$$

The Earth's Magnetic Field

The inner and outer core of the Earth are made of iron, which has a high magnetic susceptibility. The very high temperature of the inner core causes convection currents in the molten iron in the outer core.

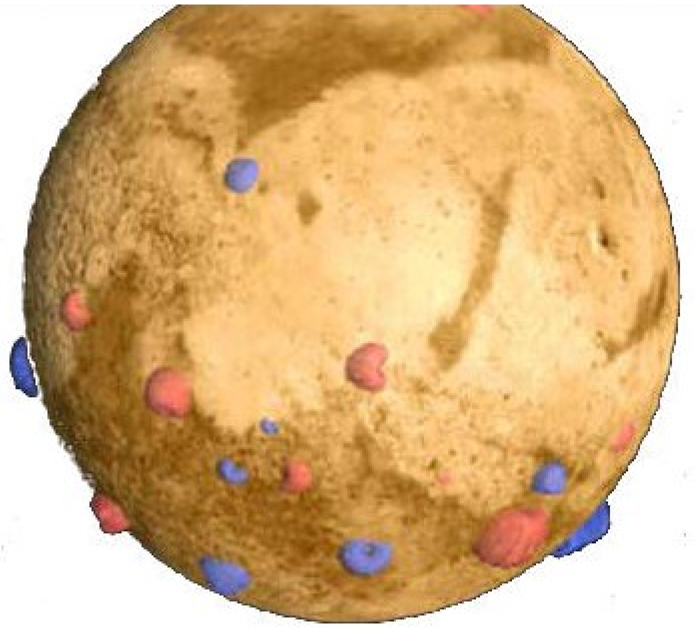
The rapid rotation of the Earth causes the molten iron in the outer core to swirl. The swirling iron causes a magnetic field over the entire planet.

Because the core of the Earth is in constant motion, the Earth's magnetic field is constantly changing. The exact location of the Earth's magnetic north and south poles varies by about 80 km over the course of each day because of the rotation of the Earth. Its average location (shown on the map of Northern Canada below) drifts by about 50 km each year:



Use this space for summary and/or additional notes:

Not all planets have a planetary magnetic field. Mars, for example, is believed to have once had a planetary magnetic field, but the planet cooled off enough to disrupt the processes that caused it. Instead, Mars has some very strong localized magnetic fields that were formed when minerals cooled down in the presence of the planetary magnetic field.



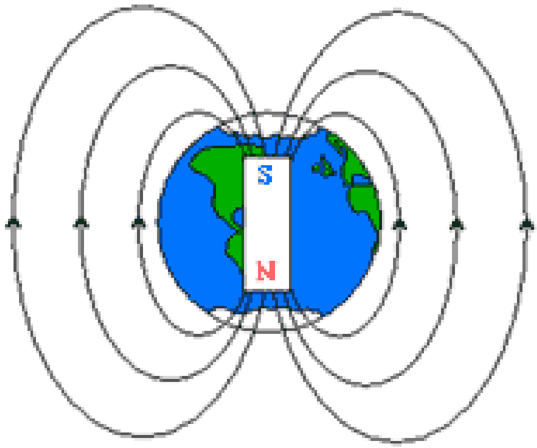
In this picture, the blue and red areas represent regions with strong localized magnetic fields. On Mars, a compass could not be used in the ways that we use a compass on Earth; if you took a compass to Mars, the needle would point either toward or away from each these regions.

Jupiter, on the other hand, has a planetary magnetic field twenty times as strong as that of Earth. This field may be caused by water with dissolved electrolytes or by liquid hydrogen.

Use this space for summary and/or additional notes:

Recall that the north pole of a magnet is the end that points toward the north on Earth. This must mean that if the Earth is a giant magnet, one of its magnetic poles must be near the geographic north pole, and the other magnetic pole must be near the geographic south pole.

For obvious reasons, the Earth's magnetic pole near the north pole is called the Earth's "north magnetic pole" or "magnetic north pole". Similarly, the Earth's magnetic pole near the south pole is called the Earth's "south magnetic pole" or "magnetic south pole".



However, because the north pole of a magnet points toward the north, the Earth's north magnetic pole (meaning its location) must therefore be the *south* pole of the giant magnet that is the Earth.

Similarly, because the south pole of a magnet points toward the south, the Earth's south magnetic pole (meaning its location) must therefore be the *north* pole of the giant Earth-magnet.

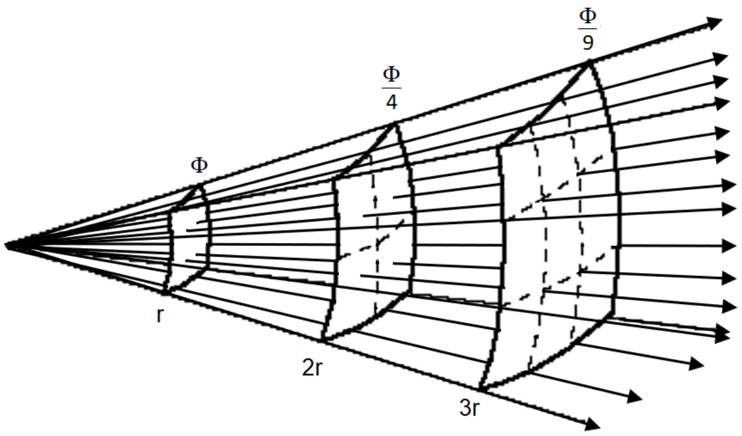
Unfortunately, the term "magnetic north pole," "north magnetic pole" or any other similar term almost always means the magnetic pole that is in the north part of the Earth. There is no universally-accepted way to name the poles of the Earth-magnet.

Use this space for summary and/or additional notes:

Magnetic Flux

flux: the flow of fluid, energy or particles across a given area.

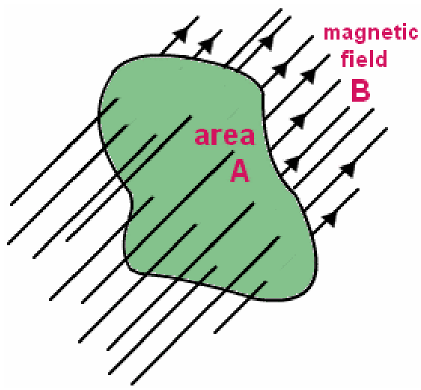
If a quantity (such as a magnetic field) originates from a point, the field spreads out and the amount of flux through a given area decreases as the square of the distance from that point.



magnetic flux (Φ): the total amount of a magnetic field that passes through a surface.

Stronger magnetic fields are generally shown with a higher density of field lines. Using this representation, you can think of the magnetic flux as the number of field lines that pass through an area.

The equation for magnetic flux is Faraday's Law, named for the English physicist Michael Faraday. The equation is usually presented as a surface integral, but in algebraic form it looks like the following:



$$\Phi = \vec{B} \cdot \vec{A}$$

where:

Φ = magnetic flux (Wb)

$\vec{B}$ = strength of magnetic field (T)

$\vec{A}$ = area of the region of interest that the magnetic field passes through (m^2)

The unit for magnetic flux is the weber (Wb). One tesla is one weber per square meter.

$$1\,\text{T} \equiv 1\,\frac{\text{Wb}}{\text{m}^2}$$

Use this space for summary and/or additional notes:

Electromagnetism

Unit: Magnetism & Electromagnetism

MA Curriculum Frameworks (2016): HS-PS2-5

AP® Physics 2 Learning Objectives: 3.C.3.1, 3.C.3.2, 4.E.2.1

Mastery Objective(s): (Students will be able to…)

- Describe and explain ways that electric and magnetic fields affect each other.
- Calculate the voltage and current changes in a step-up or step-down transformer.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Voltage and current changes are described accurately.

Language Objectives:

- Explain how various devices work including solenoids, electromagnets and electric motors.

Tier 2 Vocabulary: force, field

Labs, Activities & Demonstrations:

- current-carrying wire in a magnetic field
- electromagnet
- two coils wired together and two rare earth magnets, one on a spring
- electric motor
- magnet through copper pipe (Lenz's Law)
- wire & galvanometer jump rope
- neodymium magnet & CRT screen

Notes:

A changing electric field produces a magnetic field, and a changing magnetic field produces an electric field. These actions can produce magnetic forces or electromagnetic forces (emf).

Use this space for summary and/or additional notes:

Magnetic Fields and Moving Charges

Like gravitational and electric fields, a magnetic field is a force field. (Recall that force fields are vector quantities, meaning that they have both magnitude and direction.) The strength of a magnetic field is measured in teslas (T), named after the Serbian-American physicist Nikola Tesla.

$$1\,\text{T} = 1\,\frac{\text{N}}{\text{A}\cdot\text{m}}$$

In the 1830s, physicists Michael Faraday and Joseph Henry each independently discovered that an electric current could be produced by moving a magnet through a coil of wire, or by moving a wire through a magnetic field. This process is called electromagnetic induction.

If we move a conductive rod or wire that has length *L* at a velocity *v* through a magnetic field of strength *B*, the magnetic forces send positive charges to one end of the rod and negative charges to the other. This creates a potential difference (emf) between the ends of the rod:

$$\varepsilon = vBL$$

If the rod or wire is part of a closed loop (circuit), then the induced ε produces a current around the closed loop. From Ohm's Law $\varepsilon = IR$, we get:

$$I = \frac{\varepsilon}{R} = \frac{vBL}{R}$$

Forces and Moving Charges

The force, $\vec{\boldsymbol{F}}$ on a charge *q* moving through a magnetic field $\vec{\boldsymbol{B}}$ with a velocity $\vec{\boldsymbol{v}}$ is given by the equation:

$$\vec{\boldsymbol{F}} = q(\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}) \qquad \text{and} \qquad F = qvB\sin\theta$$

Recall that current is just a flow of charges, which means that an electric current moving through a magnetic field creates a force on the wire carrying the current.

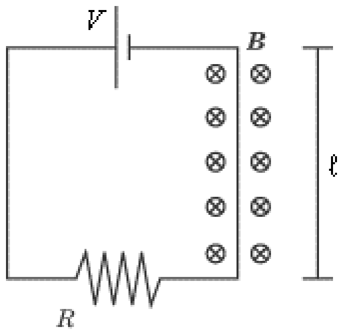
Recall that $\vec{\boldsymbol{I}} = \frac{\Delta Q}{t}$ and $\vec{\boldsymbol{v}} = \frac{\vec{\boldsymbol{d}}}{t} = \frac{\ell}{t}$, where ℓ is the length (distance) of the wire that passes through the magnetic field. This means that $q\vec{\boldsymbol{v}} = \ell\vec{\boldsymbol{I}}$, which we can use to create an equivalent equation:

$$\vec{\boldsymbol{F}} = \ell(\vec{\boldsymbol{I}} \times \vec{\boldsymbol{B}}) \qquad \text{and} \qquad F = \ell IB\sin\theta$$

Note that the direction of the cross products $\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$ and $\vec{\boldsymbol{I}} \times \vec{\boldsymbol{B}}$ can be determined using the right-hand rule.

Use this space for summary and/or additional notes:

A current passing through a magnetic field would be represented like this:



In the above diagram, the battery has voltage *V*, the resistor has resistance *R*, and the length of wire passing through the magnetic field is ℓ.

The magnetic field strength is ***B***, and the field itself is denoted by the symbols ⊗ ⊗ ⊗ ⊗ ⊗ which denote a magnetic field going _into_ the page. (A field coming out of the page would be denoted by ⊙ ⊙ ⊙ ⊙ ⊙ instead. Think of the circle as an arrow inside a tube. The dot represents the tip of the arrow facing toward you, and the "X" represents the fletches (feathers) on the tail of the arrow facing away from you.)

For example, suppose we were given the following for the above diagram:

$$\vec{\boldsymbol{B}} = 4.0\times10^{-5}\text{ T}$$
$$V = 30\text{ V}$$
$$R = 5\,\Omega$$
$$\ell = 2\text{ m}$$

Current (from Ohm's Law):

$$V = IR$$
$$30 = I(5)$$
$$I = 6\text{A}$$

Force on the wire:

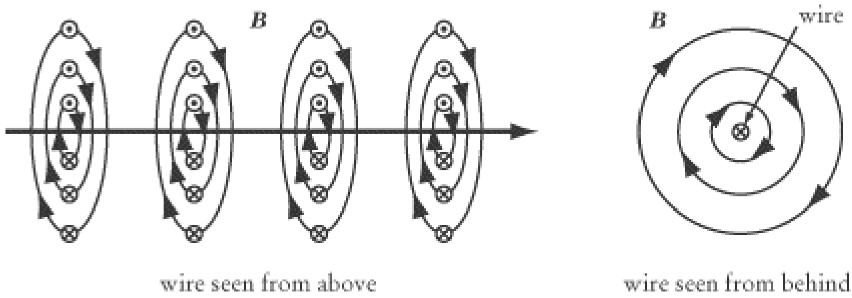
$$\vec{\boldsymbol{F}} = \ell(\vec{\boldsymbol{I}}\times\vec{\boldsymbol{B}})$$
$$F = \ell IB\sin\theta$$
$$F = (2)(6)(4.0\times10^{-5})\sin(90^\circ)$$
$$F = (2)(6)(4.0\times10^{-5})(1)$$
$$F = 4.8\times10^{-4}\text{N}$$

If the current is going upward through the magnetic field, and the magnetic field is pointing into the paper, then the right-hand rule tells us that the force would be directed to the left.

Use this space for summary and/or additional notes:

Magnetic Field Produced by Electric Current

An electric current moving through a wire also produces a magnetic field around the wire:



This time, we use the right-hand rule with our thumb pointing in the direction of the current, and our fingers curl in the direction of the magnetic field.

The strength of the magnetic field produced is given by the formula:

$$B = \frac{\mu_o}{2\pi}\frac{I}{r}$$

where B is the strength of the magnetic field, μ_o is the magnetic permeability of free space, I is the current, and r is the distance from the wire. (The variable r is used because the distance is in all directions, which means we would use polar or cylindrical coordinates.)

EMF Produced by Changing Magnetic Flux

A changing magnetic field produces an electromotive force (emf) in a loop of wire. This emf is given by the equation:

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{\Delta\Phi}{t}$$

(calculus) (algebraic)

If we replace the loop of wire with a coil that has n turns, the equation becomes:

$$\varepsilon = -n\frac{d\Phi}{dt} = -n\frac{\Delta\Phi}{t}$$

(calculus) (algebraic)

Use this space for summary and/or additional notes:

Combined Electric and Magnetic Fields

Recall that the force on a charged particle due to an electric field is:

$$\vec{F}_e = q\vec{E}$$

and that the force on a charged particle due to a magnetic field is:

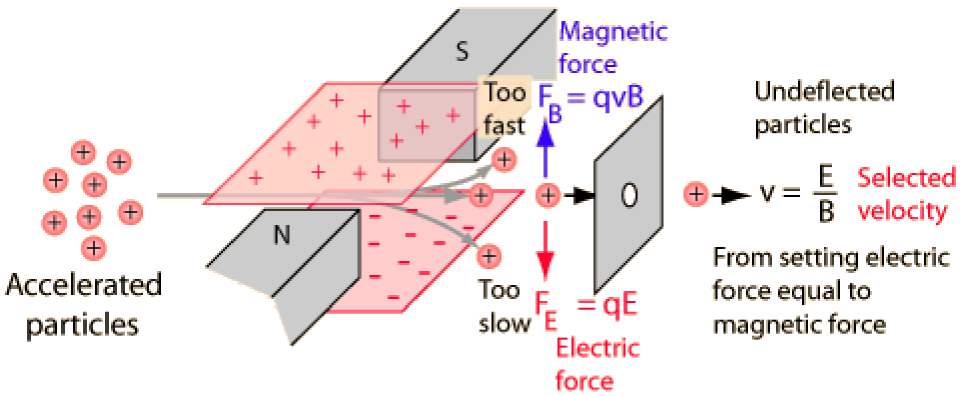
$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

Therefore, the force on a charged particle that interacts simultaneously with an electric field and a magnetic field must be the sum of the two:

$$\vec{F}_{EM} = q(\vec{E} + \vec{v} \times \vec{B})$$

An application of this combination is a particle sorter, which allows only particles with a given velocity to pass through.

Particles that enter a mass spectrometer must have the correct velocity in order for the mass spectrometer to be able to separate the particles properly. Before the particles enter the mass spectrometer, they first pass through a particle sorter, which applies opposing electric and magnetic forces to the particle:

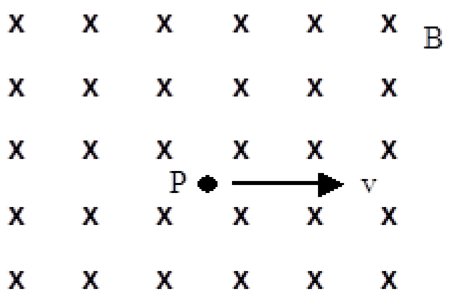


If the particles are moving too quickly, the magnetic force is stronger and the particles are deflected upwards. If the particles are moving too slowly, the electric force is stronger and the particles are deflected downwards. Particles with the desired velocity experience no net force and are not deflected.

Use this space for summary and/or additional notes:

Sample Problem:

Q: A proton has a velocity of $1\times10^{5}\ \frac{\text{m}}{\text{s}}$ to the right when it is at point P in a uniform magnetic field of 0.1 T that is directed into the page. Calculate the force (magnitude and direction) on the proton, and sketch its path.



A: The force on the proton is given by:

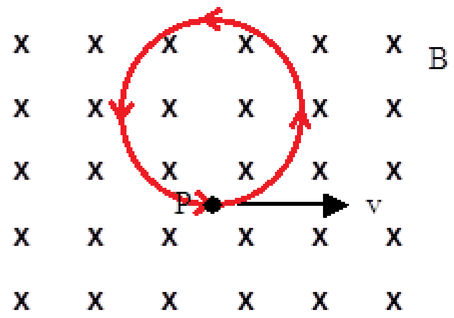
$$\vec{\boldsymbol{F}}_B = q(\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}) = qvB\sin\theta$$

Because the velocity of the particle and the direction of the magnetic field are perpendicular, $\sin\theta = \sin(90°) = 1$ and therefore the magnitude of $F_B = qvB$.

$$F_B = qvB$$
$$F_B = (1.6\times10^{-19})(1\times10^{5})(0.1)$$
$$F_B = 1.6\times10^{-15}\,\text{N}$$

The direction is given by the right-hand rule. Start with the fingers of your right hand pointing straight in the direction of velocity (to the right) and rotate your right hand until you can bend your fingers toward the magnetic field (into the page). Your thumb will be pointing upwards, which means that the force on a positively charged particle moving to the right is upwards. (Note that if the particle had been negatively charged, it would have moved in the opposite direction.)

However, note that the action of the force causes a change in the direction of the proton's velocity, and the change in the direction of the proton's velocity changes the direction of the force. This feedback loop results in the proton moving in a continuous circle.



Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** A wire 1 m long carries a current of 5 A. The wire is at right angles to a uniform magnetic field. The force on the wire is 0.2 N. What is the strength of the magnetic field?

   Answer: 0.04 T

2. **(S)** A wire 0.75 m long carries a current of 3 A. The wire is at an angle of 30° to a uniform magnetic field. The force on the wire is 0.5 N. What is the strength of the magnetic field?

   Answer: $0.\overline{4}$ T

3. **(M)** Two currents (one of 2 A and the other of 4 A) are arranged parallel to each other, with the currents flowing in the same direction. The wires are 3 m long and are separated by 8 cm (0.08 m). What is the net magnetic field at the midpoint between the two wires?

   Answer: 1×10^{-5} T

Use this space for summary and/or additional notes:

4. **(M)** A wire 2 m long moves perpendicularly through a 0.08 T field at a speed of $7\,\frac{m}{s}$.

   a. What emf is induced?

      Answer: 1.12 V

   b. If the wire has a resistance of 0.50 Ω, use Ohm's Law to find the current that flows through the wire.

      Answer: 2.24 A

Use this space for summary and/or additional notes:

Devices that Use Electromagnetism

Unit: Magnetism & Electromagnetism

MA Curriculum Frameworks (2016): HS-PS2-5

AP® Physics 2 Learning Objectives: 3.C.3.2

Mastery Objective(s): (Students will be able to…)

- Explain the basic design of solenoids, motors, generators, transformers and mass spectrometers.
- Calculate the voltage and power from a step-up or step-down transfromer.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Calculations are set up and executed correctly.

Language Objectives:

- Explain how various devices work including solenoids, electromagnets and electric motors.

Tier 2 Vocabulary: force, field

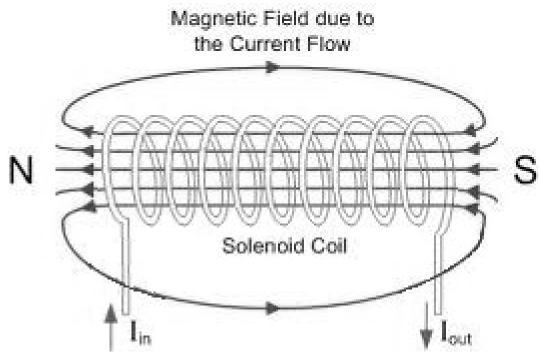
Labs, Activities & Demonstrations:

- build electromagnet
- build electric motor
- build speaker

Notes:

Solenoid

A solenoid is a coil made of fine wire. When a current is passed through the wire, it produces a magnetic field through the center of the coil.



When a current is applied, a permanent magnet placed in the center of the solenoid will be attracted or repelled and will move.

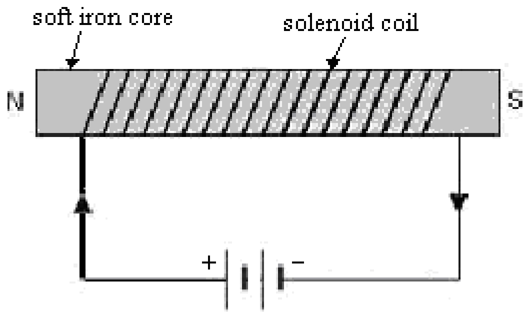
One of the most common uses of a solenoid is for electric door locks.

Use this space for summary and/or additional notes:

Electromagnet

An electromagnet is a device that acts as magnet only when electric current is flowing through it.

An electromagnet is made by placing a soft iron core in the center of a solenoid. The high magnetic permeability of iron causes the resulting magnetic field to become thousands of times stronger:



Because the iron core is not a permanent magnet, the electromagnet only works when current is flowing through the circuit. When the current is switched off, the electromagnet stops acting like a magnet and releases whatever ferromagnetic objects might have been attracted to it.

Of course, the above description is a simplification. Real ferromagnetic materials such as iron usually experience magnetic remanence, meaning that some of the electrons in the material remain aligned, and the material is weakly magnetized.

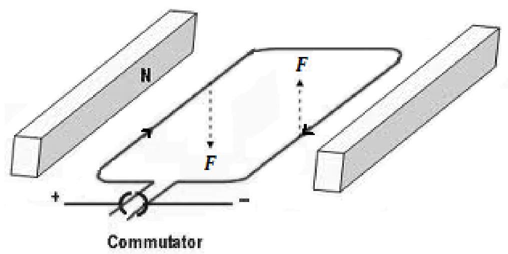
While magnetic remanence is undesirable in an electromagnet, it is the basis for magnetic computer storage media, such as audio and computer tapes and floppy and hard computer disks. To write information onto a disk, a disk head (an electromagnetic that can be moved radially) is pulsed in specific patterns as the disk spins. The patterns are encoded on the disk as locally magnetized regions.

When encoded information is read from the disk, the moving magnetic regions produce a changing electric field that causes an electric current in the disk head.

Use this space for summary and/or additional notes:

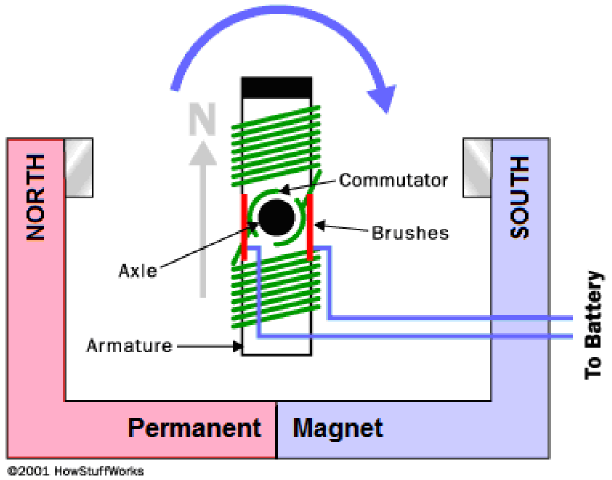
Motor

The force produced by a moving current in a magnetic field can be used to cause a loop of wire to spin:



A *commutator* is used to reverse the direction of the current as the loop turns, so that the combination of attraction and repulsion always applies force in the same direction.

If we replace the loop of wire with an electromagnet (a coil of wire wrapped around a material such as iron that has both a high electrical conductivity and a high magnetic permeability), the electromagnet will spin with a strong force.

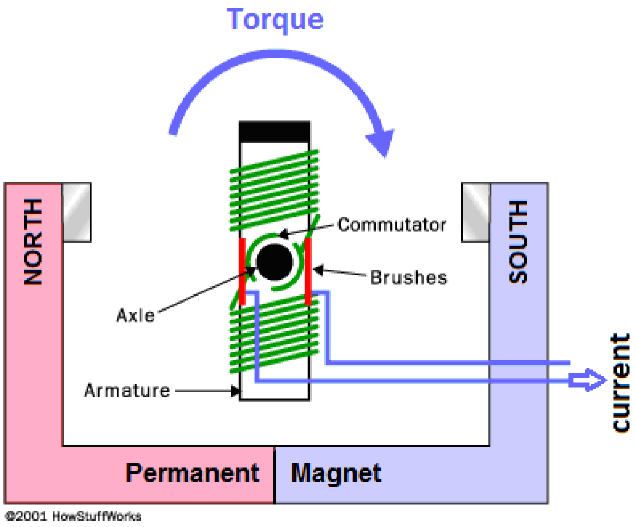


An electromagnet that spins because of its continuously switching attraction and repulsion to the magnetic field produced by a separate set of permanent magnets is called a *motor*. An electric motor turns electric current into rotational motion, which can be used to do work.

Use this space for summary and/or additional notes:

Generator

A *generator* uses the same components and operates under the same principle as a motor, except that a mechanical force is used to spin the coil. When the coil moves through the magnetic field, it produces an electric current. Thus a generator is a device that turns rotational motion (work) into an electric current.



Recall from the previous section that Lenz's Law gives the emf produced by the generator. Because the coil is rotating through a uniform magnetic field, the magnetic flux through the coil is constantly changing, which means calculus is needed to calculate the emf produced.

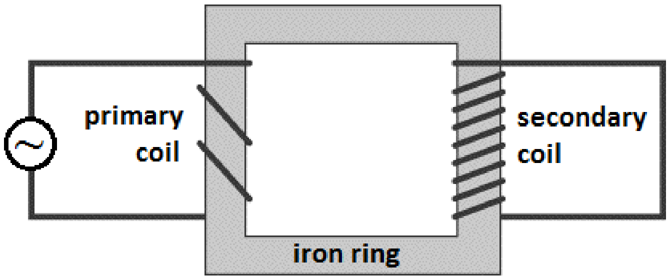
$$\varepsilon = -n\frac{d\Phi}{dt}$$

Use this space for summary and/or additional notes:

Inductor (Transformer)

Because electric current produces a magnetic field, a ring made of a ferromagnetic material can be used to move an electric current. An inductor (transformer) is a device that takes advantage of this phenomenon in order to increase or decrease the voltage in an AC circuit.

The diagram below shows an inductor or transformer.



The current on the input side (primary) generates a magnetic field in the iron ring. The magnetic field in the ring generates a current on the output side (secondary).

In this particular transformer, the coil wraps around the output side more times than the input. This means that each time the current goes through the coil, the magnetic field adds to the electromotive force (voltage). This means the voltage will increase in proportion to the increased number of coils on the output side. However, the magnetic field on the output side will produce less current with each turn, which means the current will decrease in the same proportion:

$$\frac{\#turns_{in}}{\#turns_{out}} = \frac{V_{in}}{V_{out}} = \frac{I_{out}}{I_{in}}$$

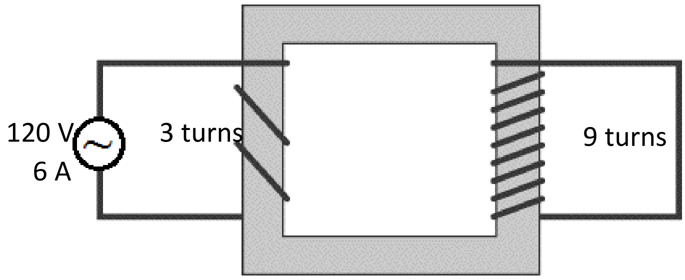
$$P_{in} = P_{out}$$

A transformer like this one, which produces an increase in voltage, is called a *step-up transformer*; a transformer that produces a decrease in voltage is called a *step-down transformer*.

Use this space for summary and/or additional notes:

Sample Problem:

If the input voltage to the following transformer is 120 V, and the input current is 6 A, what are the output voltage and current?



The voltage on either side of a transformer is proportional to the number of turns in the coil on that side. In the above transformer, the primary has 3 turns, and the secondary coil has 9 turns. This means the voltage on the right side will be $\frac{9}{3}=3$ times as much as the voltage on the left, or 360 V. The current will be $\frac{3}{9}=\frac{1}{3}$ as much, or 2 A.

We can also use:

$$\frac{\#turns_{in}}{\#turns_{out}}=\frac{V_{in}}{V_{out}}$$
$$\frac{3}{9}=\frac{120\text{ V}}{V_{out}}$$
$$V_{out}=360\text{ V}$$

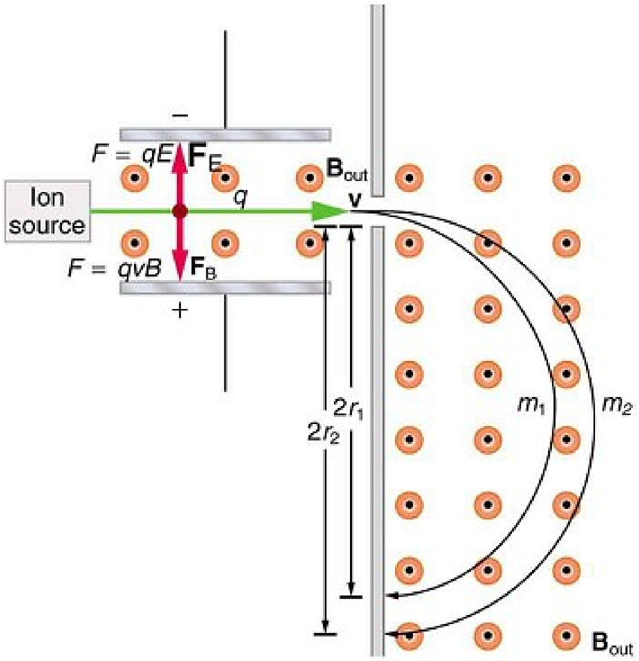
$$\frac{\#turns_{in}}{\#turns_{out}}=\frac{I_{out}}{I_{in}}$$
$$\frac{3}{9}=\frac{I_{out}}{6\text{ A}}$$
$$I_{out}=2\text{ A}$$

Use this space for summary and/or additional notes:

Mass Spectrometer

A mass spectrometer is a device uses the path of a charged particle in a magnetic field to determine its mass.

The particle is first selected for the desired velocity, as described on page 418. Then the particle enters a region where the only force on it is from the applied magnetic field. (In the example below, the magnetic field is directed out of the page.)



The magnetic field applies a force on the particle perpendicular to its path. As the particle's direction changes, the direction of the applied force changes with it, causing the particle to move in a circular path.

The path of the particle is the path for which the centripetal force (which you may recall from physics 1) is equal to the magnetic force:

$$F_B = F_c$$

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv^{\cancel{2}}}{q\cancel{v}B} = \frac{mv}{qB}$$

Thus if the particles are all ions with the same charge and are selected for having the same speed, the radius of the path will be directly proportional to the mass of the particle.

Use this space for summary and/or additional notes:

Introduction: Mechanical Waves

Unit: Mechanical Waves

Topics covered in this chapter:

This chapter discusses properties of waves that travel through a medium (mechanical waves).

- *Waves* gives general information about waves, including vocabulary and equations. *Reflection and Superposition* describes what happens when two waves share space within a medium.
- *Sound & Music* describes the properties and equations of waves that relate to music and musical instruments.
- *Sound Level* describes the decibel scale and how loudness is measured.
- *The Doppler Effect* describes the change in pitch due to motion of the source or receiver (listener).
- *Exceeding the Speed of Sound* describes the Mach scale and sonic booms.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling within various media. Recognize that electromagnetic waves can travel through empty space (without a medium) as compared to mechanical waves that require a medium.

AP® Physics 2 Learning Objectives:

This unit was removed from AP® Physics 1 starting with the 2021–22 school year. It is not part of the AP® Physics 2 curriculum as of 2021–22.

Use this space for summary and/or additional notes:

Topics from this chapter assessed on the SAT Physics Subject Test:

- **General Wave Properties**, such as wave speed, frequency, wavelength, superposition, standing wave diffraction, and the Doppler effect.

1. Wave Motion
2. Transverse Waves and Longitudinal Waves
3. Superposition
4. Standing Waves and Resonance
5. The Doppler Effect

Skills learned & applied in this chapter:

- Visualizing wave motion.

Use this space for summary and/or additional notes:

Waves

Unit: Mechanical Waves

MA Curriculum Frameworks (2016): HS-PS4-1

AP® Physics 2 Learning Objectives: 6.A.1.2, 6.A.2.2, 6.B.3.1

Mastery Objective(s): (Students will be able to…)

- Describe and explain properties of waves (frequency, wavelength, etc.)
- Differentiate between transverse, longitudinal and transverse waves.
- Calculate wavelength, frequency, period, and velocity of a wave.

Success Criteria:

- Parts of a wave are identified correctly.
- Descriptions & explanations account for observed behavior.

Language Objectives:

- Describe how waves propagate.

Tier 2 Vocabulary: wave, crest, trough, frequency, wavelength

Labs, Activities & Demonstrations:

- Show & tell: transverse waves in a string tied at one end, longitudinal waves in a spring, torsional waves.
- Buzzer in a vacuum.
- Tacoma Narrows Bridge collapse movie.
- Japan tsunami TV footage.

Notes:

<u>wave</u>: a disturbance that travels from one place to another.*

<u>medium</u>: a substance that a wave travels through.

<u>propagation</u>: the process of a wave traveling through space.

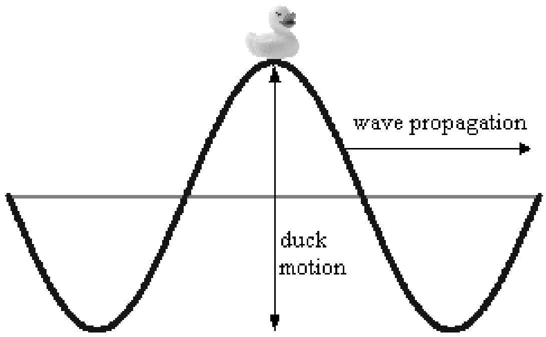
* This is my favorite definition in these notes. I jokingly suggest that I nickname some of my students "wave" based on this definition.

Use this space for summary and/or additional notes:

honors (not AP®)

mechanical wave: a wave that propagates through a medium via contact between particles of the medium. Some examples of mechanical waves include ocean waves and sound waves.

1. The energy of the wave is transmitted via the particles of the medium as the wave passes through it.
2. The wave travels through the medium. The particles of the medium are moved by the wave passing through, and then return to their original position. (The duck sitting on top of the wave below is an example.)



3. Waves generally move fastest in solids and slowest in liquids. The velocity of a mechanical wave is dependent on characteristics of the medium:

state	relevant factors	example: medium	example: velocity of sound
gas	density, pressure	air (20 °C and 1 atm)	$343\,\frac{m}{s}$ $(768\,\frac{mi}{hr})$
liquid	density, compressibility	water (20 °C)	$1\,481\,\frac{m}{s}$ $(3\,317\,\frac{mi}{hr})$
solid	stiffness	steel (longitudinal wave)	$6\,000\,\frac{m}{s}$ $(13\,000\,\frac{mi}{hr})$

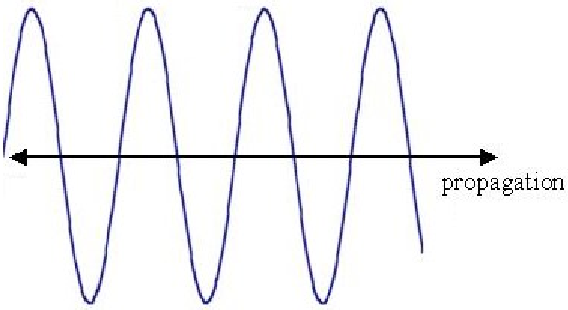
electromagnetic wave: a wave of electricity and magnetism interacting with each other. Electromagnetic waves can propagate through empty space, and are slowed down by interactions with a medium. Electromagnetic waves are discussed in more detail in the *Electromagnetic Waves* section starting on page 473.

Use this space for summary and/or additional notes:

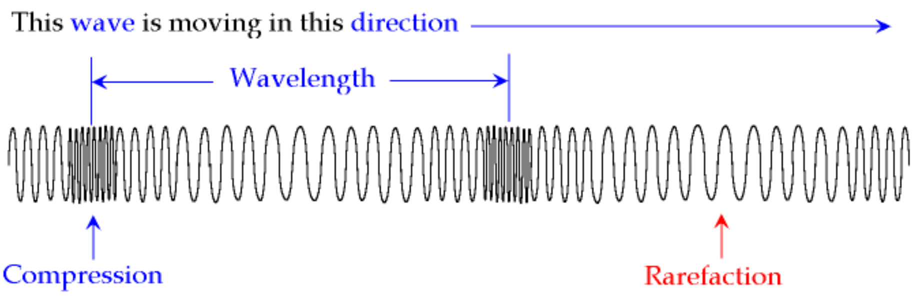
honors (not AP®)

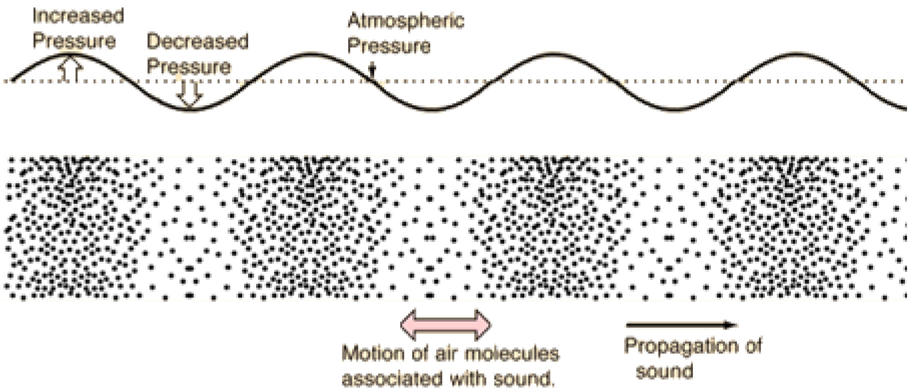
Types of Waves

transverse wave: moves its medium up & down (or back & forth) as it travels through. Examples: light, ocean waves



longitudinal wave (or compressional wave): compresses and decompresses the medium as it travels through. Examples: compression of a spring, sound.

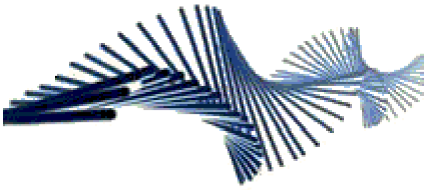




Use this space for summary and/or additional notes:

honors (not AP®)

torsional wave: a type of transverse wave that propagates by twisting about its direction of propagation.



The most famous example of the destructive power of a torsional wave was the Tacoma Narrows Bridge, which collapsed on November 7, 1940. On that day, strong winds caused the bridge to vibrate torsionally. At first, the edges of the bridge swayed about eighteen inches. (This behavior had been observed previously, earning the bridge the nickname "Galloping Gertie".) However, after a support cable snapped, the vibration increased significantly, with the edges of the bridge being displaced up to 28 feet! Eventually, the bridge started twisting in two halves, one half twisting clockwise and the other half twisting counterclockwise, and then back again. This opposing torsional motion eventually caused the bridge to twist apart and collapse.

The bridge's collapse was captured on film. Video clips of the bridge twisting and collapsing are available on the internet. There is a detailed analysis of the bridge's collapse at http://www.vibrationdata.com/Tacoma.htm

Use this space for summary and/or additional notes:

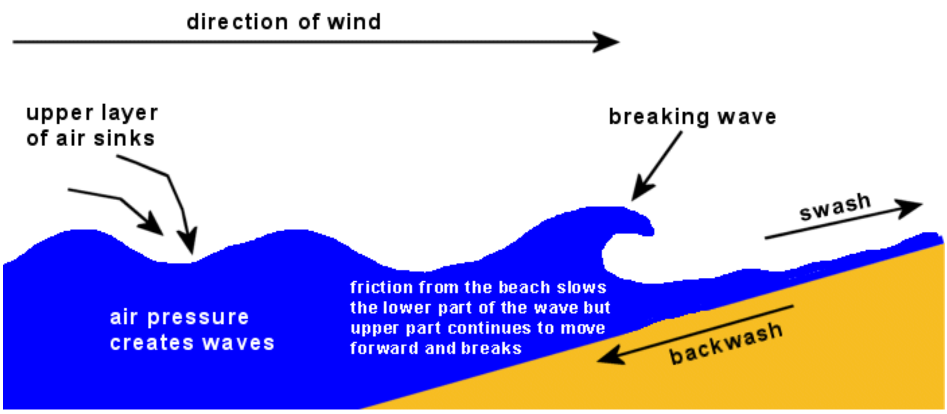
honors (not AP®)

Ocean Waves

Surface Waves

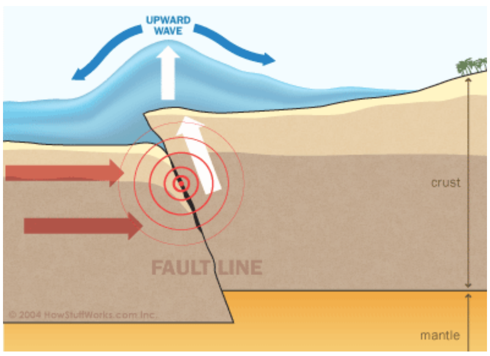
surface wave: a transverse wave that travels at the interface between two mediums.

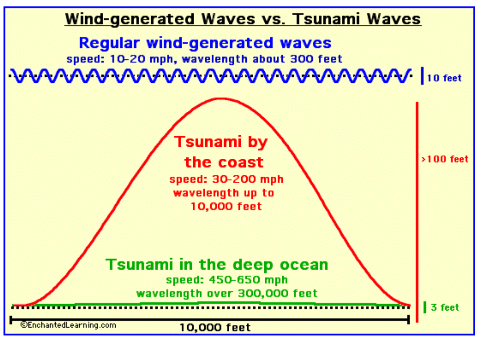
Ocean waves are an example of surface waves, because they travel at the interface between the air and the water. Surface waves on the ocean are caused by wind disturbing the surface of the water. Until the wave gets to the shore, surface waves have no effect on water molecules far below the surface.



Tsunamis

The reason tsunamis are much more dangerous than regular ocean waves is because tsunamis are created by earthquakes on the ocean floor. The tsunami wave propagates through the entire depth of the water, which means tsunamis carry many times more energy than surface waves.

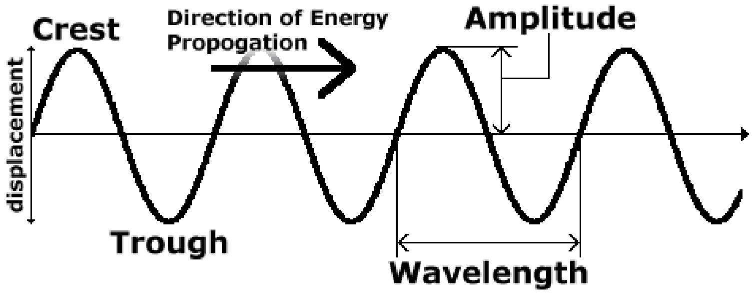




This is why a 6–12 foot high surface wave breaks harmlessly on the beach; however, a tsunami that extends 6–12 feet above the surface of the water includes a significant amount of energy throughout the entire depth of the water, and can destroy an entire city.

Use this space for summary and/or additional notes:

Properties of Waves



crest: the point of maximum positive displacement of a transverse wave. (The highest point.)

trough: the point of maximum negative displacement of a transverse wave. (The lowest point.)

amplitude (*A*): the distance of maximum displacement of a point in the medium as the wave passes through it. (The maximum height or depth.)

wavelength (λ): the length of the wave, measured from a specific point in the wave to the same point in the next wave. Unit = distance (m, cm, nm, *etc.*)

frequency (f or ν): the number of waves that travel past a point in a given time. Unit = $^{1}/_{\text{time}}$ (Hz = $^{1}/_{\text{s}}$)

Note that while high school physics courses generally use the variable f for frequency, college courses usually use ν (the Greek letter "nu", which is different from but easy to confuse with the Roman letter "v").

period or time period (T): the amount of time between two adjacent waves. Unit = time (usually seconds)

$$T = {}^{1}/_{f}$$

Use this space for summary and/or additional notes:

velocity: the velocity of a wave depends on its frequency (f) and its wavelength (λ):

$$v = \lambda f$$

The velocity of electromagnetic waves (such as light, radio waves, microwaves, X-rays, *etc.*) is called the speed of light, which is $3.00\times10^{8}\,\frac{\text{m}}{\text{s}}$ in a vacuum. The speed of light is slower in a medium that has an index of refraction* greater than 1.

The velocity of a wave traveling through a string under tension (such as a piece of string, a rubber band, a violin/guitar string, *etc.*) depends on the tension and the ratio of the mass of the string to its length:

$$v_{string} = \sqrt{\frac{F_T L}{m}}$$

where F_T is the tension in the string, L is the length, and m is the mass.

Sample Problem:

Q: The Boston radio station WZLX broadcasts waves with a frequency of 100.7 MHz. If the waves travel at the speed of light, what is the wavelength?

A: $f = 100.7\,\text{MHz} = 100\,700\,000\,\text{Hz} = 1.007\times10^{8}\,\text{Hz}$

$v = c = 3.00\times10^{8}\,\frac{\text{m}}{\text{s}}$

$v = \lambda f$

$3.00\times10^{8} = \lambda(1.007\times10^{8})$

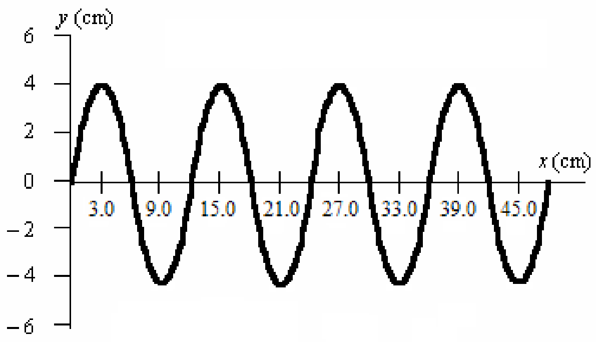
$\lambda = \dfrac{3.00\times10^{8}}{1.007\times10^{8}} = 2.98\,\text{m}$

* The index of refraction is a measure of how much light bends when it moves between one medium and another. The sine of the angle of refraction is proportional to the speed of light in that medium. Index of refraction is part of the *Refraction* topic starting on page 465.

Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** Consider the following wave:



   a. What is the amplitude of this wave?

   b. What is its wavelength?

   c. If the velocity of this wave is $30\,\frac{\text{m}}{\text{s}}$, what is its period?

2. **(M)** What is the speed of wave with a wavelength of 0.25 m and a frequency of 5.5 Hz?

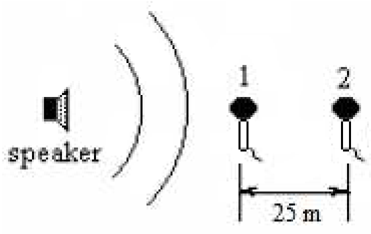
   Answer: $1.375\,\frac{\text{m}}{\text{s}}$

3. **(S)** A sound wave traveling in water at 10 °C has a wavelength of 0.65 m. What is the frequency of the wave.
   (*Note: you will need to look up the speed of sound in water at 10 °C in* Table W. Properties of Water and Air *on page 620 of your Physics Reference Tables*.)

