Linear Acceleration

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.1, 1.2

AP Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3

Knowledge/Understanding Goals:

- what linear acceleration means
- what positive *vs.* negative acceleration means

Skills:

- calculate position, velocity and acceleration for problems that involve movement in one direction

Language Objectives:

- Understand and correctly use the term "acceleration."
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- Walk with different combinations of positive/negative velocity and positive/negative acceleration.
- Drop a dollar bill or meter stick and have someone try to catch it.
- Drop two strings of beads, one spaced at equal distances and the other spaced at equal times.
- Drop a bottle of water with a hole near the bottom or bucket of ping-pong balls.

Notes:

<u>acceleration</u>: a change in velocity over a period of time.

<u>uniform acceleration</u>: when an object's rate of acceleration (*i.e.,* the rate at which its velocity changes) is constant.

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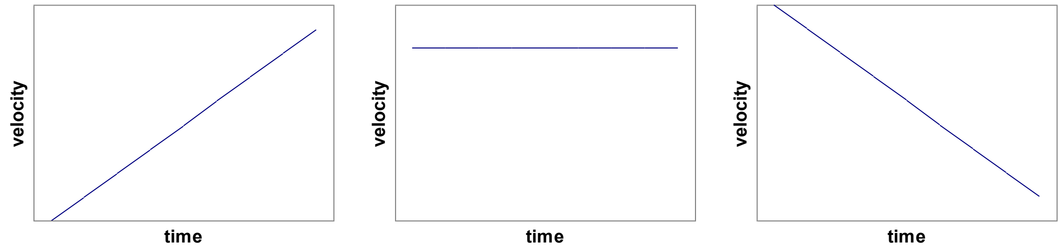
If an object's velocity is increasing, we say it has *positive* acceleration.

If an object's velocity is decreasing, we say it has *negative* acceleration.



positive acceleration — acceleration = zero — negative acceleration

Note that if the object's velocity is negative, then increasing velocity (positive acceleration) would mean that the velocity is getting *less negative*, *i.e.,* the object would be slowing down in the negative direction.

Variables Used to Describe Acceleration

Variable	Quantity	MKS Units
$\vec{a}$	acceleration	$\frac{m}{s^2}$
$\vec{g}$	acceleration due to gravity	$\frac{m}{s^2}$

By convention, physicists use the variable $\vec{g}$ to mean acceleration due to gravity, and $\vec{a}$ to mean acceleration caused by something other than gravity.

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Because acceleration is a change in velocity over a period of time, the formula for acceleration is:

$$\bar{a} = \frac{v - v_o}{t} = \frac{\Delta v}{t} = \frac{\Delta v}{\Delta t} \qquad \text{and, from calculus:} \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

The units must match the formula, which means the units for acceleration must be velocity (distance/time) divided by time, which equals distance divided by time squared.

Because $v = \frac{dx}{dt}$, this means that acceleration is the second derivative of position with respect to time: $a = \frac{dv}{dt} = \frac{d}{dt}(v) = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

However, in an algebra-based physics course, we will limit ourselves to problems in which acceleration is constant.

We can rearrange this formula to show that the change in velocity is acceleration times time:

$$\Delta v = v - v_o = at$$

Note that when an object's velocity is changing, the final velocity, v, is not the same as the average velocity, $\bar{v}$. (This is a common mistake that first-year physics students make.)