   Answer: 2 226Hz

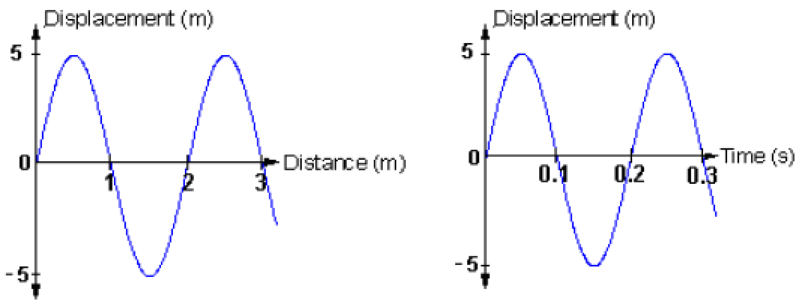
Use this space for summary and/or additional notes:

4. **(S)** Two microphones are placed in front of a speaker as shown in the diagram to the right. If the air temperature is 30 °C, what is the time delay between the two microphones?



Answer: 0.0716 s

5. **(M)** The following are two graphs of the same wave. The first graph shows the displacement *vs.* distance, and the second shows displacement *vs.* time.



a. What is the wavelength of this wave?

b. What is its amplitude?

c. What is its frequency?

d. What is its velocity?

Use this space for summary and/or additional notes:

Reflection and Superposition

Unit: Mechanical Waves

MA Curriculum Frameworks (2016): HS-PS4-1

AP® Physics 2 Learning Objectives: 6.C.1.1, 6.C.1.2

Mastery Objective(s): (Students will be able to…)

- Explain the behavior of waves when they pass each other in the same medium and when they reflect off something.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain what happens when two waves pass through each other.

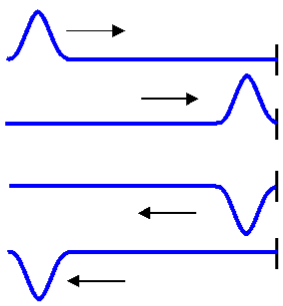
Tier 2 Vocabulary: reflection

Labs, Activities & Demonstrations:

- waves on a string or spring anchored at one end
- large Slinky with longitudinal and transverse waves passing each other

Notes:

Reflection of Waves



reflection: when a wave hits a fixed (stationary) point and "bounces" back.

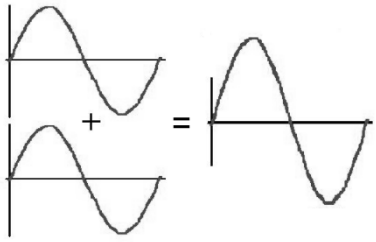
Notice that when the end of the rope is fixed, the reflected wave is inverted. (If the end of the rope were free, the wave would not invert.)

Use this space for summary and/or additional notes:

Superposition of Waves

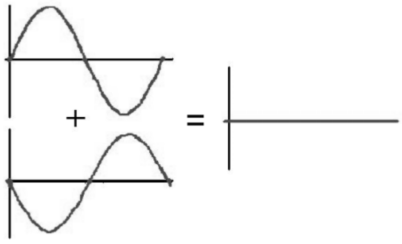
When waves are superimposed (occupy the same space), their amplitudes add.

<u>constructive interference</u>: when waves add in a way that the amplitude of the resulting wave is larger than the amplitudes of the component waves.



Because the wavelengths are the same and the maximum, minimum, and zero points all coïncide (line up), the two component waves are said to be "in phase" with each other.

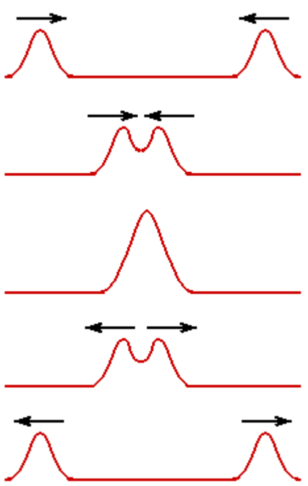
<u>destructive interference</u>: when waves add in a way that the amplitude of the resulting wave is smaller than the amplitudes of the component waves. (Sometimes we say that the waves "cancel" each other.)



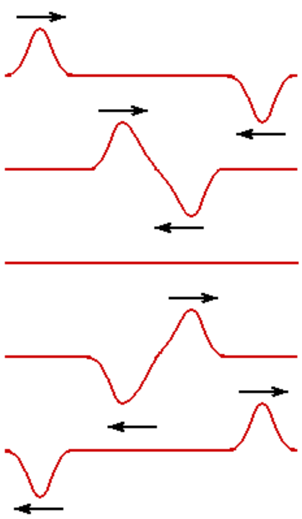
Because the wavelengths are the same but the maximum, minimum, and zero points do not coïncide, the waves are said to be "out of phase" with each other.

Note that waves can travel in two opposing directions at the same time. When this happens, the waves pass through each other, exhibiting constructive and/or destructive interference as they pass:

Constructive Interference



Destructive Interference

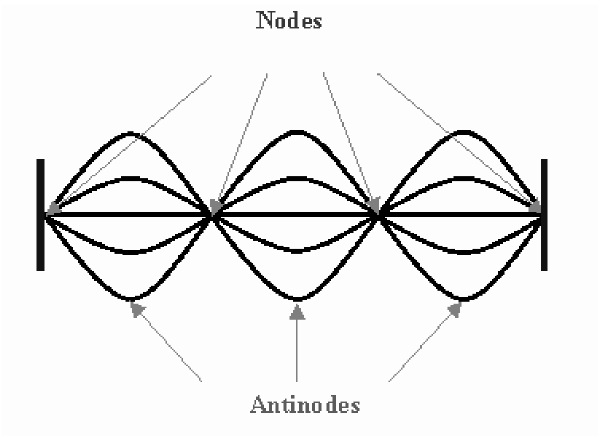


Use this space for summary and/or additional notes:

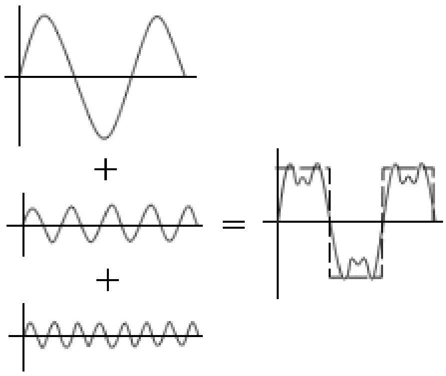
Standing Waves

standing wave: when half of the wavelength is an exact fraction of the length of a medium that is vibrating, the wave reflects back and the reflected wave interferes constructively with itself. This causes the wave to appear stationary.

Points along the wave that are not moving are called "nodes". Points of maximum displacement are called "antinodes".



When we add waves with different wavelengths and amplitudes, the result can be complex:

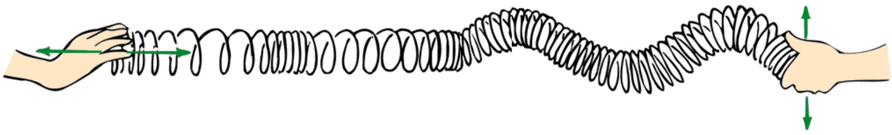


This is how radio waves encode a signal on top of a "carrier" wave. Your radio's antenna receives ("picks up") radio waves within a certain range of frequencies. Imagine that the bottom wave (the one with the shortest wavelength and highest frequency) is the "carrier" wave. If you tune your radio to its frequency, the radio will filter out other waves that don't include the carrier frequency. Then your radio subtracts the carrier wave, and everything that is left is sent to the speakers.

Use this space for summary and/or additional notes:

Homework Problem

1. **(M)** A Slinky is held at both ends. The person on the left creates a longitudinal wave, while at same time the person on the right creates a transverse wave with the same frequency. Both people stop moving their ends of the Slinky just as the waves are about to meet.



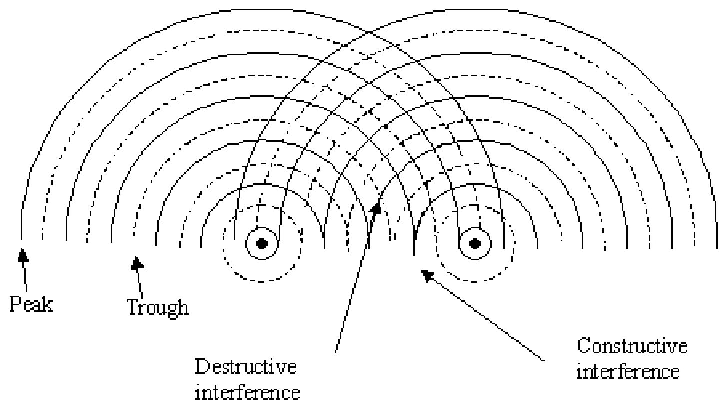
   a. Draw a picture of what the Slinky will look like when the waves completely overlap.

   b. Draw a picture of what the Slinky will look like just after the waves no longer overlap.

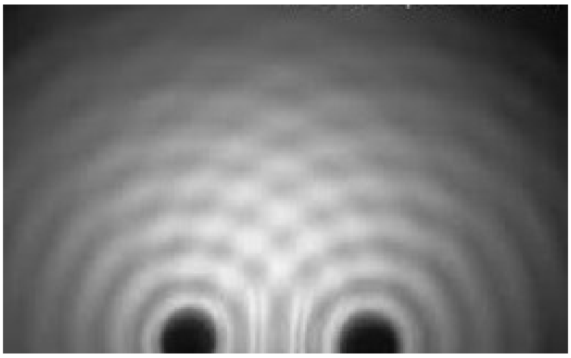
Use this space for summary and/or additional notes:

Two-Dimensional Interference Patterns

When two progressive waves propagate into each other's space, the waves produce interference patterns. This diagram shows how interference patterns form:



The resulting interference pattern looks like the following picture:

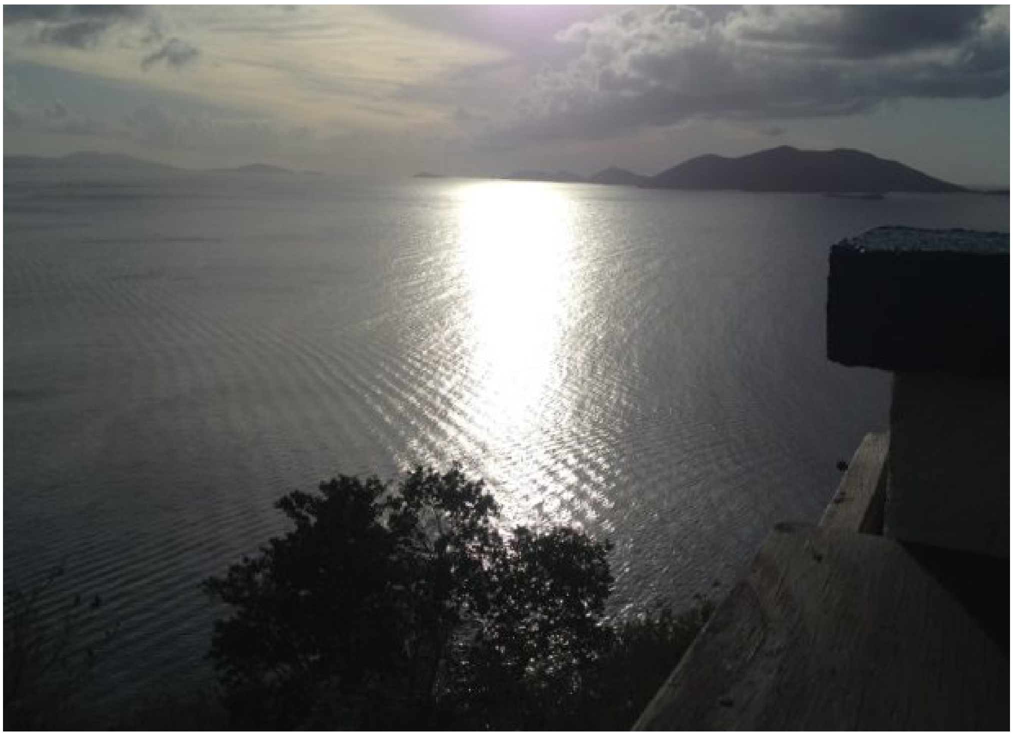


In this picture, the bright regions are wave peaks, and the dark regions are troughs. The brightest intersections are regions where the peaks interfere constructively, and the darkest intersections are regions where the troughs interfere constructively.

Use this space for summary and/or additional notes:

The following picture* shows an interference pattern created by ocean waves, one of which has been reflected off a point on the shore. The wave at the left side of the picture is traveling toward the right, and the reflected wave at the bottom right of the picture is traveling toward the top of the picture.

Because the sun is low in the sky (the picture was taken just before sunset), the light is reflected off the water, and the crests of the waves produce shadows behind them.



* Taken from Tortola in the British Virgin Islands, looking west toward Jost Van Dyke.

Use this space for summary and/or additional notes:

honors (not AP®)

Sound & Music

Unit: Mechanical Waves

MA Curriculum Frameworks (2016): HS-PS4-1

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Describe how musical instruments produce sounds.
- Describe how musical instruments vary pitch.
- Calculate frequencies of pitches produced by a vibrating string or in a pipe.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what produces the vibrations in various types of musical instruments.

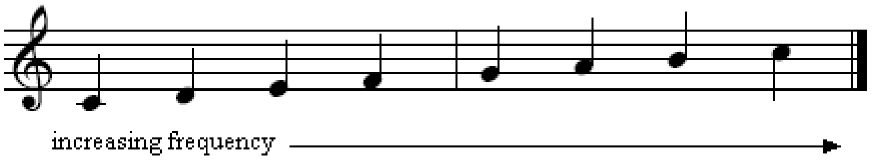
Tier 2 Vocabulary: pitch

Labs, Activities & Demonstrations:

- Show & tell: violin, penny whistle, harmonica, Boomwhackers.
- Helmholtz resonators—bottles of different sizes/air volumes, slapping your cheek with your mouth open.
- Frequency generator & speaker.
- Rubens tube (“sonic flame tube”).
- Measure the speed of sound in air using a resonance tube.

Notes:

Sound waves are caused by vibrations that create longitudinal (compressional) waves in the medium they travel through (such as air).

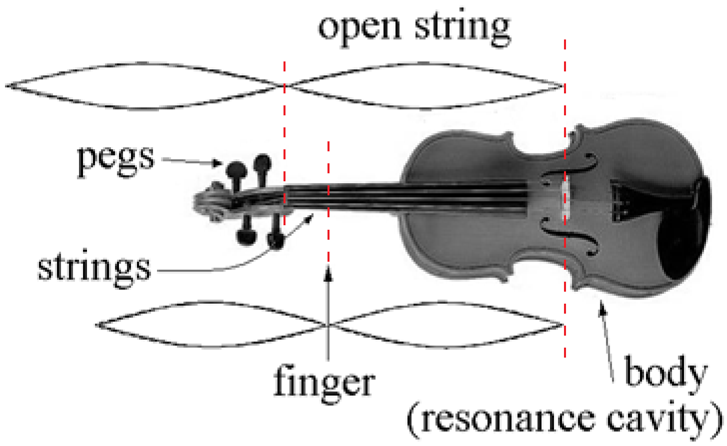
pitch: how “high” or “low” a musical note is. The pitch is determined by the frequency of the sound wave.

Use this space for summary and/or additional notes:

honors (not AP®)

resonance: when the wavelength of a half-wave (or an integer number of half-waves) coincides with one of the dimensions of an object. This creates standing waves that reinforce and amplify each other. The body of a musical instrument is an example of an object that is designed to use resonance to amplify the sounds that the instrument produces.

String Instruments

A string instrument (such as a violin or guitar) typically has four or more strings. The lower strings (strings that sound with lower pitches) are thicker, and higher strings are thinner. Pegs are used to tune the instrument by increasing (tightening) or decreasing (loosening) the tension on each string.



The vibration of the string creates a half-wave, *i.e.,* λ = 2L. The musician changes the half-wavelength by using a finger to shorten the part of the string that vibrates. (A shorter wavelength produces a higher frequency = higher pitch.)

The velocity of the wave produced on a string depends on the tension and the length and mass of the vibrating portion. The velocity is given by the equation:

$$v_{string} = \sqrt{\frac{F_T L}{m}}$$

where:

f = frequency (Hz) — F_T = tension (N)

m = mass of string (kg) — L = length of string (m) $= \frac{\lambda}{2}$

Given the velocity and wavelength, the frequency (pitch) is therefore:

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F_T L}{m}} = \sqrt{\frac{F_T}{4mL}}$$

Use this space for summary and/or additional notes:

honors (not AP®)

Pipes and Wind Instruments

A pipe (in the musical instrument sense) is a tube filled with air. The design of the mouthpiece (or air inlet) causes the air to oscillate as it enters the pipe. This causes the air molecules to compress and spread out at regular intervals based on the dimensions of the closed section of the instrument, which determines the wavelength. The wavelength and speed of sound determine the frequency.

Most wind instruments use one of three ways of causing the air to oscillate:

Brass Instruments

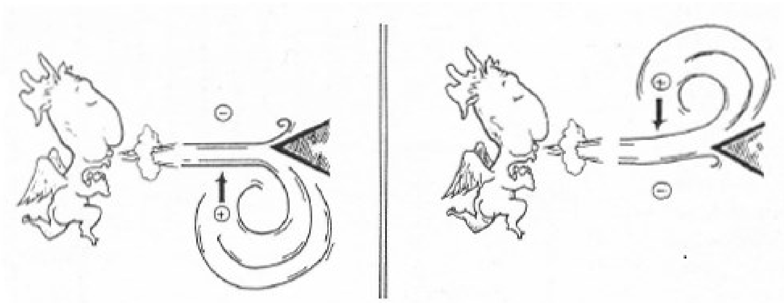
With brass instruments like trumpets, trombones, French horns, *etc.*, the player presses his/her lips tightly against the mouthpiece, and the player's lips vibrate at the appropriate frequency.

Reed Instruments

With reed instruments, air is blown past a reed (a semi-stiff object) that vibrates back and forth. Clarinets and saxophones use a single reed made from a piece of cane (a semi-stiff plant similar to bamboo). Oboes and bassoons ("double-reed instruments") use two pieces of cane that vibrate against each other. Harmonicas and accordions use reeds made from a thin piece of metal.

Whistles (Instruments with Fipples)

Instruments with fipples include recorders, whistles and flutes. A fipple is a sharp edge that air is blown past. The separation of the air going past the fipple results in a pressure difference on one side *vs.* the other. Air moves toward the lower pressure side, causing air to build up and the pressure to increase. When the pressure becomes greater than the other side, the air switches abruptly to the other side of the fipple. Then the pressure builds on the other side until the air switches back:



The frequency of this back-and-forth motion is what determines the pitch.

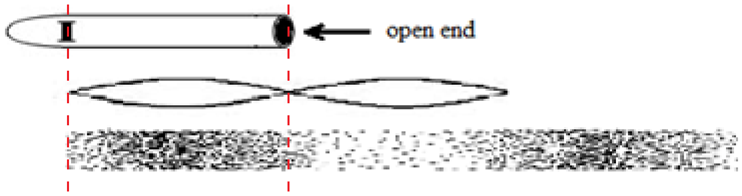
Use this space for summary and/or additional notes:

honors (not AP®)

Open vs. Closed-Pipe Instruments

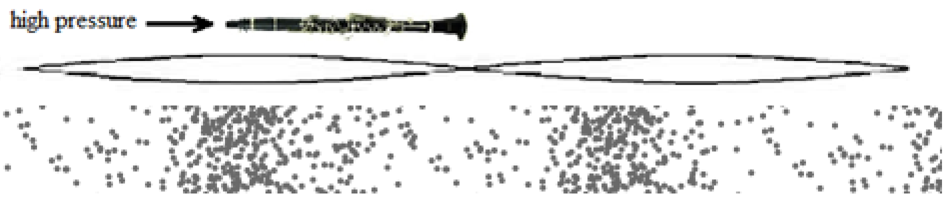
An open-pipe instrument has an opening at each end. A closed-pipe instrument has an opening at one end, and is closed at the other.

Examples of open-pipe instruments include uncapped organ pipes, whistles, recorders and flutes.



Notice that the two openings determine where the air pressure *must* be equal to atmospheric pressure (*i.e.,* the air is neither compressed nor expanded). This means that the length of the body of the instrument (L) is a half-wave, and that the wavelength (λ) of the sound produced must therefore be twice as long, *i.e.,* $\lambda = 2L$. (This is similar to string instruments, in which the length of the vibrating string is a half-wave.)

Examples of closed-pipe instruments include clarinets and all brass instruments. Air is blown in at high pressure via the mouthpiece, which means the mouthpiece is an antinode—a region of maximum displacement of the individual air molecules. This means that the body of the instrument is the distance from the antinode to a region of atmospheric pressure, *i.e.,* one-fourth of a wave. This means that for closed-pipe instruments, $\lambda = 4L$.



The difference in the resonant wavelength ($4L$ *vs.* $2L$) is why a closed-pipe instrument (*e.g.,* a clarinet) sounds an octave lower than an open-pipe instrument of similar length (*e.g.,* a flute)—twice the wavelength results in half the frequency.

Use this space for summary and/or additional notes:

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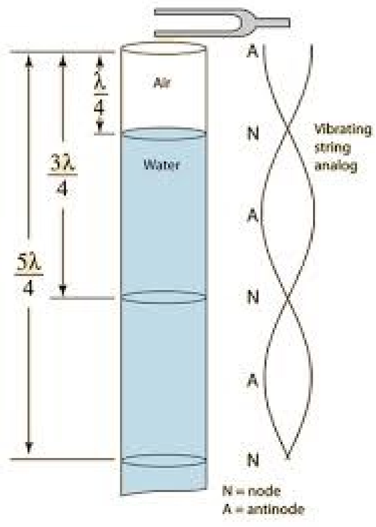
The principle of a closed-pipe instrument can be used in a lab experiment to determine the frequency of a tuning fork (or the speed of sound) using a resonance tube—an open tube filled with water to a specific depth.



A tuning fork generates an oscillation of a precise frequency at the top of the tube. Because this is a closed pipe, the source (just above the tube) is an antinode (maximum amplitude).

When the height of air above the water is exactly ¼ of a wavelength $(\frac{\lambda}{4})$, the waves that are reflected back have maximum constructive interference with the source wave, which causes the sound to be significantly amplified. This phenomenon is called resonance.

Resonance will occur at every antinode—*i.e.,* any integer plus ¼ of a wave ($\frac{\lambda}{4}$, $\frac{3\lambda}{4}$, $\frac{5\lambda}{4}$, *etc.*)

Use this space for summary and/or additional notes:

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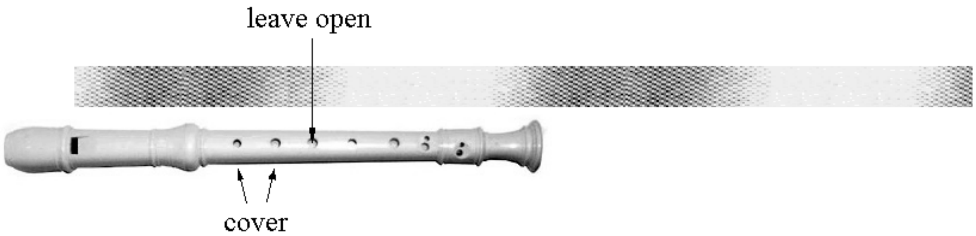
Playing Different Pitches (Frequencies)

For an instrument with holes, like a flute or recorder, the first open hole is the first place in the pipe where the pressure is equal to atmospheric pressure, which determines the half-wavelength (or quarter-wavelength):



The speed of sound in air is v_s ($343\,\frac{m}{s}$ at 20 °C and 1 atm), which means the frequency of the note (from the formula $v_s = \lambda f$) will be:

$f = \dfrac{v_s}{2L}$ for an open-pipe instrument (*e.g.,* flute, recorder, whistle)

$f = \dfrac{v_s}{4L}$ for an closed-pipe instrument (*e.g.,* clarinet, brass instrument).

Note that the frequency is directly proportional to the speed of sound in air. The speed of sound increases as the temperature increases, which means that as the air gets colder, the frequency gets lower, and as the air gets warmer, the frequency gets higher. This is why wind instruments go flat at colder temperatures and sharp at warmer temperatures. Musicians claim that the instrument is going out of tune, but actually it's not the instrument that is out of tune, but the speed of sound!

Note however, that the frequency is inversely proportional to the wavelength (which depends largely on the length of the instrument). This means that the extent to which the frequency changes with temperature will be different for different-sized instruments, which means the band will become more and more out of tune with itself as the temperature changes.

Use this space for summary and/or additional notes:

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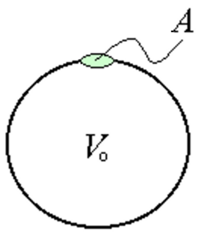
Helmholtz Resonators

The resonant frequency of a bottle or similar container (called a Helmholtz resonator, named after the German physicist Hermann von Helmholtz) is more complicated to calculate.

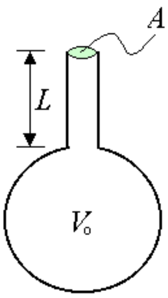
For an enclosed volume of air with a single opening, the resonant frequency depends on the resonant frequency of the air in the large cavity, and the cross-sectional area of the opening.



Resonant frequency:

$$f = \frac{v_s}{2\pi} \cdot \frac{A}{V_o}$$

For a bottle with a neck, the air in the neck behaves like a spring, with a spring constant that is proportional to the volume of air in the neck:



Resonant frequency:

$$f = \frac{v_s}{2\pi} \sqrt{\frac{A}{V_o L}}$$

where:

f = resonant frequency

v_s = speed of sound in air ($343\,\frac{\text{m}}{\text{s}}$ at 20 °C and 1 atm)

A = cross-sectional area of the neck of the bottle (m^2)

V_o = volume of the main cavity of the bottle (m^3)

L = length of the neck of the bottle (m)

(Note that it may be more convenient to use measurements in cm, cm^2, and cm^3, and use $v_s = 34\,300\,\frac{\text{cm}}{\text{s}}$.)

Blowing across the top of an open bottle is an example of a Helmholtz resonator.

You can make your mouth into a Helmholtz resonator by tapping on your cheek with your mouth open. You can change the pitch by opening or closing your mouth a little, which changes the area of the opening (A).

Use this space for summary and/or additional notes:

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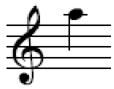
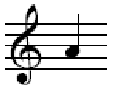
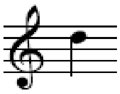
Frequencies of Music Notes

The frequencies that correspond with the pitches of the Western equal temperament scale are:

pitch	frequency (Hz)	pitch	frequency (Hz)
 A	440.0	 E	659.3
 B	493.9	 F	698.5
 C	523.3	 G	784.0
 D	587.3	 A	880.0

A note that is an octave above another note has exactly twice the frequency of the lower note. For example, the A in on the second line of the treble clef staff has a frequency of 440 Hz.* The A an octave above it (one ledger line above the staff) has a frequency of 440 × 2 = 880 Hz.

Harmonic Series

<u>harmonic series</u>: the additional, shorter standing waves that are generated by a vibrating string or column of air that correspond with integer numbers of half-waves.

<u>fundamental frequency</u>: the natural resonant frequency of a particular pitch.

<u>harmonic</u>: a resonant frequency produced by vibrations that contain an integer number of half-waves that add up to the half-wavelength of the fundamental.

The harmonics are numbered based on their pitch relative to the fundamental frequency. The harmonic that is closest in pitch is the 1st harmonic, the next closest is the 2nd harmonic, *etc.*

Any sound wave that is produced in a resonance chamber (such as a musical instrument) will produce the fundamental frequency plus all of the other waves of the harmonic series. The fundamental is the loudest, and each harmonic gets more quiet as you go up the harmonic series.

* Most bands and orchestras define the note "A" to be exactly 440 Hz, and use it for tuning.

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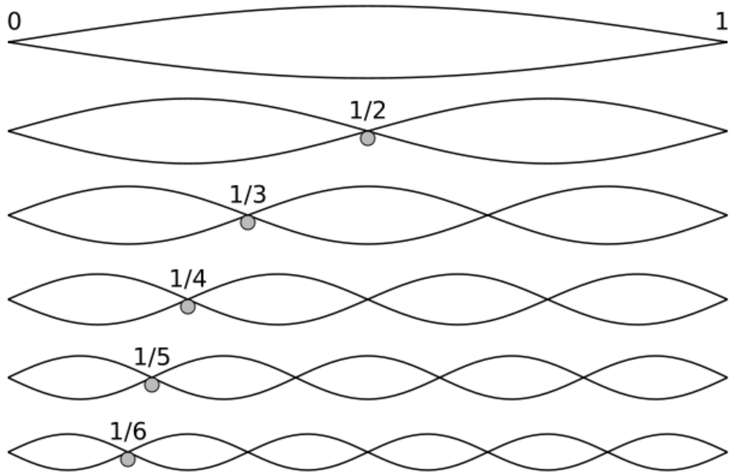
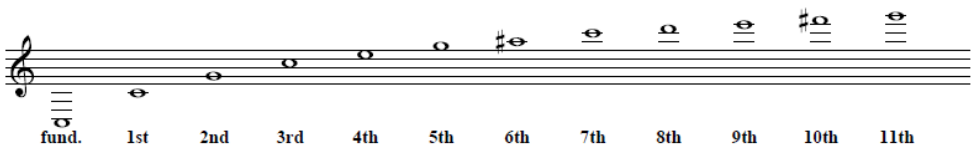
The following diagram shows the waves of the fundamental frequency and the first five harmonics in a pipe or a vibrating string:



Fraction of String	Wave-length	Harmonic	Frequency	Pitch (relative to fundamental)
1	$2L$	—	f_o	Fundamental.
$\frac{1}{2}$	$\frac{2L}{2}$	1st	$2f_o$	One octave above.
$\frac{1}{3}$	$\frac{2L}{3}$	2nd	$3f_o$	One octave + a fifth above.
$\frac{1}{4}$	$\frac{2L}{4}$	3rd	$4f_o$	Two octaves above.
$\frac{1}{5}$	$\frac{2L}{5}$	4th	$5f_o$	Two octaves + approximately a major third above.
$\frac{1}{6}$	$\frac{2L}{6}$	5th	$6f_o$	Two octaves + a fifth above.
$\frac{1}{n}$	$\frac{2L}{n}$	(n-1)th	nf_o	*etc.*



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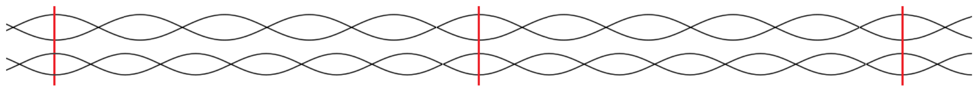
Beats

When two or more waves are close but not identical in frequency, their amplitudes reinforce each other at regular intervals.

For example, when the following pair of waves travels through the same medium, the amplitudes of the two waves have maximum constructive interference every five half-waves (2½ full waves) of the top wave and every six half-waves (3 full waves) of the bottom wave.



If this happens with sound waves, you will hear a pulse or "beat" every time the two maxima coïncide.

The closer the two wavelengths (and therefore also the two frequencies) are to each other, the more half-waves it takes before the amplitudes coïncide. This means that as the frequencies get closer, the time between beats gets longer.

Piano tuners listen for these beats, and adjust the tension of the string they are tuning until the time between beats gets longer and longer and finally disappears.

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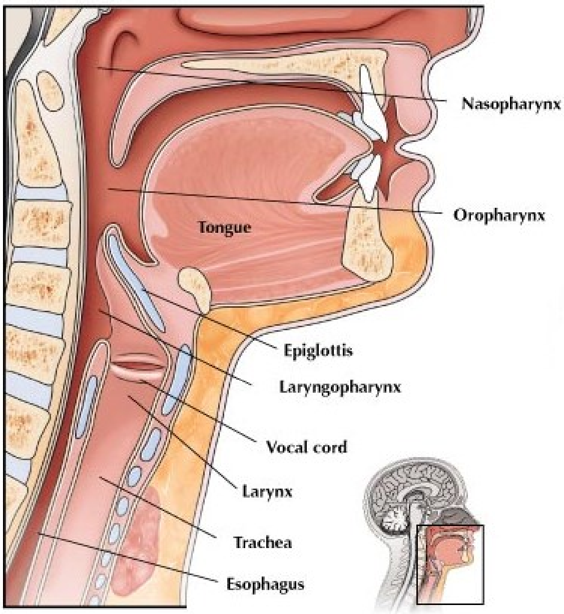
The Biophysics of Sound

When a person speaks, abdominal muscles force air from the lungs through the larynx.



The vocal cord vibrates, and this vibration creates sound waves. Muscles tighten or loosen the vocal cord, which changes the frequency at which it vibrates. Just like in a string instrument, the change in tension changes the pitch. Tightening the vocal cord increases the tension and produces a higher pitch, and relaxing the vocal cord decreases the tension and produces a lower pitch.

This process happens when you sing. Amateur musicians who sing a lot of high notes can develop laryngitis from tightening their laryngeal muscles too much for too long. Professional musicians need to train themselves to keep their larynx muscles relaxed and use other techniques (such as air pressure, which comes from breath support via the abdominal muscles) to adjust their pitch.

Use this space for summary and/or additional notes:

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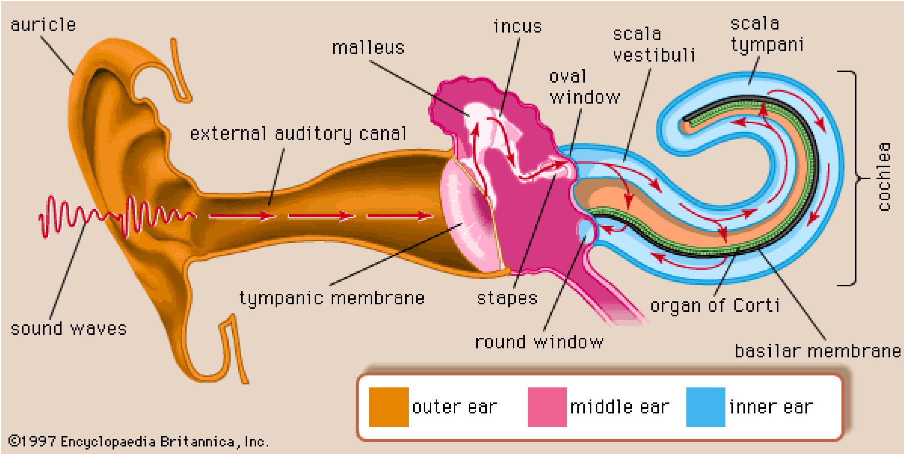
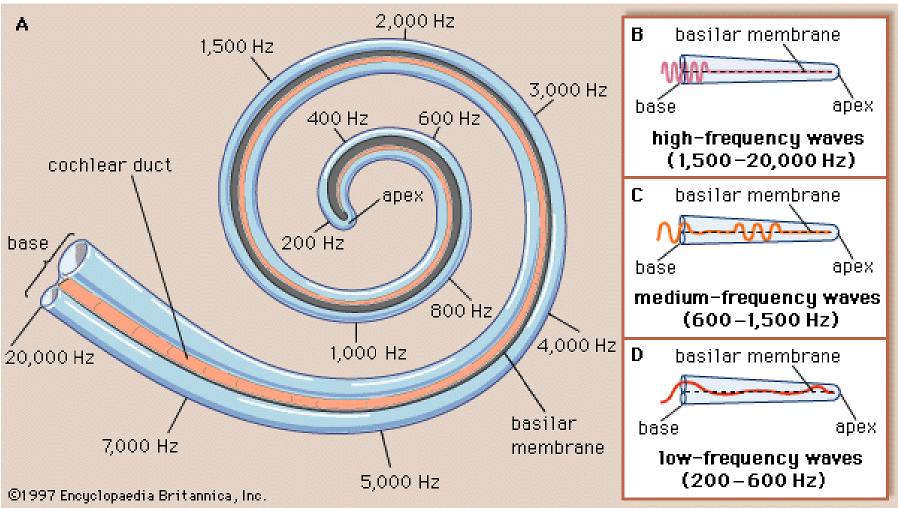
When the sound reaches the ears, it travels through the auditory canal and causes the tympanic membrane (eardrum) to vibrate. The vibrations of the tympanic membrane cause pressure waves to travel through the middle ear and through the oval window into the cochlea.



The basilar membrane in the cochlea is a membrane with cilia (small hairs) connected to it, which can detect very small movements of the membrane. As with a resonance tube, the wavelength determines exactly where the sound waves will vibrate the basilar membrane the most strongly, and the brain determines the pitch (frequency) of a sound based on the precise locations excited by these frequencies.



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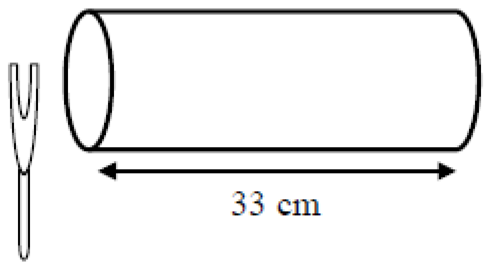
Homework Problems

A tuning fork is used to establish a standing wave in an open ended pipe filled with air at a temperature of 20°C, where the speed of sound is $343\,\frac{\text{m}}{\text{s}}$, as shown below:



The sound wave resonates at the 3nd harmonic frequency of the pipe. The length of the pipe is 33 cm.

1. **(M)** Sketch the pipe with the standing wave inside of it. (For simplicity, you may sketch a transverse wave to represent the standing wave.)

2. **(M)** Determine the wavelength of the resonating sound wave.

   Answer: 22 cm

3. **(M)** Determine the frequency of the tuning fork.

   Answer: 1 559 Hz

4. **(M)** What is the next higher frequency that will resonate in this pipe?

   Answer: 2 079 Hz

Use this space for summary and/or additional notes:

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Sound Level (Loudness)

Unit: Mechanical Waves

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain sound levels in decibels.
- Explain the Lombard Effect.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain how loudness is measured.

Tier 2 Vocabulary: level

Labs, Activities & Demonstrations:

- VU meter.

Notes:

<u>sound level</u>: the perceived intensity of a sound. Usually called "volume".

Sound level is usually measured in decibels (dB). One decibel is one tenth of one bel.

Sound level is calculated based on the logarithm of the ratio of the power (energy per unit time) causing a sound vibration to the power that causes some reference sound level.

You will not be asked to calculate decibels from an equation, but you should understand that because the scale is logarithmic, a difference of one bel (10 dB) represents a tenfold increase or decrease in sound level.

Use this space for summary and/or additional notes:

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The following table lists the approximate sound levels of various sounds:

sound level (dB)	Description
0	threshold of human hearing at 1 kHz
10	a single leaf falling to the ground
20	background in TV studio
30	quiet bedroom at night
36	whispering
40	quiet library or classroom
42	quiet voice
40–55	typical dishwasher
50–55	normal voice
60	TV from 1 m away
	normal conversation from 1 m away
60–65	raised voice
60–80	passenger car from 10 m away
70	typical vacuum cleaner from 1 m away
75	crowded restaurant at lunchtime
72–78	loud voice
85	hearing damage (long-term exposure)
84–90	shouting
80–90	busy traffic from 10 m away
100–110	rock concert, 1 m from speaker
110	chainsaw from 1 m away
110–140	jet engine from 100 m away
120	threshold of discomfort
	hearing damage (single exposure)
130	threshold of pain
140	jet engine from 50 m away
194	sound waves become shock waves

Use this space for summary and/or additional notes:

honors (not AP®)

Adjusting Sound Level in Conversation

In crowds, people unconsciously adjust the sound levels of their speech in order to be heard above the ambient noise. This behavior is called the Lombard effect, named for Étienne Lombard, the French doctor who first described it.

The Lombard coëfficient is the ratio of the increase in sound level of the speaker to the increase in sound level of the background noise:

$$L = \frac{\text{increase in speech level (dB)}}{\text{increase in background noise (dB)}}$$

Researchers have observed values of the Lombard coëfficient ranging from 0.2 to 1.0, depending on the circumstances.

When you are working in groups in a classroom, as the noise level gets louder, each person has to talk louder to be heard, which in turn makes the noise level louder. The Lombard effect creates a feedback loop in which the sound gets progressively louder and louder until your teacher complains and everyone resets to a quieter volume.

Use this space for summary and/or additional notes:

honors (not AP®)

Doppler Effect

Unit: Mechanical Waves

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain the Doppler Effect and give examples.
- Calculate the apparent shift in wavelength/frequency due to a difference in velocity between the source and receiver.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how loudness is measured.

Tier 2 Vocabulary: shift

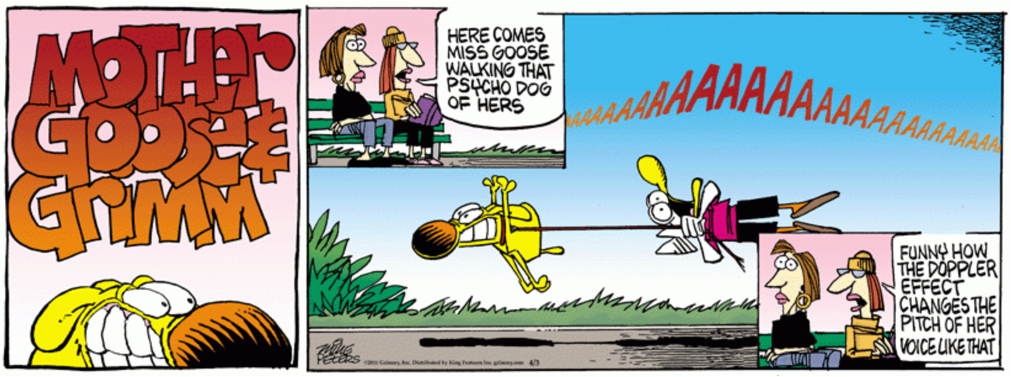
Labs, Activities & Demonstrations:

- Buzzer on a string.

Notes:

Doppler effect or Doppler shift: the apparent change in frequency/wavelength of a wave due to a difference in velocity between the source of the wave and the observer. The effect is named for the Austrian physicist Christian Doppler.

You have probably noticed the Doppler effect when an emergency vehicle with a siren drives by.



Use this space for summary and/or additional notes:

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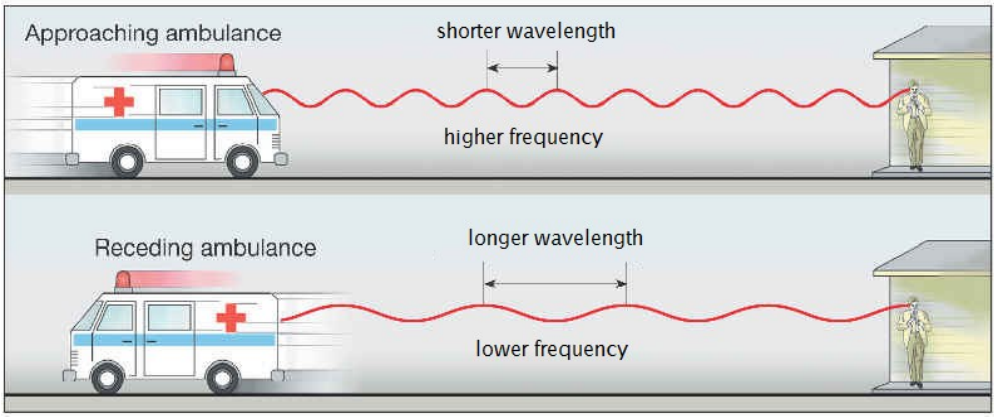
Why the Doppler Shift Happens

The Doppler shift occurs because a wave is created by a series of pulses at regular intervals, and the wave moves at a particular speed.

If the source is approaching, each pulse arrives sooner than it would have if the source had been stationary. Because frequency is the number of pulses that arrive in one second, the moving source results in an increase in the frequency observed by the receiver.

Similarly, if the source is moving away from the observer, each pulse arrives later, and the observed frequency is lower.



Use this space for summary and/or additional notes:

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Calculating the Doppler Shift

The change in frequency is given by the equation:

$$f = f_o\left(\frac{v_w \pm v_r}{v_w \pm v_s}\right)$$

where:

f = observed frequency

f_o = frequency of the original wave

v_w = velocity of the wave

v_r = velocity of the receiver (you)

v_s = velocity of the source

The rule for adding or subtracting velocities is:

- The receiver's (your) velocity is in the numerator. If you are moving toward the sound, this makes the pulses arrive sooner, which makes the frequency higher. So if you are moving ***toward*** the sound, ***add*** your velocity. If you are moving ***away*** from the sound, ***subtract*** your velocity.
- The source's velocity is in the denominator. If the source is moving toward you, this makes the frequency higher, which means the denominator needs to be smaller. This means that if the source is moving ***toward*** you, ***subtract*** its velocity. If the source is moving ***away*** from you, ***add*** its velocity.

Don't try to memorize a rule for this—you will just confuse yourself. It's safer to reason through the equation. If something that's moving would make the frequency higher, that means you need to make the numerator larger or the denominator smaller. If it would make the frequency lower, that means you need to make the numerator smaller or the denominator larger.

Use this space for summary and/or additional notes:

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Sample Problem:

Q: The horn on a fire truck sounds at a pitch of 350 Hz. What is the perceived frequency when the fire truck is moving toward you at $20\,\frac{\text{m}}{\text{s}}$? What is the perceived frequency when the fire truck is moving away from you at $20\,\frac{\text{m}}{\text{s}}$? Assume the speed of sound in air is $343\,\frac{\text{m}}{\text{s}}$.

A: The observer is not moving, so $v_r = 0$.

The fire truck is the source, so its velocity appears in the denominator.

When the fire truck is moving toward you, that makes the frequency higher. This means we need to make the denominator smaller, which means we need to ***subtract*** v_s:

$$f = f_o\left(\frac{v_w}{v_w - v_s}\right) = 350\left(\frac{343}{343-20}\right) = 350(1.062) = 372\,\text{Hz}$$

When the fire truck is moving away, the frequency will be lower, which mean we need to make the denominator larger. This means we need to ***add*** v_s:

$$f = f_o\left(\frac{v_w}{v_w + v_s}\right) = 350\left(\frac{343}{343+20}\right) = 350(0.9449) = 331\,\text{Hz}$$

Note that the pitch shift in each direction corresponds with about one half-step on the musical scale.

Use this space for summary and/or additional notes:

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Exceeding the Speed of Sound

Unit: Mechanical Waves

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain the what a "sonic boom" is.
- Calculate Mach numbers.

Success Criteria:

- Explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how a sonic boom is produced.

Tier 2 Vocabulary: sonic boom

Labs, Activities & Demonstrations:

- Crack a bullwhip.

Notes:

The speed of an object relative to the speed of sound in the same medium is called the Mach number (abbreviation Ma), named after the Austrian physicist Ernst Mach.

$$Ma = \frac{v_{object}}{v_{sound}}$$

Thus "Mach 1" or a speed of Ma = 1 is the speed of sound. An object such as an airplane that is moving at 1.5 times the speed of sound would be traveling at "Mach 1.5" or Ma = 1.5.

Use this space for summary and/or additional notes:

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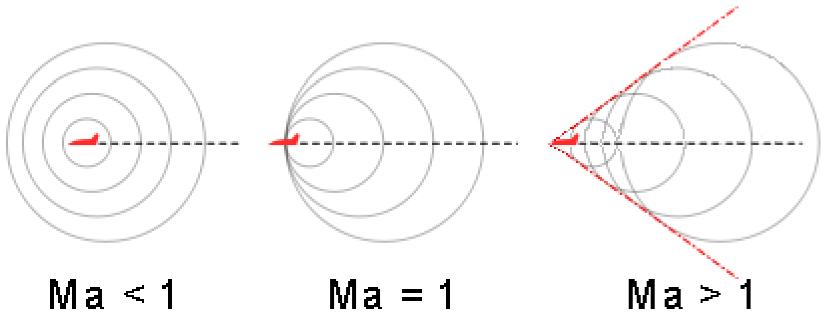
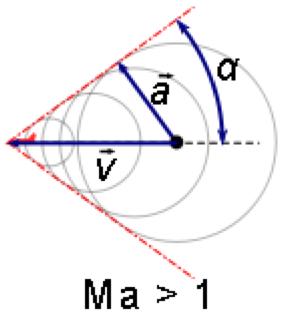
When an object such as an airplane is traveling slower than the speed of sound ($Ma < 1$), the jet engine noise is Doppler shifted just like any other sound wave. When the airplane's velocity reaches the speed of sound ($Ma = 1$), the leading edge of all of the sound waves produced by the plane coincides. These waves amplify each other, producing a loud shock wave called a "sonic boom".



When an airplane is traveling faster than sound, the sound waves coincide at points behind the airplane at a specific angle, α:



The angle α is given by the equation:

$$\sin(\alpha) = \frac{1}{Ma}$$

Note that the airplane cannot be heard at points outside of the region defined by the angle α. Note also that the faster the airplane is traveling, the smaller the angle α, and the narrower the cone.

Use this space for summary and/or additional notes:

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The shock wave temporarily increases the temperature of the air affected by it. If the air is humid enough, when it cools by returning to its normal pressure, the water vapor condenses and forms a cloud, called a vapor cone:

The "crack" of a bullwhip is also a sonic boom—when a bullwhip is snapped sharply, the end of the bullwhip travels faster than sound and creates a miniature shock wave.

Use this space for summary and/or additional notes:

Introduction: Light & Optics

Unit: Light & Optics

Topics covered in this chapter:

This chapter discusses the behavior and our perception of light.

- *Electromagnetic Waves* discusses properties and equations that are specific to electromagnetic waves (including light).
- *Color* discusses properties of visible light and how we perceive it.
- *Reflection* and *Mirrors* discuss properties of flat and curved mirrors and steps for drawing ray tracing diagrams.
- *Refraction* and *Lenses* discuss properties of convex and concave lenses and steps for drawing ray tracing diagrams.
- *Polarization, Diffraction,* and *Scattering* discuss specific optical properties of light.

One of the new skills learned in this chapter is visualizing and drawing representations of how light is affected as it is reflected off a mirror or refracted by a lens. This can be challenging because the behavior of the light rays and the size and location of the image changes depending on the location of the object relative to the focal point of the mirror or lens. Another challenge is in drawing precise, to-scale ray tracing drawings such that you can use the drawings to accurately determine properties of the image, or of the mirror or lens.

Use this space for summary and/or additional notes:

Standards addressed in this chapter:

Massachusetts Curriculum Frameworks (2016):

HS-PS4-5. Communicate technical information about how some technological devices use the principles of wave behavior and wave interactions with matter to transmit and capture information and energy.

AP® only

AP® Physics 2 Learning Objectives:

6.A.1.2: The student is able to describe representations of transverse and longitudinal waves. [**SP 1.2**]

6.A.1.3: The student is able to analyze data (or a visual representation) to identify patterns that indicate that a particular mechanical wave is polarized and construct an explanation of the fact that the wave must have a vibration perpendicular to the direction of energy propagation. [**SP 5.1**, **6.2**]

6.A.2.2: The student is able to contrast mechanical and electromagnetic waves in terms of the need for a medium in wave propagation. [**SP 6.4**, **7.2**]

6.B.3.1: The student is able to construct an equation relating the wavelength and amplitude of a wave from a graphical representation of the electric or magnetic field value as a function of position at a given time instant and vice versa, or construct an equation relating the frequency or period and amplitude of a wave from a graphical representation of the electric or magnetic field value at a given position as a function of time and vice versa. [**SP 1.5**]

6.C.1.1: The student is able to make claims and predictions about the net disturbance that occurs when two waves overlap. Examples should include standing waves. [**SP 6.4**, **7.2**]

6.C.1.2: The student is able to construct representations to graphically analyze situations in which two waves overlap over time using the principle of superposition. [**SP 1.4**]

6.C.2.1: The student is able to make claims about the diffraction pattern produced when a wave passes through a small opening, and to qualitatively apply the wave model to quantities that describe the generation of a diffraction pattern when a wave passes through an opening whose dimensions are comparable to the wavelength of the wave. [**SP 1.4**, **6.4**, **7.2**]

6.C.3.1: The student is able to qualitatively apply the wave model to quantities that describe the generation of interference patterns to make predictions about interference patterns that form when waves pass through a set of openings whose spacing and widths are small compared to the wavelength of the waves. [**SP 1.4**, **6.4**]

Use this space for summary and/or additional notes:

AP® only

6.C.4.1: The student is able to predict and explain, using representations and models, the ability or inability of waves to transfer energy around corners and behind obstacles in terms of the diffraction property of waves in situations involving various kinds of wave phenomena, including sound and light. [**SP 6.4**, **7.2**]

6.E.1.1: The student is able to make claims using connections across concepts about the behavior of light as the wave travels from one medium into another, as some is transmitted, some is reflected, and some is absorbed. [**SP 6.4**, **7.2**]

6.E.2.1: The student is able to make predictions about the locations of object and image relative to the location of a reflecting surface. The prediction should be based on the model of specular reflection with all angles measured relative to the normal to the surface. [**SP 6.4**, **7.2**]

6.E.3.1: The student is able to describe models of light traveling across a boundary from one transparent material to another when the speed of propagation changes, causing a change in the path of the light ray at the boundary of the two media. [**SP 1.1**, **1.4**]

6.E.3.2: The student is able to plan data collection strategies as well as perform data analysis and evaluation of the evidence for finding the relationship between the angle of incidence and the angle of refraction for light crossing boundaries from one transparent material to another (Snell's law). [**SP 4.1**, **5.1**, **5.2**, **5.3**]

6.E.3.3: The student is able to make claims and predictions about path changes for light traveling across a boundary from one transparent material to another at non-normal angles resulting from changes in the speed of propagation. [**SP 6.4**, **7.2**]

6.E.4.1: The student is able to plan data collection strategies, and perform data analysis and evaluation of evidence about the formation of images due to reflection of light from curved spherical mirrors. [**SP 3.2**, **4.1**, **5.1**, **5.2**, **5.3**]

6.E.4.2: The student is able to use quantitative and qualitative representations and models to analyze situations and solve problems about image formation occurring due to the reflection of light from surfaces. [**SP 1.4**, **2.2**]

6.E.5.1: The student is able to use quantitative and qualitative representations and models to analyze situations and solve problems about image formation occurring due to the refraction of light through thin lenses. [**SP 1.4**, **2.2**]

6.E.5.2: The student is able to plan data collection strategies, perform data analysis and evaluation of evidence, and refine scientific questions about the formation of images due to refraction for thin lenses. [**SP 3.2**, **4.1**, **5.1**, **5.2**, **5.3**]

6.F.1.1: The student is able to make qualitative comparisons of the wavelengths of types of electromagnetic radiation. [**SP 6.4**, **7.2**]

Use this space for summary and/or additional notes:

AP® only

6.F.2.1: The student is able to describe representations and models of electromagnetic waves that explain the transmission of energy when no medium is present. [**SP 1.1**]

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Reflection and Refraction**, such as Snell's law and changes in wavelength and speed.
- **Ray Optics**, such as image formation using pinholes, mirrors, and lenses.
- **Physical Optics**, such as single-slit diffraction, double-slit interference, polarization, and color.

1. The Electromagnetic Spectrum
2. Classical Optics
3. Optical Instruments
4. Wave Optics

Skills learned & applied in this chapter:

- Drawing images from mirrors and through lenses.

Use this space for summary and/or additional notes:

Electromagnetic Waves

Unit: Light & Optics

MA Curriculum Frameworks (2016): HS-PS4-1, HS-PS4-3, HS-PS4-5

AP® Physics 2 Learning Objectives: 6.F.1.1

Mastery Objective(s): (Students will be able to…)

- Describe the regions of the electromagnetic spectrum.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain why ultraviolet waves are more dangerous than infrared.

Tier 2 Vocabulary: wave, light, spectrum

Labs, Activities & Demonstrations:

- red *vs.* green *vs.* blue lasers on phosphorescent surface
- blue laser & tonic water
- wintergreen Life Savers™ (triboluminescence)

Notes:

electromagnetic wave: a transverse, traveling wave that is caused by oscillating electric and magnetic fields.

Electromagnetic waves travel through space and do not require a medium. The electric field creates a magnetic field, which creates an electric field, which creates another magnetic field, and so on. The repulsion from these induced fields causes the wave to propagate.

Electromagnetic waves (such as light, radio waves, etc.) travel at the speed of light. The speed of light depends on the medium it is traveling through, but it is a constant within its medium (or lack thereof), and is denoted by the letter "*c*" in equations. In a vacuum, the speed of light is:

$$c = 3.00 \times 10^{8}\ \mathrm{m/s} = 186{,}000 \text{ miles per second}$$

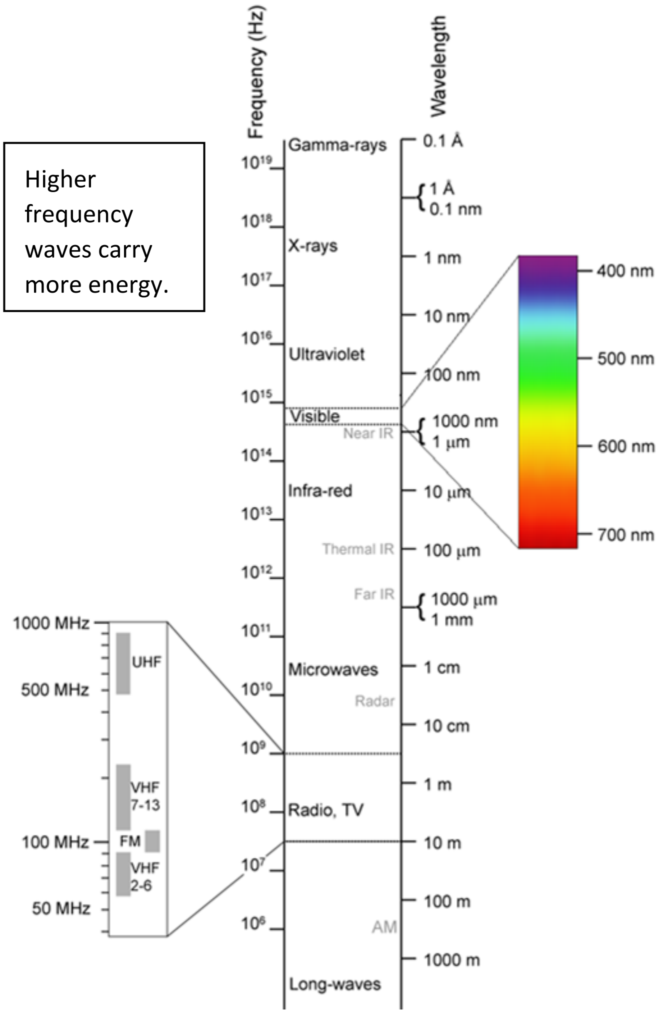
Recall that the speed of a wave equals its frequency times its wavelength:

$$c = \lambda f$$

Use this space for summary and/or additional notes:

<u>electromagnetic spectrum</u>: the entire range of possible frequencies and wavelengths for electromagnetic waves. The waves that make up the electromagnetic spectrum are shown in the diagram below:

Higher frequency waves carry more energy.



<u>visible spectrum</u>: wavelengths of light that can be detected by the human eye. These are wavelengths from roughly 400 nm to 700 nm.

The energy (E) that a wave carries is proportional to the frequency. (Think of it as the number of bursts of energy that travel through the wave every second.) For electromagnetic waves (including light), the constant of proportionality is Planck's constant (named after the physicist Max Planck), which is denoted by a script h in equations.

The energy of a wave is given by the Planck-Einstein equation:

$$E = hf = \frac{hc}{\lambda}$$

where E is the energy of the wave in Joules, f is the frequency in Hz, h is Planck's constant, which is equal to $6.626\times10^{-34}\ \text{J}\cdot\text{s}$, c is the speed of light, and λ is the wavelength in meters.

Use this space for summary and/or additional notes:

Antennas

An antenna is a piece of metal that is affected by electromagnetic waves and is used to amplify waves of specific wavelengths. The optimum length for an antenna is either the desired wavelength, or some fraction of the wavelength such that one wave is an exact multiple of the length of the antenna. (*E.g.,* good lengths for an antenna could be $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, *etc.* of the wavelength.)

Sample problem:

Q: What is the wavelength of a radio station that broadcasts at 98.5 MHz?

A:
$$c = \lambda f$$
$$3.00\times10^{8} = \lambda(9.85\times10^{7})$$
$$\lambda = \frac{3.00\times10^{8}}{9.85\times10^{7}} = 3.05\,\text{m}$$

Q: What would be a good length for an antenna that might be used to receive this radio station?

A: 3.05 m (about 10 feet) is too long to be practical for an antenna. Somewhere between half a meter and a meter is a good size.

$\frac{1}{4}$ wave would be 0.76 m (76 cm), which would be a good choice.

Use this space for summary and/or additional notes:

honors (not AP®)

Color

Unit: Light & Optics

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain how colors are produced and mixed.
- Explain why we see colors the way we do.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain how someone who is red-green color blind might see a green object.

Tier 2 Vocabulary: color, mixing

Labs, Activities & Demonstrations:

- colored light box

Notes:

Light with frequencies/wavelengths in the part of the spectrum that the eye can detect is called visible light.

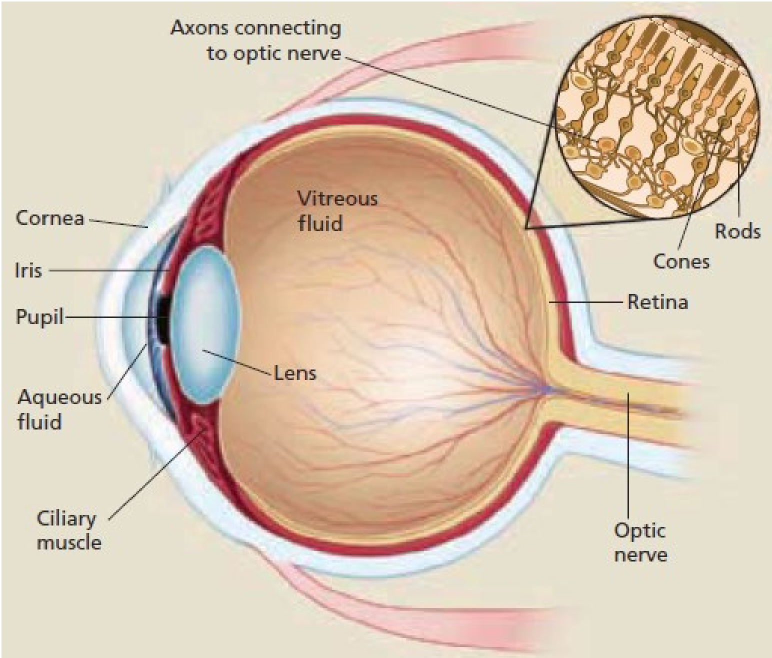
color: the perception by the human eye of how a light wave appears, based on its wavelength/frequency.

Use this space for summary and/or additional notes:

How We See Color

honors (not AP®)

Humans (and other animals) have two types of cells in our retina that respond to light:

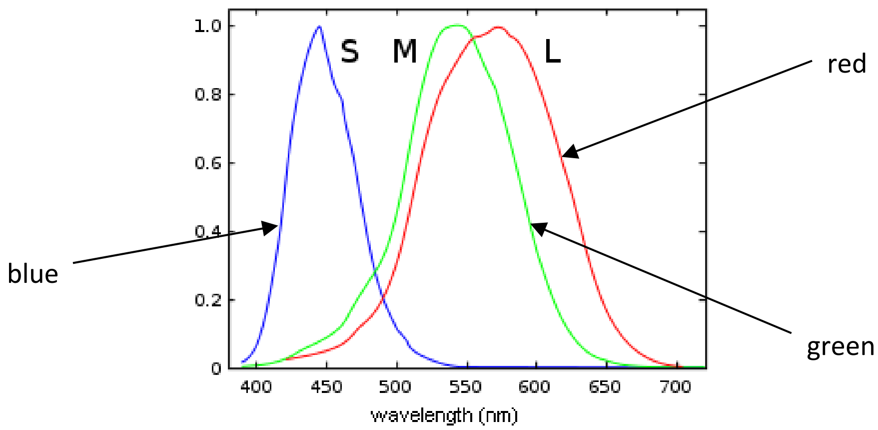


Rod cells resolve the physical details of images. Cone cells are responsible for distinguishing colors. Rod cells can operate in low light, but cone cells need much more light; this is why we cannot see colors in low light.

Use this space for summary and/or additional notes:

honors (not AP®)

There are three different types of cone cells in our eyes, called "S", "M", and "L", which stand for "short," "medium," and "long." Each type of cone cells responds to different wavelengths of light, having a peak (maximum) absorbance in a different part of the visible spectrum:



For example, light with a wavelength of 400–450 nm appears blue to us, because most of the response to this light is from the S cells, and our brains are wired to perceive this response as blue color. Light with a wavelength of around 500 nm would stimulate mostly the M cells and would appear green. Light with a wavelength of around 570 nm would stimulate the M and L cells approximately equally. When green and red receptors both respond, our brains perceive the color as yellow.

Colorblindness occurs when a genetic mutation causes a deficiency or absence of one or more types of cone cells. Most common is a deficiency in the expression of M cone cells, which causes red-green colorblindness. This means that a person with red-green colorblindness would see both colors as red.

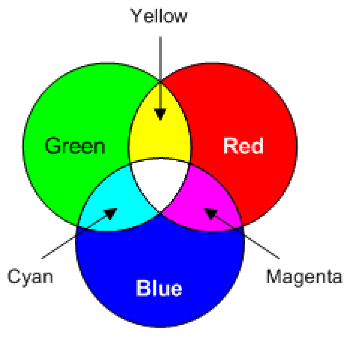
Because colorblindness is recessive and the relevant gene is on the X-chromosome, red-green colorblindness is much more common in men than in women.

Use this space for summary and/or additional notes:

honors (not AP®)

Direct Light: Additive Mixing

Because our cone cells respond to red, green, and blue light, we call these colors the primary colors of light. Other colors can be made by mixing different amounts of these colors, thereby stimulating the different types of cone cells to different degrees. When all colors are mixed, the light appears white.

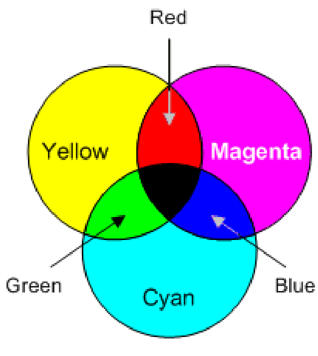


primary color: light that excites only one type of cone cell. The primary colors of light are red, green, and blue.

secondary color: light that is a combination of exactly two primary colors. The secondary colors of light are cyan, magenta, and yellow.

Reflected Light: Subtractive Mixing

When light shines on an object, properties of that object cause it to absorb certain wavelengths of light and reflect others. The wavelengths that are reflected are the ones that make it to our eyes, causing the object to appear that color.



pigment: a material that changes the color of reflected light by absorbing light with specific wavelengths.

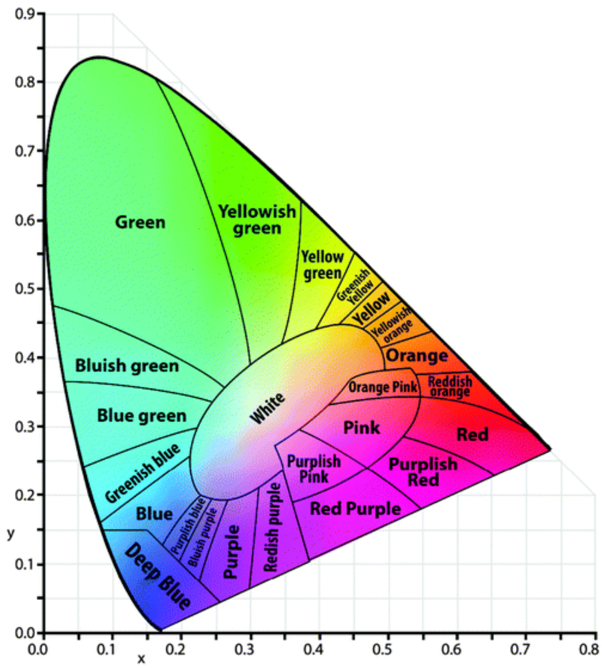
primary pigment: a material that absorbs light of only one primary color (and reflects the other two primary colors). The primary pigments are cyan, magenta, and yellow. Note that these are the secondary colors of light.

secondary pigment: a pigment that absorbs two primary colors (and reflects the other). The secondary pigments are red, green, and blue. Note that these are the primary colors of light.

Use this space for summary and/or additional notes:

honors (not AP®)

Of course, our perception of color is biological, so mixing primary colors is not a simple matter of taking a weighted average of positions on the color wheel. The relationship between the fractions of primary colors used to produce a color and the color perceived is called chromaticity. The following diagram shows the colors that would be produced by varying the intensities of red, green, and blue light.



On this graph, the *x*-axis is the fraction (from 0 – 1) of red light, the *y*-axis is the fraction of green light, and the fraction of blue is implicit [1 – (red + green)].

Notice that equal fractions (0.33) of red, green and blue light would produce white light.

To show the effects of mixing two colors, plot each color's position on the graph and connect them with a line. The linear distance along that line shows the proportional effects of mixing. (*E.g.,* the midpoint would represent the color generated by 50% of each of the source colors.) This method is how fireworks manufacturers determine the mixtures of different compounds that will produce the desired colors.

Use this space for summary and/or additional notes:

Reflection

Unit: Light & Optics

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 6.E.2.1

Mastery Objective(s): (Students will be able to…)

- Explain why light is reflected off smooth surfaces.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain why light is reflected off smooth surfaces.

Tier 2 Vocabulary: light, reflection, virtual image, real image

Labs, Activities & Demonstrations:

- full length mirror on the wall (does amount of image visible change with distance?)
- Mirascope ("hologram maker")

Notes:

reflection: when a wave "bounces" off an object and changes direction.

specular reflection: reflection from a smooth surface.

diffuse reflection: reflection from a rough surface.

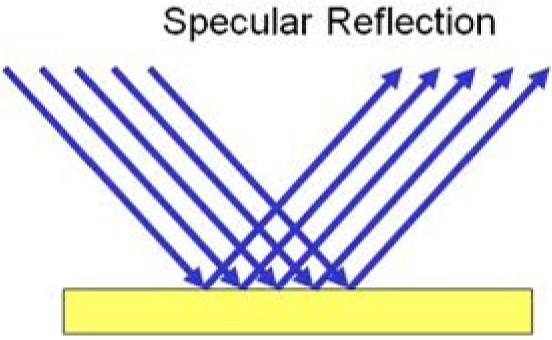
virtual image: a perceived image that appears to be the point of origin of photons (rays of light) that diverge. Because light is reflected back from a mirror (*i.e.,* light cannot pass through it), a ***virtual image*** is one that appears ***behind*** (or "inside") ***the mirror***. A virtual image is what you are used to seeing in a mirror.

real image: a reflected image that is created by photons (rays of light) that converge. Because light is reflected back from a mirror (*i.e.,* light cannot pass through it), a ***real image*** is one that appears ***in front of the mirror***. A real image created by a mirror looks like a hologram.

A rule of thumb that works for both mirrors and lenses is that a real image is produced by the convergence of actual rays of light. A virtual image is our perception of where the rays of light appear to have come from.

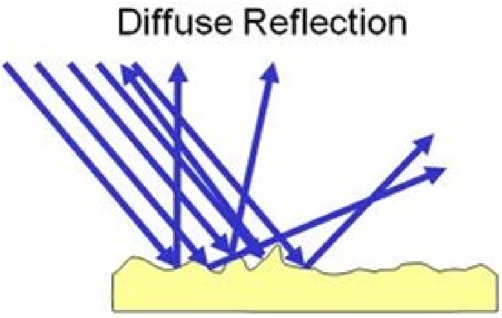
Use this space for summary and/or additional notes:

Specular reflection: reflection from a smooth surface, such as a mirror.



If the photons of light from the source are parallel when they strike the surface, they will also be parallel when they reflect from the surface. This results in a reflected image that appears to be the same size, shape, and distance from the surface as the original object.

Diffuse reflection : reflection from a rough surface, such as a wall.

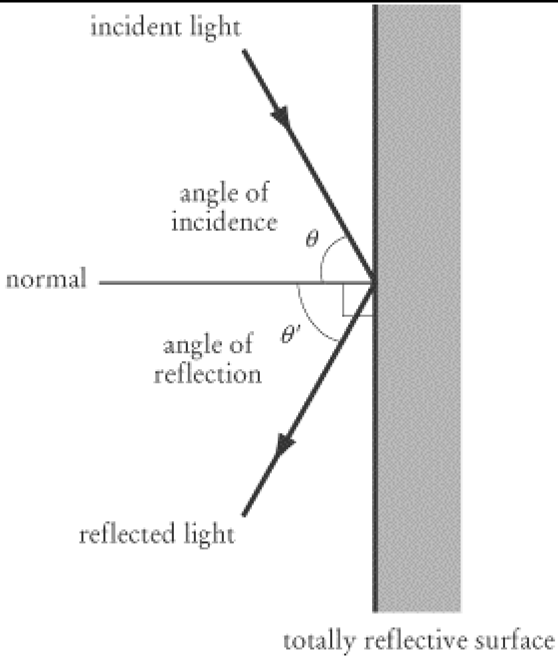


Light striking a rough surface will illuminate the surface. However, because the reflected light rays are not parallel, the reflected light does not create a reflected image of the object.

mirror: a surface that causes specular reflection. An object that was not made to be a mirror but behaves like one is often called a mirrored surface.

Use this space for summary and/or additional notes:

When light waves strike a mirrored surface at an angle (measured from the perpendicular or "normal" direction), they are reflected at the same angle away from the perpendicular. The most common statement of this concept is "The angle of incidence equals the angle of reflection."



This can be stated mathematically as either $\theta = \theta'$ or $\theta_i = \theta_r$.

Use this space for summary and/or additional notes:

Mirrors

Unit: Light & Optics

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 6.E.4.1, 6.E.4.2

Mastery Objective(s): (Students will be able to…)

- Draw ray tracing diagrams for reflection from flat and curved (spherical) mirrors.
- Numerically calculate the distance from the mirror to its focus and the mirror to the image.

Success Criteria:

- Ray diagrams correctly show location of object, focus and image.
- Calculations are correct with correct algebra.

Language Objectives:

- Explain when and why images are inverted (upside-down) *vs.* upright.

Tier 2 Vocabulary: light, reflection, virtual image, real image, mirror, focus

Labs, Activities & Demonstrations:

- Mirascope
- turn a glove inside-out

Notes:

mirror: a surface that light rays reflect from at the same angle the light rays came from.

convex: an object that curves outward.

concave: an object that curves inward.

flat: an object that is neither convex nor concave.

focal point: the point at which parallel rays striking a mirror converge.

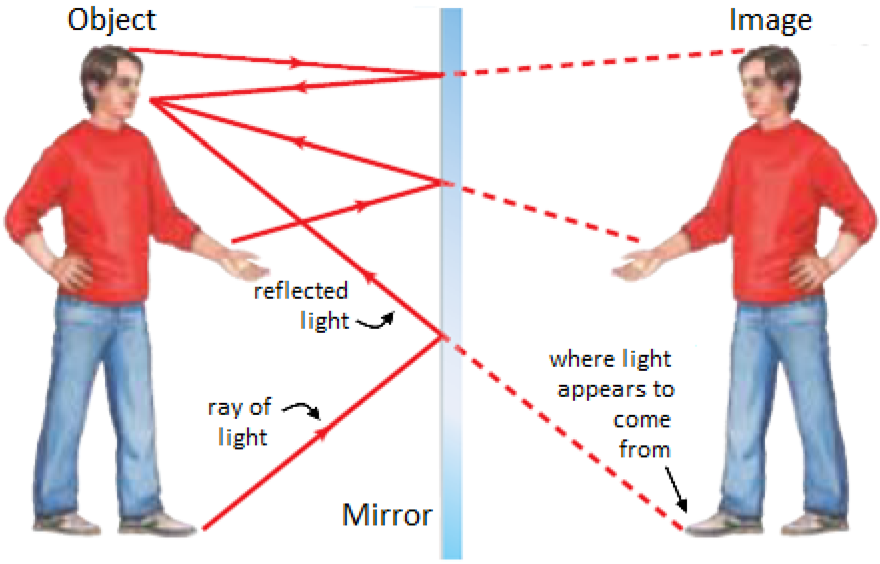
principal axis: a line perpendicular to a mirror (*i.e.,* with an angle of incidence of 0°) such that a ray of light is reflected back along its incident (incoming) path.

The principal axis is often shown as a single horizontal line, but every point on a mirror has a principal axis.

Use this space for summary and/or additional notes:

Flat Mirrors

With a flat mirror, the light reflected off the object (such as the person in the picture below) bounces off the mirror and is reflected back. Because our eyes and the part of our brains that decode visual images can't tell that the light has been reflected, we "see" the reflection of the object in the mirror.



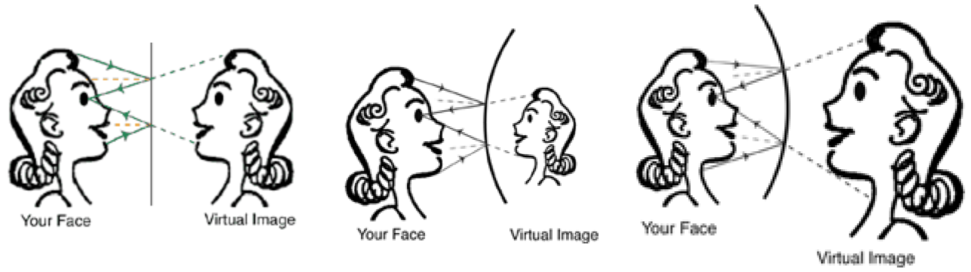
If the mirror is flat, the reflection is the same size and the same distance from the mirror as the actual object. However, the image looks like it is reversed horizontally, but not vertically.

It would seem that the mirror "knows" to reverse the image horizontally but not vertically. (Of course this is not true. If you want the mirror to reverse the image vertically, all you need to do is put the mirror on the floor.) What is actually happening is that light is reflected straight back from the mirror. Anything that is on your right will also be on the right side of the image (from your point of view; if the image were actually a person, this would be the other person's left). Anything that is on top of you will also be on top of the image as you look at it.

What the mirror is doing is the same transformation as flipping a polygon over the *y*-axis. ***The reversal is actually front-to-back*** (where "front" means closer to the mirror and "back" means farther away from it).

Use this space for summary and/or additional notes:

Convex and Concave Mirrors



flat mirror — same-size image
convex mirror — smaller image
concave mirror — larger image

With a convex mirror (curved outwards), the reflected rays diverge (get farther apart). When this happens, it makes the reflection appear smaller.

In a concave mirror (curved inwards), the reflected rays converge (get closer together). When this happens, it makes the reflection appear larger.

One place you have probably seen convex mirrors is the passenger-side mirrors in cars. The mirror is slightly convex in order to show a wider field of view. However, this makes the image smaller and appear farther away.

If you wear makeup, you may have used a concave mirror. The larger image makes it easier to see small details. (However, it is important to remember that those details are smaller than they appear!)

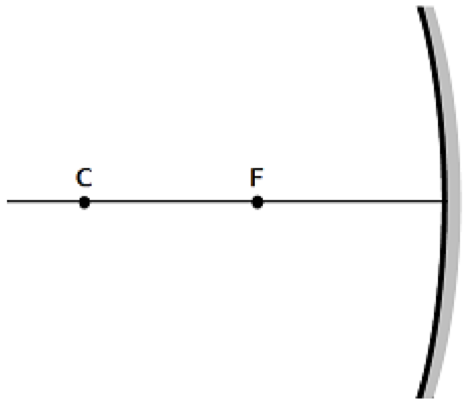
Use this space for summary and/or additional notes:

Focal Point

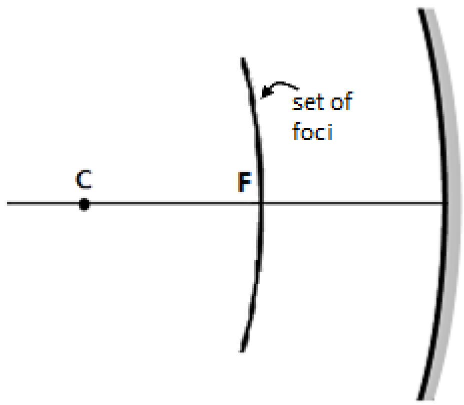
The focal point or focus of a mirror is the point where the rays of reflected light converge. For a spherical mirror (one in which the shape of the mirror is the surface of a sphere), the focus is halfway between the surface of the mirror and the center of the sphere. This means the distance from the mirror to the focus (f)* is half of the radius of curvature (r_c):

$$f = \frac{r_c}{2} \quad \text{or} \quad r_c = 2f$$

In an introductory physics class, the focus of a curved mirror is often described as a single point, as in the following diagram.



However, is important to remember that a principal axis (a line perpendicular to the mirror) can be drawn from any point along the surface of the mirror. This means that the focus is not a single point, but rather the ***set of all points*** that are halfway between the center of the sphere and the surface of the mirror:



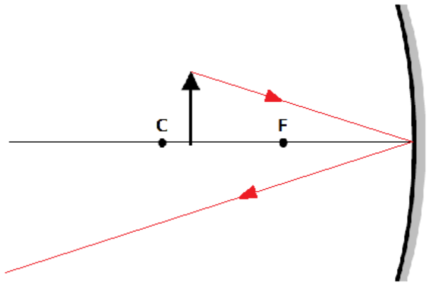
* Some physics textbooks use the variables d_o, d_i, and d_f for distances to the object, image, and focus, respectively. These notes use the variables s_o, s_i, and f in order to be consistent with the equation sheet provided by the College Board for the AP® Physics 2 exam.

Use this space for summary and/or additional notes:

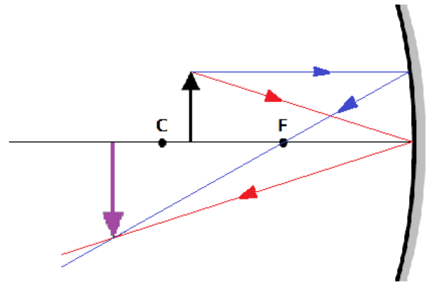
Ray Tracing

An intuitive way of finding the location, size and orientation of an image in a mirror is to draw (trace) the rays of light to see where they converge.

1. A ray of light that hits the mirror anywhere on a principal axis is reflected back at the same angle relative to that principal axis. (The angle of incidence relative to the principal axis equals the angle of reflection.)



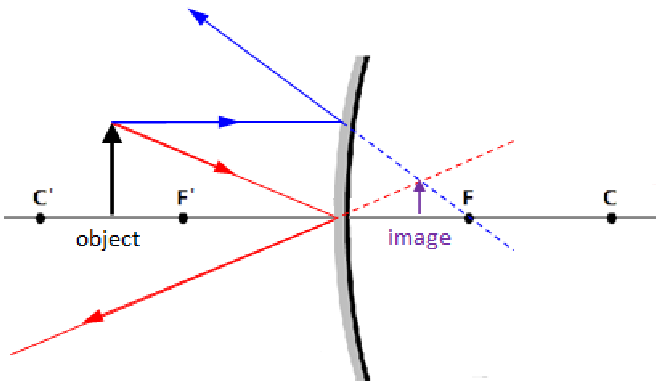
2. A ray of light that hits the mirror parallel to the principal axis is reflected directly toward or away from the focus.



3. If you draw a pair of rays from the top of the object as described by #1 and #2 above, the intersection will be at the top of the image of the object.

Convex Mirrors

For a convex mirror, the image is always virtual (behind the mirror) and is always smaller than the object:

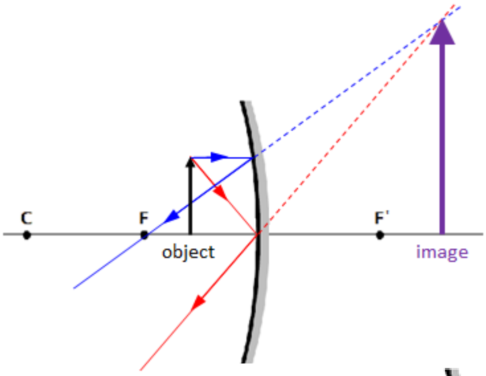


Use this space for summary and/or additional notes:

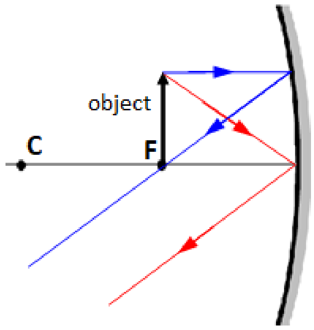
Concave Mirrors

For a concave mirror, what happens with the image changes depending on where the object is relative to the center of curvature and the focus, as shown by ray tracing in each the following cases.

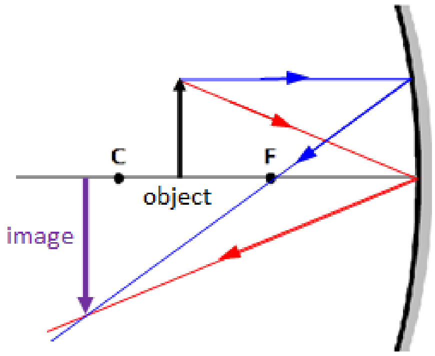
1. If the object is closer to the mirror than the focus, you see a virtual image (behind the mirror) that is upright (right-side-up), and larger than the original.



2. If the object is at the focus, there is no image because the rays do not converge.

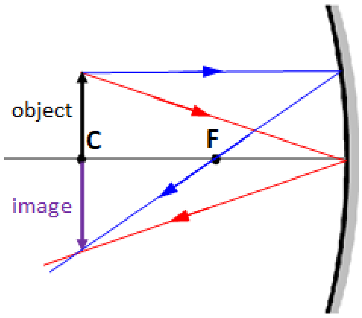


3. If the object is between the focus and the center of curvature, you see a real image (in front of the mirror) that is behind the object, inverted (upside-down), and larger.

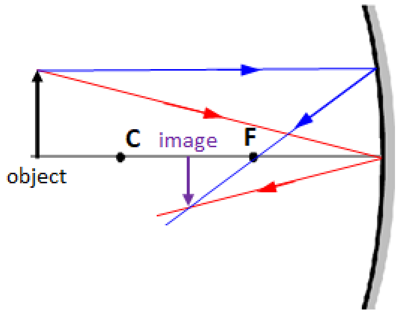


Use this space for summary and/or additional notes:

4. If the object is at the center of curvature, you see a real, inverted image that is the same size and same distance from the mirror as the object.



5. If the object is farther from the mirror than the center of curvature, you see a real, inverted image that is smaller and closer to the mirror than the object.



Equations

The distance from the mirror to the focus (f) can be calculated from the distance to the object (s_o) and the distance to the image (s_i), using the following equation:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Distances for the image (s_i) and focus (f) are positive in front of the mirror (where a real image would be), and negative behind the mirror (where a virtual image would be).

The height of the image (h_i) can be calculated from the height of the object (h_o) and the two distances (s_i and s_o), using the following equation:

$$M = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

A positive value for h_i means the image is upright (right-side-up), and a negative value for h_i means the image is inverted (upside-down).

<u>magnification</u>: the ratio of the size of the image to the size of the object.

If $M > 1$, the image is larger than the object. (For example, if $M = 2$, then the image is twice as large as the object.) If $M = 1$, the object and image are the same size. If $M < 1$, the image is smaller. Finally, note that in a mirror, virtual images are always upright, and real images are always inverted.

Use this space for summary and/or additional notes:

Sample Problem:

Q: An object that is 5 cm high is placed 9 cm in front of a spherical convex mirror. The radius of curvature of the mirror is 10 cm. Find the height of the image and its distance from the mirror. State whether the image is real or virtual, and upright or inverted.

A: The mirror is convex, which means the focus is behind the mirror. This is the side where a ***virtual*** image would be, so the distance to the focus is therefore negative. The distance to the focus is half the radius of curvature, which means $f = -5$ cm. From this information, we can find the distance from the mirror to the image (s_i):

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$\frac{1}{9} + \frac{1}{s_i} = \frac{1}{-5}$$

$$\frac{5}{45} + \frac{1}{s_i} = -\frac{9}{45}$$

$$\frac{1}{s_i} = -\frac{14}{45}$$

$$s_i = -\frac{45}{14} = -3.2\,\text{cm}$$

The value of −3.2 cm means the image is a virtual image located 3.2 cm behind the mirror.

Now that we know the distance from the mirror to the image, we can calculate the height of the image (h_i):

$$\frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

$$\frac{h_i}{5} = -\frac{-3.2}{9}$$

$$(5)(3.2) = 9\,h_i$$

$$h_i = \frac{16}{9} = +1.8\,\text{cm}$$

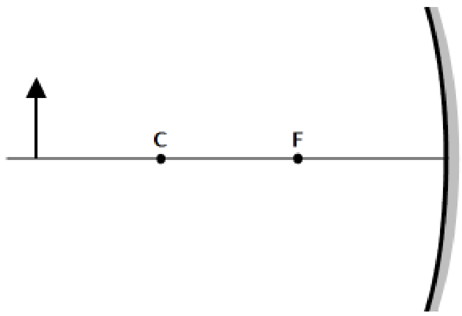
The image is 1.8 cm high. Because the height is a positive number, this means the image is upright (right-side-up).

Use this space for summary and/or additional notes:

Homework Problems

In each of the following problems, an object that is 12 cm tall is placed in front of a curved, spherical mirror with a focal length of 18 cm.

1. **(M)** The mirror is concave and the object is placed 58 cm from the mirror.
    a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.



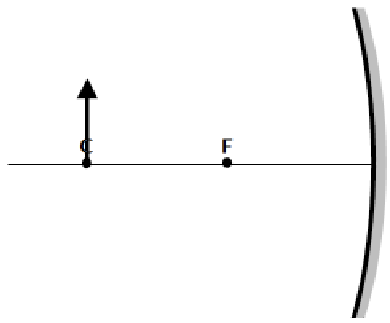
    b. Calculate the height and orientation (upright or inverted) of the image, and its distance from the mirror.

Answers: $s_i = 26.1$ cm; $h_i = -5.4$ cm

Use this space for summary and/or additional notes:

2. **(S)** The mirror is concave and the object is placed 36 cm from the mirror.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.



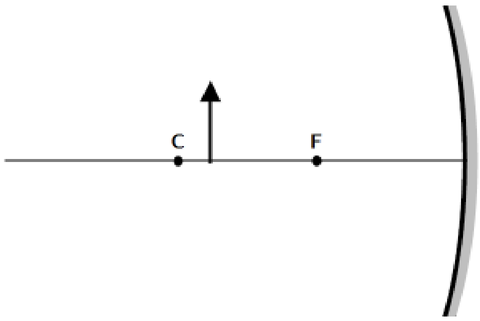
   b. Calculate the height and orientation of the image, and its distance from the mirror.

Answers: s_i = 36 cm; h_i = −12 cm

Use this space for summary and/or additional notes:

3. **(S)** The mirror is concave and the object is placed 32 cm from the mirror.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.



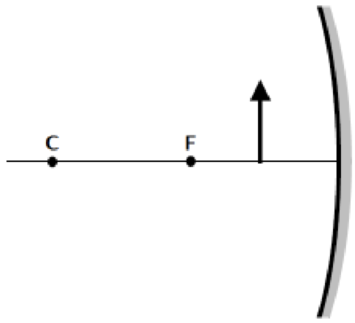
   b. Calculate the height and orientation of the image, and its distance from the mirror.

Answers: s_i = 41.1 cm; h_i = −15.4 cm

Use this space for summary and/or additional notes:

4. **(M)** The mirror is concave and the object is placed 6 cm from the mirror.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.



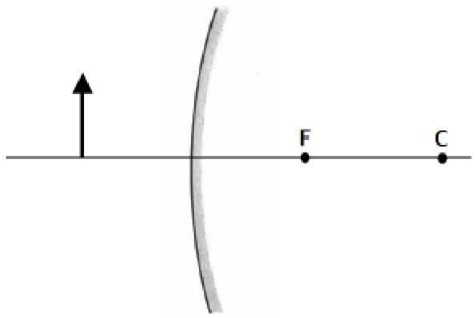
   b. Calculate the height and orientation of the image, and its distance from the mirror.

Answers: $s_i = -9$ cm; $h_i = 18$ cm

Use this space for summary and/or additional notes:

5. **(M)** The mirror is convex and the object is placed 15 cm from the mirror.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.



   b. Calculate the height and orientation of the image, and its distance from the mirror.

Answers: $s_i = -8.2$ cm; $h_i = 6.5$ cm

Use this space for summary and/or additional notes:

Refraction

Unit: Light & Optics

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 6.E.1.1, 6.E.2.1, 6.E.3.1, 6.E.3.2, 6.E.3.3

Mastery Objective(s): (Students will be able to…)

- Explain how and why refraction happens.
- Solve problems using Snell's Law.

Success Criteria:

- Explanation accounts for the size, location and orientation of the image.
- Calculations are correct with correct algebra and trigonometry.

Language Objectives:

- Explain why we see the image of an object through a magnifying glass but not the object in its actual location.

Tier 2 Vocabulary: light, reflection, virtual image, real image, lens, focus

Labs, Activities & Demonstrations:

- laser through clear plastic
- laser through bent plastic (total internal reflection)
- laser through falling stream of water (with 1 drop milk)
- Pyrex stirring rod in vegetable oil (same index of refraction)
- penny in cup of water

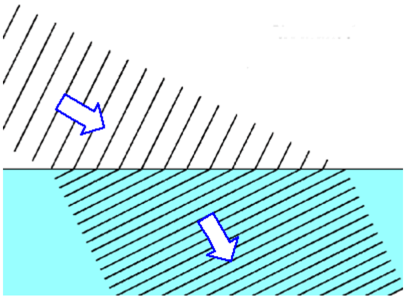
Notes:

<u>refraction</u>: a change in the velocity and direction of a wave as it passes from one medium to another. The change in direction occurs because the wave travels at different velocities in the different media.

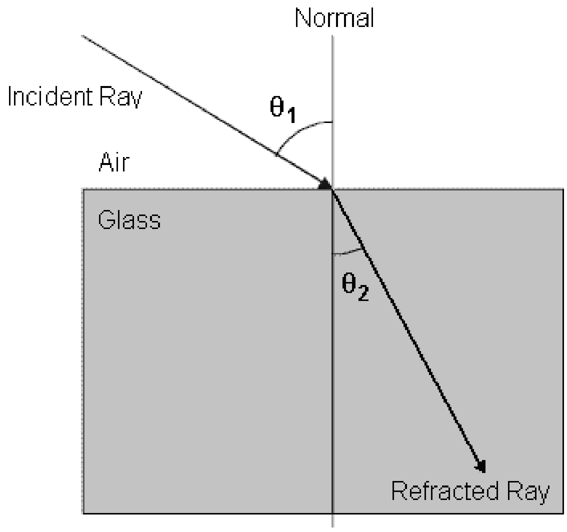
<u>index of refraction</u>: a number that relates the velocity of light in a medium to the velocity of light in a vacuum.

Use this space for summary and/or additional notes:

When light crosses from one medium to another, the difference in velocity of the waves causes the wave to bend. For example, in the picture below, the waves are moving faster in the upper medium. As they enter the lower medium, they slow down. Because the part of the wave that enters the medium soonest slows down first, the angle of the wave changes as it crosses the boundary.



When the waves slow down, they are bent toward the normal (perpendicular), as in the following diagram:



Use this space for summary and/or additional notes:

The index of refraction of a medium is the velocity of light in a vacuum divided by the velocity of light in the medium:

$$n = \frac{c}{v}$$

Thus the larger the index of refraction, the more the medium slows down light as it passes through.

The index of refraction for some substances is given below.

Substance	Index of Refraction	Substance	Index of Refraction
vacuum	1.00000	quartz	1.46
air (0°C and 1 atm)	1.00029	glass (typical)	1.52
water (20°C)	1.333	NaCl (salt) crystals	1.54
acetone	1.357	polystyrene (#6 plastic)	1.55
ethyl alcohol	1.362	diamond	2.42

These values are for yellow light with a wavelength of 589 nm.