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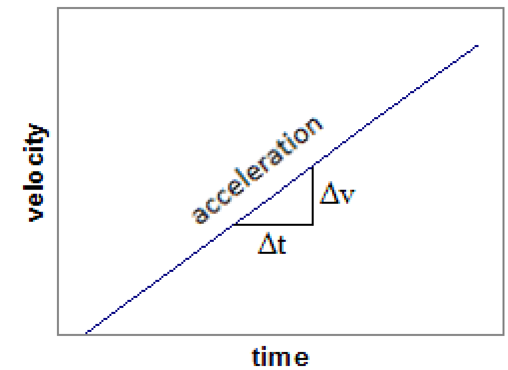
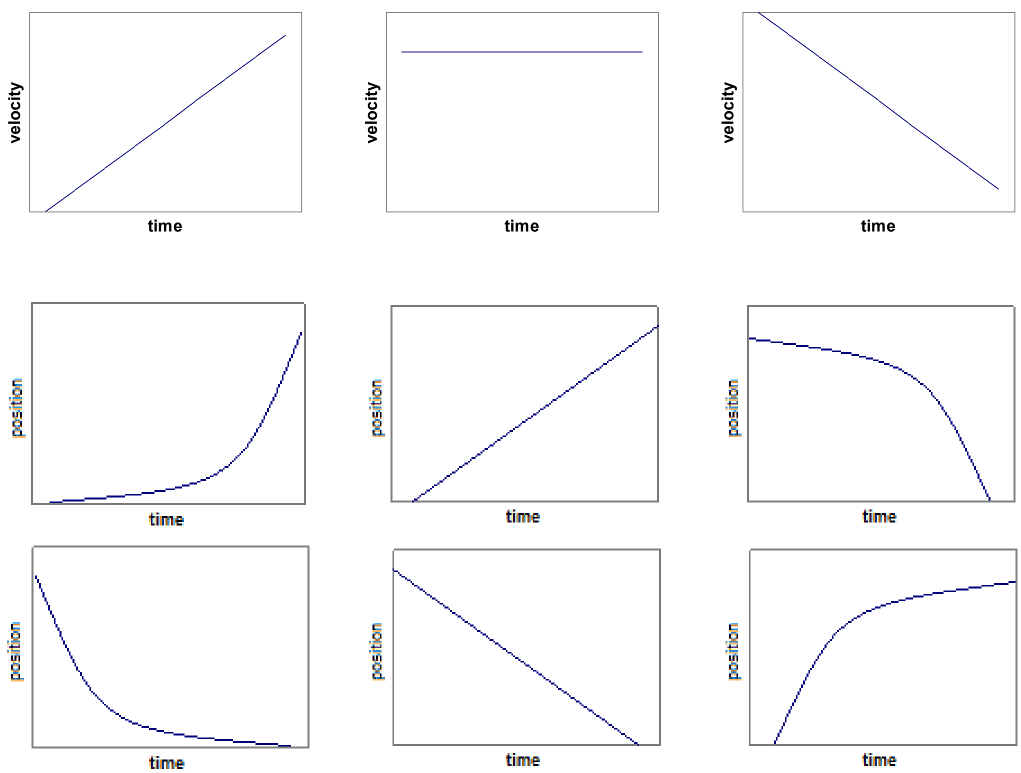
$\frac{\Delta v}{\Delta t}$ is the slope of a graph of velocity (v) *vs.* time (t). Because $\frac{\Delta v}{\Delta t} = a$, this means that acceleration is the ***slope*** of a graph of velocity *vs.* time:



Note the relationship between velocity-time graphs and position-time graphs.

positive acceleration | acceleration = zero | negative acceleration



concave up | linear | concave down

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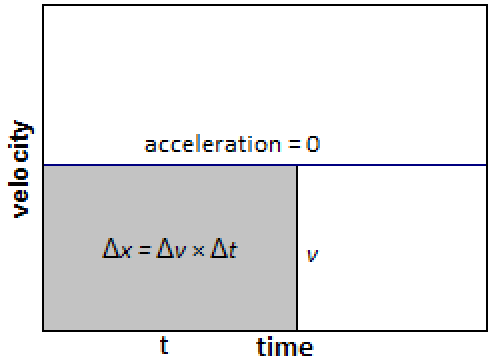
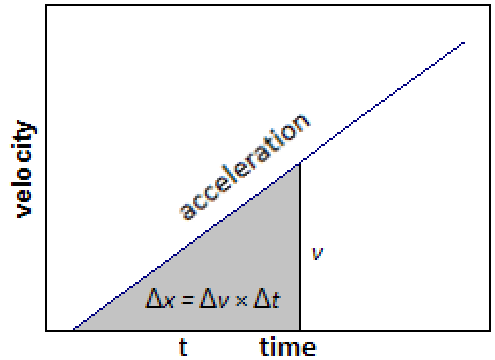
Note also that $\bar{v}t$ is the area under a graph (*i.e.,* the area between the curve and the x-axis) of velocity (v) *vs.* time (t). Because $\bar{v}t = d$, this means the ***area under*** a graph of velocity *vs.* time is the displacement (Δx). Note that this works both for constant velocity (the graph on the left) and changing velocity (as shown in the graph on the right).