For light traveling from one medium into another, the ratio of the speeds of light is related inversely to the ratio of the indices of refraction, as described by Snell's Law (named for the Dutch astronomer Willebrord Snellius):

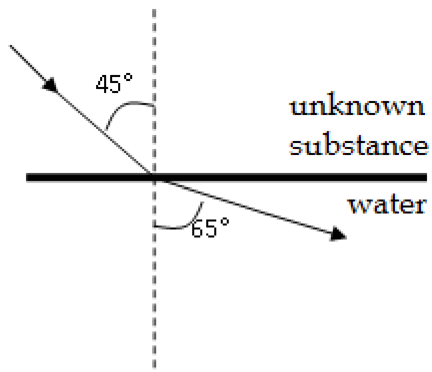
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

The more familiar presentation of Snell's Law is:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Sample Problem:

Q: Incident light coming from an unknown substance strikes water at an angle of 45°. The light refracted by the water at an angle of 65°, as shown in the diagram at the right. What is the index of refraction of the unknown substance?



A: Applying Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$n_1 \sin(45°) = (1.33)\sin(65°)$$
$$n_1 = \frac{(1.33)\sin 65°}{\sin 45°} = \frac{(1.33)(0.906)}{0.707} = 1.70$$

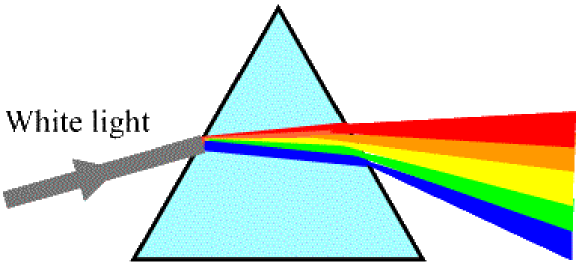
Use this space for summary and/or additional notes:

Prisms

The index of refraction of a medium varies with the wavelength of light passing through it. The index of refraction is greater for shorter wavelengths (toward the violet end of the spectrum) and less (closer to 1) for longer wavelengths (toward the red end of the spectrum.

<u>prism</u>: an object that refracts light

If light passes through a prism (from air into the prism and back out) and the two interfaces are not parallel, the different indices of refraction for the different wavelengths will cause the light to spread out.

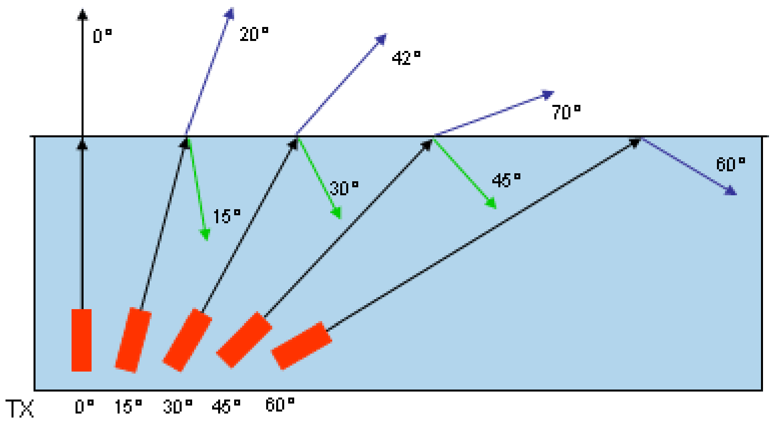


When light is bent by a prism, the ratio of indices of refraction is the inverse of the ratio of wavelengths. Thus we can expand Snell's Law as follows:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

Total Internal Reflection

If a light wave is traveling from a slower medium to a faster one and the angle is so steep that the refracted angle would be 90° or greater, the boundary acts as a mirror and the light ray reflects off of it. This phenomenon is called *<u>total internal reflection</u>:*

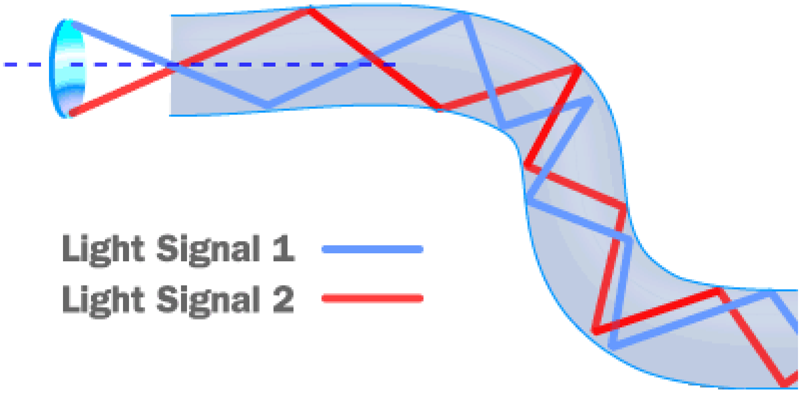


<u>critical angle</u> (θ_c): the angle beyond which total internal reflection occurs.

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Use this space for summary and/or additional notes:

Total internal reflection is how optical fibers (long strands of optically pure glass with a high index of refraction) are used to transmit information over long distances, using pulses of light.



Total internal reflection is also the principle behind speech teleprompters:

The speaker stands behind a clear piece of glass. The image of the speech is projected onto the glass. The text is visible to the speaker, but not to the audience.

Use this space for summary and/or additional notes:

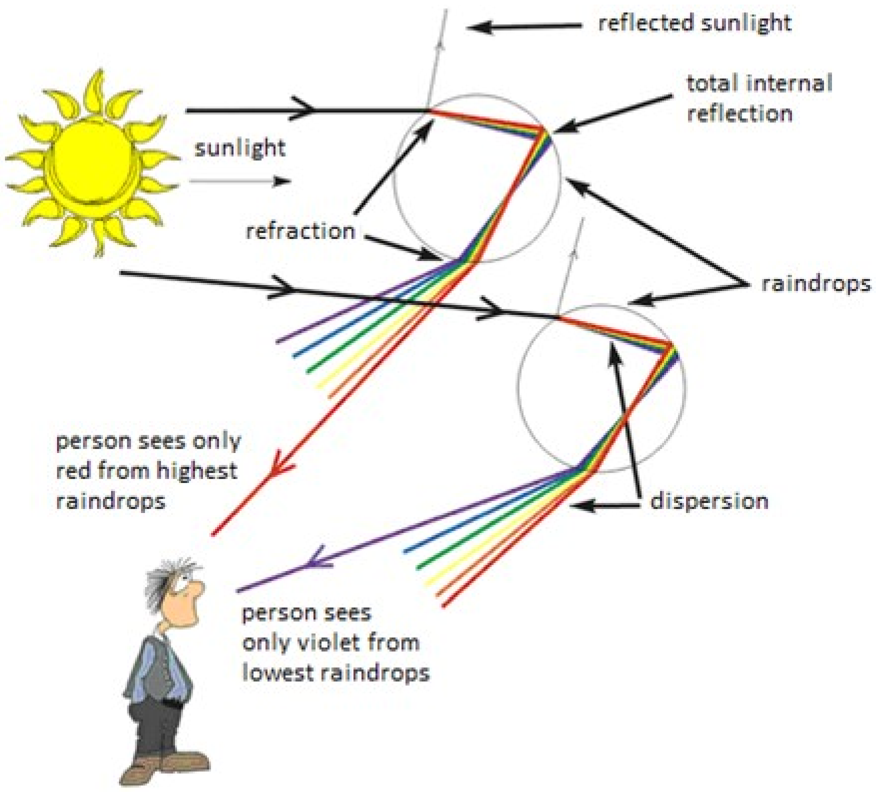
honors (not AP®)

Rainbows

A rainbow occurs from a combination of refraction, total internal reflection, and a second refraction, with raindrops acting as the prisms.

When this process occurs, different wavelengths of are refracted at different angles. Because colors near the red end of the spectrum have a lower index of refraction, the critical angle is shallower for these wavelengths, and they are reflected at a shallower angle than colors closer to the violet end of the spectrum.

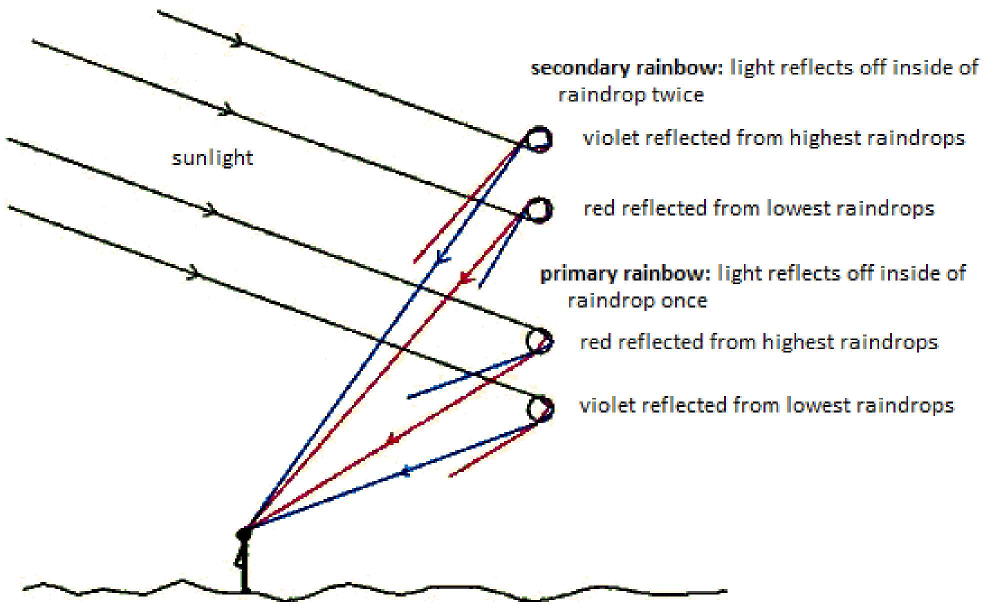
The overall change in the direction of the light after this combination of refraction–reflection–refraction (including both refractions as well as the reflection) ranges from approximately 40° for violet light to approximately 42° for red light. This difference is what produces the spread of colors in a rainbow, and is why red is always on the outside of the rainbow and violet is always on the inside.



Use this space for summary and/or additional notes:

honors (not AP®)

When internal reflection occurs twice on the inside of a raindrop, the result is a second rainbow.



The second rainbow appears above the first because the angle of light exiting the raindrop is greater—varying from 50° for red light to 52.5° for violet light. The second internal reflection reverses the colors, which is why violet is on the outside and red is on the inside in the second rainbow.

Use this space for summary and/or additional notes:

honors (not AP®)

This is a picture of a double rainbow in Lynn, Massachusetts. Note that the order of the colors in the second rainbow is reversed.

Note also that the sky is brighter inside the primary rainbow. There are two reasons for this. First, it's not actually true that each band is only one color of light. Because red light reflects at *all* angles greater than or equal to 40°, red light is therefore a component of all of the colors inside the red band of the rainbow. The same is true for each of the other colors; inside of the violet band, all wavelengths of visible light are present, and the result is white light. Outside of the red band, no visible light is refracted, which causes the sky outside the rainbow to appear darker.

Second, raindrops scatter light at all wavelengths, and light scattering is also a significant contributing factor to the brightness inside. (See the *Scattering* topic starting on page 528 for more information.)

You may also notice that because the second rainbow is reversed, the sky is slightly brighter outside of the second rainbow.

Use this space for summary and/or additional notes:

Homework Problems

You will need to look up indices of refraction in Table Q on page 587 of your Physics Reference Tables in order to answer these questions.

1. **(M)** A ray of light traveling from air into borosilicate glass strikes the surface at an angle of 30°. What will be the angle of refraction?

   Answer: 19.8°

2. **(S)** Light traveling through air encounters a second medium which slows the light to $2.7\times10^{8}\ \frac{m}{s}$. What is the index of refraction of the second medium?

   Answer: 1.11

3. **(M)** What is the velocity of light as it passes through a diamond?

   Answer: $1.24\times10^{8}\ \frac{m}{s}$

Use this space for summary and/or additional notes:

4. **(M)** A diver in a freshwater lake shines a flashlight toward the surface of the water. What is the minimum angle (from the vertical) that will cause beam of light to be reflected back into the water (total internal reflection)?

   Answer: 48.6°

5. **(S)** A graduated cylinder contains a layer of silicone oil floating on water. A laser beam is shone into the silicone oil from above (in air) at an angle of 25° from the vertical. What is the angle of the beam in the water?

   Answer: 18.5°

6. **(S)** A second graduated cylinder contains only a layer of water. The same laser beam is shone into the water from above (in air) at the same angle of 25° from the vertical. What is the angle of the beam in the water?

   Answer: 18.5°

Use this space for summary and/or additional notes:

Polarization

Unit: Light & Optics

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 6.A.1.3

Mastery Objective(s): (Students will be able to…)

- Explain how and under which circumstances light can be polarized.

Success Criteria:

- Explanation accounts for the filtering of waves of other orientations and for the specific direction.

Language Objectives:

- Explain how polarized sunglasses work.

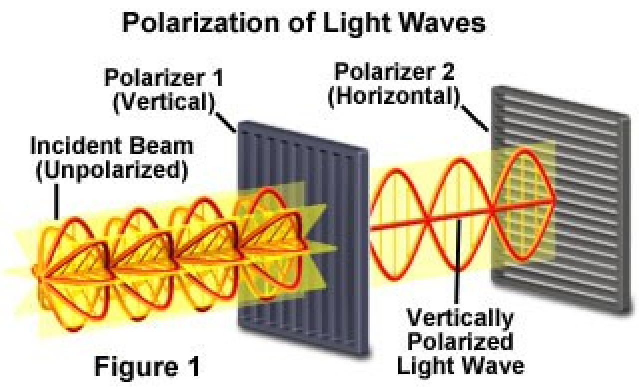
Tier 2 Vocabulary: polarized

Labs, Activities & Demonstrations:

- polarizing filters

Notes:

Normally, light (and other electromagnetic waves) propagate in all directions. When the light is passed through a special filter, called a polarizer, it blocks light waves in all but one plane (direction), as shown in the following diagram:



Light that is polarized in this manner is called plane-polarized light.

Note that if you place two polarizers on top of each other and turn them so they polarize in different directions, no light can get through. This is called crossed polarization.

Use this space for summary and/or additional notes:

A flat surface can act as a polarizer at certain angles. The Scottish physicist Sir David Brewster derived a formula for the angle of maximum polarization based on the indices of refraction of the two substances:

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

where:

θ_B = Brewster's angle, the angle of incidence at which unpolarized light striking a surface is perfectly polarized when reflected.

n_1 & n_2 = indices of refraction of the two substances

The two pictures below were taken with the same camera and a polarizing filter. In the picture on the left, the polarizing filter is aligned with the light reflected off the window. In the picture on the right, the polarizing filter is rotated 90° so that none of the reflected light from the window can get to the camera lens.


Another example is light reflecting off a wet road. When the sun shines on a wet road at a low angle, the reflected light is polarized parallel to the surface (*i.e.,* horizontally). Sunglasses that are polarized vertically (*i.e.,* that allow only vertically polarized light to pass through) will effectively block most or all of the light reflected from the road.

Yet another example is the light that creates a rainbow. When sunlight reflects off the inside of a raindrop, the angle of incidence is very close to Brewster's angle. This causes the light that exits the raindrop to be polarized in the same direction as the bows of the rainbow (*i.e.,* horizontally at the top). This is why you cannot see a rainbow through polarized sunglasses!

Use this space for summary and/or additional notes:

Lenses

Unit: Light & Optics

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 6.E.5.1, 6.E.5.2

Mastery Objective(s): (Students will be able to…)

- Draw ray tracing diagrams for refraction through convex and concave lenses.
- Numerically calculate the distance from the lens to its focus and the lens to the image.

Success Criteria:

- Ray diagrams correctly show location of object, focus and image.
- Calculations are correct with correct algebra.

Language Objectives:

- Explain when and why images are inverted (upside-down) *vs.* upright.

Tier 2 Vocabulary: light, refraction, virtual image, real image, lens, focus

Labs, Activities & Demonstrations:

- Fresnel lens
- optics bench lab

Notes:

Lenses are similar to curved mirrors in that they change the direction of light rays to produce an image of an object that can have a different size, orientation and distance from the mirror relative to the object.

Lenses are different from mirrors in that light passes through them, which means they operate by refraction instead of reflection.

<u>lens</u>: a usually-symmetrical optical device which refracts light in a way that makes the rays of light either converge or diverge.

<u>convex lens</u>: a lens that refracts light so that it converges as it passes through.

<u>concave lens</u>: a lens that refracts light so that it diverges as it passes through.

<u>focus or focal point</u>: the point at which light rays converge after passing through the lens.

Use this space for summary and/or additional notes:

principal axis: a line perpendicular to the surface of the lens, such that light passing through it is refracted at an angle of 0° (*i.e.*, the direction is not changed).

The principal axis is often shown as a single horizontal line, but every point on the surface of a lens has a principal axis. Note also that if a lens is asymmetrical, its principal axis may be different on each side.

vertex: the point where the principal axis passes through the center of the lens.

real image: an image produced by light rays that pass through the lens. A ***real image*** will appear on the ***opposite side of the lens*** from the object. A real image is what you are used to seeing through a magnifying glass.

virtual image: an apparent image produced at the point where diverging rays appear to originate. A ***virtual image*** will appear on the ***same side of the lens*** as the object.

A rule of thumb that works for both mirrors and lenses is that a ***real image*** is produced by the convergence of the ***actual rays of light***. A virtual image is our perception of where the rays of light would have come from.

upright image: an image that is oriented in the same direction as the object. ("right-side-up")

inverted image; an image that is oriented in the opposite direction from the object. ("upside-down")

Use this space for summary and/or additional notes:

Calculations

The equations for lenses are the same as the equations for curved mirrors. Distances are measured from the vertex.

The magnification (M) is the ratio of the height of the image (h_i) to the height of the object (h_o), which is equal to the ratio of the distance of the image (s_i) to distance of the object (s_o).

$$M = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

As with mirrors, the distance to the image is defined to be positive for a real image, and negative for a virtual image. However, note that with lenses the real image is caused by the rays of light that pass through the lens, which means a real image is behind a lens, where as a real image is in front of a mirror.

Note also that for lenses, this means that the positive direction for the object and the positive for the image are opposite.

As with mirrors, the distance from the vertex of the lens to the focus (f) is defined by the equation:

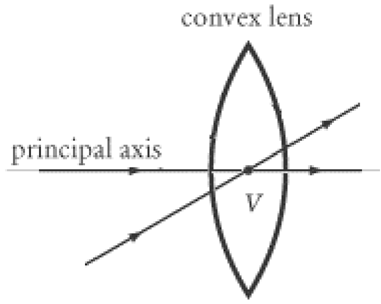
$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

Use this space for summary and/or additional notes:

Ray Tracing

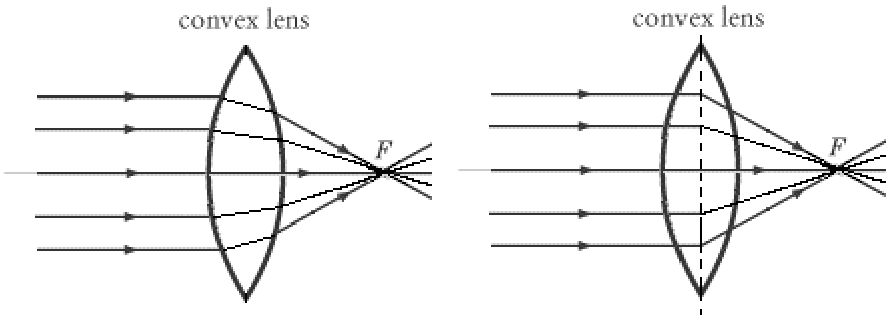
In all lenses:

1. Light passing through the *vertex* of the lens comes out of the lens in the same direction as it entered, as if the lens were not there.



2. Light passing through any part of the lens other than the vertex is refracted through the *focus*.

   Notice that the light is refracted twice, once upon entering the lens and a second time upon entering the air when it exits. For convenience, we usually draw the ray trace as if the light is refracted once when it crosses the center of the lens.



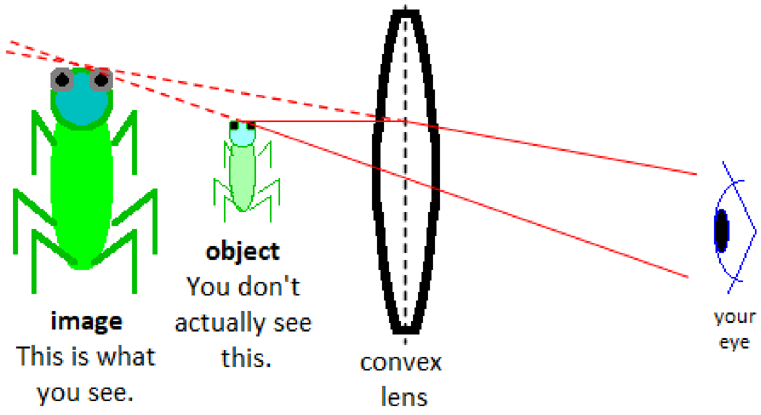
what actually happens how we usually draw it

Use this space for summary and/or additional notes:

Convex Lenses

A convex lens causes light rays to converge (bend towards each other) as they pass through.

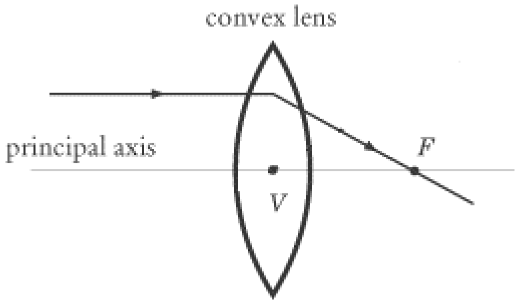
The most familiar use of convex lenses is as a magnifying glass. Note how the bending of the light rays makes the object appear larger



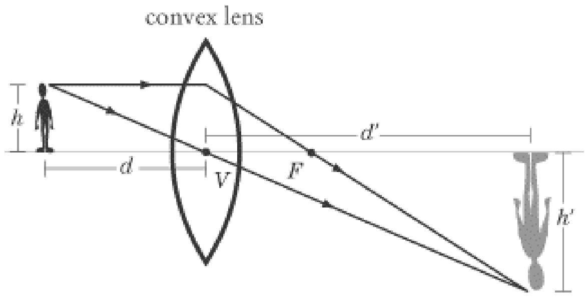
Note also that the lens bends ***all*** of the light. Your eyes cannot see the unbent light rays, which means you cannot see the actual object in its actual location; you *only* see the image.

Use this space for summary and/or additional notes:

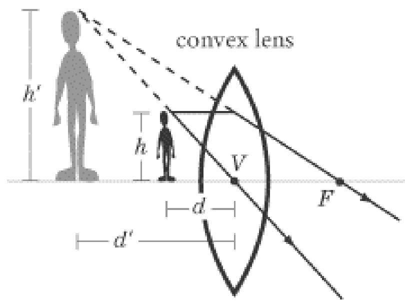
For a convex lens, the focus is always on the opposite side of the lens from the object:



1. If the object is farther away from the lens than the focus, the image is real (on the opposite side of the lens) and inverted (upside-down).



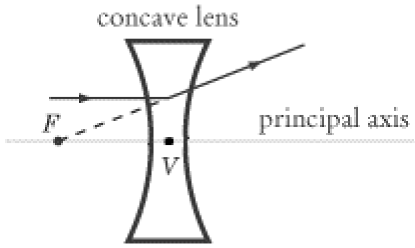
2. If the object is closer than the focus, the image is virtual (on the same side of the lens) and upright (right-side-up).



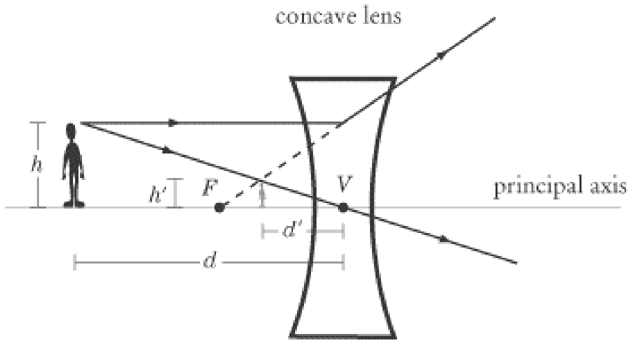
Use this space for summary and/or additional notes:

Concave Lenses

A concave lens causes light rays to diverge (bend away from each other). For a concave lens, the focus is on the same side of the lens as the object.



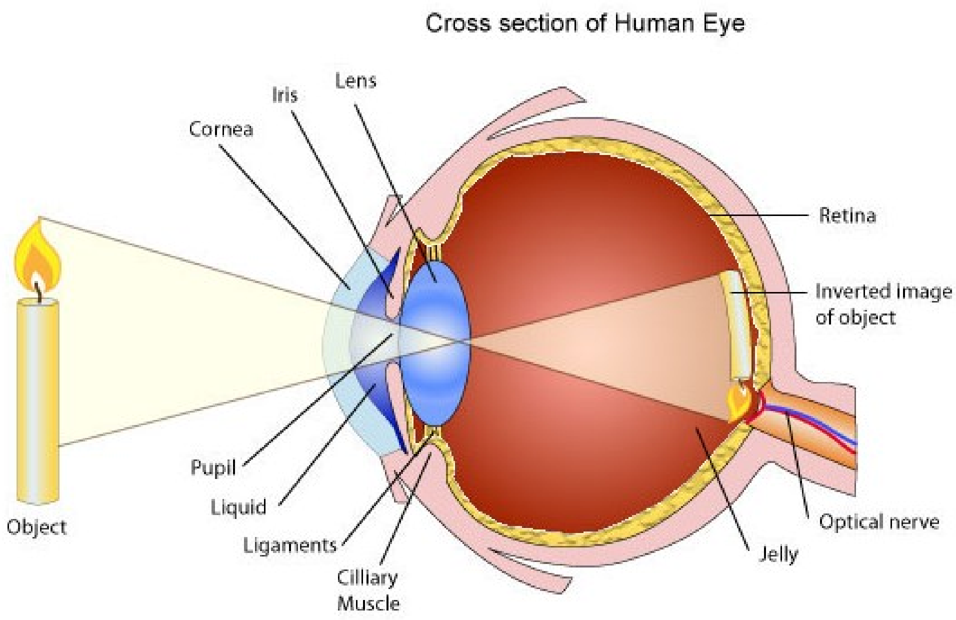
For a concave lens, the image is always virtual (on the same side of the lens) and upright (right-side-up):



Use this space for summary and/or additional notes:

Physiology

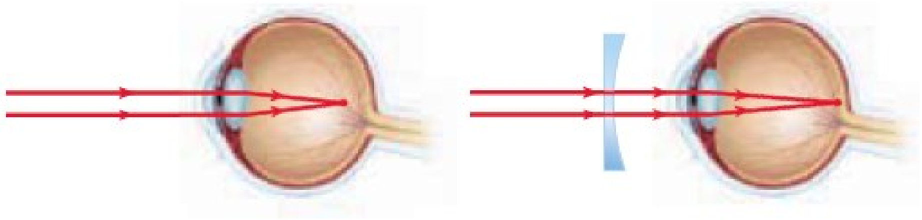
In the human eye, the cornea and lens both act as lenses. However, because the action of the ciliary muscles changes the shape of the lens, the lens is responsible for the exact focal point, which determines what we are focusing our eyes on. When the ciliary muscles relax, the images of distant objects are focused on the retina. When these muscles contract, the focal point moves and closer objects come into focus.



Use this space for summary and/or additional notes:

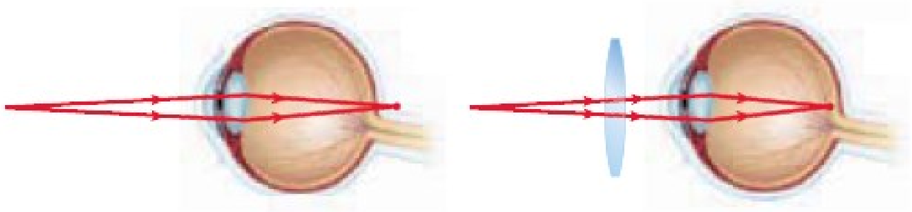
Nearsightedness and Farsightedness

"Nearsighted" means only objects near the eye are in focus; the viewer is unable to focus on distant objects. This happens because the focus of the lens when the ciliary muscles are fully relaxed is in front of the retina. Nearsightedness is corrected by eyeglasses with concave lenses, which move the focal point back to the retina.



Notice that lenses that correct nearsightedness are concave only on the inside. This helps the lenses avoid the "Coke bottle" look.

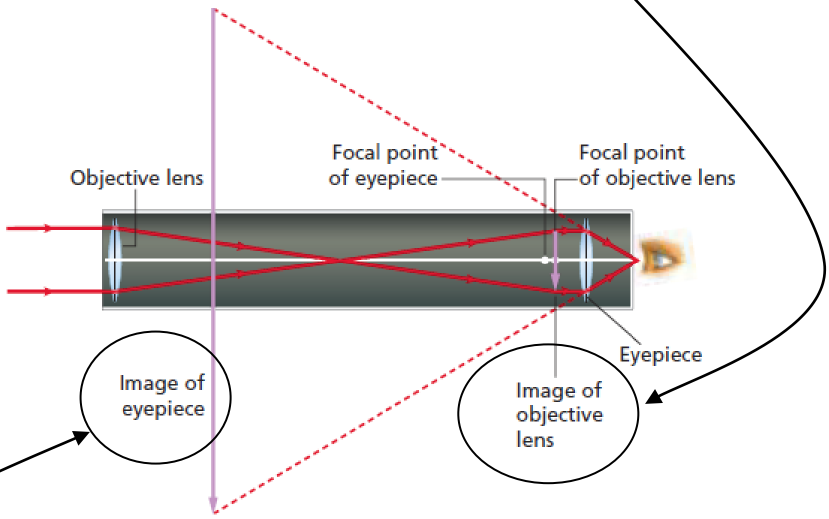
"Farsighted" means only objects far away from the eye are in focus; the viewer is unable to focus on close objects. This happens because the ciliary muscles cannot contract enough to bring the focal point of the lens for light coming from nearby objects onto the retina. Farsightedness is corrected by eyeglasses with convex lenses, which move the focal point forward to the retina.



Use this space for summary and/or additional notes:

Telescopes

A telescope performs two tasks. The objective lens focuses light from a distant object and creates a virtual image in front of the eyepiece.



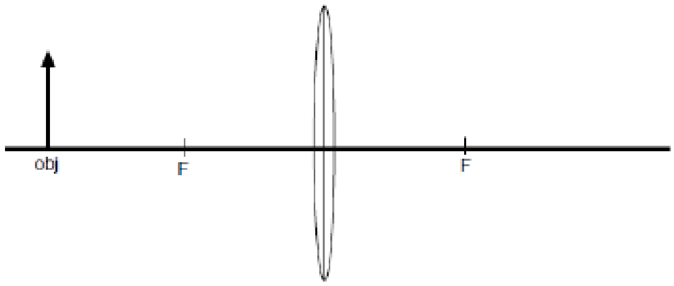
The image from the objective lens is then refracted by the eyepiece. The eyepiece creates a much larger virtual image, which is what the eye sees.

Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** A 4.2 cm tall object is placed 12 cm from a convex lens that has a focal length of 6.0 cm.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.



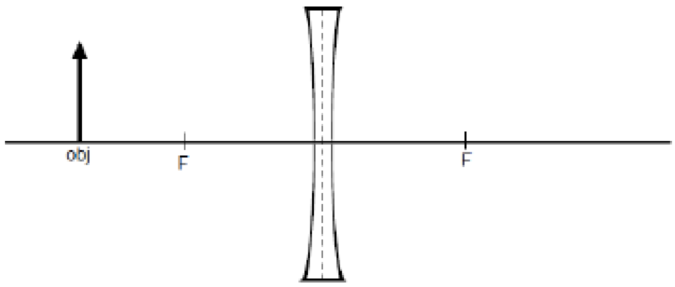
   b. Calculate the height and orientation of the image, and its distance from the lens.

Answers: $s_i = +12$ cm; $h_i = -4.2$ cm

Use this space for summary and/or additional notes:

2. **(M)** A 4.2 cm tall object is placed 12 cm from a concave lens that has a focal length of 6.0 cm.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.



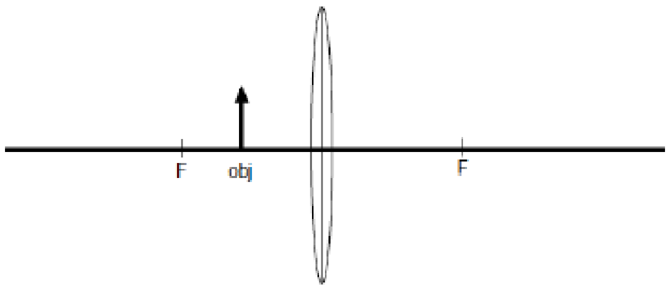
   b. Calculate the height and orientation of the image, and its distance from the lens.

Answers: $s_i = -4$ cm; $h_i = +1.4$ cm

Use this space for summary and/or additional notes:

3. **(M)** A 2.7 cm tall object is placed 3.4 cm from a convex lens that has a focal length of 6.0 cm.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.



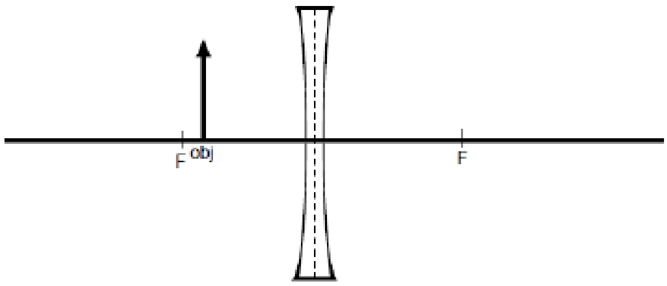
   b. Calculate the height and orientation of the image, and its distance from the lens.

Answers: $s_i = -7.84$ cm; $h_i = +6.23$ cm

Use this space for summary and/or additional notes:

4. **(M)** A 2.7 cm tall object is placed 5.1 cm from a concave lens that has a focal length of 6.0 cm.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.



   b. Calculate the height and orientation of the image, and its distance from the lens.

Answers: $s_i = -2.76$ cm; $h_i = +1.46$ cm

Use this space for summary and/or additional notes:

Diffraction

Unit: Light & Optics

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 6.C.2.1, 6.C.3.1, 6.C.4.1

Mastery Objective(s): (Students will be able to…)

- Explain how light "spreads" beyond an opening or around an obstacle.
- Perform calculations relating to the location of bright and dim regions when light passes through a diffraction grating.

Success Criteria:

- Explanations account for observed behavior.
- Calculations are correct with correct algebra.

Language Objectives:

- Explain why looking through a diffraction grating produces a "rainbow".

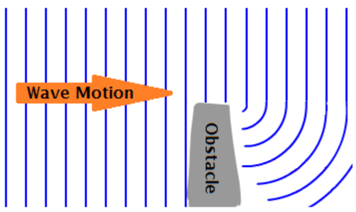
Tier 2 Vocabulary: light, diffraction, slit

Labs, Activities & Demonstrations:

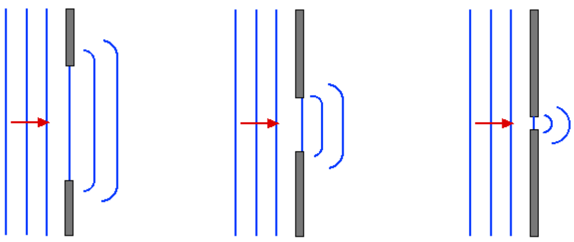
- thickness of human hair
- double slit experiment with laser & diffraction grating

Notes:

diffraction: the slight bending of a wave as it passes around the edge of an object or through a slit:



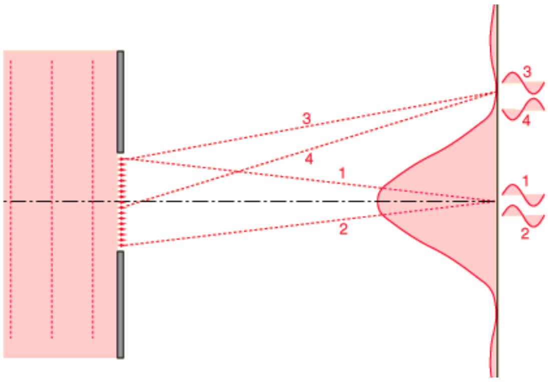
When light passes straight through a wide opening, the rays continue in a straight line. However, if we make the slit so narrow that the width is approximately equal to the wavelength, then the slit effectively becomes a point, and diffraction occurs in all directions from it.



Use this space for summary and/or additional notes:

If we shine light through a slit whose thickness is approximately the same order of magnitude as the wavelength, the light can only hit the wall in specific locations.

In the this diagram, light travels the same distance for paths 1 and 2—the same number of wavelengths. Light waves hitting this point will add constructively, which makes the light brighter.



However, for paths 3 and 4, path 4 is ½ wavelength longer than path 3. Light taking path 4 is ½ wavelength out of phase with light from path 3. The waves add destructively (cancel), and there is no light:

Farther up or down on the right side will be alternating locations where the difference in path length results in waves that are different by an exact multiple of the wavelength (in phase = constructive interference = bright spots), *vs.* by a multiple of the wavelength plus ½ (out-of-phase = destructive interference = dark spots).

The equation that relates the distance between these regions of constructive interference to the distance between the slits in a diffraction grating is:

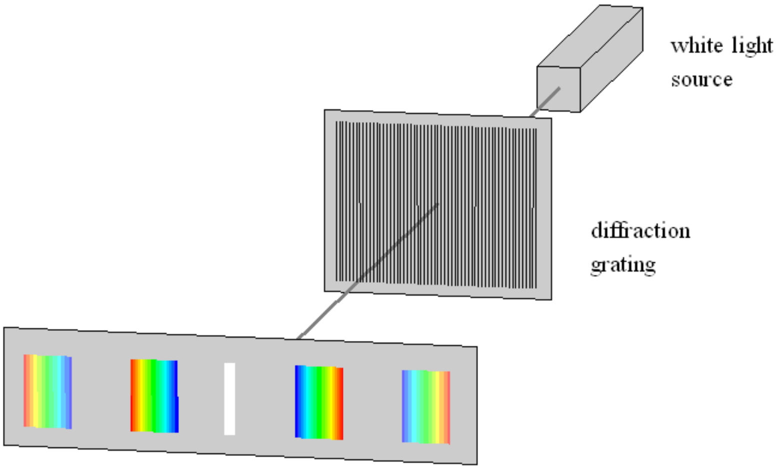
$$d\sin\theta_m = m\lambda$$

where:

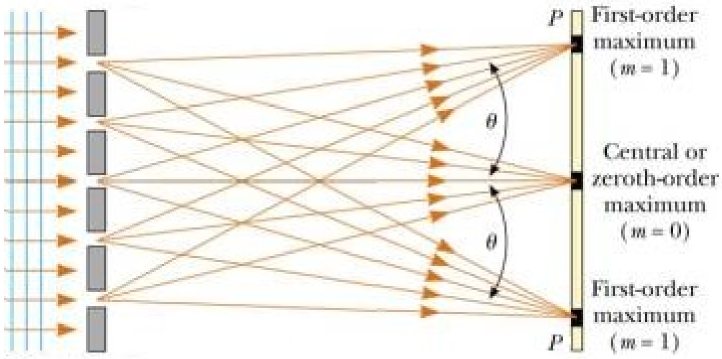
- m = the number of waves that equals the difference in the lengths of the two paths (integer)
- θ_m = the angle of emergence (or angle of deviation) in order for light from one slit to add constructively to light from a neighboring slit that is m wavelengths away.
- d = the distance between the slits
- λ = the wavelength of the light

Use this space for summary and/or additional notes:

diffraction grating: a screen with a series of evenly-spaced slits that scatters light in a repeating, predictable pattern.



When light shines through a diffraction grating, the following happens:



The patterns of light surrounding the center are the points where the waves of light add constructively.

Notice that blue and violet light (with the shortest wavelengths) is diffracted the least and appears closest to the center, whereas red light is diffracted more and appears farther away. This is because shorter waves bend more around the edges of a slit, because they need to turn less far to fit through the slits than longer waves do.

Use this space for summary and/or additional notes:

Sample Problem:

Q: Consider three laser pointers: a red laser with a wavelength of 650 nm, a green laser with a wavelength of 532 nm, and a blue laser with a wavelength of 405 nm. If each of these is shone through a diffraction grating with 5 000 lines per cm, what will be the angle of emergence for each color?

A: For our diffraction grating, 5 000 lines per cm equals 500 000 lines per meter.

$$d = \frac{1}{500\,000} = 2\times10^{-6}\text{ m}$$

For the red laser, 650 nm equals $\lambda = 650\text{ nm} = 6.50\times10^{-7}\text{m}$

The equation is:

$$d\sin\theta_m = m\lambda$$

For the red laser at m = 1, this becomes:

$$\begin{aligned}
&(2\times10^{-6})\sin\theta = (1)(6.50\times10^{-7})\\
&\sin\theta = \frac{6.50\times10^{-7}}{2\times10^{-6}} = 0.325\\
&\theta = \sin^{-1}(0.325) = 19.0°
\end{aligned}$$

For the green laser $(\lambda = 532\text{ nm} = 5.32\times10^{-7}\text{m})$ and the blue laser also at m = 1 $(\lambda = 405\text{ nm} = 4.05\times10^{-7}\text{m})$:

$$\begin{aligned}
&\sin\theta = \frac{5.32\times10^{-7}}{2\times10^{-6}} = 0.266\\
&\theta = \sin^{-1}(0.266) = 15.4°
\end{aligned}$$

and

$$\begin{aligned}
&\sin\theta = \frac{4.05\times10^{-7}}{2\times10^{-6}} = 0.203\\
&\theta = \sin^{-1}(0.203) = 11.7°
\end{aligned}$$

Use this space for summary and/or additional notes:

Homework Problems

In a Young's double slit experiment using yellow light of wavelength 550 nm, the fringe separation (separation between bright spots) is 0.275 mm.

1. **(M)** Find the slit separation if the fringes are 2.0 m from the slit.

   Answer: 0.004 m (= 4 mm)

2. **(M)** The yellow lamp is replaced with a purple one whose light is made of two colors, red light with a wavelength of 700 nm, and violet light with a wavelength of 400 nm.

   a. Find the distance between the violet fringes.

      Answer: 0.000 2 m (= 0.2 mm)

   b. Find the distance between the red fringes.

      Answer: 0.000 35 m (= 0.35 mm)

Use this space for summary and/or additional notes:

honors (not AP®)

Scattering

Unit: Light & Optics

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain why the sky is blue and the sun looks red at sunset.

Success Criteria:

- Explanations account for observed behavior.

Language Objectives:

- Explain why the beaker in the "sunset in a beaker" demo looks light blue, but the light coming through it looks yellow or red.

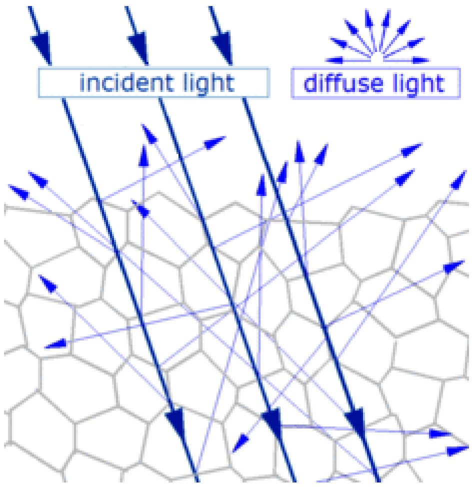
Tier 2 Vocabulary: light, scatter

Labs, Activities & Demonstrations:

- sunset in a beaker

Notes:

scattering: a change in the direction of rays of light caused by irregularities in the propagation medium, collisions with small particles, or at the interface between two media.

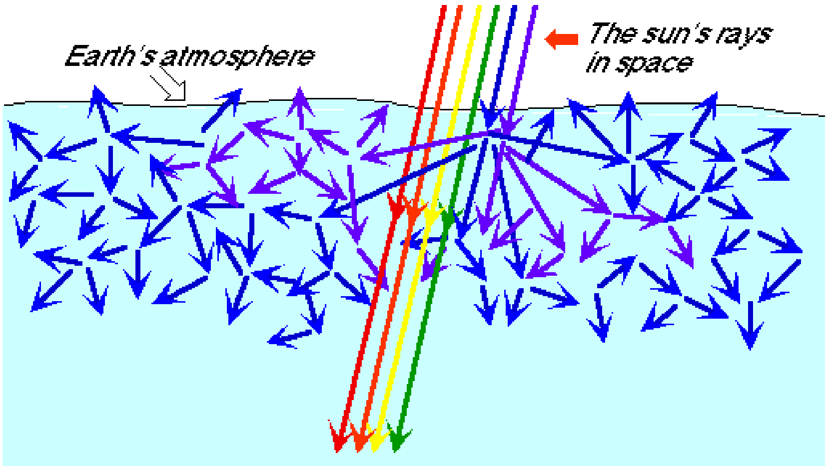


Use this space for summary and/or additional notes:

honors (not AP®)

Rayleigh scattering: scattering of light because of collisions with small particles in the medium. Rayleigh scattering is named after the British physicist Lord Rayleigh.

Rayleigh scattering is responsible for the color of the sky. Small particles (0.5–1 micron) scatter visible light as it passes through Earth's atmosphere. Because light at the blue and violet end of the spectrum is about five times as likely to be scattered as light at the red end of the spectrum, the majority of the scattered light in our atmosphere is blue.



This is also why the sun appears yellow during the day—the combination of red, orange, yellow and green light appears yellow to us.

Water vapor molecules are much larger—ranging in size from 2–5 microns. For these larger particles, the probability of scattering is approximately the same for all wavelengths, which is why clouds appear white.

At sunset, because the angle of the sun is much lower, the light must pass through much more of the atmosphere before we see it. By the time the light gets to our eyes, all of the colors are removed by scattering except for the extreme red end of the spectrum, which is why the sun appears red when it sets.

Use this space for summary and/or additional notes:

Introduction: Special Relativity

Unit: Special Relativity

Topics covered in this chapter:

This chapter describes changes to the properties of objects when they are moving at speeds near the speed of light.

- *Relative Motion* and *Relative Velocities* describes relationships between objects that are moving with different velocities.
- *Speed of Light* describes some familiar assumptions we have about our universe that do not apply at speeds near the speed of light.
- *Length Contraction & Time Dilation* and the *Energy-Momentum Relation* describe calculations involving changes in the length, time, mass, and momentum of objects as their speeds approach the speed of light.

New challenges in this chapter involve determining and understanding the changing relationships between two objects, both of which are moving in different directions and at different speeds.

Textbook:

- *Physics Fundamentals* Ch. 27: Relativity (pp. 765–797)

Standards addressed in this chapter:

Massachusetts Curriculum Frameworks (2016):

No MA curriculum frameworks are addressed in this chapter.

Use this space for summary and/or additional notes:

AP® only

AP® Physics 2 Learning Objectives:

1.D.3.1: The student is able to articulate the reasons that classical mechanics must be replaced by special relativity to describe the experimental results and theoretical predictions that show that the properties of space and time are not absolute. [Students will be expected to recognize situations in which nonrelativistic classical physics breaks down and to explain how relativity addresses that breakdown, but students will not be expected to know in which of two reference frames a given series of events corresponds to a greater or lesser time interval, or a greater or lesser spatial distance; they will just need to know that observers in the two reference frames can "disagree" about some time and distance intervals.] [**SP 6.3**, **7.1**]

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Relativity**, such as time dilation, length contraction, and mass-energy equivalence.

1. Special Relativity

Skills learned & applied in this chapter:

- keeping track of the changing relationships between two objects

Use this space for summary and/or additional notes:

Relative Motion

Unit: Special Relativity

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Describe how a situation appears differently in different reference frames.

Success Criteria:

- Explanations account for observed behavior.

Language Objectives:

- Describe a situation when you thought you were moving but you weren't (or *vice versa*).

Tier 2 Vocabulary: relative, reference frame

Vocabulary:

relativity: the concept that motion can be described only with respect to an observer, who may be moving or not moving relative to the object under consideration.

reference frame: the position and velocity of an observer watching an object that is moving relative to himself/herself.

Use this space for summary and/or additional notes:

Notes:

Consider the following picture, taken from a moving streetcar in New Orleans:

"New Orleans Streetcar." Photo by Don Chamblee.

If the streetcar is moving at a constant velocity and the track is smooth, the passengers may not notice that they are moving until they look out of the window.

In the reference frame of a person standing on the ground, the trolley and the passengers on it are moving at approximately 30 miles per hour.

In the reference frame of the trolley, the passengers sitting in the seats are stationary (not moving), and the ground is moving past the trolley at approximately 30 miles per hour.

Of course, you might want to say that the person on the ground has the "correct" reference frame. However, despite what you might prefer, neither answer is more correct than the other. Either reference frame is valid, which means either description of what is moving and what is stationary is equally valid.

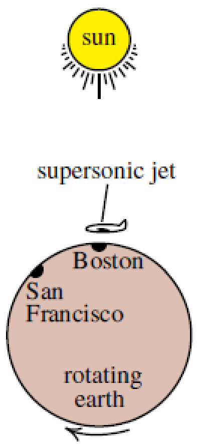
Use this space for summary and/or additional notes:

Principle of Relativity

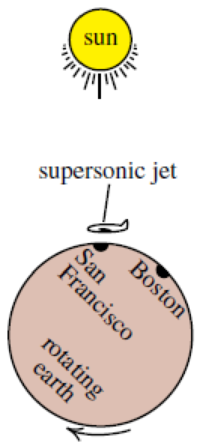
There is no experiment you can do that would allow you to determine conclusively whether or not you are moving uniformly.

Recall that "uniform motion" means moving with constant velocity, which means with a constant speed and direction. If velocity is constant, there is no acceleration, which means there is no net force.

Consider a fast airplane (such as a supersonic jet) flying from Boston to San Francisco. Imagine that the plane takes exactly three hours to fly to San Francisco, which is the same as the time difference between the two locations. Seen from outside the Earth, the situation might look like this:



Supersonic jet over Boston just after takeoff.



Supersonic jet three hours later over San Francisco.

You could argue that either:

1. The jet was moving from the airspace over Boston to the airspace over San Francisco.
2. The jet was stationary and the Earth rotated underneath it. (The jet needed to burn fuel to overcome the drag from the Earth's atmosphere as the Earth rotated, pulling its atmosphere and the jet with it.)

Use this space for summary and/or additional notes:

Of course, there are other reference frames you might consider as well.

3. Both the supersonic jet and the Earth are moving, because the Earth is revolving around the Sun with a speed of about $30\,000\,\frac{m}{s}$.

4. The jet, the Earth and the Sun are all moving, because the sun is revolving around the Milky Way galaxy with a speed of about $220\,000\,\frac{m}{s}$.

5. The jet, the Earth, the Sun, and the entire Milky Way galaxy are all moving through space toward the Great Attractor (a massive region of visible and dark matter about 150 million light-years away from us) with a speed of approximately $1\,000\,000\,\frac{m}{s}$.

6. It is not clear whether there might be multiple Great Attractors, and what their motion might be relative to each other, or relative to some yet-to-be-discovered entity.

Regardless of which objects are moving with which velocities, if you are on the airplane and you drop a ball, you would observe that it falls straight down. In relativistic terms, we would say "In the reference frame of the moving airplane, the ball has no initial velocity, so it falls straight down."

Use this space for summary and/or additional notes:

Relative Velocities

Unit: Special Relativity

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain how relative velocity depends on both the motion of an object and the motion of the observer
- Calculate relative velocities.

Success Criteria:

- Explanations account for observed behavior.

Language Objectives:

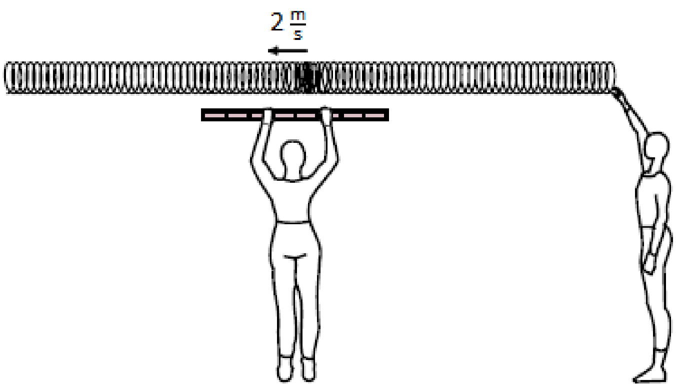
- Explain why velocities are different in different reference frames.

Tier 2 Vocabulary: relative, reference frame

Notes:

Because the observation of motion depends on the reference frames of the observer and the object, calculations of velocity need to take these into account.

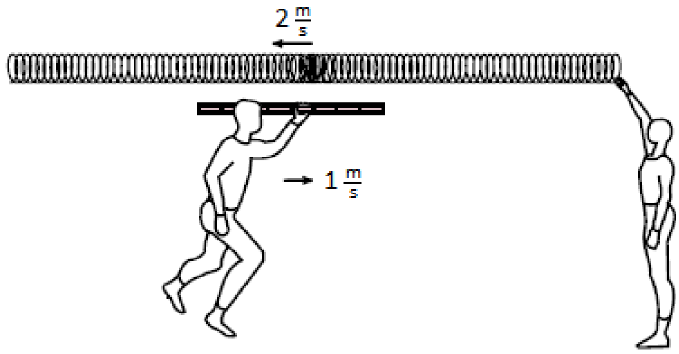
Suppose we set up a Slinky and a student sends a compression wave that moves with a velocity of $2\,\frac{\text{m}}{\text{s}}$ along its length:



A second student holds a meter stick and times how long it takes the wave to travel from one end of the meter stick to the other. The wave would take 0.5 s to travel the length of the meter stick, and the student would calculate a velocity of $\frac{1\,\text{m}}{0.5\,\text{s}} = 2\,\frac{\text{m}}{\text{s}}$.

Use this space for summary and/or additional notes:

Suppose instead that the student with the meter stick is running with a velocity of $1\,\frac{m}{s}$ toward the point of origin of the wave:



In this situation, you could use the velocities of the moving student and the wave and solve for the amount of time it would take for the wave and the end of the ruler to reach the same point. The calculation for this would be complicated, and the answer works out to be 0.33 s, which gives a velocity of $3\,\frac{m}{s}$.

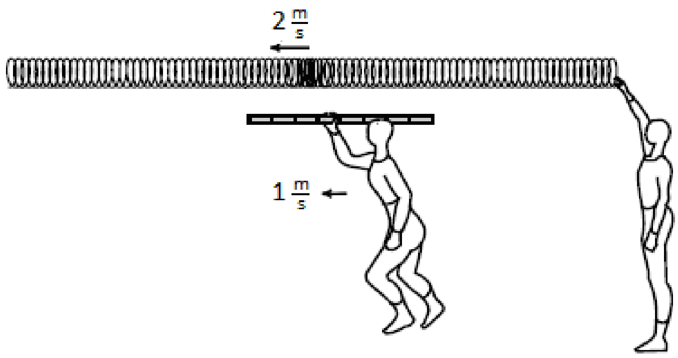
The easier way to calculate this number is to realize that the velocity of the wave relative to the moving student is simply the sum of the velocity vectors. The velocity of the wave relative to the moving student is therefore $2\,\frac{m}{s}+1\,\frac{m}{s}=3\,\frac{m}{s}$.

This $3\,\frac{m}{s}$ is called the relative velocity, specifically the velocity of the wave relative to the moving student.

<u>relative velocity</u>: the apparent velocity of an object relative to an observer, which takes into account the velocities of both the object and the observer. When the object and the observer are both moving, the relative velocity is sometimes called the *approach velocity*.

Use this space for summary and/or additional notes:

Suppose instead that the student is running away from the point of origin (*i.e.,* in the same direction as the wave is traveling) with a velocity of $1\,\frac{\text{m}}{\text{s}}$:



Now the relative velocity of the wave is $2\,\frac{\text{m}}{\text{s}} - 1\,\frac{\text{m}}{\text{s}} = 1\,\frac{\text{m}}{\text{s}}$ relative to the moving student.

If the student and the wave were moving with the same velocity (magnitude and direction), the relative velocity would be zero and the wave would appear stationary to the moving student.

Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** A river is flowing at a rate of $2\,\frac{\text{m}}{\text{s}}$ to the south. Jack is swimming downstream (southward) at $2\,\frac{\text{m}}{\text{s}}$ relative to the current, and Jill is swimming upstream (northward) at $2\,\frac{\text{m}}{\text{s}}$ relative to the current.

    a. What is Jack's velocity relative to Jill?

    Answer: $4\,\frac{\text{m}}{\text{s}}$ southward

    b. What is Jill's velocity relative to Jack?

    Answer: $4\,\frac{\text{m}}{\text{s}}$ northward

    c. What is Jack's velocity relative to a stationary observer on the shore?

    Answer: $4\,\frac{\text{m}}{\text{s}}$ southward

    d. What is Jill's velocity relative to a stationary observer on the shore?

    Answer: zero

2. **(S)** A small airplane is flying due east with an airspeed (*i.e.,* speed relative to the air) of $125\,\frac{\text{m}}{\text{s}}$. The wind is blowing toward the north at $40\,\frac{\text{m}}{\text{s}}$. What is the airplane's speed and heading relative to a stationary observer on the ground? (*Hint: this is a vector problem.*)

    Answer: $131\,\frac{\text{m}}{\text{s}}$ in a direction of 17.7° north of due east

Use this space for summary and/or additional notes:

3. **(M)** A ship is heading 30° north of east at a velocity of $10\,\frac{\text{m}}{\text{s}}$. The ocean current is flowing north at $1\,\frac{\text{m}}{\text{s}}$. A man walks across the ship at $2\,\frac{\text{m}}{\text{s}}$ in a direction perpendicular to the ship (30° west of north).

   Add the velocity vectors by drawing them on the grid below to show the velocity of the man relative to a stationary observer. (*Note: you do not have to calculate the numerical value.*)

Use this space for summary and/or additional notes:

Speed of Light

Unit: Special Relativity

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 1.D.3.1

Mastery Objective(s): (Students will be able to…)

- Understand that the speed of light is constant in all reference frames.

Success Criteria:

- Explanations account for observed behavior.

Language Objectives:

- Explain why scientists hesitated to accept the idea that the speed of light does not depend on the reference frame.

Tier 2 Vocabulary: reference frame

Notes:

If the principle of relativity is true, it must be true for all measurements and all reference frames, including those involving light.

In 1864, physicist James Clerk Maxwell united four calculus equations involving magnetic and electric fields into one unified theory of light. The four equations were:

1. Gauss's Law (which describes the relationship between an electric field and the electric charges that cause it).
2. Gauss's Law for Magnetism (which states that there are no discrete North and South magnetic charges).
3. Faraday's Law (which describes how a changing magnetic field creates an electric field).
4. Ampère's Law (which describes how an electric current can create a magnetic field), including Maxwell's own correction (which describes how a changing electric field can also create a magnetic field).

According to Maxwell's theory, light travels as an electromagnetic wave, *i.e.,* a wave of both electrical and magnetic energy. The moving electric field produces a magnetic field, and the moving magnetic field produces an electric field. Thus the electric and magnetic fields of the electromagnetic wave reinforce each other as they travel through space.

Use this space for summary and/or additional notes:

From Maxwell's equations, starting from the measured values for two physical constants: the electric permittivity of free space (ε_o) and the magnetic permeability of a vacuum (μ_o), Maxwell determined that the speed of light in a vacuum must be:

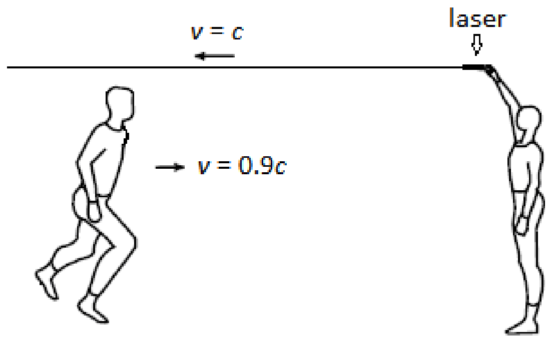
$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = 2.997\,924\,58 \times 10^8 \, \tfrac{\text{m}}{\text{s}} \, ^*$$

Both μ_o and ε_0 are physical constants, which do not depend on the reference frame. Maxwell theorized that the speed of light in a vacuum must therefore also be a physical constant, and it therefore cannot depend on the reference frame that is used to measure it.

This means:

1. Light travels at a constant velocity, regardless of whether the light is produced by something that is moving or stationary.
2. The velocity of light is the same in all reference frames. This means a photon of light moves at the same velocity, regardless of whether that velocity is measured by an observer who is stationary or by an observer who is moving.

If the wave in the above relative velocity examples was a beam of laser light instead of a Slinky, and the observer was running at a relativistic speed (meaning a speed close to the speed of light), the velocity of the light, both students would measure exactly the same velocity for the light!



Because the speed of light (in a vacuum) is a constant, we use the variable *c* (which stands for "constant") to represent it in equations.

* This is an exact value as defined by the International Committee for Weights and Measures in 2019 and is one of the seven Defining Constants used to determine exact values of other constants in the International System of Units (SI).

Use this space for summary and/or additional notes:

This idea seemed just as strange to 19th century physicists as it does today, and most physicists did not believe Maxwell for more than 45 years, when Albert Einstein published his theory of special relativity in 1905. However, several experiments have confirmed Maxwell's conclusion, and no experiment has ever successfully refuted it.

Light travels through a vacuum (empty space) with a velocity of exactly $2.997\,924\,58\times10^{8}\,\frac{\text{m}}{\text{s}}$. However while the speed of light does not depend on the reference frame, it does depend on the medium it is traveling through. (For more information, see the *Refraction* section on page 497.) When light travels through matter, (*e.g.,* air, glass, plastic, *etc.*), the velocity can be substantially slower.

Use this space for summary and/or additional notes:

Length Contraction & Time Dilation

Unit: Special Relativity

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 1.D.3.1

Mastery Objective(s): (Students will be able to…)

- Explain how and why distance and time change at relativistic speeds.

Success Criteria:

- Explanations account for observed behavior.

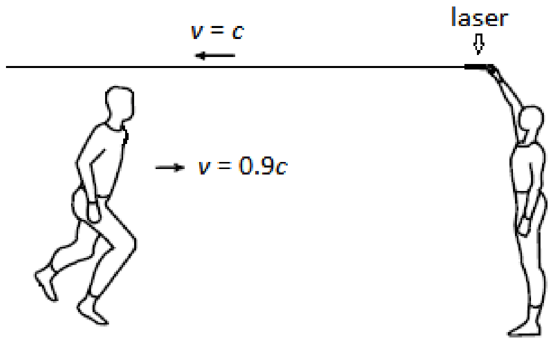
Language Objectives:

- Discuss how length contraction and/or time dilation can lead to a paradox.

Tier 2 Vocabulary: reference frame, contraction, dilation

Notes:

Based on Maxwell's conclusions, if an observer were somehow running with a relativistic speed of light toward an oncoming beam of light, the student should measure the same velocity of light as a stationary observer:



However, the amount of time it takes for a photon of light to pass the moving observer must be significantly less than the amount of time it would take a photon to pass a stationary observer.

Because velocity depends on distance and time, if the velocity of light cannot change, then as the observer approaches the speed of light, this means the distance and/or time must change!

For most people, the idea that distance and time depend on the reference frame is just as strange and uncomfortable as the idea that the speed of light cannot depend on the reference frame.

Use this space for summary and/or additional notes:

Length Contraction

If an object is moving at relativistic speeds and the velocity of light must be constant, then distances must become shorter as velocity increases. This means that as the velocity of an object approaches the speed of light, distances in its reference frame approach zero.

The Dutch physicist Hendrick Lorentz determined that the apparent change in length should vary according to the formula:

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$

where:

L = length of moving object

L_o = "proper length" of object (length of object at rest)

v = velocity of object

c = velocity of light

The ratio of L_o to L is named after Lorentz and is called the Lorentz factor (γ):

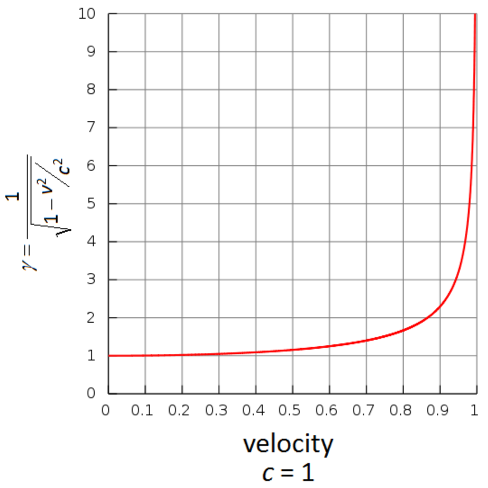
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The contracted length is therefore given by the equations:

$$L = \frac{L_o}{\gamma} \qquad \text{or} \qquad \frac{L_o}{L} = \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

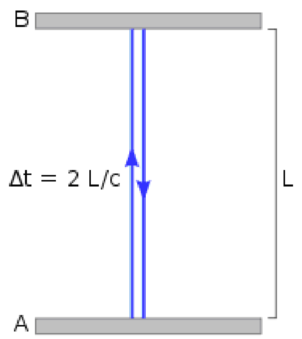
Use this space for summary and/or additional notes:

The Lorentz factor, γ, is 1 at rest and approaches infinity as the velocity approaches the speed of light:



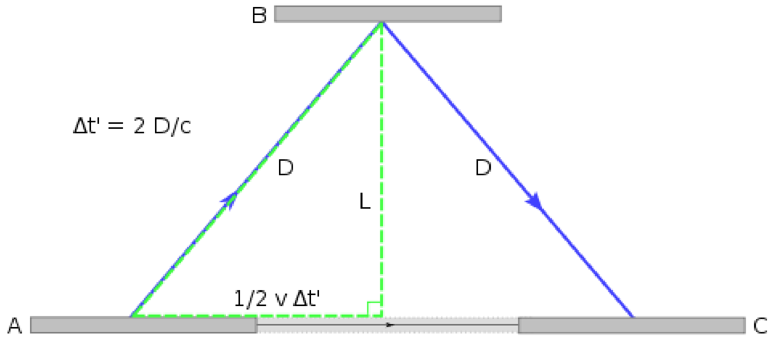
Time Dilation

In order to imagine how time is affected at relativistic speeds, imagine a clock that keeps time by sending a pulse of light from point (A), bouncing it off a mirror at point (B) and then measuring the time it takes to reach a detector back at point (A). The distance between the two surfaces is L, and the time for a pulse of light to travel to the mirror and back is therefore $\Delta t = \frac{2L}{c}$.



However, suppose the clock is moving at a relativistic speed. In the moving reference frame the situation looks exactly like the situation above. However, from an inertial (stationary) reference frame, the situation would look like the following:

Use this space for summary and/or additional notes:



Notice that in the stationary reference frame, the pulses of light must travel farther because of the motion of the mirror and detector. Because the speed of light is constant, the longer distance takes a longer time.

In other words, time is longer in the inertial (stationary) reference frame than it is in the moving reference frame!

This conclusion has significant consequences. For example, events that happen in two different locations could be simultaneous in one reference frame, but occur at different times in another reference frame!

Using arguments similar to those for length contraction, the equation for time dilation turns out to be:

$$\Delta t' = \gamma\,\Delta t \qquad \text{or} \qquad \frac{\Delta t'}{\Delta t} = \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

where:

$\Delta t'$ = time difference between two events in stationary reference frame

Δt = time difference between two events in moving reference frame

v = velocity of moving reference frame

c = velocity of light

Effect of Gravity on Time

Albert Einstein first postulated the idea that gravity slows down time in his paper on special relativity. This was confirmed experimentally in 1959.

Use this space for summary and/or additional notes:

As with relativistic time dilation, gravitational time dilation relates a duration of time in the absence of gravity ("proper time") to a duration in a gravitational field. The equation for gravitational time dilation is:

$$\Delta t' = \Delta t\sqrt{1-\frac{2GM}{rc^2}}$$

where:

$\Delta t'$ = time difference between two events in stationary reference frame

Δt = time difference between two events in moving reference frame

G = universal gravitational constant $(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})$

M = mass of the object creating the gravitational field

r = observer's distance (radius) from the center of the massive object

c = velocity of light

In 2014, a new atomic clock was built at the University of Colorado at Boulder, based on the vibration of a lattice of strontium atoms in a network of crisscrossing laser beams. The clock has been improved even since its invention, and is now accurate to better than one second per fifteen billion years (the approximate age of the universe). This clock is precise enough to measure differences in time caused by differences in the gravitational pull of the Earth near Earth surface. This clock would run measurably faster on a shelf than on the floor, because of the differences in time itself due to the Earth's gravitational field.

Black Holes

A black hole is an object that is so dense that its escape velocity (from physics 1) is faster than the speed of light, which means even light cannot escape.

For this to happen, the radius of the black hole needs to be smaller than $\frac{2GM}{c^2}$.

This results in a negative value for $\sqrt{1-\frac{2GM}{rc^2}}$, which makes $\Delta t'$ imaginary.

The consequence of this is that time is imaginary (does not pass) on a black hole, and therefore light cannot escape. This critical value for the radius is called the Schwarzschild radius, named for the German astronomer Karl Schwarzschild who first solved Einstein's field equations exactly in 1916 and postulated the existence of black holes.

The sun is too small to be able to form a black hole, but if it could, the Schwarzschild radius would be approximately 3.0 km for the Sun, and approximately 9.0 mm for the Earth.

Use this space for summary and/or additional notes:

Sample problem:

Q: In order to avoid detection by the Borg, the starship Enterprise must make itself appear to be less than 25 m long. If the rest length of the Enterprise is 420 m, how fast must it be traveling? What fraction of the speed of light is this?

A: $L = 25\text{ m}$

$L_o = 420\text{ m}$

$c = 2.998\times10^8\ \tfrac{\text{m}}{\text{s}}$

$$\gamma = \frac{L_o}{L} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{420}{25} = \frac{1}{\sqrt{1-\frac{v^2}{(2.998\times10^8)^2}}}$$

$$\frac{25}{420} = 0.0595 = \sqrt{1-\frac{v^2}{(2.998\times10^8)^2}}$$

$$(0.0595)^2 = 0.00354 = 1-\frac{v^2}{8.988\times10^{16}}$$

$$\frac{v^2}{8.988\times10^{16}} = 1-0.00354 = 0.99646$$

$$v^2 = (0.99646)(8.988\times10^{16})$$

$$v^2 = 8.956\times10^{16}$$

$$v = \sqrt{8.956\times10^{16}} = 2.993\times10^8\ \tfrac{\text{m}}{\text{s}}$$

$$\frac{2.993\times10^8}{2.998\times10^8} = 0.998\,c$$

Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** A spaceship is travelling at 0.7*c* on a trip to the Andromeda galaxy and returns to Earth 25 years later (from the reference frame of the people who remain on Earth). How many years have passed for the people on the ship?

   Answer: 17.85 years

2. **(S)** A 16-year-old girl sends her 48-year-old parents on a vacation trip to the center of the universe. When they return, the parents have aged 10 years, and the girl is the same age as her parents. How fast was the ship going? (*Give your answer in terms of a fraction of the speed of light.*)

   Answer: 0.971*c*

3. **(M)** The starship Voyager has a length of 120 m and a mass of 1.30×10^6 kg at rest. When it is travelling at $2.88 \times 10^8\ \frac{\text{m}}{\text{s}}$, what is its apparent length according to a stationary observer?

   Answer: 33.6 m

Use this space for summary and/or additional notes:

honors (not AP®)

Energy-Momentum Relation

Unit: Special Relativity

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain how and why mass and momentum change at relativistic speeds.

Success Criteria:

- Explanations account for observed behavior.

Language Objectives:

- Discuss how length contraction and/or time dilation can lead to a paradox.

Tier 2 Vocabulary: reference frame, contraction, dilation

Notes:

The momentum of an object also changes according to the Lorentz factor as it approaches the speed of light:

$$p = \gamma\, p_o \qquad \text{or} \qquad \frac{p}{p_o} = \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where:

p = momentum of object in moving reference frame

p_o = momentum of object in stationary reference frame

v = velocity of moving reference frame

c = velocity of light

Because momentum is conserved, an object's momentum in its own reference frame must remain constant. Therefore, at relativistic speeds the object's mass must change!

The equation for relativistic mass is:

$$m = \gamma\, m_o \qquad \text{or} \qquad \frac{m}{m_o} = \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where:

m = mass of object in moving reference frame

m_o = mass of object at rest

Therefore we can write the momentum equation as:

$$p = \gamma\, m_o v$$

Use this space for summary and/or additional notes:

honors (not AP®)

Note that as the velocity of the object approaches the speed of light, the denominator of the Lorentz factor, $\sqrt{1-\frac{v^2}{c^2}}$ approaches zero, which means that the Lorentz factor approaches infinity.

Therefore the momentum of an object must also approach infinity as the velocity of the object approaches the speed of light.

This relationship creates a potential problem. An object with infinite momentum must have infinite kinetic energy, but Einstein's equation $E = mc^2$ is finite. While it is true that the relativistic mass becomes infinite as velocity approaches the speed of light, there is still a discrepancy. Recall from mechanics that:

$$E_k = \frac{p^2}{2m}$$

According to this formula, the energy predicted using relativistic momentum should increase faster than the energy predicted by using $E = mc^2$ with relativistic mass. Obviously the amount of energy cannot depend on how the calculation is performed; the problem must therefore be that Einstein's equation needs a correction for relativistic speeds.

The solution is to modify Einstein's equation by adding a momentum term. The resulting energy-momentum relation is:

$$E^2 = (pc)^2 + (mc^2)^2$$

This equation gives results that are consistent with length contraction, time dilation and relativistic mass.

For an object at rest, its momentum is zero, and the equation reduces to the familiar form:

$$E^2 = 0 + (mc^2)^2$$
$$E = mc^2$$

Use this space for summary and/or additional notes:

Introduction: Quantum and Particle Physics

Unit: Quantum and Particle Physics

Topics covered in this chapter:

This chapter discusses the particles that atoms and other matter are made of, how those particles interact, and the process by which radioactive decay can change the composition of a substance from one element into another.

- *Photoelectric Effect* describes the observation that light of a sufficiently high frequency can remove electrons from an atom.
- *Bohr Model of the Hydrogen Atom* describes the development of quantum theory to describe the behavior of the electrons in an atom.
- *Wave-Particle Duality* and *Quantum Mechanical Model of the Atom* describe the idea that matter can behave like a wave as well as a particle, and the application of that idea to the modern quantum mechanical model of the atom.
- *Fundamental Forces* describes the four natural forces that affect everything in the universe: the strong and weak nuclear forces, the electromagnetic force, and the gravitational force.
- *The Standard Model* describes and classifies the particles that make up atoms.
- *Particle Interactions* describes interactions between subatomic particles.

One of the challenging aspects of this chapter is that it describes process that happen on a scale that is much too small to observe directly. Another challenge is the fact that the Standard Model continues to evolve. Many of the connections between concepts that make other topics easier to understand have yet to be made in the realm of quantum & particle physics.

Use this space for summary and/or additional notes:

Standards addressed in this chapter:

Massachusetts Curriculum Frameworks (2016):

HS-PS4-3. Evaluate the claims, evidence, and reasoning behind the idea that electromagnetic radiation can be described by either a wave model or a particle model, and that for some situations involving resonance, interference, diffraction, refraction, or the photoelectric effect, one model is more useful than the other.

AP® only

AP® Physics 2 Learning Objectives:

1.A.2.1: The student is able to construct representations of the differences between a fundamental particle and a system composed of fundamental particles and to relate this to the properties and scales of the systems being investigated. [**SP 1.1**, **7.1**]

1.A.4.1: The student is able to construct representations of the energy-level structure of an electron in an atom and to relate this to the properties and scales of the systems being investigated. [**SP 1.1**, **7.1**]

1.C.4.1: The student is able to articulate the reasons that the theory of conservation of mass was replaced by the theory of conservation of mass-energy. [**SP 6.3**]

1.D.1.1: The student is able to explain why classical mechanics cannot describe all properties of objects by articulating the reasons that classical mechanics must be refined and an alternative explanation developed when classical particles display wave properties. [**SP 6.3**]

1.D.3.1: The student is able to articulate the reasons that classical mechanics must be replaced by special relativity to describe the experimental results and theoretical predictions that show that the properties of space and time are not absolute. [Students will be expected to recognize situations in which nonrelativistic classical physics breaks down and to explain how relativity addresses that breakdown, but students will not be expected to know in which of two reference frames a given series of events corresponds to a greater or lesser time interval, or a greater or lesser spatial distance; they will just need to know that observers in the two reference frames can "disagree" about some time and distance intervals.] [**SP 6.3**, **7.1**]

3.G.3.1: The student is able to identify the strong force as the force that is responsible for holding the nucleus together. [**SP 7.2**]

4.C.4.1: The student is able to apply mathematical routines to describe the relationship between mass and energy and apply this concept across domains of scale. [**SP 2.2**, **2.3**, **7.2**]

Use this space for summary and/or additional notes:

AP® only

5.B.8.1: The student is able to describe emission or absorption spectra associated with electronic or nuclear transitions as transitions between allowed energy states of the atom in terms of the principle of energy conservation, including characterization of the frequency of radiation emitted or absorbed. [**SP 1.2**, **7.2**]

5.B.11.1: The student is able to apply conservation of mass and conservation of energy concepts to a natural phenomenon and use the equation $E = mc^2$ to make a related calculation. [**SP 2.2**, **7.2**]

5.D.1.6: The student is able to make predictions of the dynamical properties of a system undergoing a collision by application of the principle of linear momentum conservation and the principle of the conservation of energy in situations in which an elastic collision may also be assumed. [**SP 6.4**]

5.D.1.7: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values. [**SP 2.1**, **2.2**]

5.D.2.5: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum as the appropriate solution method for an inelastic collision, recognize that there is a common final velocity for the colliding objects in the totally inelastic case, solve for missing variables, and calculate their values. [**SP 2.1**, **2.2**]

5.D.2.6: The student is able to apply the conservation of linear momentum to a closed system of objects involved in an inelastic collision to predict the change in kinetic energy. [**SP 6.4**, **7.2**]

5.D.3.2: The student is able to make predictions about the velocity of the center of mass for interactions within a defined one-dimensional system. [**SP 6.4**]

5.D.3.3: The student is able to make predictions about the velocity of the center of mass for interactions within a defined two-dimensional system. [**SP 6.4**]

6.F.3.1: The student is able to support the photon model of radiant energy with evidence provided by the photoelectric effect. [**SP 6.4**]

6.F.4.1: The student is able to select a model of radiant energy that is appropriate to the spatial or temporal scale of an interaction with matter. [**SP 6.4**, **7.1**]

6.G.1.1: The student is able to make predictions about using the scale of the problem to determine at what regimes a particle or wave model is more appropriate. [**SP 6.4**, **7.1**]

Use this space for summary and/or additional notes:

AP® only

6.G.2.1: The student is able to articulate the evidence supporting the claim that a wave model of matter is appropriate to explain the diffraction of matter interacting with a crystal, given conditions where a particle of matter has momentum corresponding to a de Broglie wavelength smaller than the separation between adjacent atoms in the crystal. [**SP 6.1**]

6.G.2.2: The student is able to predict the dependence of major features of a diffraction pattern (*e.g.,* spacing between interference maxima), based upon the particle speed and de Broglie wavelength of electrons in an electron beam interacting with a crystal. (de Broglie wavelength need not be given, so students may need to obtain it.) [**SP 6.4**]

7.C.1.1: The student is able to use a graphical wave function representation of a particle to predict qualitatively the probability of finding a particle in a specific spatial region. [**SP 1.4**]

7.C.2.1: The student is able to use a standing wave model in which an electron orbit circumference is an integer multiple of the de Broglie wavelength to give a qualitative explanation that accounts for the existence of specific allowed energy states of an electron in an atom. [**SP 1.4**]

7.C.4.1: The student is able to construct or interpret representations of transitions between atomic energy states involving the emission and absorption of photons. [For questions addressing stimulated emission, students will not be expected to recall the details of the process, such as the fact that the emitted photons have the same frequency and phase as the incident photon; but given a representation of the process, students are expected to make inferences such as figuring out from energy conservation that since the atom loses energy in the process, the emitted photons taken together must carry more energy than the incident photon.] [**SP 1.1**, **1.2**]

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Quantum Phenomena**, such as photons and the photoelectric effect.
- **Atomic Physics**, such as the Rutherford and Bohr models, atomic energy levels, and atomic spectra.

1. The Discovery of the Atom
2. Quantum Physics

Use this space for summary and/or additional notes:

Photoelectric Effect

Unit: Quantum and Particle Physics

MA Curriculum Frameworks (2016): HS-PS4-3

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain the photoelectric effect.
- Calculate the work function of an atom and the kinetic energy of electrons emitted.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct with correct units and reasonable rounding.

Language Objectives:

- Explain why a minimum amount of energy is needed in order to emit an electron.

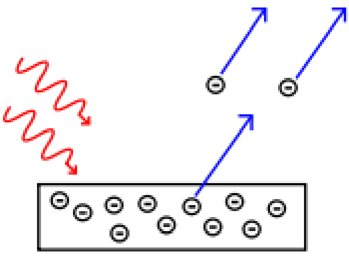
Tier 2 Vocabulary: work function

Labs, Activities & Demonstrations:

- threshold voltage to light an LED
- glow-in-the-dark substance and red *vs.* green *vs.* blue laser pointer

Notes:

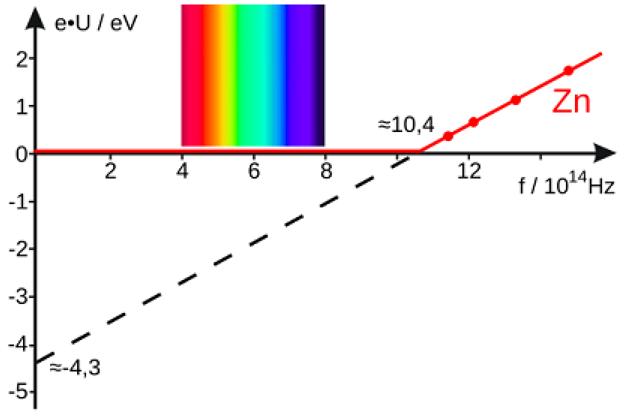
The photoelectric effect was discovered in 1887 when Heinrich Hertz discovered that electrodes emitted sparks more effectively when ultraviolet light was shone on them. We now know that the particles are electrons, and that ultraviolet light of sufficiently high frequency (which varies from element to element) causes the electrons to be emitted from the surface of the element:



Use this space for summary and/or additional notes:

The photoelectric effect requires light with a sufficiently high frequency, because the frequency of the light is related to the amount of energy it carries. The energy of the photons needs to be above a certain threshold frequency in order to have enough energy to ionize the atom.

For example, a minimum frequency of 10.4×10^{14} Hz is needed to dislodge electrons from a zinc atom:



The maximum kinetic energy of the emitted electron is equal to Planck's constant times the difference between the frequency of incident light (f) and the minimum threshold frequency of the element (f_0):

$$K_{max} = h(f - f_o)$$

The quantity hf_0 is called the "work function" of the atom, and is denoted by the variable ϕ. Thus the kinetic energy equation can be rewritten as:

$$K_{max} = hf - \phi$$

Values of the work function for different elements range from about 2.3–6 eV. ($1\text{ eV} = 1.6\times10^{-19}$ J)

The importance of this discovery was that it gave rise to the idea that light can behave both as a wave and as a particle.

In 1905, Albert Einstein published a paper explaining that the photoelectric effect was evidence that energy from light was carried in discrete, quantized packets. This discovery, for which Einstein was awarded the Nobel prize in physics in 1921, led to the birth of the field of quantum physics.

Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** The work function for gold is 4.8 eV.

   a. What is the minimum frequency of light required to remove electrons from gold?

   Answer: 1.16×10^{15} Hz

   b. What is the wavelength of this frequency of light?

   Answer: 2.58×10^{-7} m or 258 nm

   c. In what part of the spectrum is light of this frequency & wavelength?

   Answer: ultraviolet

Use this space for summary and/or additional notes:

2. **(S)** A beam of light from a 445 nm blue laser pointer contains how much energy?

   Answer: 4.45×10^{-19} J or 2.78 eV

3. **(M)** Photons of energy 6 eV cause electrons to be emitted from an unknown metal with a kinetic energy of 2 eV. If photons of twice the wavelength are incident on this metal, what will be the energy of the emitted electrons? (If no electrons are emitted, explain why.)

   Answer: no electrons will be emitted.

Use this space for summary and/or additional notes:

honors (not AP®)

Bohr Model of the Hydrogen Atom

Unit: Quantum and Particle Physics

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 5.B.8.1

Mastery Objective(s): (Students will be able to…)

- Explain how Bohr's model unified recent developments in the fields of spectroscopy, atomic theory and early quantum theory.
- Calculate the frequency/wavelength of light emitted using the Rydberg equation.
- Calculate the energy associated with a quantum number using Bohr's equation.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct with correct rounding and reasonable units.

Language Objectives:

- Explain why the Bohr Model was such a big deal.

Tier 2 Vocabulary: model, quantum

Notes:

Significant Developments Prior to 1913

Discovery of the Electron (1897): English physicist J.J. Thomson determined that cathode rays were actually particles emitted from atoms that the cathode was made of. These particles had an electrical charge, so they were named "electrons" (though Thomson called them "corpuscles").

Planetary Model of the Atom (1903): Japanese physicist Hantaro Nagaoka first proposed a model of the atom in which a small nucleus was surrounded by a ring of electrons.

Discovery of the Atomic Nucleus (1909): English physicist Ernest Rutherford's famous "gold foil experiment" determined that atoms contained a dense, positively-charged nucleus that comprised most of the atom's mass.

Use this space for summary and/or additional notes:

honors (not AP®)

Spectroscopy

Balmer Formula (1885): Swiss mathematician Johann Balmer devised an empirical equation to relate the emission lines in the visible spectrum for the hydrogen atom.

Rydberg Formula (1888): Swedish physicist Johannes Rydberg developed a generalized formula that could describe the wave numbers of all of the spectral lines in hydrogen (and similar elements).

There are several series of spectral lines for hydrogen, each of which converge at different wavelengths. Rydberg described the Balmer series in terms of a pair of integers (n_1 and n_2, where $n_1 < n_2$), and devised a single formula with a single constant (now called the Rydberg constant) that relates them.

$$\frac{1}{\lambda_{vac}} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

The value of Rydberg's constant is $\dfrac{m_e e^4}{8\varepsilon_o^2 h^3 c} = 10\,973\,731.6\ \text{m}^{-1} \approx 1.1\times10^7\ \text{m}^{-1}$

where m_e is the rest mass of the electron, e is the elementary charge, ε_o is the permittivity of free space, h is Planck's constant, and c is the speed of light in a vacuum.

Rydberg's equation was later found to be consistent with other series discovered later, including the Lyman series (in the ultraviolet region; first discovered in 1906) and the Paschen series (in the infrared region; first discovered in 1908).

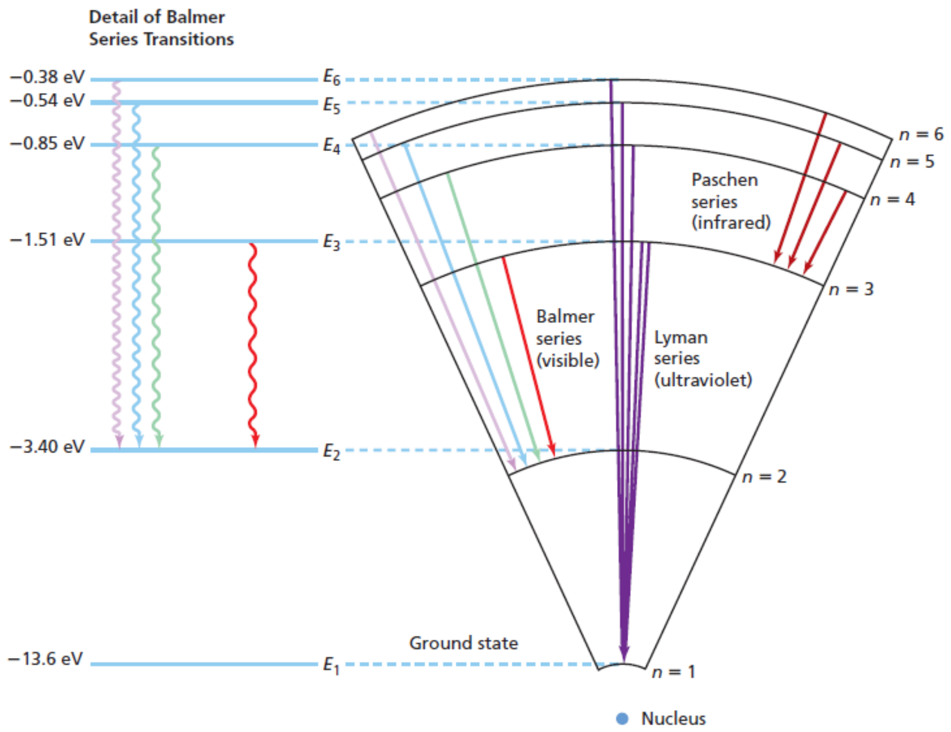
Those series and their converging wavelengths are:

Series	**Wavelength**	n_1	n_2
Lyman	91 nm	1	$2 \rightarrow \infty$
Balmer	365 nm	2	$3 \rightarrow \infty$
Paschen	820 nm	3	$4 \rightarrow \infty$

Use this space for summary and/or additional notes:

honors (not AP®)

The following diagram shows Lyman, Balmer and Paschen series transitions form higher energy levels (n = 2 through n = 6) back to lower ones (n = 1 through n = 3):



Early Quantum Theory

quantum: an elementary unit of energy.

In 1900, German physicist Max Planck published the Planck postulate, stating that electromagnetic energy could be emitted only in quantized form, *i.e.,* only certain "allowed" energy states are possible.

Plack determined the constant that bears his name as the relationship between the frequency of one quantum unit of electromagnetic wave and its energy. This relationship is the equation:

$$E = hf$$

where:

E = energy (J)

h = Planck's constant = $6.626\times10^{-34}\,\mathrm{J\cdot s}$

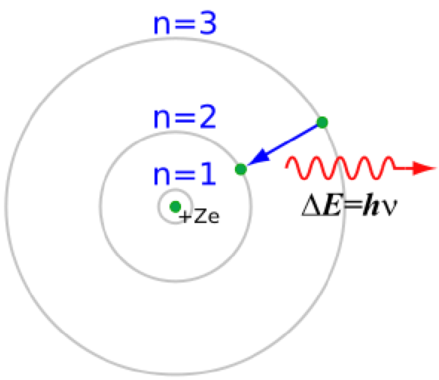
f = frequency* (Hz ≡ s^{-1})

* Most physics texts use the Greek letter *ν* (nu) as the variable for frequency. However, high school texts and the College Board use *f*, presumably to avoid confusion with the letter "v".

Use this space for summary and/or additional notes:

Bohr's Model of the Atom (1913)

In 1913, Danish physicist Niels Bohr combined atomic, quantum and spectroscopy theories into a single unified theory. Bohr hypothesized that electrons moved around the nucleus as in Rutherford's model, but that these electrons had only certain allowed quantum values of energy, which could be described by a quantum number (n). The value of that quantum number was the same n as in Rydberg's equation, and that using quantum numbers in Rydberg's equation could predict the wavelengths of light emitted when the electrons gained or lost energy by moved from one quantum level to another.



Bohr's model gained wide acceptance, because it related several prominent theories of the time. He received a Nobel Prize in physics in 1922 for his work.

honors (not AP®)

The theory worked well for hydrogen, giving a theoretical basis for Rydberg's equation. Bohr defined the energy associated with a quantum number (n) in terms of Rydberg's constant:

$$E_n = -\frac{R_H}{n^2}$$

Although the Bohr model worked well for hydrogen, the equations could not be solved exactly for atoms with more than one electron, because of the additional effects that electrons exert on each other (*e.g.,* the Coulomb force, $F_e = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$).

Use this space for summary and/or additional notes:

Wave-Particle Duality

Unit: Quantum and Particle Physics

MA Curriculum Frameworks (2016): HS-PS4-3

AP® Physics 2 Learning Objectives: 6.G.1.1, 6.G.2.1, 6.G.2.2, 7.C.1.1, 7.C.2.1

Mastery Objective(s): (Students will be able to…)

- Explain the de Broglie model of the atom.
- Calculate the de Broglie wavelength of a moving particle such as an electron.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

- Explain the important features of each model of the atom.

Tier 2 Vocabulary: model, quantum

Notes:

In 1924, French physicist Louis de Broglie determined that quanta of light could be considered to particles with very small mass moving at relativistic speeds (*i.e.,* close to the speed of light. See the section on *Introduction:* Special Relativity starting on page 530.)

From this, de Broglie concluded that any moving particle or object must therefore be able to be characterized with some periodic frequency, $f=\frac{E}{h}$, from Planck's equation. This means that the wavelength of any moving object is therefore:

$$\lambda=\frac{h}{p}=\frac{h}{mv}$$

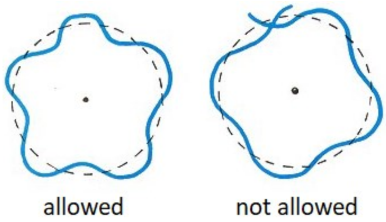
where:

λ = de Broglie wavelength (m)
h = Planck's constant = $6.626\times10^{-34}\ \text{J}\cdot\text{s}$
p = momentum (N·s)
m = mass (kg)
v = velocity $\left(\frac{\text{m}}{\text{s}}\right)$

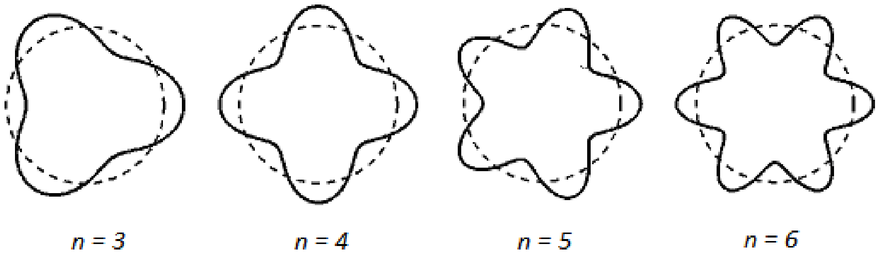
Every moving object has a de Broglie wavelength, though wavelengths of large objects are too small to be detectable.

Use this space for summary and/or additional notes:

de Broglie theorized that if waves were considered to be electrons, then the reason that only certain wavelengths were possible was because the wave produced by an electron would only be stable if the path length as it orbited the nucleus was an integer multiple of the wavelength.



Different quantum amounts of energy were possible with de Broglie's theory, but were restricted to amounts that produced an integer number of wavelengths.



Homework Problems

1. **(M)** What is the de Broglie wavelength associated with an electron moving at 0.5*c*? (You will need to look up the mass of the electron and the speed of light in a vacuum in *Table FF. Constants Used in Nuclear Physics* on page 624 of your Physics Reference Tables.)

   Answer: $4.8\times 10^{-12}\,\text{m} = 0.0048\,\text{nm}$

2. **(M)** How fast would that same electron need to be moving in order to produce a wavelength of visible light of 500 nm (which equals $5\times 10^{-7}\ \text{m}$)?

   Answer: $1450\,\frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

Quantum Mechanical Model of the Atom

Unit: Quantum and Particle Physics

MA Curriculum Frameworks (2016): HS-PS4-3

AP® Physics 2 Learning Objectives: 6.G.1.1, 6.G.2.1, 6.G.2.2, 7.C.1.1, 7.C.2.1

Mastery Objective(s): (Students will be able to…)

- Explain the de Broglie model of the atom.
- Explain the Schrödinger model of the atom.
- Explain the wave-particle duality of nature.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

- Explain the important features of each model of the atom.

Tier 2 Vocabulary: model, quantum

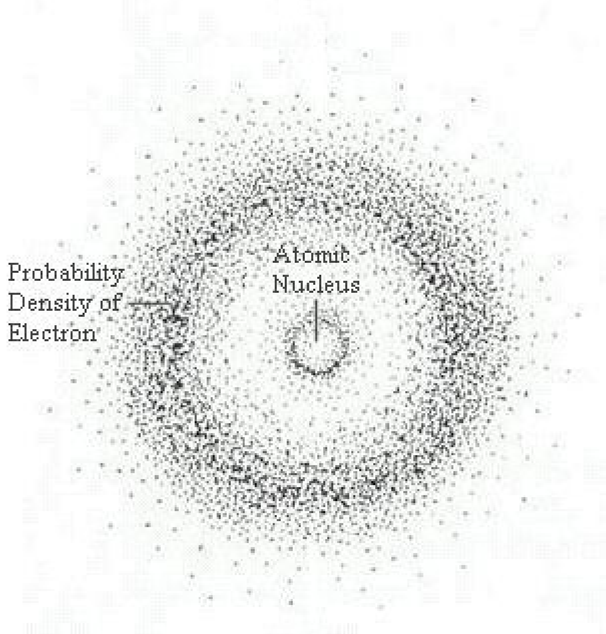
Notes:

In 1925, following de Broglie's research, Austrian physicist Erwin Schrödinger found that by treating each electron as a unique wave function, the energies of the electrons could be predicted by the mathematical solutions to the wave equation*. Schrödinger used the wave equation to construct a probability map for where the electrons can be found in an atom. Schrödinger's work is the basis for the modern quantum-mechanical model of the atom.