In fact, *on any graph, the quantity you get when you multiply the quantities on the* x- *and* y-*axes is, by definition, the area under the graph.*

In calculus, the area under a curve is the integral of the equation for the curve. This means:

$$d = \int_0^t v\,dt$$

where v can be any function of t.

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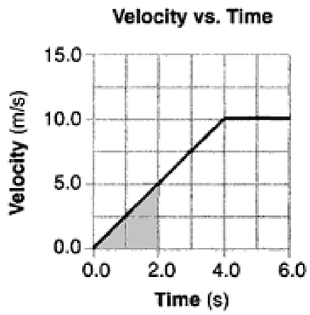
In the graph below, between 0 s and 4 s the object is accelerating at a rate of $+2.5\,\frac{\text{m}}{\text{s}^2}$.

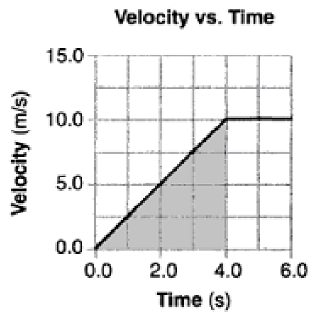
Between 4 s and 6 s the object is moving at a constant velocity (of $+10\,\frac{\text{m}}{\text{s}}$), so the acceleration is zero.



$a = 2.5\,\frac{\text{m}}{\text{s}^2}$

$d = \frac{1}{2}(2.5)(2^2) = 5\text{ m}$

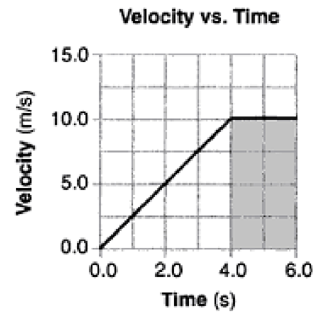
$A = \frac{1}{2}(2)(5) = 5\text{ m}$



$a = 2.5\,\frac{\text{m}}{\text{s}^2}$

$d = \frac{1}{2}(2.5)(4^2) = 20\text{ m}$

$A = \frac{1}{2}(4)(10) = 20\text{ m}$



$a = 0$

$d = \bar{v}t = (10)(2) = 20\text{ m}$

$A = (2)(10) = 20\text{ m}$

In each case, the area under the velocity-time graph equals the total distance traveled.

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To show the relationship between v and $\bar{v}$, we can combine the formula for average velocity with the formula for acceleration in order to get a formula for the position of an object that is accelerating.

$$d = \bar{v}t$$

$$v = at$$

However, the problem is that v in the formula $v = at$ is the velocity at the *end*, which is not the same as the *average* velocity $\bar{v}$.

If the velocity of an object is changing (*i.e.,* the object is accelerating), the average velocity, $\bar{v}$ (the line over the v means "average"), is given by the formula:

$$\bar{v} = \frac{v_o + v}{2}$$

If the object starts at rest (not moving, which means $v_o = 0$) and it accelerates at a constant rate, the average velocity is therefore the average of the initial velocity and the final velocity:

$$\bar{v} = \frac{v_o + v}{2} = \frac{0 + v}{2} = \frac{v}{2} = \tfrac{1}{2}v$$

Combining all of these gives, for an object starting from rest:

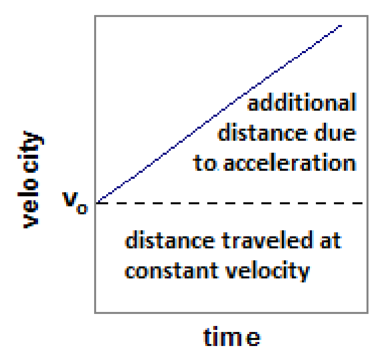
$$d = \bar{v}t = \tfrac{1}{2}vt = \tfrac{1}{2}(at)\,t = \tfrac{1}{2}at^2$$

If an object was moving before it started to accelerate, it had an initial velocity, or a velocity at time = 0. We will represent this initial velocity as v_o^*. Now, the formula becomes:

$$x - x_o = d = v_o t + \tfrac{1}{2}at^2$$

distance the object would travel at its initial velocity

additional distance the object will travel because it is accelerating



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This equation can be combined with the equation for velocity to give the following equation, which relates initial and final velocity and distance:

$$v^2 - v_o^2 = 2ad$$

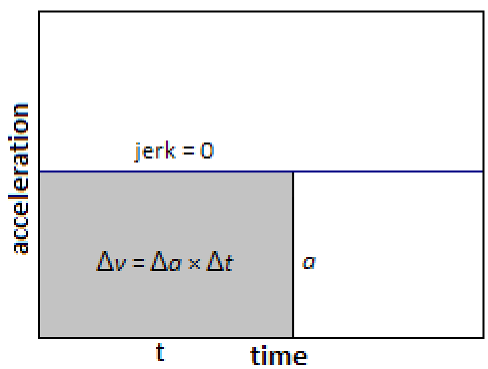
Finally, when an object is accelerating because of gravity, we say that the object is in "free fall".

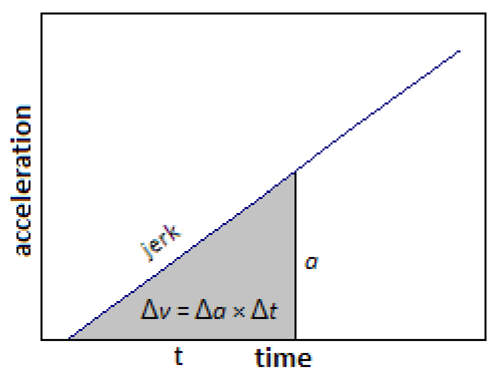
On earth, the average acceleration due to gravity is approximately $9.807\,\frac{\text{m}}{\text{s}^2}$ at sea level (which we will usually round to $10\,\frac{\text{m}}{\text{s}^2}$). Any time gravity is involved (and the problem takes place on Earth), assume that $a = g = 10\,\frac{\text{m}}{\text{s}^2}$.

Extensions

Just as a change in velocity is called acceleration, a change in acceleration with respect to time is called "jerk": $\vec{j} = \frac{\Delta \vec{a}}{\Delta t}$.

While questions about jerk have not been seen on the AP exam, some AP problems do require you to understand that the area under a graph of acceleration *vs.* time would be the change in velocity (Δv), just as the area under a graph of velocity *vs.* time is the change in position.





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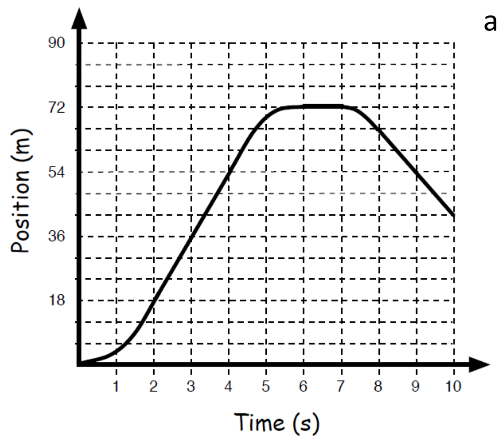
Homework Problems: Motion Graphs

1. An object's motion is described by the following graph of position *vs.* time:



   a. What is the object doing between 2 s and 4 s? What is its velocity during that interval?

   b. What is the object doing between 6 s and 7 s? What is its velocity during that interval?

   c. What is the object doing between 8 s and 10 s? What is its velocity during that interval?