* The wave equation in physics is a second-order partial differential equation that mathematically describes the behavior of waves in space and time. The mathematics required are well beyond the scope of a high school physics class.

Use this space for summary and/or additional notes:

To understand the probability map, it is important to realize that because the electron acts as a wave, it is detectable when the amplitude of the wave is nonzero, but not detectable when the amplitude is zero. This makes it appear as if the electron is teleporting from place to place around the atom. If you were somehow able to take a time-lapse picture of an electron as it moves around the nucleus, the picture might look something like the diagram to the right.



Notice that there is a region close to the nucleus where the electron is unlikely to be found, and a ring a little farther out where there is a high probability of finding the electron.

As you get farther and farther from the nucleus, Schrödinger's equation predicts different shapes for these probability distributions. These regions of high probability are called "orbitals," because of their relation to the orbits originally suggested by the planetary model.

Schrödinger was awarded the Nobel prize in physics in 1933 for this discovery.

The implications of quantum theory are vast. Among other things, the energies, shapes and numbers of orbitals in an atom is responsible for each atom's chemical and physical properties and its location on the Periodic Table of the Elements, which means quantum mechanics is responsible for pretty much all of chemistry!

Some principles of quantum theory that are studied explicitly in chemistry include:

- atomic & molecular orbitals
- electron configurations
- the aufbau principle

Use this space for summary and/or additional notes:

Fundamental Forces

Unit: Quantum and Particle Physics

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 3.G.3.1

Mastery Objective(s): (Students will be able to…)

- Name, describe, and give relative magnitudes of the four fundamental forces of nature.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

- Explain why the gravitational force is more relevant than the electromagnetic force in astrophysics.

Tier 2 Vocabulary: model, quantum

Notes:

All forces in nature ultimately come from one of the following four forces:

strong force (or "strong nuclear force" or "strong interaction"): an attractive force between quarks. The strong force holds the nuclei of atoms together. The energy comes from converting mass to energy.
Effective range: about the size of the nucleus of an average-size atom.

weak force (or "weak nuclear force" or "weak interaction"): the force that causes protons and/or neutrons in the nucleus to become unstable and leads to beta nuclear decay. This happens because the weak force causes an up or down quark to change its flavor. (This process is described in more detail in the section on the *Standard Model* of Particle Physics, starting on page 570.)
Relative Strength: 10^{-6} to 10^{-7} times the strength of the strong force.
Effective range: about $^{1}/_{3}$ the diameter of an average nucleus.

electromagnetic force: the force between electrical charges. If the charges are the same ("like charges")—both positive or both negative—the particles repel each other. If the charges are different ("opposite charges")—one positive and one negative—the particles attract each other.
Relative Strength: about $^{1}/_{137}$ as strong as the strong force.
Effective range: ∞, but gets smaller as $(\text{distance})^2$.

gravitational force: the force that causes masses to attract each other. Usually only observable if one of the masses is very large (like a planet).
Relative Strength: only 10^{-39} times as strong as the strong force.
Effective range: ∞, but gets smaller as $(\text{distance})^2$.

Use this space for summary and/or additional notes:

Standard Model

Unit: Quantum and Particle Physics

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: 1.A.2.1

Mastery Objective(s): (Students will be able to…)

- Name and describe the particles of the Standard Model.
- Describe interactions between particles, according to the Standard Model.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

- Explain the important features of each model of the atom.

Tier 2 Vocabulary: model, quantum

Notes:

The Standard Model is a theory of particle physics that:

- identifies the particles that matter is ultimately comprised of
- describes properties of these particles, including their mass, charge, and spin
- describes interactions between these particles

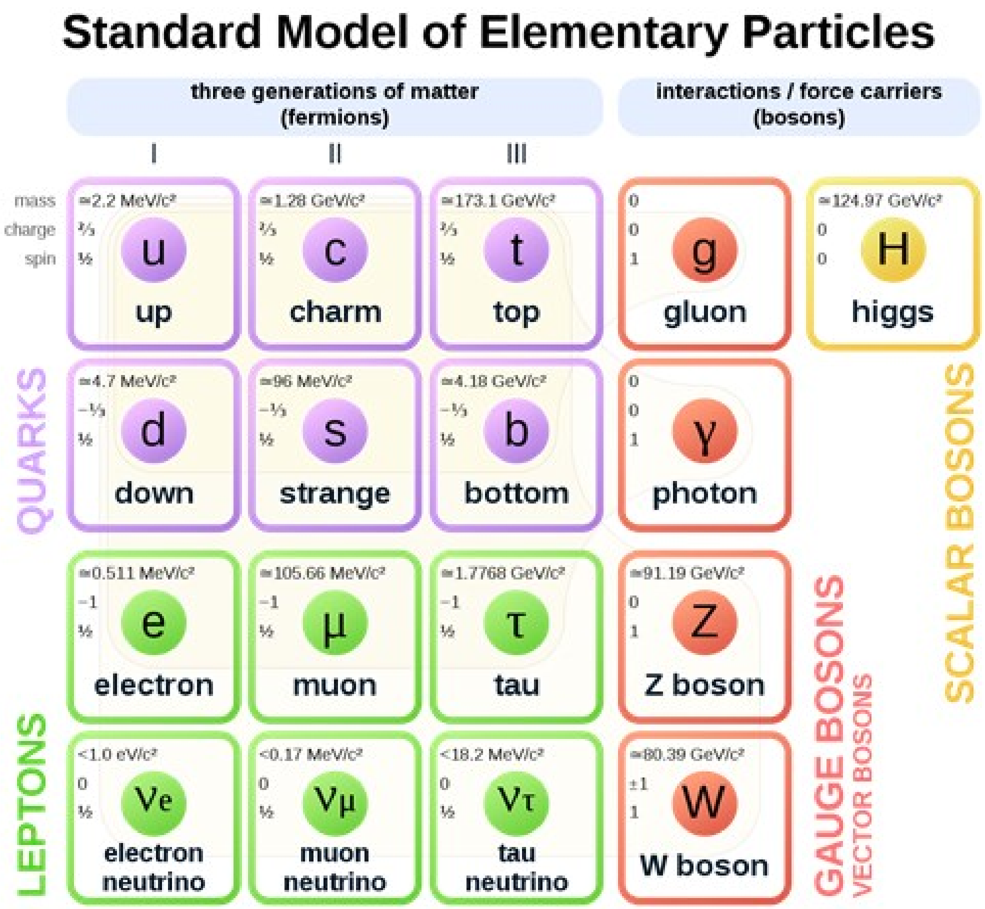
The Standard Model dates to the mid-1970s, when the existence of quarks was first experimentally confirmed. Physicists are still discovering new particles and relationships between particles, so the model and the ways it is represented are evolving, much like atomic theory and the Periodic Table of the Elements was evolving at the turn of the twentieth century. The table and the model described in these notes represent our understanding, as of 2024. By the middle of this century, the Standard Model may evolve to a form that is substantially different from the way we represent it today.

The Standard Model in its present form does not incorporate dark matter, dark energy, or gravitational attraction.

Use this space for summary and/or additional notes:

The Standard Model is often presented in a table, with rows, columns, and color-coded sections used to group subsets of particles according to their properties.

As of 2021, the Standard Model is represented by a table similar to this one:



Properties shown in the table include mass, charge, and spin.

- **Mass** is shown in units of electron volts divided by the speed of light squared $\left(\frac{\text{eV}}{\text{c}^2}\right)$. An electron volt (eV) is the energy acquired by an electric potential difference of one volt applied to one electron. (Recall that the metric prefix "M" stands for mega (10^6) and the metric prefix "G" stands for giga (10^9).) The c^2 in the denominator comes from Einstein's equation, $E = mc^2$, solved for m.
- **Charge** is the same property that we studied in the electricity unit. The magnitude and sign of charge is relative to the charge of an electron, which is defined to be −1.
- **Spin** is the property that is believed to be responsible for magnetism. (The name is because magnetism was previously thought to come from a magnetic field produced by electrons spinning within their orbitals.)

Use this space for summary and/or additional notes:

Fundamental Particles

Quarks

Quarks are particles that participate in strong interactions (sometimes called the "strong force") through the action of "color charge" (which will be described later). Because protons and neutrons (which make up most of the mass of an atom) are made of three quarks each, quarks are the subatomic particles that make up most of the ordinary matter* in the universe.

- quarks have color charge (*i.e.,* they interact via the strong force)
- quarks have spin of $\pm \frac{1}{2}$
- "up-type" quarks carry a charge of $+\frac{2}{3}$; "down-type" quarks carry a charge of $-\frac{1}{3}$.

There are six flavors† of quarks: up and down, charm and strange, and top and bottom. (Originally, top and bottom quarks were called truth and beauty.)

Leptons

Leptons are the smaller particles that make up most matter. The most familiar lepton is the electron. Leptons participate in "electroweak" interactions, meaning combinations of the electromagnetic and weak forces.

- leptons do not have color charge (*i.e.,* they do not interact via the strong force)
- leptons have spins of $+\frac{1}{2}$
- electron-type leptons have a charge of −1; neutrinos do not have a charge.
- neutrinos oscillate, which makes their mass indefinite.

Gauge Bosons

Gauge bosons are the particles that carry force—their interactions are responsible for the fundamental forces of nature: the strong force, the weak force, the electromagnetic force and the gravitational force. The hypothetical particle responsible for the gravitational force is the graviton, which has not yet been detected (as of 2024).

- photons are responsible for the electromagnetic force.
- gluons are responsible for the strong interaction (strong force)
- W and Z bosons are responsible for the weak interaction (weak force)

---

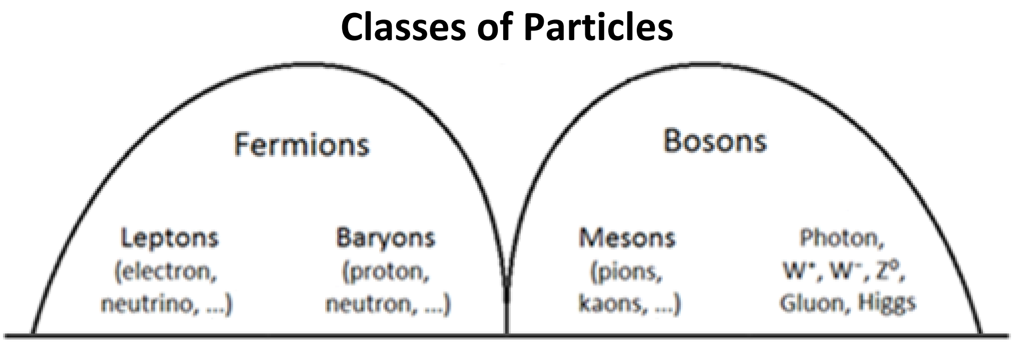
* Matter that is not "ordinary matter" is called "dark matter", whose existence is theorized but not yet proven.

† Yes, "flavors" really is the correct term.

Use this space for summary and/or additional notes:

Scalar Bosons

At present, the only scalar boson we know of is the Higgs boson, discovered in 2012, which is responsible for mass.



Fermions

Quarks and leptons (the left columns in the table of the Standard Model) are fermions. Fermions are described by Fermi-Dirac statistics and obey the Pauli exclusion principle (which states that no two particles in an atom may have the same exact set of quantum numbers—numbers that describe the energy states of the particle).

Fermions are the building blocks of matter. They have a spin of ½, and each fermion has its own antiparticle (see below).

Bosons

Bosons (the right columns in the table of the Standard Model) are described by Bose-Einstein statistics, have integer spins and do not obey the Pauli Exclusion Principle. Interactions between boson are responsible for forces and mass.

Use this space for summary and/or additional notes:

Antiparticles

Each particle in the Standard Model has a corresponding antiparticle. Like chemical elements in the Periodic Table of the Elements, fundamental particles are designated by their symbols in the table of the Standard Model. Antiparticles are designated by the same letter, but with a line over it. For example, an up quark would be designated "u", and an antiup quark would be designated "$\overline{\mathrm{u}}$".

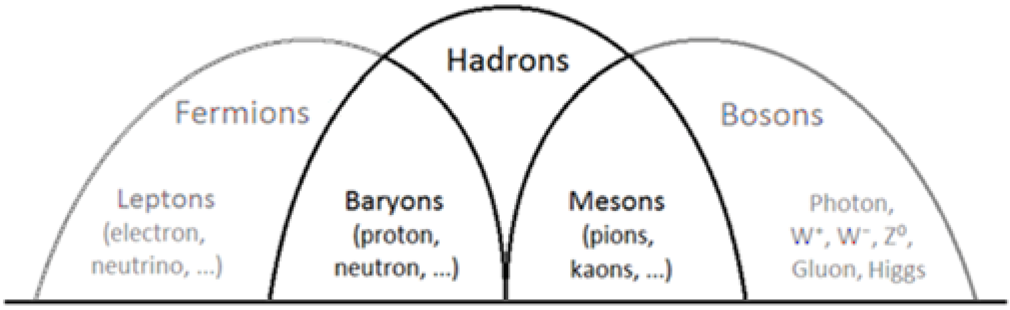
The antiparticle of a fermion has the same name as the corresponding particle, with the prefix "anti-", and has the opposite charge. *E.g.,* the antiparticle of a tau neutrino is a tau antineutrino. (However, for historical reasons an antielectron is usually called a positron.) *E.g.,* up quark carries a charge of $+\frac{2}{3}$, which means an antiup quark carries a charge of $-\frac{2}{3}$.

Each of the fundamental bosons ***is*** its own antiparticle, except for the W^- boson, whose antiparticle is the W^+ boson.

When a particle collides with its antiparticle, the particles annihilate each other, and their mass is converted to energy ($E = mc^2$) and released.

Use this space for summary and/or additional notes:

Composite Particles



Hadrons

Hadrons are a special class of strongly-interacting composite particles (meaning that they are comprised of multiple individual particles). Hadrons can be bosons or fermions. Hadrons composed of strongly-interacting fermions are called baryons; hadrons composed of strongly-interacting bosons are called mesons.

Baryons

The most well-known baryons are protons and neutrons, which each comprised of three quarks. Protons are made of two up quarks and one down quark ("uud"), and carry a charge of +1. Neutrons are made of one up quark and two down quarks ("udd"), and carry a charge of zero.

Some of the better-known baryons include:

- nucleons (protons & neutrons).
- hyperons, *e.g.,* the Λ, Σ, Ξ, and Ω particles. These contain one or more strange quarks, and are much heavier than nucleons.
- various charmed and bottom baryons.
- pentaquarks, which contain four quarks and an antiquark.

Mesons

Ordinary mesons are comprised of a quark plus an antiquark. Examples include the pion, kaon, and the J/Ψ. Mesons mediate the residual strong force between nucleons.

Some of the exotic mesons include:

- tetraquarks, which contain two quarks and two antiquarks.
- glueball, a bound set of gluons with no quarks.
- hybrid mesons, which contain one or more quark/antiquark pairs and one or more gluons.

Use this space for summary and/or additional notes:

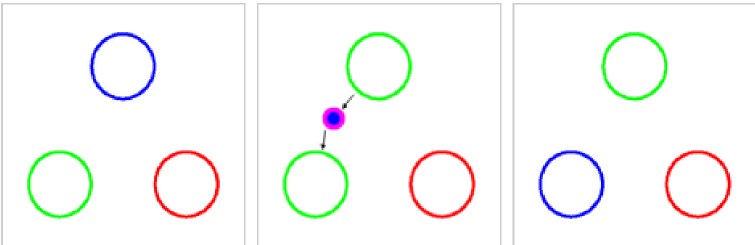
Color Charge

Color charge is the property that is responsible for the strong nuclear interaction. All electrons and fermions (particles that have half-integer spin quantum numbers) must obey the Pauli Exclusion Principle, which states that no two particles within the same larger particle (such as a hadron or atom) can have identical sets of quantum numbers.

For electrons, (as you learned in chemistry), if two electrons share the same orbital, they need to have opposite spins. In the case of quarks, all quarks have a spin of $+\frac{1}{2}$, so in order to satisfy the Pauli Exclusion Principle, if a proton or neutron contains three quarks, there has to be some other quantum property that has different values for each of those quarks. This property is called "color charge" (or sometimes just "color*").

The "color" property has three values, which are called "red," "green," and "blue" (named after the primary colors of light). When there are three quarks in a subatomic particle, the colors have to be different, and have to add up to "colorless". (Recall that combining each of the primary colors of light produces white light, which is colorless.)

Quarks can exchange color charge by emitting a gluon that contains one color and one anticolor. Another quark absorbs the gluon, and both quarks undergo color change. For example, suppose a blue quark emits a blue antigreen gluon:



You can imagine that the quark sent away its own blue color (the "blue" in the "blue antigreen" gluon). Because it also sent out antigreen, it was left with green so it became a green quark. Meanwhile, the antigreen part of the gluon finds the green quark and cancels its color. The blue from the blue antigreen gluon causes the receiving quark to become blue. After the interaction, the particle once again has one red, one green, and one blue quark, which means color charge is conserved.

* Just like "spin" is the name of a property of energy that has nothing to do with actual spinning, "color" is a property that has nothing to do with actual color. In fact, quarks couldn't possibly have actual color—the wavelengths of visible light are thousands of times larger than quarks!

Use this space for summary and/or additional notes:

honors (not AP®)

Particle Interactions

Unit: Quantum and Particle Physics

MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Fully describe an interaction between particles based on a Feynman diagram.
- Draw a Feynman diagram representing an interaction between particles.

Success Criteria:

- Descriptions & explanations are accurate.
- Diagrams correctly show all parts of the interaction.

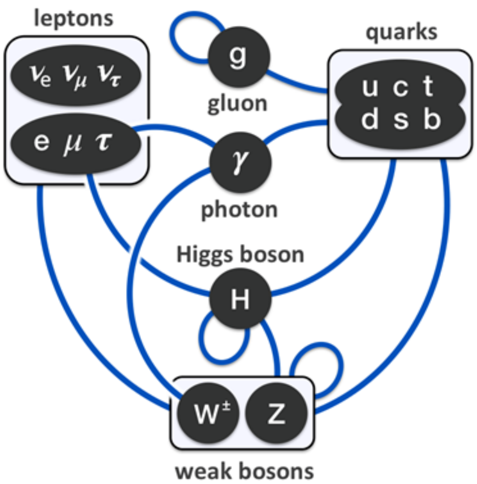
Language Objectives:

- Describe a particle interaction as a narrative.

Tier 2 Vocabulary: interaction, particle

Notes:

In particle physics, the Standard Model describes the types of particles found in nature, their properties, and how they interact. The following diagram shows which types of particles can interact with which other types.

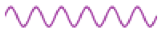


Use this space for summary and/or additional notes:

honors (not AP®)

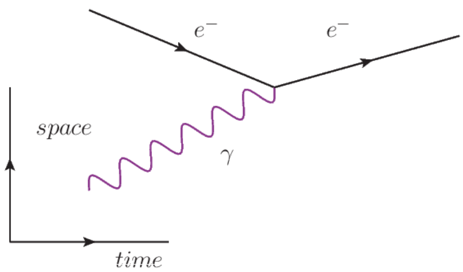
The interactions between particles can be shown pictorially in diagram called a Feynman diagram, named for American physicist Richard Feynman. The Feynman diagram tells the "story" of the interaction.

The characteristics of a Feynman diagram are:

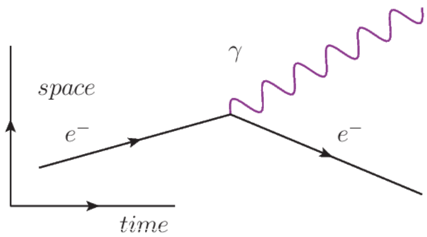
1. Straight lines represent the motion of a particle. The arrow points to the right for a negatively-charged particle, and to the left for a positively-charged particle.
2. A wavy line represents a photon (γ). 
3. A coiled line (like a spring) represents a gluon (g). 
4. The x-axis is time. The interaction starts (in terms of time) on the left and proceeds from left to right.
5. The y-axis represents space. Lines coming together represent particles coming together. Lines moving apart represent particles moving away from each other. (Note that ***the diagram is not a map***; particles can move together or apart in any direction.)
6. Each vertex, where two or more lines come together, represents an interaction.

Probably the best way to explain the diagrams is with examples.

In this diagram, we start (at the left) with an electron (e^-) and photon (γ). The two come together, and the electron absorbs the photon.



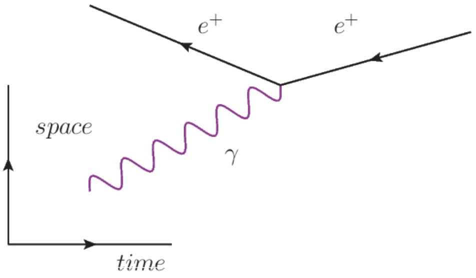
In this diagram, we start (at the left) with an electron (e^-) by itself. The electron emits a photon (γ), but is otherwise unchanged.



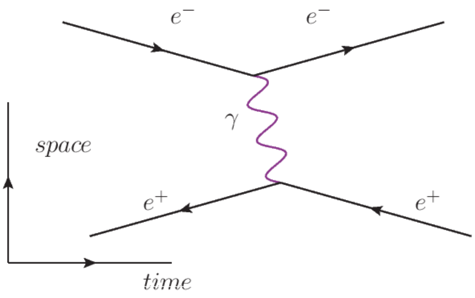
Use this space for summary and/or additional notes:

honors (not AP®)

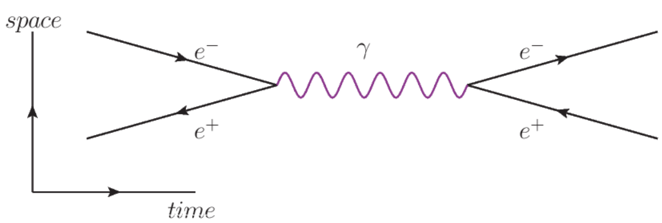
In this diagram, we start (at the left) with a positron (e^+) and photon (γ). (Note that the arrow pointing to the left indicates a positively-charged particle.) The two come together, and the positron absorbs the photon.



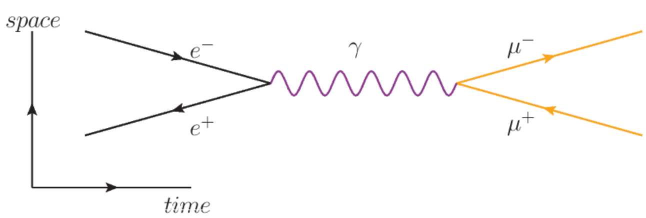
In this diagram, we start with an electron (e^-) and positron (e^+) (coming in from the left). They exchange a photon (γ) between them. (Note that the diagram does not make it clear which particle emits the photon and which one absorbs it.) Then the two particles exit.



In the following diagram, we start with an electron (e^-) and positron (e^+). They come together and annihilate each other, producing a photon (γ). (You can tell this because for a length of time, nothing else exists except for the photon.) Then the photon pair-produces a new electron/positron pair.



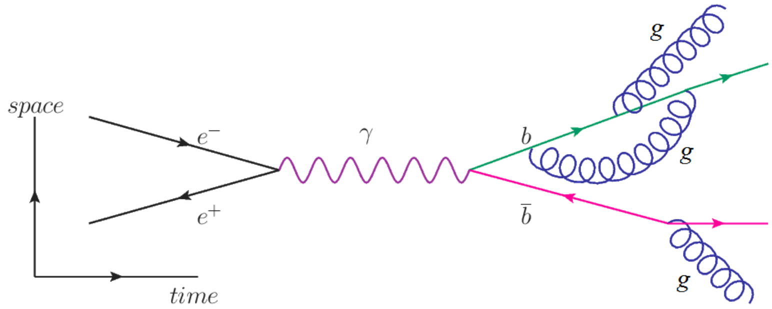
In the following diagram, an electron (e^-) and positron (e^+) annihilate each other as above, but this time the photon produces a muon (μ^-)/antimuon (μ^+) pair. (Again, note that the muon, which has a negative charge, has the arrow pointing to the right. The antimuon, which has a positive charge, has the arrow pointing to the left.)



Use this space for summary and/or additional notes:

honors (not AP®)

Finally, in the following diagram, an electron (e^-) and positron (e^+) annihilate each other, producing a photon (γ). The photon pair-produces a bottom quark (b) and an antibottom quark ($\bar{b}$), which radiate gluons (g).



Use this space for summary and/or additional notes:

Introduction: Atomic and Nuclear Physics

Unit: Atomic and Nuclear Physics

Topics covered in this chapter:

This chapter discusses the particles that atoms and other matter are made of, how those particles interact, and the process by which radioactive decay can change the composition of a substance from one element into another.

- *Radioactive Decay* and *Nuclear Equations* describe the process of radioactive decay and how to predict the results.
- *Mass Defect & Binding Energy* uses Einstein's equation $E = mc^2$ to determine the energy that was converted to mass in order to hold the nucleus of an atom together.
- *Half-Life* explains how to calculate the rate at which radioactive decay happens and the amount of material remaining.
- *Nuclear Fission & Fusion* and *Practical Uses for Nuclear Radiation* describe ways that radioactive materials are used to produce energy or otherwise provide benefits to society.

One of the challenges of this chapter is remembering concepts from chemistry, including numbers of protons, neutrons and electrons, and how to use the Periodic Table of the Elements.

Standards addressed in this chapter:

Massachusetts Curriculum Frameworks (2016):

HS-PS1-8: Develop a model to illustrate the energy released or absorbed during the processes of fission, fusion, and radioactive decay.

Use this space for summary and/or additional notes:

AP® only

AP® Physics 2 Learning Objectives:

5.C.1.1: The student is able to analyze electric charge conservation for nuclear and elementary particle reactions and make predictions related to such reactions based upon conservation of charge. [**SP 6.4**, **7.2**]

5.G.1.1: The student is able to apply conservation of nucleon number and conservation of electric charge to make predictions about nuclear reactions and decays such as fission, fusion, alpha decay, beta decay, or gamma decay. [**SP 6.4**]

7.C.3.1: The student is able to predict the number of radioactive nuclei remaining in a sample after a certain period of time, and also predict the missing species (alpha, beta, gamma) in a radioactive decay. [**SP 6.4**]

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Nuclear and Particle Physics**, such as radioactivity, nuclear reactions, and fundamental particles.

  1. Nuclear Physics

Use this space for summary and/or additional notes:

Radioactive Decay

Unit: Atomic, Particle, and Nuclear Physics

MA Curriculum Frameworks (2016): HS-PS1-8

AP® Physics 2 Learning Objectives: 7.C.3.1

Mastery Objective(s): (Students will be able to…)

- Explain the causes of nuclear instability.
- Explain the processes of α, $\beta-$, and $\beta+$ decay and electron capture.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

- Explain what happens in each of the four types of radioactive decay.

Tier 2 Vocabulary: decay, capture

Labs, Activities & Demonstrations:

- (old) smoke detector & Geiger counter

Notes:

<u>nuclear instability</u>: When something is unstable, it is likely to change. If the nucleus of an atom is unstable, changes can occur that affect the number of protons and neutrons in the atom.

Note that when this happens, the nucleus ends up with a different number of protons. This causes the atom to literally turn into an atom of a different element. When this happens, the physical and chemical properties instantaneously change into the properties of the new element!

<u>radioactive decay</u>: the process by which the nucleus of an atom changes, transforming the element into a different element or isotope.

<u>nuclear equation</u>: an equation describing (through chemical symbols) what happens to an atom as it undergoes radioactive decay.

Use this space for summary and/or additional notes:

Causes of Nuclear Instability

Two of the causes of nuclear instability are:

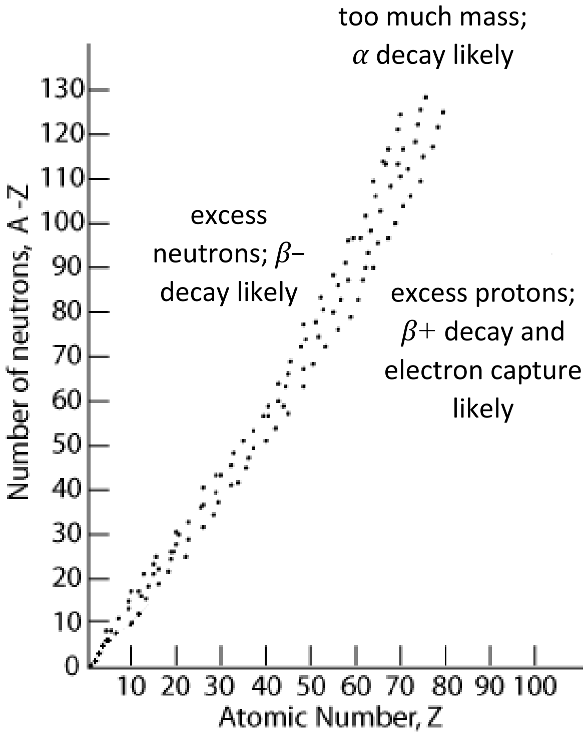
Size

Because the strong force acts over a limited distance, when nuclei get too large (more than 82 protons), it is no longer possible for the strong force to keep the nucleus together indefinitely. The form of decay that results from an atom exceeding its stable size is called alpha (α) decay.

The Weak Nuclear Force

The weak force is caused by the exchange (absorption and/or emission) of W and Z bosons. This causes a down quark to change to an up quark or vice-versa. The change of quark flavor has the effect of changing a proton to a neutron, or a neutron to a proton. (Note that the action of the weak force is the only known way of changing the flavor of a quark.) The form of decay that results from the action of the weak force is called beta (β) decay.

band of stability: isotopes with a ratio of protons to neutrons that results in a stable nucleus (one that does not spontaneously undergo radioactive decay). This observation suggests that the ratio of up to down quarks within the nucleus is somehow involved in preventing the weak force from causing quarks to change flavor.



Use this space for summary and/or additional notes:

alpha (α) decay: a type of radioactive decay in which the nucleus loses two protons and two neutrons (an alpha particle). An alpha particle is a ${}^{4}_{2}He^{2+}$ ion (the nucleus of a helium-4 atom), with two protons, a mass of 4 amu, and a charge of +2. For example:

$$ {}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He $$

Atoms are most likely to undergo alpha decay if they have an otherwise stable proton/neutron ratio but a large atomic number.

Alpha decay has never been observed in atoms with an atomic number less than 52 (tellurium), and is rare in elements with an atomic number less than 73 (tantalum).

Net effects of α decay:

- Atom loses 2 protons and 2 neutrons (atomic number goes down by 2 and mass number goes down by 4)
- An α particle (a ${}^{4}_{2}He^{+2}$ ion) is ejected from the nucleus at high speed.

beta minus ($\beta-$) decay: a type of radioactive decay in which a neutron is converted to a proton and the nucleus ejects a high speed electron (${}^{0}_{-1}e$).

Note that a neutron consists of one up quark and two down quarks (udd), and a proton consists of two up quarks and one down quark (uud). When $\beta-$ decay occurs, the weak force causes one of the quarks changes its flavor from down to up, which causes the neutron (uud) to change into a proton (udd). Because a proton was gained, the atomic number increases by one. However, because the proton used to be a neutron, the mass number does not change. For example:

$$ {}^{32}_{15}P \rightarrow {}^{32}_{16}S + {}^{0}_{-1}e $$

Atoms are likely to undergo $\beta-$ decay if they have too many neutrons and not enough protons to achieve a stable neutron/proton ratio. Almost all isotopes that are heavier than isotopes of the same element within the band of stability (because of the "extra" neutrons) undergo $\beta-$ decay.

Net effects of $\beta-$ decay:

- Atom loses 1 neutron and gains 1 proton (atomic number goes up by 1; mass number does not change)
- A $\beta-$ particle (an electron) is ejected from the nucleus at high speed.

Note that a $\beta-$ particle is assigned an atomic number of −1. *This does not mean an electron is some sort of "anti-proton".* The −1 is just used to make the equation for the number of protons work out in the nuclear equation.

Use this space for summary and/or additional notes:

beta plus (β+) decay: a type of radioactive decay in which a proton is converted to a neutron and the nucleus ejects a high speed antielectron (positron, ${}^{0}_{+1}e$).

With respect to the quarks, β+ decay is the opposite of β− decay When β+ decay occurs, one of the quarks changes its flavor from up to down, which changes the proton (uud) into a neutron (udd). Because a proton was lost, the atomic number decreases by one. However, because the neutron used to be a proton, the mass number does not change. For example:

$$^{23}_{12}\text{Mg} \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{+1}e$$

Atoms are likely to undergo β+ decay if they have too many protons and not enough neutrons to achieve a stable neutron/proton ratio. Almost all isotopes that are lighter than the isotopes of the same element that fall within the band of stability ("not enough neutrons") undergo β+ decay.

Net effects of β+ decay:

- Atom loses 1 proton and gains 1 neutron (atomic number goes down by 1; mass number does not change)
- A β+ particle (an antielectron or positron) is ejected from the nucleus at high speed.

electron capture (sometimes called "K-capture"): when the nucleus of the atom "captures" an electron from the innermost shell (the K-shell) and incorporates it into the nucleus. This process is exactly the reverse of β− decay; during electron capture, a quark changes flavor from up to down, which changes a proton (uud) into a neutron (udd):

$$^{23}_{12}\text{Mg} + {}^{0}_{-1}e \rightarrow {}^{23}_{11}\text{Na}$$

Note that β+ decay and electron capture produce the same products. Electron capture can sometimes (but not often) occur without β+ decay. However, β+ decay is ***always*** accompanied by electron capture.

Atoms are likely to undergo electron capture (and usually also β+ decay) if they have too many protons and not enough neutrons to achieve a stable neutron/proton ratio. Almost all isotopes that are lighter than the isotopes of the same element that fall within the band of stability undergo electron capture, and usually also β+ decay.

Net effects of electron capture:

- An electron is absorbed by the nucleus.
- Atom loses 1 proton and gains 1 neutron (atomic number goes down by 1; mass number does not change)

Use this space for summary and/or additional notes:

gamma (γ) rays: most radioactive decay produces energy. Some of that energy is emitted in the form of gamma rays, which are very high energy photons of electromagnetic radiation. (Radio waves, visible light, and X-rays are other types of electromagnetic radiation.) Because gamma rays are waves (which have no mass), they can penetrate far into substances and can do a lot of damage. Because gamma rays are not particles, emission of gamma rays does not change the composition of the nucleus.

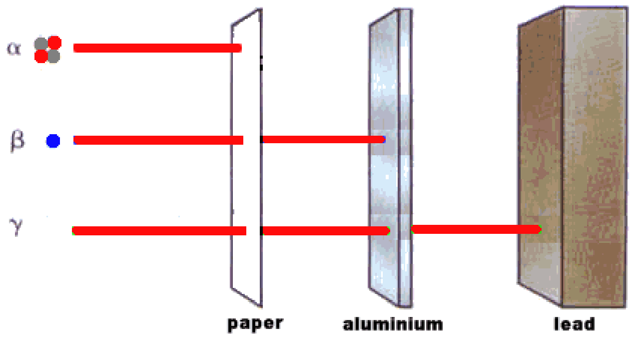
All of the types of radioactive decay mentioned in these notes also produce γ rays. This means to be complete, we would add gamma radiation to each of the radioactive decay equations described above:

$${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He} + {}^{0}_{0}\gamma \qquad {}^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + {}^{0}_{-1}e + {}^{0}_{0}\gamma$$

$${}^{23}_{12}\text{Mg} \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{+1}e + {}^{0}_{0}\gamma \qquad {}^{23}_{12}\text{Mg} + {}^{0}_{-1}e \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{0}\gamma$$

penetrating power: the distance that radioactive particles can penetrate into/through another substance is directly related to the velocity of the emission (faster = more penetrating) and inversely related to the mass of the emission (heaver = less penetrating):

gamma rays	**beta particles**	**alpha particles**
lightest	⟵⟶	heaviest
fastest	⟵⟶	slowest
most penetrating power	⟵⟶	least penetrating power



Note also that denser substances (such as lead) do a better job of blocking and absorbing radioactive emissions. This is why lead is commonly used as shielding for experiments involving radioactive substances.

Use this space for summary and/or additional notes:

Nuclear Equations

Unit: Atomic and Nuclear Physics

MA Curriculum Frameworks (2016): HS-PS1-8

AP® Physics 2 Learning Objectives: 5.C.1.1, 7.C.3.1

Mastery Objective(s): (Students will be able to…)

- Determine the products of α, β−, and β+ decay and electron capture.

Success Criteria:

- Equations give the correct starting material and products.

Language Objectives:

- Describe the changes to the nucleus during radioactive decay.

Tier 2 Vocabulary: decay, capture

Notes:

nuclear equation: a chemical equation describing the process of an isotope undergoing radioactive decay. For example:

$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$$

In a nuclear equation, the number of protons (atomic number) and the total mass (mass number) are conserved on both sides of the arrow. If you look at the bottom (atomic) numbers, and replace the arrow with an = sign, you would have the following:

$$92 = 90 + 2$$

Similarly, if you look at the top (mass) numbers, and replace the arrow with an = sign, you would have:

$$238 = 234 + 4$$

Use this space for summary and/or additional notes:

Sample problems:

Q: What are the products of beta-minus ($\beta-$) decay of ^{131}I?

A: A $\beta-$ particle is an electron, which we write as ${}^{0}_{-1}e$ in a nuclear equation. This means ^{131}I decays into some unknown particle plus ${}^{0}_{-1}e$. The equation is:

$$ {}^{131}_{53}\text{I} \rightarrow {}^{m}_{p}X + {}^{0}_{-1}e $$

We can write the following equations for the atomic and mass numbers:

Atomic #s: $53 = p + -1 \;\rightarrow\; p = 54$; therefore X is Xe

Mass #s: $131 = m + 0 \;\rightarrow\; m = 131$

Therefore, particle X is ${}^{131}_{54}\text{Xe}$ So our final answer is:

The two products of decay in this reaction are ${}^{131}_{54}\text{Xe}$ and ${}^{0}_{-1}e$.

Q: Which particle was produced in the following radioactive decay reaction:

$$ {}^{212}_{86}\text{Rn} \rightarrow {}^{208}_{84}\text{Po} + {}^{m}_{p}X $$

A: The two equations are:

Atomic #s: $86 = 84 + p \;\rightarrow\; p = 2$; therefore X is He

Mass #s: $212 = 208 + m \;\rightarrow\; m = 4$

Therefore, particle X is ${}^{4}_{2}\text{He}$, which means it is an α particle.

Use this space for summary and/or additional notes:

Homework Problems

For these problems, you will need to use a Figure CC. Periodic Table of the Elements (on page 623 of your Physics Reference Tables) and radioactive decay information from *Table EE. Selected Radioisotopes* on page 624 of your Physics Reference Tables.

Give the nuclear equation(s) for radioactive decay of the following:

1. **(M)** ^{222}Rn

2. **(M)** ^{85}Kr

3. **(S)** ^{220}Fr

4. **(S)** ^{37}K

5. **(S)** ^{3}H

Give the starting material for the following materials produced by radioactive decay:

6. **(M)** Alpha (α) decay resulting in $^{267}_{108}Hs$

7. **(M)** Beta-minus ($\beta-$) decay resulting in $^{185}_{75}Re$

Use this space for summary and/or additional notes:

Mass Defect & Binding Energy

Unit: Atomic and Nuclear Physics

MA Curriculum Frameworks (2016): HS-PS1-8

AP® Physics 2 Learning Objectives: 1.4.C.1, 5.B.11.1

Mastery Objective(s): (Students will be able to…)

- Calculate the binding energy of an atom.
- Calculate the energy given off by a radioactive decay based on the binding energies before and after.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain where the energy behind the strong force (which holds the nucleus together) comes from.

Tier 2 Vocabulary: defect

Notes:

mass defect: the difference between the actual mass of an atom, and the sum of the masses of the protons, neutrons, and electrons that it contains. The mass defect is the amount of "missing" mass that was turned into binding energy.

- A proton has a mass of 1.6726×10^{-27} kg = 1.0073 amu
- A neutron has a mass of 1.6749×10^{-27} kg = 1.0087 amu
- An electron has a mass of 9.1094×10^{-31} kg = 0.0005486 amu

To calculate the mass defect, total up the masses of each of the protons, neutrons, and electrons in an atom. The actual (observed) atomic mass of the atom is always *less* than this number. The "missing mass" is called the mass defect.

binding energy: the energy that holds the nucleus of an atom together through the strong nuclear force

The binding energy comes from the small amount of mass (the mass defect) that was released as energy when the nucleus was formed, given by the equation:

$$E = mc^2$$

where E is the binding energy, m is the mass defect, and c is the speed of light ($3\times10^8\ \frac{\text{m}}{\text{s}}$), which means c^2 is $9\times10^{16}\ \frac{\text{m}^2}{\text{s}^2}$ (a very large number)!

Use this space for summary and/or additional notes:

You can figure out how much energy is produced by spontaneous radioactive decay by calculating the difference in the sum of the binding energies of the atoms before and after the decay.

Sample problem:

Q: Calculate the mass defect of 1 mole of uranium-238.

A: ${}^{238}_{92}U$ has 92 protons, 146 neutrons, and 92 electrons. This means the total mass of one atom of ${}^{238}_{92}U$ should theoretically be:

92 protons × 1.0073 amu = 92.6704 amu

146 neutrons × 1.0087 amu = 147.2661 amu

92 electrons × 0.000 5486 amu = 0.0505 amu

92.6704 + 147.2661 + 0.0505 = 239.9870 amu

The actual observed mass of one atom of ${}^{238}_{92}U$ is 238.0003 amu.

The mass defect of one atom of ${}^{238}_{92}U$ is therefore
239.9870 − 238.0003 = 1.9867 amu.

One mole of ${}^{238}_{92}U$ would have a mass of 238.0003 g, and therefore a total mass defect of 1.9867 g, or 0.001 9867 kg.

Because $E = mc^2$, that means the binding energy of one mole of ${}^{238}_{92}U$ is:

$$0.001\,9867\,\text{kg} \times (3.00 \times 10^8)^2 = 1.79 \times 10^{14}\ \text{J}$$

In case you don't realize just how large that number is, the binding energy of just 238 g (1 mole) of ${}^{238}_{92}U$ would be enough energy to heat every house on Earth for an entire winter!

Use this space for summary and/or additional notes:

Half-Life

Unit: Atomic and Nuclear Physics

MA Curriculum Frameworks (2016): N/A

AP Physics 2 Learning Objectives: 7.C.3.1

Mastery Objective(s): (Students will be able to…)

- Calculate the amount of material remaining after an amount of time.
- Calculate the elapsed time based on the amount of material remaining.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why the *mass* of material that decays keeps decreasing.

Tier 2 Vocabulary: life, decay

Labs, Activities & Demonstrations:

- half-life of dice or M & M candies

Notes:

The atoms of radioactive elements are unstable, and they spontaneously decay (change) into atoms of other elements.

For any given atom, there is a certain probability, *P*, that it will undergo radioactive decay in a given amount of time. The half-life, τ, is how much time it would take to have a 50% probability of the atom decaying. If you start with *n* atoms, after one half-life, half of them (0.5*n*) will have decayed.

If we start with 32 g of ^{53}Fe, which has a half-life (τ) of 8.5 minutes, we would observe the following:

# minutes	0	8.5	17	25.5	34
# half lives	0	1	2	3	4
amount left	32 g	16 g	8 g	4 g	2 g

Use this space for summary and/or additional notes:

Amount of Material Remaining

Most half-life problems in a first-year high school physics course involve a whole number of half-lives and can be solved by making a table like the one above. However, on the AP® exam you can expect problems that do not involve a whole number of half-lives, and you need to use the exponential decay equation.

Because *n* is decreasing, the number of atoms (and consequently also the mass) remaining after any specific period of time follows the exponential decay function:

$$A = A_o (\tfrac{1}{2})^n$$

where *A* is the amount you have now, A_o is the amount you started with, and *n* is the number of half-lives that have elapsed.

Because the number of half-lives equals the total time elapsed (t) divided by the half-life (τ), we can replace $n = t/\tau$ and rewrite the equation as:

$$A = A_o \left(\frac{1}{2}\right)^{t/\tau} \quad \text{or} \quad \frac{A}{A_o} = \left(\frac{1}{2}\right)^{t/\tau}$$

If you want to find either *A* or A_o, you can plug the values for *t* and τ into the above equation.

Sample Problem:

Q: If you start with 228 g of ^{90}Sr, how much would remain after 112.4 years?

A: A_0 = 228 g

A = A

τ = 28.1 years (from the "Selected Radioisotopes" table in your reference tables)

t = 112.4 years

$$A = A_0 \left(\frac{1}{2}\right)^{t/\tau}$$

$$A = (228)\left(\frac{1}{2}\right)^{112.4/28.1} = (228)\left(\frac{1}{2}\right)^{4} = (228)\left(\frac{1}{16}\right) = 14.25\text{ g}$$

Or, if the decay happens to occur over an integer number of half-lives (as in this example), you can use a chart:

# years	0	28.1	56.2	84.3	112.4
# half lives	0	1	2	3	4
amount left	228 g	114 g	57 g	28.5 g	14.25 g

Use this space for summary and/or additional notes:

Finding the Time that has Passed

Integer Number of Half-Lives

If the amount you started with divided by the amount left is an exact power of two, you have an integer number of half-lives and you can just make a table.

Sample problem:

Q: If you started with 64 g of ^{131}I, how long would it take until there was only 4 g remaining? The half-life (τ) of ^{131}I is 8.07 days.

A: $\frac{64}{4} = 16$ which is a power of 2, so we can simply make a table:

# half lives	0	1	2	3	4
amount remaining	64 g	32 g	16 g	8 g	4 g

From the table, after 4 half-lives, we have 4 g remaining.

The half-life (τ) of ^{131}I is 8.07 days.

$$8.07 \times 4 = 32.3 \text{ days}$$

Use this space for summary and/or additional notes:

Non-Integer Number of Half-Lives

If you need to find the elapsed time and it is not an exact half-life, you need to use logarithms.

In mathematics, *the only reason you ever need to use logarithms is when you need to solve for a variable that's in an exponent*. For example, suppose we have the expression of the form $a^b = c$.

If b is a constant, we can solve for either a or c, as in the expressions:

$$a^3 = 21 \quad (\sqrt[3]{a^3} = \sqrt[3]{21} = 2.76)$$
$$6^2 = c \quad (6^2 = 36)$$

However, we can't do this if a and c are constants and we need to solve for b, as in the expression:

$$3^b = 17$$

To solve for b, we need to get b out of the exponent. We do this by taking the logarithm of both sides:

$$b\log(3) = \log(17)$$
$$b = \frac{\log(17)}{\log(3)} = \frac{1.23}{0.477} = 2.58$$

It doesn't matter which base you use. For example, using ln instead of log gives the same result:

$$b\ln(3) = \ln(17)$$
$$b = \frac{ln(17)}{ln(3)} = \frac{2.83}{1.10} = 2.58$$

We can apply this same logic to the half-life equation:

$$\frac{A}{A_o} = \left(\frac{1}{2}\right)^{t/\tau}$$
$$\log A - \log A_o = \frac{t}{\tau}\log\left(\frac{1}{2}\right)$$

Use this space for summary and/or additional notes:

Sample problem:

Q: If you started with 64 g of ^{131}I, how long would it take until there was only 5.75 g remaining? The half-life (τ) of ^{131}I is 8.07 days.

A: We have 5.75 g remaining. However, $\frac{64}{5.75} = 11.13$, which is not a power of two.

This means we don't have an integer number of half-lives, so we need to use logarithms:

$$\frac{A}{A_o} = \left(\frac{1}{2}\right)^{t/\tau}$$

$$\log A - \log A_o = \frac{t}{\tau}\log\left(\frac{1}{2}\right)$$

$$\log 5.75 - \log 64 = \frac{t}{8.07}\log\left(\frac{1}{2}\right)$$

$$0.7597 - 1.8062 = \frac{t}{8.07}(-0.3010)$$

$$-1.0465 = -0.03730\,t$$

$$28.1\ \text{days} = t$$

Homework Problems

For these problems, you will need to use half-life information from *Table EE. Selected Radioisotopes* on page 624 of your physics reference tables.