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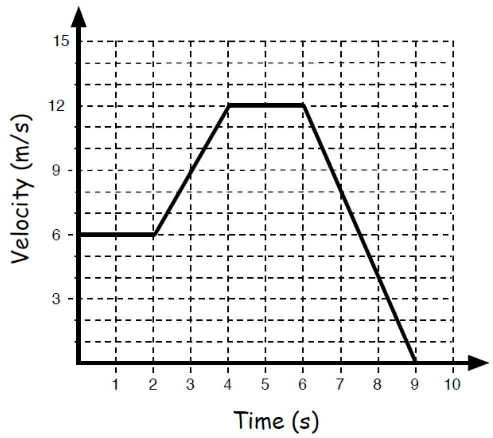
2. An object's motion is described by the following graph of velocity *vs.* time:



a. What is the object doing between 0 s and 2 s? What are its velocity and acceleration during that interval?

b. What is the object doing between 2 s and 4 s? What is its acceleration during that interval?

c. What is the object doing between 6 s and 9 s? What is its acceleration during that interval?

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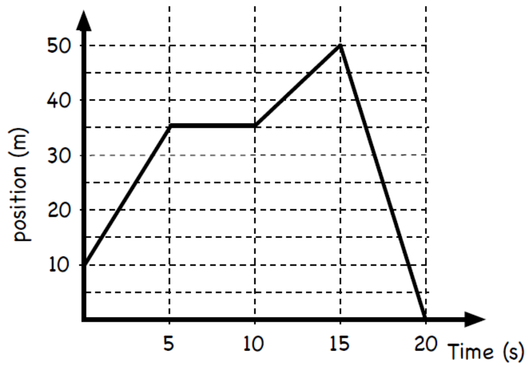
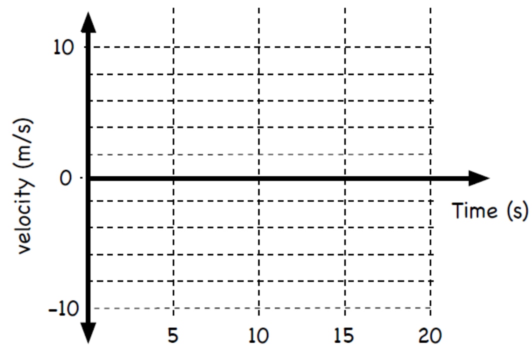
3. The graph on the left below shows the position of an object *vs.* time. Sketch a graph of velocity *vs.* time for the same object on a graph similar to the one on the right.

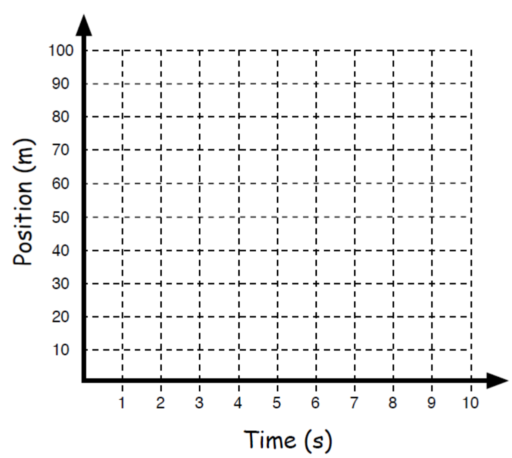


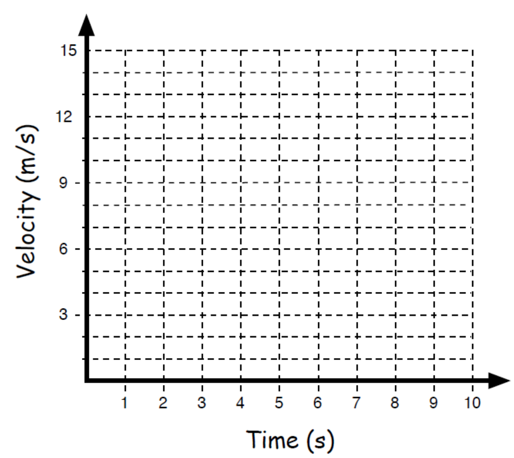


4. In 1991, Carl Lewis became the first sprinter to break the 10-second barrier for the 100 m dash, completing the event in 9.86 s. The chart below shows his time for each 10 m interval.

distance (m)	0	10	20	30	40	50	60	70	80	90	100
time (s)	0	1.88	2.96	3.88	4.77	5.61	6.45	7.29	8.12	8.97	9.86

Plot Lewis's displacement vs. time and velocity *vs.* time on graphs similar to the ones below.





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