1. **(M)** If a lab had 128 g of ^{3}H waste 49 years ago, how much of it would be left today? (*Note: you may round off to a whole number of half-lives.*)

   Answer: 8 g

Use this space for summary and/or additional notes:

2. **(S)** Suppose you set aside a 20. g sample of ^{42}K at 5:00pm on a Friday for an experiment, but you are not able to perform the experiment until 9:00am on Monday (64 hours later). How much of the ^{42}K will be left?

   Answer: 0.56 g

3. **(M)** If a school wants to dispose of small amounts of radioactive waste, they can store the materials for ten half-lives, and then dispose of the materials as regular trash.

   a. If we had a sample of ^{32}P, how long would we need to store it before disposing of it?

      Answer: 143 days

   b. If we had started with 64 g of ^{32}P, how much ^{32}P would be left after ten half-lives? Approximately what fraction of the original amount would be left?

      Answer: 0.063 g; approximately $\frac{1}{1000}$ of the original amount.

4. **(M)** If the carbon in a sample of human bone contained 30. % of the expected amount of ^{14}C, approximately how old is the sample?

   Answer: 9 950 years

Use this space for summary and/or additional notes:

honors (not AP®)

Nuclear Fission & Fusion

Unit: Atomic and Nuclear Physics

MA Curriculum Frameworks (2016): HS-PS1-8

AP® Physics 2 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Identify nuclear processes as “fission” or “fusion”.
- Describe the basic construction and operation of fission-based and fusion-based nuclear reactors.

Success Criteria:

- Descriptions account for how the energy is produced and how the radiation is contained.

Language Objectives:

- Explain how fission-based and fusion-based nuclear reactors work.

Tier 2 Vocabulary: fusion, nuclear

Notes:

Fission

fission: splitting of the nucleus of an atom, usually by bombarding it with a high-speed neutron.

When atoms are split by bombardment with neutrons, they can divide in hundreds of ways. For example, when ^{235}U is hit by a neutron, it can split more than 200 ways. Three examples that have been observed are:

$$\begin{aligned}
{}^{1}_{0}n + {}^{235}_{92}\text{U} &\rightarrow {}^{90}_{37}\text{Rb} + {}^{144}_{55}\text{Cs} + 2\,{}^{1}_{0}n \\
{}^{1}_{0}n + {}^{235}_{92}\text{U} &\rightarrow {}^{87}_{35}\text{Br} + {}^{146}_{57}\text{La} + 3\,{}^{1}_{0}n \\
{}^{1}_{0}n + {}^{235}_{92}\text{U} &\rightarrow {}^{72}_{30}\text{Br} + {}^{160}_{62}\text{Sm} + 4\,{}^{1}_{0}n
\end{aligned}$$

Note that each of these bombardments produces more neutrons. A reaction that produces more fuel (in this case, neutrons) than it consumes will accelerate. This self-propagation is called a chain reaction.

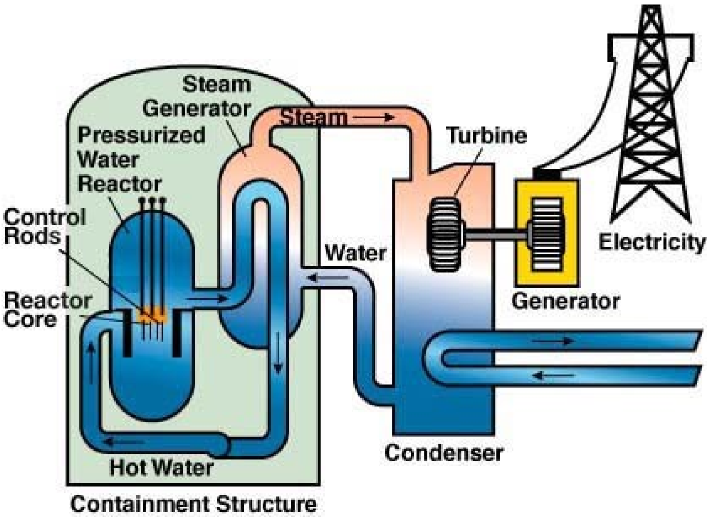
Note also that the neutron/proton ratio of ^{235}U is about 1.5. The stable neutron/proton ratio of each of the products would be approximately 1.2. This means that almost all of the products of fission reactions have too many neutrons to be stable, which means they will themselves undergo $\beta-$ decay.

Use this space for summary and/or additional notes:

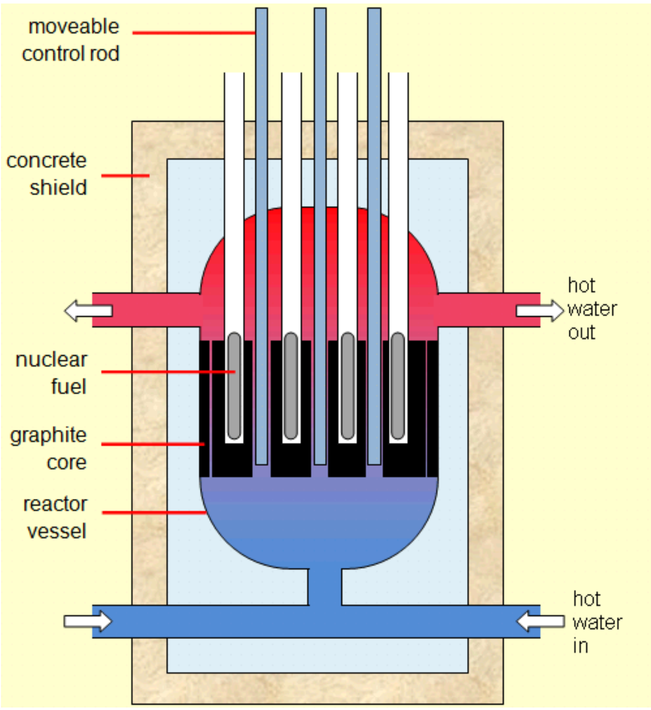
honors (not AP®)

Nuclear Fission Reactors

In a nuclear reactor, the heat from a fission reaction is used to heat water. The radioactive hot water from the reactor (under pressure, so it can be heated well above 100 °C without boiling) is used to boil clean (non-radioactive) water. The clean steam is used to turn a turbine, which generates electricity.



The inside of the reactor looks like this:



The fuel is the radioactive material (such as ^{235}U) that is undergoing fission. The graphite in the core of the reactor is used to absorb some of the neutrons. The moveable control rods are adjusted so they can absorb some or all of the remaining neutrons as desired. If the control rods are all the way down, all of the neutrons are absorbed and no heating occurs. When the reactor is in operation, the control rods are raised just enough to make the reaction proceed at the desired rate.

Use this space for summary and/or additional notes:

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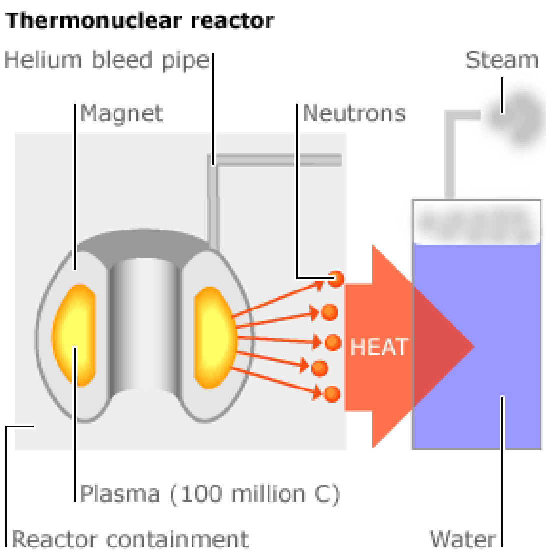
Fusion

<u>fusion</u>: the joining together of the nuclei of two atoms, accomplished by colliding them at high speeds.

Nuclear fusion reactions occur naturally on stars (such as the sun), and are the source of the heat and energy that stars produce.

On the sun, fusion occurs between atoms of deuterium (^2H) to produce helium:

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{4}_{2}He$$



The major challenge in building nuclear fusion reactors is the high temperatures produced—on the order of $10^6 - 10^9$ °C. In a tokamak fusion reactor, the starting materials are heated until they become plasma—a sea of highly charged ions and electrons. The highly charged plasma is kept away from the sides by powerful electromagnets.

At the left is a schematic of the ITER tokamak reactor currently under construction in southern France.

MIT has a smaller tokamak reactor at its Plasma Science & Fusion Center. The MIT reactor is able to conduct fusion reactions lasting for only a few seconds; if the reaction continued beyond this point, the current in the electromagnets that is necessary to generate the high magnetic fields required to confine the reaction would become hot enough to melt the copper wire and fuse the coils of the electromagnet together.

After each "burst" (short fusion reaction), the electromagnets in the MIT reactor need to be cooled in a liquid nitrogen bath (−196 °C) for fifteen minutes before the reactor is ready for the next burst.

Use this space for summary and/or additional notes:

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Practical Uses for Nuclear Radiation

Unit: Atomic and Nuclear Physics

MA Curriculum Frameworks (2016): HS-PS1-8

MA Curriculum Frameworks (2006): N/A

Mastery Objective(s): (Students will be able to…)

- Identify & describe practical (peaceful) uses for nuclear radiation.

Success Criteria:

- Descriptions give examples and explain how radiation is essential to the particular use.

Language Objectives:

- Explain how radiation makes certain scientific procedures possible.

Tier 2 Vocabulary: radiation

Notes:

While most people think of the dangers and destructive power of nuclear radiation, there are a lot of other uses of radioactive materials:

Power Plants: nuclear reactors can generate electricity in a manner that does not produce CO_2 and other greenhouse gases.

Cancer Therapy: nuclear radiation can be focused in order to kill cancer cells in patients with certain forms of cancer. Radioprotective drugs are now available that can help shield non-cancerous cells from the high-energy gamma rays.

Radioactive Tracers: chemicals made with radioactive isotopes can be easily detected in complex mixtures or even in humans. This enables doctors to give a patient a chemical with a small amount of radioactive material and track the progress of the material through the body and determine where it ends up. It also enables biologists to grow bacteria with radioactive isotopes and follow where those isotopes end up in subsequent experiments.

Irradiation of Food: food can be exposed to high-energy gamma rays in order to kill germs. These gamma rays kill all of the bacteria in the food, but do not make the food itself radioactive. (Gamma rays cannot build up inside a substance.) This provides a way to create food that will not spoil for months on a shelf in a store. There is a lot of irrational fear of irradiated food in the United States, but irradiation is commonly used in Europe. For example, irradiated milk will keep for months on a shelf at room temperature without spoiling.

Use this space for summary and/or additional notes:

honors (not AP®)

Carbon Dating: Because ^{14}C is a long-lived isotope (with a half-life of 5 700 years), the amount of ^{14}C in archeological samples can give an accurate estimate of their age. One famous use of carbon dating was its use to prove that the Shroud of Turin (the supposed burial shroud of Jesus Christ) was fake, because it was actually made between 1260 C.E. and 1390 C.E.

Smoke Detectors: In a smoke detector, ^{241}Am emits positively-charged alpha particles, which are directed towards a metal plate. This steady flow of positive charges completes an electrical circuit. If there is a fire, smoke particles neutralize positive charges. This makes the flow of charges through the electrical circuit stop, which is used to trigger the alarm.

Use this space for summary and/or additional notes:

Appendix: AP® Physics 2 Equation Tables

ADVANCED PLACEMENT PHYSICS PHYSICS 2 IN PLAIN ENGLISH, EFFECTIVE 2017

CONSTANTS AND CONVERSION FACTORS			
Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg	1 electron volt,	$1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg	Speed of light,	$c = 3.00 \times 10^8\ \text{m/s}$
Avogadro's number,	$N_o = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant,	$G = 6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2$
Universal gas constant,	$R = 8.31\ \frac{\text{J}}{\text{mol}\cdot\text{K}}$	Acceleration due to gravity at Earth's surface,	$g = 9.8\ \text{m/s}^2$
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}\ \frac{\text{J}}{\text{K}}$		

1 unified atomic mass unit,	$1\ \text{u} = 1.66 \times 10^{-27}\ \text{kg} = 931\ \frac{\text{MeV}}{\text{c}^2}$
Planck's constant,	$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s} = 4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25}\ \text{J}\cdot\text{m} = 1.24 \times 10^{-3}\ \text{eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_o = 8.85 \times 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$
Coulomb's law constant,	$k = \frac{1}{4\pi\varepsilon_o} = 9.0 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$
Vacuum permeability,	$\mu_o = 4\pi \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}$
Magnetic constant,	$k' = \frac{\mu_o}{4\pi} = 1 \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}$
1 atmosphere pressure,	$1\ \text{atm} = 1.0 \times 10^5\ \frac{\text{N}}{\text{m}^2} = 1.0 \times 10^5\ \text{Pa}$

UNIT SYMBOLS								
	meter,	m	mole	mol	watt,	W	farad,	F
	kilogram,	k	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbo l
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	1/2	3/5	$\sqrt{2}/2$	4/5	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	4/5	$\sqrt{2}/2$	3/5	1/2	0
$\tan\theta$	0	$\sqrt{3}/3$	3/4	1	4/3	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. The frame of reference of any problem is assumed to be inertial unless otherwise stated.
II. In all situations, positive work is defined as work done **on** a system.
III. The direction of current is conventional current: the direction in which positive charge would drift.
IV. Assume all batteries and meters are ideal unless otherwise stated.
V. Assume edge effects for the electric field of a parallel plate capacitor unless otherwise stated.
VI. For any isolated electrically charged object, the electric potential is defined as zero at infinite distance from the charged object.

MECHANICS

$v_x = v_{xo} + a_x t$

$x = x_o + v_{xo}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{xo}^2 + 2a_x(x - x_o)$

$\vec{a} = \dfrac{\sum \vec{F}}{m} = \dfrac{\vec{F}_{net}}{m}$

$\left|\vec{F}_f\right| \le \mu\left|\vec{F}_n\right|$

$a_c = \dfrac{v^2}{r}$

$\vec{p} = m\vec{v}$

$\Delta\vec{p} = \vec{F}\Delta t$

$K = \frac{1}{2}mv^2$

$\Delta E = W = F_{\parallel}d = Fd\cos\theta$

$P = \dfrac{\Delta E}{\Delta t}$

$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$

$\omega = \omega_o + \alpha t$

$x = A\cos(\omega t) = A\cos(2\pi ft)$

$x_{cm} = \dfrac{\sum m_i x_i}{\sum m_i}$

$\vec{\alpha} = \dfrac{\sum \vec{\tau}}{I} = \dfrac{\vec{\tau}_{net}}{I}$

$\tau = r_{\perp}F = rF\sin\theta$

$L = I\omega$

$\Delta L = \tau\Delta t$

$K = \frac{1}{2}I\omega^2$

$\left|\vec{F}_s\right| = k\left|\vec{x}\right|$

$U_s = \frac{1}{2}kx^2$

$\Delta U_g = mg\Delta y$

$T = \dfrac{2\pi}{\omega} = \dfrac{1}{f}$

$T_s = 2\pi\sqrt{\dfrac{m}{k}}$

$T_p = 2\pi\sqrt{\dfrac{\ell}{g}}$

$\left|\vec{F}_g\right| = G\dfrac{m_1 m_2}{r^2}$

$\vec{g} = \dfrac{\vec{F}_g}{m}$

$U_g = G\dfrac{m_1 m_2}{r}$

a = acceleration
A = amplitude
d = distance
E = energy
f = frequency
F = force
I = rotational inertia
K = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
r = radius or separation
T = period
t = time
U = potential energy
V = volume
v = speed
W = work done on a system
x = position
y = height
α = angular acceleration
μ = coefficient of friction
θ = angle
ρ = density
τ = torque
ω = angular speed

ELECTRICITY AND MAGNETISM

$\left|\vec{\boldsymbol{F}}_E\right| = \dfrac{1}{4\pi\varepsilon_o}\dfrac{\left|q_1 q_2\right|}{r^2}$

$\vec{\boldsymbol{E}} = \dfrac{\vec{\boldsymbol{F}}_E}{q}$

$\left|\vec{\boldsymbol{E}}\right| = \dfrac{1}{4\pi\varepsilon_o}\dfrac{|q|}{r^2}$

$\Delta U_E = q\Delta V$

$V = \dfrac{1}{4\pi\varepsilon_o}\dfrac{q}{r}$

$\left|\vec{\boldsymbol{E}}\right| = \left|\dfrac{\Delta V}{\Delta r}\right|$

$\Delta V = \dfrac{Q}{C}$

$C = \kappa\varepsilon_o\dfrac{A}{d}$

$E = \dfrac{Q}{\varepsilon_o A}$

$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

$I = \dfrac{\Delta Q}{\Delta t}$

$R = \dfrac{\rho\ell}{A}$

$P = I\Delta V$

$I = \dfrac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\dfrac{1}{R_p} = \sum_i \dfrac{1}{R_i}$

$C_p = \sum_i C_i$

$\dfrac{1}{C_s} = \sum_i \dfrac{1}{C_i}$

$B = \dfrac{\mu_o}{2\pi}\dfrac{I}{R}$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
ℓ = length
P = power
Q = charge
q = point charge
R = resistance
r = separation
t = time
U = potential (stored) energy
V = electric potential
v = speed
κ = dielectric constant
ρ = resistivity
θ = angle
Φ = flux

$\vec{\boldsymbol{F}}_M = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$

$\left|\vec{\boldsymbol{F}}_M\right| = \left|q\vec{\boldsymbol{v}}\right|\left|\sin\theta\right|\left|\vec{\boldsymbol{B}}\right|$

$\vec{\boldsymbol{F}}_M = \vec{\boldsymbol{I}}\ell \times \vec{\boldsymbol{B}}$

$\left|\vec{\boldsymbol{F}}_M\right| = \left|\vec{\boldsymbol{I}}\ell\right|\left|\sin\theta\right|\left|\vec{\boldsymbol{B}}\right|$

$\Phi_B = \vec{\boldsymbol{B}} \bullet \vec{\boldsymbol{A}}$

$\Phi_B = \left|\vec{\boldsymbol{B}}\right|\cos\theta\left|\vec{\boldsymbol{A}}\right|$

$\mathcal{E} = -\dfrac{\Delta\Phi_B}{\Delta t}$

$\mathcal{E} = B\ell v$

FLUID MECHANICS AND THERMAL PHYSICS

$$\rho = \frac{m}{V}$$

$$P = \frac{F}{A}$$

$$P = P_o + \rho g h$$

$$F_b = \rho V g$$

$$A_1 v_1 = A_2 v_2$$

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

$$\frac{Q}{\Delta t} = \frac{kA\,\Delta T}{L}$$

$$PV = nRT = Nk_B T$$

$$K = \tfrac{3}{2}k_B T$$

$$W = -P\Delta V$$

$$\Delta U = Q + W$$

A = area
F = force
h = depth
k = thermal conductivity
K = kinetic energy
L = thickness
m = mass
n = number of moles
N = number of molecules
P = pressure
Q = energy transferred to a system by heating
T = temperature
t = time
U = internal energy
V = volume
v = speed
W = work done on a system
y = height
ρ = density

MODERN PHYSICS

$$E = hf$$

$$K_{\max} = hf - \phi$$

$$\lambda = \frac{h}{p}$$

$$E = mc^2$$

E = energy
f = frequency
K = kinetic energy
m = mass
p = momentum
λ = wavelength
ϕ = work function

WAVES AND OPTICS

$$\lambda = \frac{v}{f}$$

$$n = \frac{c}{v}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

$$|M| = \left|\frac{h_i}{h_o}\right| = \left|\frac{s_i}{s_o}\right|$$

$$\Delta L = m\lambda$$

$$d\sin\theta = m\lambda$$

d = separation
f = frequency or focal length
h = height
L = distance
M = magnification
m = an integer
n = index of refraction
s = distance
v = speed
λ = wavelength
θ = angle

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = bh$

Circle
$A = \tfrac{1}{2}bh$

Rectangular solid
$V = \ell wh$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + 2\pi r^2$

Sphere
$V = \tfrac{4}{3}\pi r^3$
$S = 4\pi r^2$

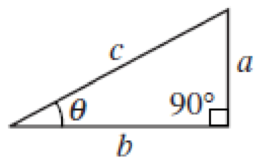
A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

Right triangle
$c^2 = a^2 + b^2$

$$\sin\theta = \frac{a}{c}$$

$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{a}{b}$$



Appendix: Physics Reference Tables

Contents:

Table A. Metric Prefixes

Factor		Prefix	Symbol
1 000 000 000 000 000 000 000 000 000 000	10^{30}	quetta	Q
1 000 000 000 000 000 000 000 000 000	10^{27}	ronna	R
1 000 000 000 000 000 000 000 000	10^{24}	yotta	Y
1 000 000 000 000 000 000 000	10^{21}	zeta	Z
1 000 000 000 000 000 000	10^{18}	exa	E
1 000 000 000 000 000	10^{15}	peta	P
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto	h
10	10^{1}	deca	da
1	10^{0}	—	—
0.1	10^{-1}	deci	d
0.01	10^{-2}	centi	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p
0.000 000 000 000 001	10^{-15}	femto	f
0.000 000 000 000 000 001	10^{-18}	atto	a
0.000 000 000 000 000 000 001	10^{-21}	zepto	z
0.000 000 000 000 000 000 000 001	10^{-24}	yocto	y
0.000 000 000 000 000 000 000 000 001	10^{-27}	ronto	r
0.000 000 000 000 000 000 000 000 000 001	10^{-30}	quecto	q

↓ Move Decimal Point to the Right ↓

↑ Move Decimal Point to the Left ↑

Table B. Physical Constants

Description	Symbol	Precise Value	Common Approximation
acceleration due to gravity on Earth strength of gravity field on Earth	g	$9.7639\,\frac{m}{s^2}$ to $9.8337\,\frac{m}{s^2}$ average value at sea level is $9.806\,65\,\frac{m}{s^2}$	$9.8\,\frac{m}{s^2} \equiv 9.8\,\frac{N}{kg}$ or $10\,\frac{m}{s^2} \equiv 10\,\frac{N}{kg}$
universal gravitational constant	G	$6.673\,84(80)\times10^{-11}\,\frac{N\cdot m^2}{kg^2}$	$6.67\times10^{-11}\,\frac{N\cdot m^2}{kg^2}$
speed of light in a vacuum	c	$299\,792\,458\,\frac{m}{s}$ *	$3.00\times10^{8}\,\frac{m}{s}$
elementary charge (proton or electron)	e	$\pm1.602\,176\,634\times10^{-19}$ C*	$\pm1.60\times10^{-19}$ C
1 coulomb (C)		$6.241\,509\,074\times10^{18}$ elementary charges	6.24×10^{18} elementary charges
(electric) permittivity of a vacuum	ε_o	$8.854\,187\,82\times10^{-12}\,\frac{A^2\cdot s^4}{kg\cdot m^3}$	$8.85\times10^{-12}\,\frac{A^2\cdot s^4}{kg\cdot m^3}$
(magnetic) permeability of a vacuum	μ_o	$4\pi\times10^{-7} = 1.256\,637\,06\times10^{-6}\,\frac{T\cdot m}{A}$	$1.26\times10^{-6}\,\frac{T\cdot m}{A}$
electrostatic constant	k	$\frac{1}{4\pi\varepsilon_o} = 8.987\,551\,787\,368\,176\,4\times10^{9}\,\frac{N\cdot m^2}{C^2}$ *	$8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}$
1 electron volt (eV)		$1.602\,176\,565(35)\times10^{-19}$ J	1.60×10^{-19} J
Planck's constant	h	$6.626\,070\,15\times10^{-34}$ J·s*	6.63×10^{-34} J·s
1 universal (atomic) mass unit (u)		$931.494\,061(21)$ MeV/ c^2 $1.660\,538\,921(73)\times10^{-27}$ kg	931 MeV/c^2 1.66×10^{-27} kg
Avogadro's constant	N_A	$6.022\,140\,76\times10^{23}$ mol^{-1} *	6.02×10^{23} mol^{-1}
Boltzmann constant	k_B	$1.380\,649\times10^{-23}\,\frac{J}{K}$ *	$1.38\times10^{-23}\,\frac{J}{K}$
universal gas constant	R	$8.314\,4621(75)\,\frac{J}{mol\cdot K}$	$8.31\,\frac{J}{mol\cdot K}$
Rydberg constant	R_H	$\frac{m_e e^4}{8\varepsilon_o^2 h^3 c} = 10\,973\,731.6\,\frac{1}{m}$	1.10×10^{7} m^{-1}
Stefan-Boltzmann constant	σ	$\frac{2\pi^5 R^4}{15h^3c^2} = 5.670\,374\,419\times10^{-8}\,\frac{J}{m^2\cdot s\cdot K^4}$	$5.67\times10^{-8}\,\frac{J}{m^2\cdot s\cdot K^4}$
standard atmospheric pressure at sea level		101 325 Pa ≡ 1.01325 bar*	100 000 Pa ≡ 1.0 bar
rest mass of an electron	m_e	$9.109\,382\,15(45)\times10^{-31}$ kg	9.11×10^{-31} kg
mass of a proton	m_p	$1.672\,621\,777(74)\times10^{-27}$ kg	1.67×10^{-27} kg
mass of a neutron	m_n	$1.674\,927\,351(74)\times10^{-27}$ kg	1.67×10^{-27} kg

*denotes an exact value (by definition)

Table C. Quantities, Variables and Units

Quantity	Variable	MKS Unit Name	MKS Unit Symbol	S.I. Base Unit
position	$\vec{\boldsymbol{x}}$	meter*	m	m
distance/displacement, (length, height)	$d, \vec{\boldsymbol{d}}, (L, h)$	meter*	m	m
angle	θ	radian, degree	—, °	—
area	A	square meter	m^2	m^2
volume	V	cubic meter, liter	m^3	m^3
time	t	second*	s	s
velocity	$\vec{\boldsymbol{v}}$	meter/second	$\frac{m}{s}$	$\frac{m}{s}$
speed of light	c			
angular velocity	$\vec{\boldsymbol{\omega}}$	radians/second	$\frac{1}{s^2}$, s^{-1}	$\frac{1}{s^2}$, s^{-1}
acceleration	$\vec{\boldsymbol{a}}$	meter/second²	$\frac{m}{s^2}$	$\frac{m}{s^2}$
acceleration due to gravity	$\vec{\boldsymbol{g}}$			
angular acceleration	$\vec{\boldsymbol{\alpha}}$	radians/second²	$\frac{1}{s^2}$, s^{-2}	$\frac{1}{s^2}$, s^{-2}
mass	m	kilogram*	kg	kg
force	$\vec{\boldsymbol{F}}$	newton	N	$\frac{kg \cdot m}{s^2}$
gravitational field	$\vec{\boldsymbol{g}}$	newton/kilogram	$\frac{N}{kg}$	$\frac{m}{s^2}$
pressure	P	pascal	Pa	$\frac{kg}{m \cdot s^2}$
energy (generic)	E	joule	J	$\frac{kg \cdot m^2}{s^2}$
potential energy	U			
kinetic energy	K, E_k			
heat	Q			
work	W	joule , newton-meter	J , N·m	$\frac{kg \cdot m^2}{s^2}$
torque	$\vec{\boldsymbol{\tau}}$	newton-meter	N·m	$\frac{kg \cdot m^2}{s^2}$
power	P	watt	W	$\frac{kg \cdot m^2}{s^3}$
momentum	$\vec{\boldsymbol{p}}$	newton-second	N·s	$\frac{kg \cdot m}{s}$
impulse	$\vec{\boldsymbol{J}}$			
moment of inertia	I	kilogram-meter²	kg·m²	kg·m²
angular momentum	$\vec{\boldsymbol{L}}$	newton-meter-second	N·m·s	$\frac{kg \cdot m^2}{s}$
frequency	f	hertz	Hz	s^{-1}
wavelength	λ	meter	m	m
period	T	second	s	s
index of refraction	n	—	—	—
electric current	$\vec{\boldsymbol{I}}$	ampere*	A	A
electric charge	q	coulomb	C	A·s
electric potential potential difference (voltage) electromotive force (emf)	V ΔV ε	volt	V	$\frac{kg \cdot m^2}{A \cdot s^3}$
electrical resistance	R	ohm	Ω	$\frac{kg \cdot m^2}{A^2 \cdot s^3}$
capacitance	C	farad	F	$\frac{A^2 \cdot s^4}{m^2 \cdot kg}$
electric field	$\vec{\boldsymbol{E}}$	netwon/coulomb volt/meter	$\frac{N}{C}$, $\frac{V}{m}$	$\frac{kg \cdot m}{A \cdot s^3}$
magnetic field	$\vec{\boldsymbol{B}}$	tesla	T	$\frac{kg}{A \cdot s^2}$
temperature	T	kelvin*	K	K
amount of substance	n	mole*	mol	mol
luminous intensity	I_v	candela*	cd	cd

Variables representing vector quantities are typeset in ***bold italics*** with ***arrows***. * = S.I. base unit

Table D. Mechanics Formulas and Equations

Kinematics (Distance, Velocity & Acceleration)	$\vec{d} = \Delta\vec{x} = \vec{x} - \vec{x}_o$ $\frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2} (= \vec{v}_{ave.})$ $\vec{v} - \vec{v}_o = \vec{a}t$ $\vec{d} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2$ $\vec{v}^2 - \vec{v}_o^2 = 2\vec{a}\vec{d}$
Forces & Dynamics	$\sum \vec{F} = \vec{F}_{net} = m\vec{a}$ $F_f \le \mu_s F_N \qquad F_f = \mu_k F_N$ $\vec{F}_g = m\vec{g} = \frac{Gm_1 m_2}{r^2}$
Circular/ Centripetal Motion & Force	$a_c = \frac{v^2}{r}$ $F_c = ma_c$
Simple Harmonic Motion	$T = \frac{1}{f}$ $T_s = 2\pi\sqrt{\frac{m}{k}} \qquad T_p = 2\pi\sqrt{\frac{L}{g}}$ $\vec{F}_s = -k\vec{x}$ $U_s = \frac{1}{2}kx^2$
Energy, Work & Power	$U_g = mgh = \frac{Gm_1 m_2}{r}$ $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ $W = \Delta E = \Delta(U_g + K)$ $W = F_{\parallel}d = \vec{F}_{net} \bullet \vec{d} = Fd\cos\theta$ $TME = U_g + K$ $TME_i + W = TME_f$ $P = \frac{W}{t} = \vec{F} \bullet \vec{v} = Fv\cos\theta$
Momentum	$\vec{p} = \sum m\vec{v}$ $\Sigma m_i \vec{v}_i + \vec{J} = \Sigma m_f \vec{v}_f$ $\vec{J} = \Delta\vec{p} = \vec{F}_{net}t$

var. = name of quantity (unit)

- Δ = change in something (*E.g.,* Δx means change in x)
- Σ = sum
- d = distance (m)
- $\vec{d}$ = displacement (m)
- $\vec{x}$ = position (m)
- t = time (s)
- $\vec{v}$ = velocity $\left(\frac{\text{m}}{\text{s}}\right)$
- $\vec{v}_{ave.}$ = average velocity $\left(\frac{\text{m}}{\text{s}}\right)$
- $\vec{a}$ = acceleration $\left(\frac{\text{m}}{\text{s}^2}\right)$
- f = frequency $\left(\text{Hz} = \frac{1}{\text{s}}\right)$
- $\vec{F}$ = force (N)
- $\vec{F}_{net}$ = net force (N)
- F_f = force due to friction (N)
- $\vec{F}_g$ = force due to gravity (N)
- $\vec{F}_N$ = normal force (N)
- m = mass (kg)
- $\vec{g}$ = strength of gravity field = aceleration due to gravity $= 10\,\frac{\text{N}}{\text{kg}} = 10\,\frac{\text{m}}{\text{s}^2}$ on Earth
- G = gravitational constant $= 6.67 \times 10^{-11}\,\frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}$
- r = radius (m)
- μ = coëfficient of friction* (*dimensionless*)
- θ = angle (°, radians)
- k = spring constant $\left(\frac{\text{N}}{\text{m}}\right)$
- $\vec{x}$ = displacement of spring (m)
- L = length of pendulum (m)
- E = energy (J)
- $K = E_k$ = kinetic energy (J)
- U = potential energy (J)
- TME = total mechanical energy (J)
- h = height (m)
- Q = heat (J)
- P = power (W)
- W = work (J, N·m)
- T = (time) period (Hz)
- $\vec{p}$ = momentum (N·s)
- $\vec{J}$ = impulse (N·s)
- π = pi (mathematical constant) = 3.14159 26535 89793...

*characteristic property of a substance (to be looked up)

Table E. Approximate Coëfficients of Friction

Substance	Static (μ_s)	Kinetic (μ_k)	Substance	Static (μ_s)	Kinetic (μ_k)
rubber on concrete (dry)	0.90	0.68	wood on wood (dry)	0.42	0.30
rubber on concrete (wet)		0.58	wood on wood (wet)	0.2	
rubber on asphalt (dry)	0.85	0.67	wood on metal	0.3	
rubber on asphalt (wet)		0.53	wood on brick	0.6	
rubber on ice		0.15	wood on concrete	0.62	
steel on ice	0.03	0.01	Teflon on Teflon	0.04	0.04
waxed ski on snow	0.14	0.05	Teflon on steel	0.04	0.04
aluminum on aluminum	1.2	1.4	graphite on steel	0.1	
cast iron on cast iron	1.1	0.15	leather on wood	0.3–0.4	
steel on steel	0.74	0.57	leather on metal (dry)	0.6	
copper on steel	0.53	0.36	leather on metal (wet)	0.4	
diamond on diamond	0.1		glass on glass	0.9–1.0	0.4
diamond on metal	0.1–0.15		metal on glass	0.5–0.7	

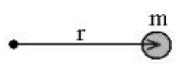
Table F. Angular/Rotational Mechanics Formulas and Equations

Angular Kinematics (Distance, Velocity & Acceleration)	$\Delta\vec{\boldsymbol{\theta}}=\vec{\boldsymbol{\theta}}-\vec{\boldsymbol{\theta}}_o$ $\frac{\Delta\vec{\boldsymbol{\theta}}}{t}=\frac{\vec{\boldsymbol{\omega}}_o+\vec{\boldsymbol{\omega}}}{2}(=\vec{\boldsymbol{\omega}}_{ave.})$ $\vec{\boldsymbol{\omega}}-\vec{\boldsymbol{\omega}}_o=\vec{\boldsymbol{\alpha}}t$ $\Delta\vec{\boldsymbol{\theta}}=\vec{\boldsymbol{\omega}}_o t+\frac{1}{2}\vec{\boldsymbol{\alpha}}t^2$ $\vec{\boldsymbol{\omega}}^2-\vec{\boldsymbol{\omega}}_o^2=2\vec{\boldsymbol{\alpha}}(\Delta\vec{\boldsymbol{\theta}})$
Circular/ Centripetal Motion	$s=r\Delta\theta \quad v_T=r\omega \quad a_T=r\alpha$ $a_c=\frac{v^2}{r}=\omega^2 r$
Rotational Dynamics	$x_{cm}=\frac{\sum m_i x_i}{\sum m_i}$ $I=\_\_mr^2=\int_0^m r^2 dm$ $F_c=ma_c=mr\omega^2$ $\vec{\boldsymbol{\tau}}=\vec{\boldsymbol{r}}\times\vec{\boldsymbol{F}} \qquad \tau=rF\sin\theta=r_\perp F$ $\sum\vec{\boldsymbol{\tau}}=\vec{\boldsymbol{\tau}}_{net}=I\vec{\boldsymbol{\alpha}}$
Simple Harmonic Motion	$T=\frac{1}{f}=\frac{2\pi}{\omega}$ $x=A\cos(2\pi ft)+\phi$
Angular Momentum	$\vec{\boldsymbol{L}}=\vec{\boldsymbol{r}}\times\vec{\boldsymbol{p}}=I\vec{\boldsymbol{\omega}} \qquad L=rp\sin\theta=I\omega$ $\Delta\vec{\boldsymbol{L}}=\vec{\boldsymbol{\tau}}\Delta t$
Angular/ Rotational Energy, Work & Power	$K_r=\frac{1}{2}I\omega^2$ $K=K_t+K_r=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$ $W_r=\tau\Delta\theta$ $P=\frac{W}{t}=\tau\omega$

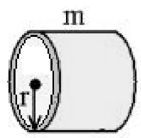
var. = name of quantity (unit)

- Δ = change in something (*E.g.,* Δx = change in *x*)
- Σ = sum
- s = arc length (m)
- t = time (s)
- a_c = centripetal acceleration $\left(\frac{\text{m}}{\text{s}^2}\right)$
- F_c = centripetal force (N)
- m = mass (kg)
- r = radius (m)
- $\vec{\boldsymbol{r}}$ = radius (vector)
- θ = angle (°, radians)
- $\vec{\boldsymbol{\omega}}$ = angular velocity $\left(\frac{\text{rad}}{\text{s}}\right)$
- $\vec{\boldsymbol{\alpha}}$ = angular velocity $\left(\frac{\text{rad}}{\text{s}^2}\right)$
- $\vec{\boldsymbol{\tau}}$ = torque (N·m)
- x = position (m)
- f = frequency (Hz)
- A = amplitude (m)
- ϕ = phase offset (°, rad)
- E = energy (J)
- $K=E_k$ = kinetic energy (J)
- K_t = translational kinetic energy (J)
- K_r = rotational kinetic energy (J)
- P = power (W)
- W = work (J, N·m)
- $\vec{\boldsymbol{p}}$ = momentum (N·s)
- $\vec{\boldsymbol{L}}$ = angular momentum (N·m·s)

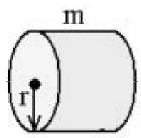
Table G. Moments of Inertia



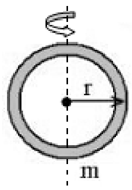
Point Mass:
$I=mr^2$



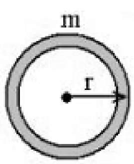
Hollow Cylinder: $I=mr^2$



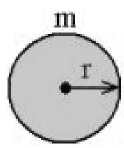
Solid Cylinder:
$I=\frac{1}{2}mr^2$



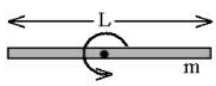
Hoop About Diameter:
$I=\frac{1}{2}mr^2$



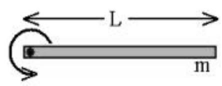
Hollow Sphere:
$I=\frac{2}{3}mr^2$



Solid Sphere:
$I=\frac{2}{5}mr^2$



Rod About the Middle:
$I=\frac{1}{12}mL^2$



Rod About the End:
$I=\frac{1}{3}mL^2$

Table H. Heat and Thermal Physics Formulas and Equations

Temperature	$T_{°F} = 1.8(T_{°C}) + 32$ $T_K = T_{°C} + 273.15$
Heat	$Q = mC\Delta T$ $Q_{melt} = m\Delta H_{fus}$ $Q_{boil} = m\Delta H_{vap}$ $C_p - C_v = R$ $\Delta L = \alpha L_i \Delta T$ $\Delta V = \beta V_i \Delta T$ $P = \frac{Q}{t} = (\pm) kA\frac{\Delta T}{L}$ $P = \frac{Q}{t} = \varepsilon\sigma A T^4$ (*in this section, P = power*)
Thermo-dynamics	$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ $PV = nRT = Nk_BT$ $P\Delta V = nR\Delta T = Nk_B\Delta T$ $\Delta U = Q + W$ $U = \frac{3}{2}nRT \quad \Delta U = \frac{3}{2}nR\Delta T$ $W = -P\Delta V = -\int_{V_1}^{V_2} P\,dV$ $K_{(molecular)} = \frac{3}{2}RT$ $U = \frac{3}{2}nRT = \frac{3}{2}Nk_BT$ $\Delta U = \frac{3}{2}nR\Delta T = \frac{3}{2}Nk_B\Delta T$ $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_BT}{\mu}}$ $\Delta S = \frac{Q_{rev}}{T} \geq \frac{Q}{T}$ $A = U - TS$ $\Delta A = \Delta U - T\Delta S$ (*in this section, P = pressure*)

var. = name of quantity (unit)

- Δ = change in something (*E.g.*, Δx = change in x)
- $T = T_K$ = Kelvin temperature (K)
- $T_{°F}$ = Fahrenheit temperature (°F)
- $T_{°C}$ = Celsius temperature (°C)
- Q = heat (J, kJ)
- m = mass (kg)
- C = specific heat capacity* $\left(\frac{kJ}{kg\cdot°C}, \frac{J}{g\cdot°C}\right)$
- t = time (s)
- L = length (m)
- k = coëfficient of thermal conductivity* $\left(\frac{J}{m\cdot s\cdot°C}, \frac{W}{m\cdot°C}\right)$
- ε = emissivity* (*dimensionless*)
- H_{fus} = latent heat of fusion $\left(\frac{kJ}{kg}, \frac{J}{g}\right)$
- H_{vap} = heat of vaporization $\left(\frac{kJ}{kg}, \frac{J}{g}\right)$
- σ = Stefan-Boltzmann constant $= 5.67\times10^{-8}\ \frac{J}{m^2\cdot s\cdot K^4}$
- V = volume (m^3)
- α = linear coëfficient of thermal expansion* ($°C^{-1}$)
- β = volumetric coëfficient of thermal expansion* ($°C^{-1}$)
- P = power (W)
- P = pressure (Pa)
- n = number of moles (mol)
- N = number of molecules
- R = gas constant $= 8.31\ \frac{J}{mol\cdot K}$
- k_B = Boltzmann constant $= 1.38\times10^{-23}\ \frac{J}{K}$
- U = internal energy (J)
- W = work (J, N·m)
- v_{rms} = root mean square speed $\left(\frac{m}{s}\right)$
- μ = molecular mass* (kg)
- M = molar mass* $\left(\frac{kg}{mol}\right)$
- K = kinetic energy (J)
- Q_{rev} = "reversible" heat (J)
- S = entropy $\left(\frac{J}{K}\right)$
- A = Helmholtz free energy (J)

*characteristic property of a substance (to be looked up)

Table I. Thermodynamics Equation Map

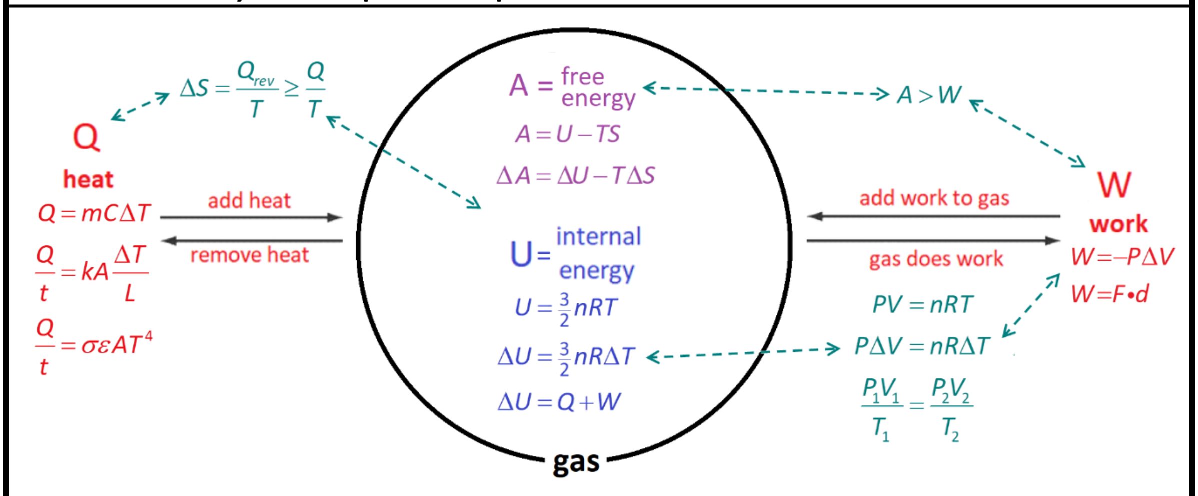


Table J. Pressure-Volume Curves and Equations

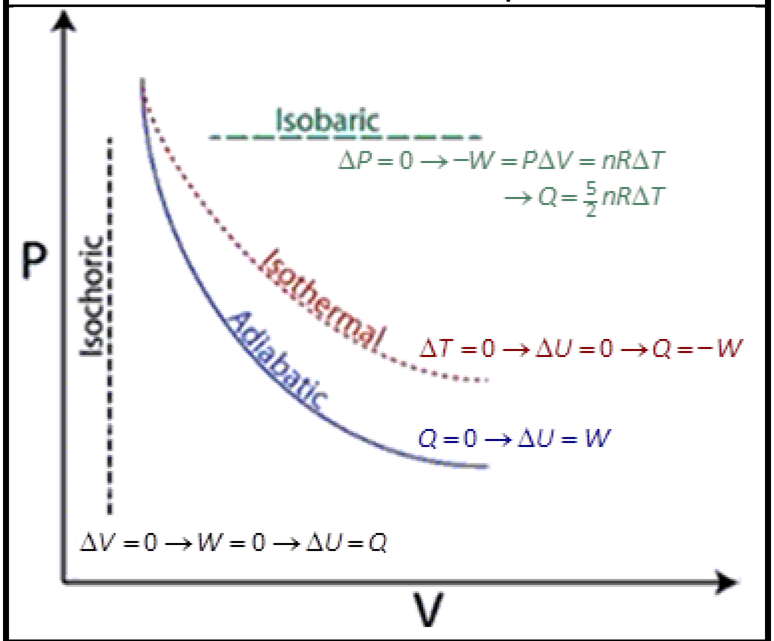


Table K. Thermal Properties of Selected Materials

Substance	Melting Point (°C)	Boiling Point (°C)	Heat of Fusion ΔH_{fus} $\left(\frac{kJ}{kg}, \frac{J}{g}\right)$	Heat of Vaporization ΔH_{vap} $\left(\frac{kJ}{kg}, \frac{J}{g}\right)$	Specific Heat Capacity C $\left(\frac{kJ}{kg \cdot °C}\right)$ at 25°C	Thermal Conductivity k $\left(\frac{J}{m \cdot s \cdot °C}\right)$ at 25°C	Emissivity ε black body = 1	Coefficients of Expansion at 20°C: Linear α (°C^{-1})	Coefficients of Expansion at 20°C: Volumetric β (°C^{-1})
air (gas)	—	—	—	—	1.012	0.024	—	—	—
aluminum (solid)	659	2467	395	10460	0.897	250	0.09*	2.3×10^{-5}	6.9×10^{-5}
ammonia (gas)	−75	−33.3	339	1369	4.7	0.024	—	—	—
argon (gas)	−189	−186	29.5	161	0.520	0.016	—	—	—
carbon dioxide (gas)	−78		574		0.839	0.0146	—	—	—
copper (solid)	1086	1187	134	5063	0.385	401	0.03*	1.7×10^{-5}	5.1×10^{-5}
brass (solid)	—	—	—	—	0.380	120	0.03*	1.9×10^{-5}	5.6×10^{-5}
diamond (solid)	3550	4827	10 000	30 000	0.509	2200	—	1×10^{-6}	3×10^{-6}
ethanol (liquid)	−117	78	104	858	2.44	0.171	—	2.5×10^{-4}	7.5×10^{-4}
glass (solid)	—	—	—	—	0.84	0.96–1.05	0.92	8.5×10^{-6}	2.55×10^{-5}
gold (solid)	1063	2660	64.4	1577	0.129	310	0.025*	1.4×10^{-5}	4.2×10^{-5}
granite (solid)	1240	—	—	—	0.790	1.7–4.0	0.96	—	—
helium (gas)	—	−269	—	21	5.193	0.142	—	—	—
hydrogen (gas)	−259	−253	58.6	452	14.30	0.168	—	—	—
iron (solid)	1535	2750	289	6360	0.450	80	0.31	1.18×10^{-5}	3.33×10^{-5}
lead (solid)	327	1750	24.7	870	0.160	35	0.06	2.9×10^{-5}	8.7×10^{-5}
mercury (liquid)	−39	357	11.3	293	0.140	8	—	6.1×10^{-5}	1.82×10^{-4}
paraffin wax (solid)	46–68	~300	~210	—	2.5	0.25	—	—	—
silver (solid)	962	2212	111	2360	0.233	429	0.025*	1.8×10^{-5}	5.4×10^{-5}
zinc (solid)	420	906	112	1760	0.387	120	0.05*	$\sim 3\times10^{-5}$	8.9×10^{-5}
steam (gas) @ 100°C			—	2260	2.080	0.016	—	—	—
water (liq.) @ 25°C	0	100	334		4.181	0.58	0.95	6.9×10^{-5}	2.07×10^{-4}
ice (solid) @ −10°C				—	2.11	2.18	0.97	—	—

*polished surface

Table L. Electricity Formulas & Equations

Electrostatic Charges & Electric Fields	$\boldsymbol{F}_e = \frac{kq_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}$ $\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}_e}{q} = \frac{Q}{\varepsilon_o A}$ $\quad \vec{\boldsymbol{E}} = \frac{kq}{r^2} = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{r^2} = \frac{\Delta V}{\Delta r}$ $W = q\vec{\boldsymbol{E}}\bullet\vec{\boldsymbol{d}} = qEd_{\parallel} = qEd\cos\theta$ $\Delta V = \frac{W}{q} = \vec{\boldsymbol{E}}\bullet\vec{\boldsymbol{d}} = Ed_{\parallel} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$ $\Delta U_E = q\Delta V \qquad U_E = \frac{kq_1q_2}{r}$
Circuits and Electrical Components	$\Delta V = IR \qquad I = \frac{\Delta Q}{\Delta t} = \frac{\Delta V}{R}$ $\mathcal{E} = IR$ $P = I\Delta V = I^2R = \frac{(\Delta V)^2}{R}$ $W = Pt = I\Delta Vt$ $R = \frac{\rho L}{A}$ $C = \kappa\varepsilon_o\frac{A}{d}$ $Q = C\Delta V$ $U_{capacitor} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$ $P_{total} = P_1 + P_2 + P_3 + \ldots = \sum P_i$ $U_{total} = U_1 + U_2 + U_3 + \ldots = \sum U_i$
Series Circuits (or Series Sections of Circuits)	$I_{total} = I_1 = I_2 = I_3 = \ldots$ $\Delta V_{total} = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots = \sum \Delta V_i$ $R_{equiv.} = R_1 + R_2 + R_3 + \ldots = \sum R_i$ $Q_{total} = Q_1 = Q_2 = Q_3 = \ldots$ $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots = \sum \frac{1}{C_i}$
Parallel Circuits (or Parallel Sections of Circuits)	$I_{total} = I_1 + I_2 + I_3 + \ldots = \sum I_i$ $\Delta V_{total} = \Delta V_1 = \Delta V_2 = \Delta V_3 = \ldots$ $\frac{1}{R_{equiv.}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots = \sum \frac{1}{R_i}$ $Q_{total} = Q_1 + Q_2 + Q_3 + \ldots = \sum Q_i$ $C_{total} = C_1 + C_2 + C_3 + \ldots = \sum C_i$
Resistor-Capacitor (RC) Circuits	charging: $\frac{I}{I_o} = e^{-t/RC}$ charging: $\frac{Q}{Q_{max}} = 1 - e^{-t/RC}$ discharging: $\frac{I}{I_o} = \frac{V}{V_o} = \frac{Q}{Q_{max}} = e^{-t/RC}$

var. = name of quantity (unit)

- Δ = change in something. (*E.g.,* Δx = change in x)
- $\vec{\boldsymbol{F}}_e$ = force due to electric field (N)
- ε_o = electric permittivity of a vacuum
  $= 8.85\times10^{-12}\ \frac{\text{A}^2\cdot\text{s}^4}{\text{kg}\cdot\text{m}^3}$
- k = electrostatic constant
  $= \frac{1}{4\pi\varepsilon_o} = 9.0\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$
- q = point charge (C)
- Q = charge (C)
- $\vec{\boldsymbol{E}}$ = electric field $\left(\frac{\text{N}}{\text{C}}, \frac{\text{V}}{\text{m}}\right)$
- V = electric potential (V)
- ΔV = voltage = electric potential difference (V)
- $\mathcal{E}$ = emf = electromotive force (V)
- W = work (J, N·m)
- $\kappa = \varepsilon_r$ = relative permittivity* (*dimensionless*)
- d = distance (m)
- r = radius (m)
- $\vec{\boldsymbol{I}}$ = current (A)
- t = time (s)
- R = resistance (Ω)
- P = power (W)
- ρ = resistivity ($\Omega\cdot$m)
- L = length (m)
- A = cross-sectional area (m^2)
- C = capacitance (F)
- U = potential energy (J)
- π = pi (mathematical constant)
  = 3.14159 26535 89793...
- e = Euler's number (mathematical constant)
  = 2.78182 81812 84590...

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*characteristic property of a substance (to be looked up)

Table M. Electricity & Magnetism Formulas & Equations

Magnetism and Electro-magnetism	$\vec{F}_M = q\,(\vec{v}\times\vec{B})$ $\quad F_M = qvB\sin\theta$ $\vec{F}_M = \ell(\vec{I}\times\vec{B})$ $\quad F_M = \ell\, I\, B\sin\theta$ $\Delta V = \ell(\vec{v}\times\vec{B})$ $\quad \Delta V = \ell vB\sin\theta$ $B = \dfrac{\mu_o}{2\pi}\dfrac{I}{r}$ $\Phi_B = \vec{B}\bullet\vec{A} = BA\cos\theta$ $\mathcal{E} = \dfrac{\Delta\Phi_B}{\Delta t} = BLv$	*var.* = name of quantity (unit) Δ = change in something. (*E.g.,* Δx = change in x) $\vec{F}_e$ = force due to electric field (N) $\vec{v}$ = velocity (of moving charge or wire) $\left(\frac{\text{m}}{\text{s}}\right)$ q = point charge (C) ΔV = voltage = electric potential difference (V) $\mathcal{E}$ = emf = electromotive force (V) r = radius (m) = distance from wire $\vec{I}$ = current (A) L = length (m) t = time (s) A = cross-sectional area (m^2) $\vec{B}$ = magnetic field (T) μ_o = magnetic permeability of a vacuum $= 4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}$ Φ_B = magnetic flux ($\text{T}\cdot\text{m}^2$)
Electro-magnetic Induction	$\dfrac{\#turns_{in}}{\#turns_{out}} = \dfrac{V_{in}}{V_{out}} = \dfrac{I_{out}}{I_{in}}$ $P_{in} = P_{out}$	

Table N. Resistor Color Code

Color	Digit	Multiplier
black	0	$\times 10^0$
brown	1	$\times 10^1$
red	2	$\times 10^2$
orange	3	$\times 10^3$
yellow	4	$\times 10^4$
green	5	$\times 10^5$
blue	6	$\times 10^6$
violet	7	$\times 10^7$
gray	8	$\times 10^8$
white	9	$\times 10^9$
gold		± 5%
silver		± 10%

Table O. Symbols Used in Electrical Circuit Diagrams

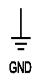
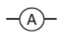
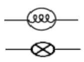
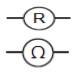
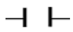
Component	Symbol	Component	Symbol
wire		battery	
switch		ground	
fuse		resistor	
voltmeter		variable resistor (rheostat, potentiometer, dimmer)	
ammeter		lamp (light bulb)	
ohmmeter		capacitor	
		diode	

Table P. Resistivities at 20°C

Conductors		Semiconductors		Insulators	
Substance	**Resistivity $(\Omega\cdot\text{m})$**	**Substance**	**Resistivity $(\Omega\cdot\text{m})$**	**Substance**	**Resistivity $(\Omega\cdot\text{m})$**
silver	1.59×10^{-8}	germanium	0.001 to 0.5	deionized water	1.8×10^{5}
copper	1.72×10^{-8}	silicon	0.1 to 60	glass	1×10^{9} to 1×10^{13}
gold	2.44×10^{-8}	sea water	0.2	rubber, hard	1×10^{13} to 1×10^{13}
aluminum	2.82×10^{-8}	drinking water	20 to 2 000	paraffin (wax)	1×10^{13} to 1×10^{17}
tungsten	5.60×10^{-8}			air	1.3×10^{16} to 3.3×10^{16}
iron	9.71×10^{-8}			quartz, fused	7.5×10^{17}
nichrome	1.50×10^{-6}				
graphite	3×10^{-5} to 6×10^{-4}				

Table Q. Waves & Optics Formulas & Equations

Waves	$v = \lambda f$ $f = \frac{1}{T}$ $v_{\text{wave on a string}} = \sqrt{\frac{F_T}{\mu}}$ $f_{\text{doppler shifted}} = f\left(\frac{\vec{\boldsymbol{v}}_{\text{wave}} + \vec{\boldsymbol{v}}_{\text{detector}}}{\vec{\boldsymbol{v}}_{\text{wave}} + \vec{\boldsymbol{v}}_{\text{source}}}\right)$ $x = A\cos(2\pi ft + \phi)$	*var. = name of quantity (unit)* Δ = change in something (*E.g.,* Δx = change in x) v = velocity of wave $\left(\frac{m}{s}\right)$ $\vec{\boldsymbol{v}}$ = velocity of source or detector $\left(\frac{m}{s}\right)$ f = frequency (Hz) λ = wavelength (m) A = amplitude (m) x = position (m) T = period (of time) (s) F_T = tension (force) on string (N) μ = elastic modulus of string $\left(\frac{kg}{m}\right)$ θ = angle (°, rad) ϕ = phase offset (°, rad) θ_i = angle of incidence (°, rad) θ_r = angle of reflection (°, rad) θ_c = critical angle (°, rad) n = index of refraction* (*dimensionless*) c = speed of light in a vacuum $= 3.00\times10^8\ \frac{m}{s}$ $f = s_f = d_f$ = distance to focus of mirror/lens (m) r_c = radius of curvature of spherical mirror (m) $s_i = d_i$ = distance from mirror/lens to image (m) $s_o = d_o$ = distance from mirror/lens to object (m) h_i = height of image (m) h_o = height of object (m) M = magnification (*dimensionless*) d = separation (m) L = distance from the opening (m) m = an integer ____________ *characteristic property of a substance (to be looked up)
Reflection, Refraction & Diffraction	$\theta_i = \theta_r$ $n = \frac{c}{v}$ $n_1 \sin\theta_1 = n_2 \sin\theta_2$ $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ $\frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$ $\Delta L = m\lambda = d\sin\theta$	
Mirrors & Lenses	$f = \frac{r_c}{2}$ $\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$ $M = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$	

Table R. Absolute Indices of Refraction

Measured using $f = 5.89\times10^{14}$ Hz (yellow light) at 20 °C unless otherwise specified

Substance	Index of Refraction	Substance	Index of Refraction
air (0 °C and 1 atm)	1.000293	silica (quartz), fused	1.459
ice (0 °C)	1.309	Plexiglas	1.488
water	1.3330	Lucite	1.495
ethyl alcohol	1.36	glass, borosilicate (Pyrex)	1.474
human eye, cornea	1.38	glass, crown	1.50–1.54
human eye, lens	1.41	glass, flint	1.569–1.805
safflower oil	1.466	sodium chloride, solid	1.516
corn oil	1.47	PET (#1 plastic)	1.575
glycerol	1.473	zircon	1.777–1.987
honey	1.484–1.504	cubic zirconia	2.173–2.21
silicone oil	1.52	diamond	2.417
carbon disulfide	1.628	silicon	3.96

Figure S. The Electromagnetic Spectrum

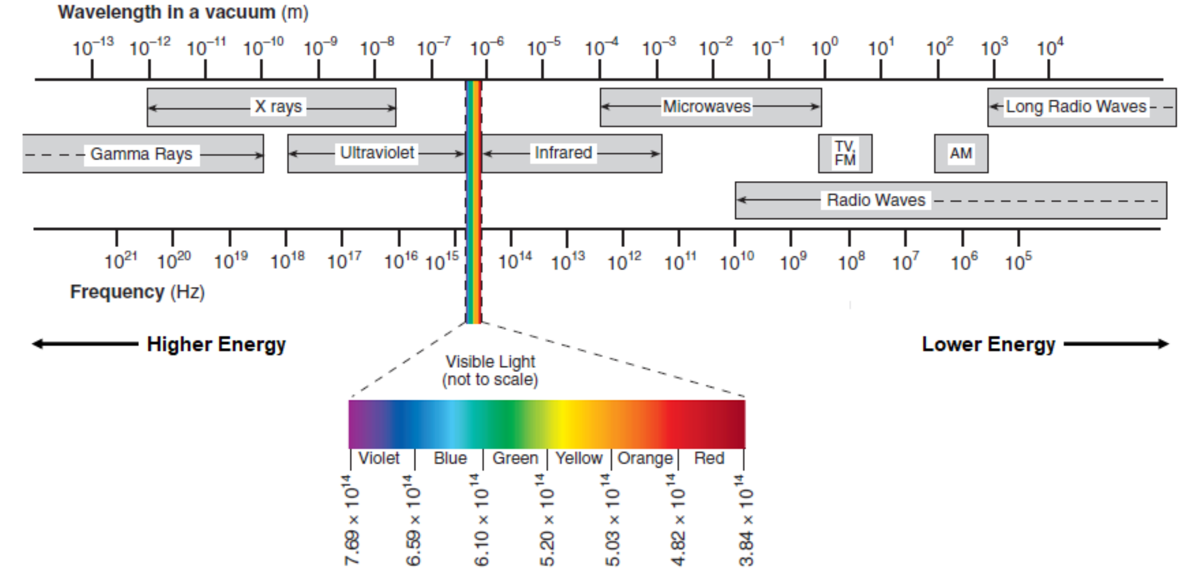


Table T. Planetary Data

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Distance from Sun (m)	5.79×10^{10}	1.08×10^{11}	1.50×10^{11}	2.28×10^{11}	7.79×10^{11}	1.43×10^{12}	2.87×10^{12}	4.52×10^{12}	5.91×10^{12}
Radius (m)	2.44×10^{6}	6.05×10^{6}	6.38×10^{6}	3.40×10^{6}	7.15×10^{7}	6.03×10^{7}	2.56×10^{7}	2.48×10^{7}	1.19×10^{6}
Mass (kg)	3.30×10^{23}	4.87×10^{24}	5.97×10^{24}	6.42×10^{23}	1.90×10^{27}	5.68×10^{26}	8.68×10^{25}	1.02×10^{26}	1.30×10^{22}
Density $\left(\frac{kg}{m^3}\right)$	5429	5243	5514	3934	1326	687	1270	1638	1850
Orbit (years)	0.24	0.61	1.00	1.88	11.8	29	84	164	248
Rotation Period (hours)	1408	−5833	23.9	24.6	9.9	10.7	−17.2	16.1	−153.3
Tilt of axis	0.034°	177.4°	23.4°	25.2°	3.1°	26.7°	97.8°	28.3°	122.5°
# of observed satellites	0	0	1	2	92	83	27	14	5
Mean temp. (°C)	167	464	15	−65	−110	−140	−195	−200	−225
Global magnetic field	Yes	No	Yes	No	Yes	Yes	Yes	Yes	Yes

Data from NASA Planetary Fact Sheet, https://nssdc.gsfc.nasa.gov/planetary/factsheet/ last updated 11 February 2023.

Table U. Sun & Moon Data

Radius of the sun (m)	6.96×10^{8}
Mass of the sun (kg)	1.99×10^{30}
Radius of the moon (m)	1.74×10^{6}
Mass of the moon (kg)	7.35×10^{22}
Distance of moon from Earth (m)	3.84×10^{8}

Table V. Fluids Formulas and Equations		
Fluids	$\rho = \dfrac{m}{V}$ $P = \dfrac{F}{A}$ $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$ $P_{\text{hydrostatic}} = P_H = \rho g h$ $F_B = \rho V_d g$ $P_{\text{dynamic}} = P_D = \frac{1}{2}\rho v^2$ $A_1 v_1 = A_2 v_2$ $P_{total} = P_{ext.} + P_H + P_D$ $P_1 + P_{H,1} + P_{D,1} = P_2 + P_{H,2} + P_{D,2}$ $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$	$var.$ = name of quantity (unit) Δ = change in something. (*E.g.,* Δx = change in x) ρ = density $\left(\frac{\text{kg}}{\text{m}^3}\right)$ m = mass (kg) V = volume (m^3) P = presure (Pa) g = gravitational field = $9.8\,\frac{\text{N}}{\text{kg}} \approx 10\,\frac{\text{N}}{\text{kg}}$ h = height or depth (m) A = area (m^2) v = velocity (of fluid) $\left(\frac{\text{m}}{\text{s}}\right)$ F = force (N) ______________ *characteristic property of a substance (to be looked up)

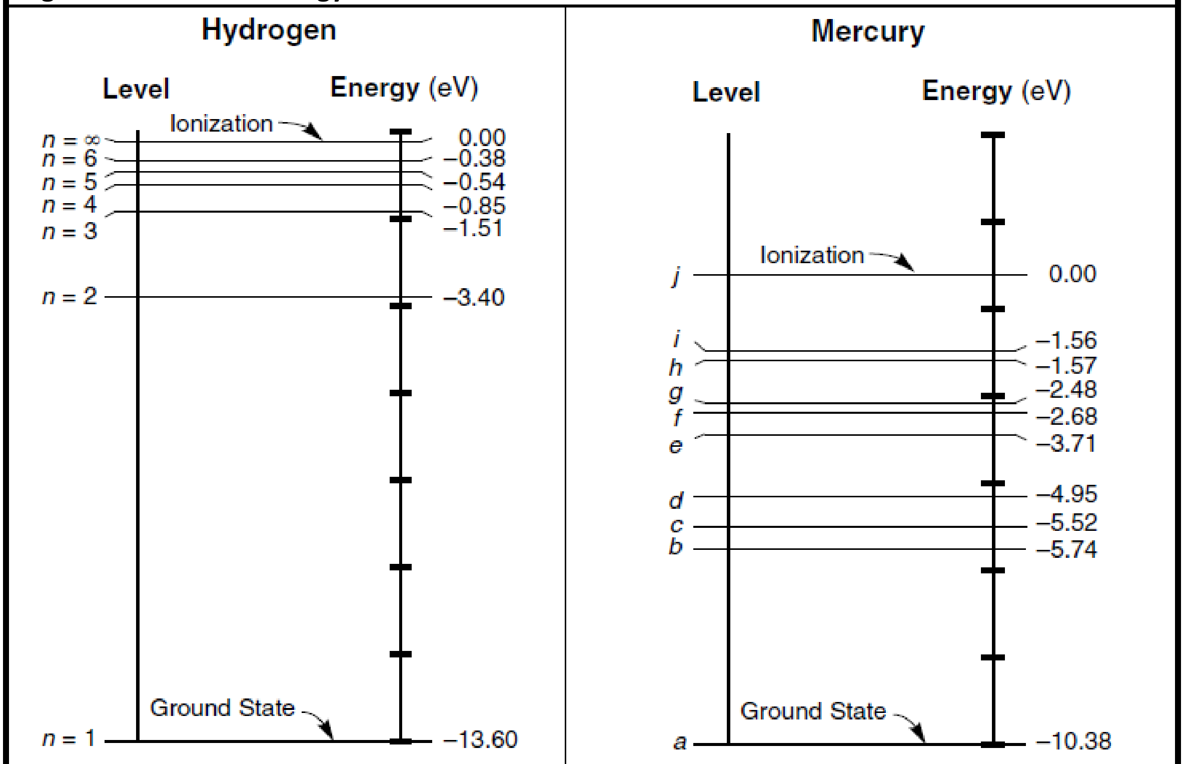
Table W. Properties of Water and Air

Temp. (°C)	Water			Air	
	Density $\left(\frac{\text{kg}}{\text{m}^3}\right)$	Speed of Sound $\left(\frac{\text{m}}{\text{s}}\right)$	Vapor Pressure (Pa)	Density $\left(\frac{\text{kg}}{\text{m}^3}\right)$	Speed of Sound $\left(\frac{\text{m}}{\text{s}}\right)$
0	999.78	1 403	611.73	1.288	331.30
5	999.94	1 427	872.60	1.265	334.32
10	999.69	1 447	1 228.1	1.243	337.31
20	998.19	1 481	2 338.8	1.200	343.22
25	997.02	1 496	3 169.1	1.180	346.13
30	995.61	1 507	4 245.5	1.161	349.02
40	992.17	1 526	7 381.4	1.124	354.73
50	990.17	1 541	9 589.8	1.089	360.35
60	983.16	1 552	19 932	1.056	365.88
70	980.53	1 555	25 022	1.025	371.33
80	971.79	1 555	47 373	0.996	376.71
90	965.33	1 550	70 117	0.969	382.00
100	954.75	1 543	101 325	0.943	387.23

Table X. Atomic & Particle Physics (Modern Physics)

Energy	$E_{photon} = hf = \frac{hc}{\lambda} = pc = \hbar\omega$ $E_{k,\max} = hf - \phi$ $\lambda = \frac{h}{p}$ $E_{photon} = E_i - E_f$ $E^2 = (pc)^2 + (mc^2)^2$ $\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$	*var.* = name of quantity (unit) Δ = change in something. (*E.g.*, Δx = change in x) E = energy (J) h = Planck's constant $= 6.63\times10^{-34}\ \text{J}\cdot\text{s}$ $\hbar$ = reduced Planck's constant $= \frac{h}{2\pi} = 1.05\times10^{-34}\ \text{J}\cdot\text{s}$ f = frequency (Hz) v = velocity $\left(\frac{\text{m}}{\text{s}}\right)$ c = speed of light $= 3.00\times10^{8}\ \frac{\text{m}}{\text{s}}$ λ = wavelength (m) p = momentum (N·s) m = mass (kg) K = kinetic energy (J) ϕ = work function* (J) R_H = Rydberg constant $= 1.10\times10^{7}\ \text{m}^{-1}$
Special Relativity	$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ $\gamma = \frac{L_o}{L} = \frac{\Delta t'}{\Delta t} = \frac{m_{rel}}{m_o}$	γ = Lorentz factor (dimensionless) L = length in moving reference frame (m) L_o = length in stationary reference frame (m) $\Delta t'$ = time in stationary reference frame (s) Δt = time in moving reference frame (s) m_o = mass in stationary reference frame (kg) m_{rel} = apparent mass in moving reference frame (kg) *characteristic property of a substance (to be looked up)

Figure Y. Quantum Energy Levels



Energy Levels for the Hydrogen Atom

A Few Energy Levels for the Mercury Atom

Figure Z. Particle Sizes

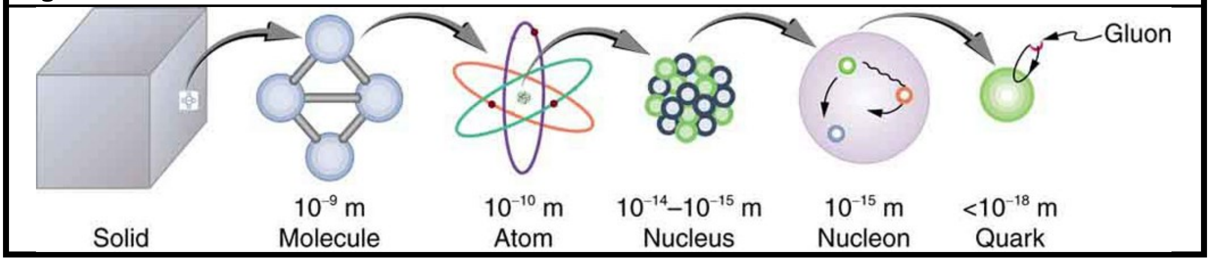


Figure AA. Classification of Matter

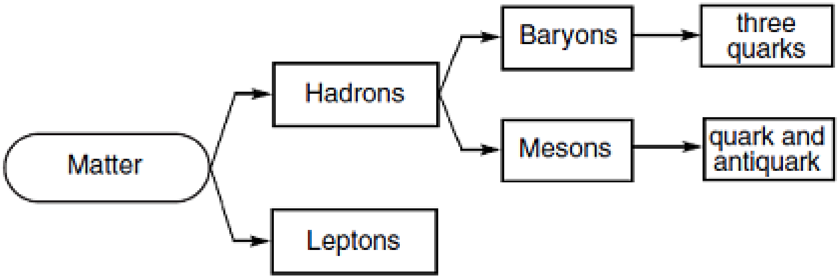


Table BB. The Standard Model of Elementary Particles

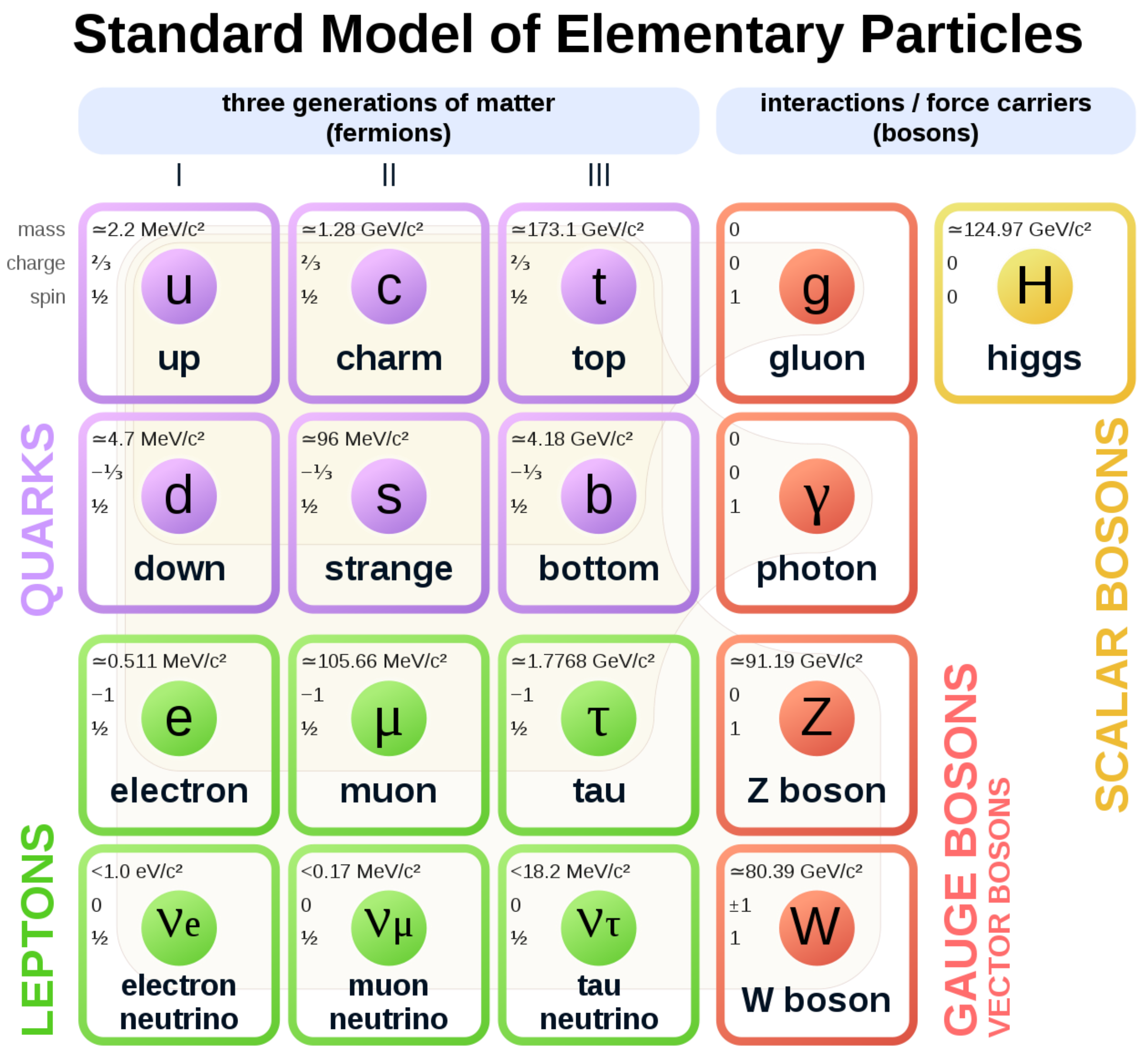


Figure CC. Periodic Table of the Elements

Period	1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8 VIIIB	9 VIIIB	10 VIIIB	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
1	1 **H** hydrogen 1.008																	2 **He** helium 4.003
2	3 **Li** lithium 6.968	4 **Be** beryllium 9.012											5 **B** boron 10.81	6 **C** carbon 12.01	7 **N** nitrogen 14.01	8 **O** oxygen 16.00	9 **F** fluorine 19.00	10 **Ne** neon 20.18
3	11 **Na** sodium 22.99	12 **Mg** magnesium 24.31											13 **Al** aluminum 26.98	14 **Si** silicon 28.09	15 **P** phosphorus 30.97	16 **S** sulfur 32.07	17 **Cl** chlorine 35.45	18 **Ar** argon 39.95
4	19 **K** potassium 39.10	20 **Ca** calcium 40.08	21 **Sc** scandium 44.96	22 **Ti** titanium 47.87	23 **V** vanadium 50.94	24 **Cr** chromium 52.00	25 **Mn** manganese 54.94	26 **Fe** iron 55.85	27 **Co** cobalt 58.93	28 **Ni** nickel 58.69	29 **Cu** copper 63.55	30 **Zn** zinc 65.38	31 **Ga** gallium 69.72	32 **Ge** germanium 72.63	33 **As** arsenic 74.92	34 **Se** selenium 78.97	35 **Br** bromine 79.90	36 **Kr** krypton 83.80
5	37 **Rb** rubidium 85.47	38 **Sr** strontium 87.62	39 **Y** yttrium 88.91	40 **Zr** zirconium 91.22	41 **Nb** niobium 92.91	42 **Mo** molybdenum 95.95	43 **Tc** technetium 98	44 **Ru** ruthenium 101.1	45 **Rh** rhodium 102.9	46 **Pd** palladium 106.4	47 **Ag** silver 107.9	48 **Cd** cadmium 112.4	49 **In** indium 114.8	50 **Sn** tin 118.7	51 **Sb** antimony 121.8	52 **Te** tellurium 127.6	53 **I** iodine 126.9	54 **Xe** xenon 131.3
6	55 **Cs** cesium 132.9	56 **Ba** barium 137.3	71 **Lu** lutetium 175.0	72 **Hf** hafnium 178.5	73 **Ta** tantalum 180.9	74 **W** tungsten 183.8	75 **Re** rhenium 186.2	76 **Os** osmium 190.2	77 **Ir** iridium 192.2	78 **Pt** platinum 195.1	79 **Au** gold 197.0	80 **Hg** mercury 200.6	81 **Tl** thallium 204.4	82 **Pb** lead 207.2	83 **Bi** bismuth 209.0	84 **Po** polonium 209	85 **At** astatine 210	86 **Rn** radon 222
7	87 **Fr** francium 223	88 **Ra** radium 226	103 **Lr** lawrencium 262	104 **Rf** rutherfordium 267	105 **Db** dubnium 268	106 **Sg** seaborgium 271	107 **Bh** bohrium 272	108 **Hs** hassium 270	109 **Mt** meitnerium 276	110 **Ds** darmstadtium 281	111 **Rg** roentgenium 280	112 **Cn** copernicium 285	113 **Nh** nihonium 284	114 **Fl** flerovium 289	115 **Mc** moscovium 288	116 **Lv** livermorium 293	117 **Ts** tennessine 292	118 **Og** oganesson 294
lanthanides (rare earth metals)			57 **La** lanthanum 138.9	58 **Ce** cerium 140.1	59 **Pr** praseodymium 140.9	60 **Nd** neodymium 144.2	61 **Pm** promethium 145	62 **Sm** samarium 150.4	63 **Eu** europium 152.0	64 **Gd** gadolinium 157.3	65 **Tb** terbium 158.9	66 **Dy** dysprosium 162.5	67 **Ho** holmium 164.9	68 **Er** erbium 167.3	69 **Tm** thulium 168.9	70 **Yb** ytterbium 173.1		
actinides			89 **Ac** actinium 227	90 **Th** thorium 232.0	91 **Pa** protactinium 231.0	92 **U** uranium 238.0	93 **Np** neptunium 237	94 **Pu** plutonium 244	95 **Am** americium 243	96 **Cm** curium 247	97 **Bk** berkelium 247	98 **Cf** californium 251	99 **Es** einsteinium 252	100 **Fm** fermium 257	101 **Md** mendelevium 258	102 **No** nobelium 259		

Table DD. Symbols Used in Nuclear Physics

Name	Notation	Symbol
alpha particle	${}^{4}_{2}\text{He}$ or ${}^{4}_{2}\alpha$	⍰
beta particle (electron)	${}^{0}_{-1}e$ or ${}^{0}_{-1}\beta$	⍰⁻
gamma radiation	${}^{0}_{0}\gamma$	⍰
neutron	${}^{1}_{0}n$	n
proton	${}^{1}_{1}\text{H}$ or ${}^{1}_{1}p$	p
positron	${}^{0}_{+1}e$ or ${}^{0}_{+1}\beta$	⍰⁺

Table FF. Constants Used in Nuclear Physics

Constant	Value
mass of an electron (m_e)	0.00055 amu
mass of a proton (m_p)	1.00728 amu
mass of a neutron (m_n)	1.00867 amu
Bequerel (Bq)	1 disintegration/second
Curie (Ci)	3.7 x 10^{10} Bq

Figure GG. Neutron/Proton Stability Band

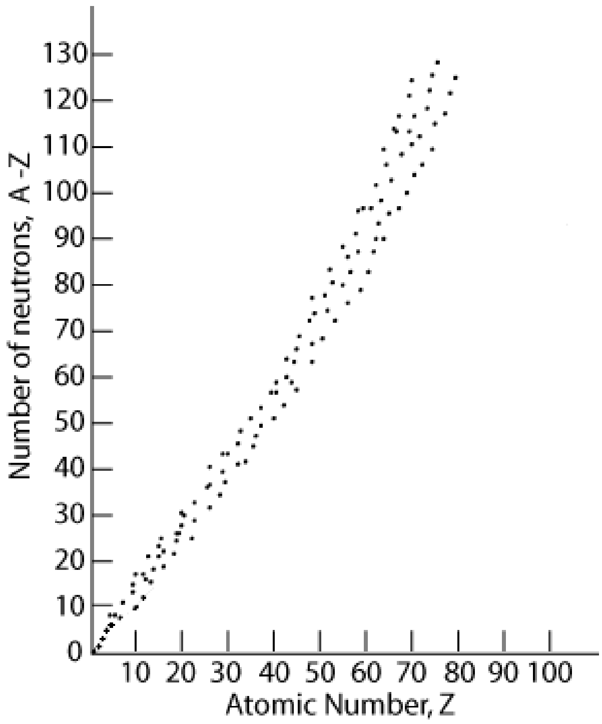


Table EE. Selected Radioisotopes

Nuclide	Half-Life	Decay Mode
^{3}H	12.26 y	⍰⁻
^{14}C	5730 y	⍰⁻
^{16}N	7.2 s	⍰⁻
^{19}Ne	17.2 s	⍰⁺
^{24}Na	15 h	⍰⁻
^{27}Mg	9.5 min	⍰⁻
^{32}P	14.3 d	⍰⁻
^{36}Cl	3.01 × 10^{5} y	⍰⁻
^{37}K	1.23 s	⍰⁺
^{40}K	1.26 × 10^{9} y	⍰⁺
^{42}K	12.4 h	⍰⁻
^{37}Ca	0.175 s	⍰⁻
^{51}Cr	27.7 d	⍰
^{53}Fe	8.51 min	⍰⁻
^{59}Fe	46.3 d	⍰⁻
^{60}Co	5.26 y	⍰⁻
^{85}Kr	10.76 y	⍰⁻
^{87}Rb	4.8 × 10^{10} y	⍰⁻
^{90}Sr	28.1 y	⍰⁻
^{99}Tc	2.13 x 10^{5} y	⍰⁻
^{131}I	8.07 d	⍰⁻
^{137}Cs	30.23 y	⍰⁻
^{153}Sm	1.93 d	⍰⁻
^{198}Au	2.69 d	⍰⁻
^{222}Rn	3.82 d	⍰
^{220}Fr	27.5 s	⍰
^{226}Ra	1600 y	⍰
^{232}Th	1.4 x 10^{10} y	⍰
^{233}U	1.62 x 10^{5} y	⍰
^{235}U	7.1 x 10^{8} y	⍰
^{238}U	4.51 x 10^{9} y	⍰
^{239}Pu	2.44 x 10^{4} y	⍰
^{241}Am	432 y	⍰

Table HH. Mathematics Formulas

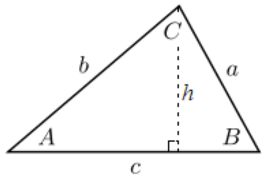
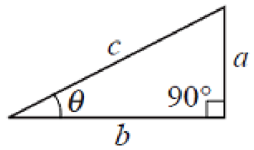
Scientific Notation	$3\times10^4 = 3\times10\,000 = 30\,000 \qquad 2\times10^{-3} = 2\times0.001 = 0.002$ $(3\times10^4)(2\times10^{-3}) = (3\cdot2)(10^4\cdot10^{-3}) = 6\times10^{4+(-3)} = 6\times10^1 = 60$	
Rounding **(to underlined place)**	$15\,\underline{3}54 \rightarrow 15\,\underline{4}00 \qquad 27\,\underline{2}49.99 \rightarrow 27\,\underline{2}00$ $0.037\,\underline{5}00 \rightarrow 0.037\underline{5}$	
Algebra with Fractions	$\frac{a}{b}+\frac{c}{d} = \frac{a}{b}\cdot\frac{d}{d}+\frac{c}{d}\cdot\frac{b}{b} = \frac{ad+cb}{bd}$ $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd} \qquad \frac{a}{b/c}=a\cdot\frac{c}{b}$ $\frac{a}{x}=b \rightarrow \cancel{x}\cdot\frac{a}{\cancel{x}}=b\cdot x \rightarrow a=bx \rightarrow \frac{a}{b}=\frac{bx}{\cancel{b}} \rightarrow \frac{a}{b}=x$	
Quadratic Equation	$ax^2+bx+c=0 \quad\rightarrow\quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	
All Triangles	$A=\tfrac{1}{2}bh$ $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $c^2=a^2+b^2-2ab\cos C$	
Right Triangles	$c^2=a^2+b^2$ $\sin\theta=\frac{a}{c}=\frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta=\frac{b}{c}=\frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{a}{b}=\frac{\text{opposite}}{\text{adjacent}}$ $b=c\cos\theta$ $a=c\sin\theta$	
Rectangles, Parallelograms and Trapezoids	$A=\bar{b}h$	$a,b,c=$ length of a side of a triangle $\theta=$ angle $A=$ area $C=$ circumference $S=$ surface area $V=$ volume $b=$ base $\bar{b}=$ average base $=\frac{b_1+b_2}{2}$ $h=$ height $L=$ length $w=$ width $r=$ radius
Rectangular Solids	$V=Lwh$	
Circles	$C=2\pi r$ $A=\pi r^2$	
Cylinders	$S=2\pi rL+2\pi r^2=2\pi r(L+r)$ $V=\pi r^2L$	
Spheres	$S=4\pi r^2$ $V=\frac{4}{3}\pi r^3$	

Table II. Values of Trigonometric Functions

degree	radian	sine	cosine	tangent	degree	radian	sine	cosine	tangent
0°	0.000	0.000	1.000	0.000					
1°	0.017	0.017	1.000	0.017	46°	0.803	0.719	0.695	1.036
2°	0.035	0.035	0.999	0.035	47°	0.820	0.731	0.682	1.072
3°	0.052	0.052	0.999	0.052	48°	0.838	0.743	0.669	1.111
4°	0.070	0.070	0.998	0.070	49°	0.855	0.755	0.656	1.150
5°	0.087	0.087	0.996	0.087	50°	0.873	0.766	0.643	1.192
6°	0.105	0.105	0.995	0.105	51°	0.890	0.777	0.629	1.235
7°	0.122	0.122	0.993	0.123	52°	0.908	0.788	0.616	1.280
8°	0.140	0.139	0.990	0.141	53°	0.925	0.799	0.602	1.327
9°	0.157	0.156	0.988	0.158	54°	0.942	0.809	0.588	1.376
10°	0.175	0.174	0.985	0.176	55°	0.960	0.819	0.574	1.428
11°	0.192	0.191	0.982	0.194	56°	0.977	0.829	0.559	1.483
12°	0.209	0.208	0.978	0.213	57°	0.995	0.839	0.545	1.540
13°	0.227	0.225	0.974	0.231	58°	1.012	0.848	0.530	1.600
14°	0.244	0.242	0.970	0.249	59°	1.030	0.857	0.515	1.664
15°	0.262	0.259	0.966	0.268	60°	1.047	0.866	0.500	1.732
16°	0.279	0.276	0.961	0.287	61°	1.065	0.875	0.485	1.804
17°	0.297	0.292	0.956	0.306	62°	1.082	0.883	0.469	1.881
18°	0.314	0.309	0.951	0.325	63°	1.100	0.891	0.454	1.963
19°	0.332	0.326	0.946	0.344	64°	1.117	0.899	0.438	2.050
20°	0.349	0.342	0.940	0.364	65°	1.134	0.906	0.423	2.145
21°	0.367	0.358	0.934	0.384	66°	1.152	0.914	0.407	2.246
22°	0.384	0.375	0.927	0.404	67°	1.169	0.921	0.391	2.356
23°	0.401	0.391	0.921	0.424	68°	1.187	0.927	0.375	2.475
24°	0.419	0.407	0.914	0.445	69°	1.204	0.934	0.358	2.605
25°	0.436	0.423	0.906	0.466	70°	1.222	0.940	0.342	2.747
26°	0.454	0.438	0.899	0.488	71°	1.239	0.946	0.326	2.904
27°	0.471	0.454	0.891	0.510	72°	1.257	0.951	0.309	3.078
28°	0.489	0.469	0.883	0.532	73°	1.274	0.956	0.292	3.271
29°	0.506	0.485	0.875	0.554	74°	1.292	0.961	0.276	3.487
30°	0.524	0.500	0.866	0.577	75°	1.309	0.966	0.259	3.732
31°	0.541	0.515	0.857	0.601	76°	1.326	0.970	0.242	4.011
32°	0.559	0.530	0.848	0.625	77°	1.344	0.974	0.225	4.331
33°	0.576	0.545	0.839	0.649	78°	1.361	0.978	0.208	4.705
34°	0.593	0.559	0.829	0.675	79°	1.379	0.982	0.191	5.145
35°	0.611	0.574	0.819	0.700	80°	1.396	0.985	0.174	5.671
36°	0.628	0.588	0.809	0.727	81°	1.414	0.988	0.156	6.314
37°	0.646	0.602	0.799	0.754	82°	1.431	0.990	0.139	7.115
38°	0.663	0.616	0.788	0.781	83°	1.449	0.993	0.122	8.144
39°	0.681	0.629	0.777	0.810	84°	1.466	0.995	0.105	9.514
40°	0.698	0.643	0.766	0.839	85°	1.484	0.996	0.087	11.430
41°	0.716	0.656	0.755	0.869	86°	1.501	0.998	0.070	14.301
42°	0.733	0.669	0.743	0.900	87°	1.518	0.999	0.052	19.081
43°	0.750	0.682	0.731	0.933	88°	1.536	0.999	0.035	28.636
44°	0.768	0.695	0.719	0.966	89°	1.553	1.000	0.017	57.290
45°	0.785	0.707	0.707	1.000	90°	1.571	1.000	0.000	∞

Table JJ. Some Exact and Approximate Conversions

Length	1 cm	≈	width of a small paper clip		
	1 inch (in.)	≡	2.54 cm		
	length of a US dollar bill	=	6.14 in.	=	15.6 cm
	12 in.	≡	1 foot (ft.)	≈	30 cm
	3 ft.	≡	1 yard (yd.)	≈	1 m
	1 m	≡	0.3048 ft.	=	39.37 in.
	1 km	≈	0.6 mi.		
	5,280 ft.	≡	1 mile (mi.)	≈	1.6 km
Mass / Weight	1 small paper clip	≈	0.5 g		
	US 1¢ coin (1983–present)	=	2.5 g		
	US 5¢ coin	=	5 g		
	1 oz.	≈	30 g		
	one medium-sized apple	≈	1 N	≈	3.6 oz.
	1 pound (lb.)	≡	16 oz.	≈	454 g
	1 pound (lb.)	≈	4.45 N		
	1 ton	≡	2000 lb.	≈	0.9 tonne
	1 tonne	≡	1000 kg	≈	1.1 ton
Volume	1 pinch	≈	$^1/_{16}$ teaspoon (tsp.)		
	1 dash	≈	$^1/_8$ teaspoon (tsp.)		
	1 mL	≈	10 drops		
	1 tsp.	≈	5 mL	≈	60 drops
	3 tsp.	≡	1 tablespoon (Tbsp.)	≈	15 mL
	2 Tbsp.	≡	1 fluid ounce (fl. oz.)	≈	30 mL
	8 fl. oz.	≡	1 cup (C)	≈	250 mL
	16 fl. oz.	≡	1 U.S. pint (pt.)	≈	500 mL
	20 fl. oz.	≡	1 Imperial pint (UK)	≈	600 mL
	2 pt. (U.S.)	≡	1 U.S. quart (qt.)	≈	1 L
	4 qt. (U.S.)	≡	1 U.S. gallon (gal.)	≈	3.8 L
	4 qt. (UK) ≡ 5 qt. (U.S.)	≡	1 Imperial gal. (UK)	≈	4.7 L
Speed / Velocity	1 $^{m}/_{s}$	=	3.6 $^{km}/_{h}$	≈	2.24 $^{mi.}/_{h}$
	60 $^{mi.}/_{h}$	≈	100 $^{km}/_{h}$	≈	27 $^{m}/_{s}$
Energy	1 cal	≈	4.18 J		
	1 Calorie (food)	≡	1 kcal	≈	4.18 kJ
	1 BTU	≈	1.06 kJ		
Power	1 hp	≈	746 W		
	1 kW	≈	1.34 hp		
Temper-ature	0 K	≡	−273.15 °C	=	absolute zero
	0 °R	≡	−459.67 °F	=	absolute zero
	0 °F	≈	−18 °C ≡ 459.67 °R		
	32 °F	=	0 °C ≡ 273.15 K	=	water freezes
	70 °F	≈	21 °C	≈	room temperature
	212 °F	=	100 °C	=	water boils
Speed of light	300 000 000 $^{m}/_{s}$	≈	186 000 $^{mi.}/_{s}$	≈	1 $^{ft.}/_{ns}$

Table KK. Greek Alphabet

Α	α	alpha
Β	β	beta
Γ	γ	gamma
Δ	δ	delta
Ε	ε	epsilon
Ζ	ζ	zeta
Η	η	eta
Θ	θ	theta
Ι	ι	iota
Κ	κ	kappa
Λ	λ	lambda
Μ	μ	mu
Ν	ν	nu
Ξ	ξ	xi
Ο	ο	omicron
Π	π	pi
Ρ	ρ	rho
Σ	σ	sigma
Τ	τ	tau
Υ	υ	upsilon
Φ	φ	phi
Χ	χ	chi
Ψ	ψ	psi
Ω	ω	omega

Table LL. Decimal Equivalents

$1/2 = 0.5$	$1/5 = 0.2$
$1/3 = 0.33\overline{3}$	$2/5 = 0.4$
$2/3 = 0.66\overline{6}$	$3/5 = 0.6$
$1/4 = 0.25$	$4/5 = 0.8$
$3/4 = 0.75$	$1/8 = 0.125$
$1/6 = 0.166\overline{6}$	$3/8 = 0.375$
$5/6 = 0.833\overline{3}$	$5/8 = 0.625$
$1/7 = 0.\overline{142857}$	$7/8 = 0.875$
$2/7 = 0.\overline{285714}$	$1/9 = 0.11\overline{1}$
$3/7 = 0.\overline{428571}$	$2/9 = 0.22\overline{2}$
$4/7 = 0.\overline{571428}$	$4/9 = 0.44\overline{4}$
$5/7 = 0.\overline{714285}$	$5/9 = 0.55\overline{5}$
$6/7 = 0.\overline{857142}$	$7/9 = 0.77\overline{7}$
$1/11 = 0.09\overline{09}$	$8/9 = 0.88\overline{8}$
$2/11 = 0.18\overline{18}$	$1/16 = 0.0625$
$3/11 = 0.27\overline{27}$	$3/16 = 0.1875$
$4/11 = 0.36\overline{36}$	$5/16 = 0.3125$
$5/11 = 0.45\overline{45}$	$7/16 = 0.4375$
$6/11 = 0.54\overline{54}$	$9/16 = 0.5625$
$7/11 = 0.63\overline{63}$	$11/16 = 0.6875$
$8/11 = 0.72\overline{72}$	$13/16 = 0.8125$
$9/11 = 0.81\overline{81}$	$15/16 = 0.9375$
$10/11 = 0.90\overline{90}$	

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