Angular Acceleration

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.1, 1.2

AP Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3

Knowledge/Understanding Goals:

- what angular acceleration means

Skills:

- calculate angle, angular velocity and angular acceleration for problems that involve rotational motion.

Language Objectives:

- Understand and correctly use the term "angular acceleration."
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- Swing an object on a string and then change its angular velocity.

Notes:

If a rotating object starts rotating faster or slower, this means its rotational velocity is changing.

angular acceleration (α): the change in angular velocity with respect to time. (Again, the definition is presented with the linear equation for comparison.)

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_o}{t} \qquad \vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t} = \frac{\vec{\omega} - \vec{\omega}_o}{t}$$

linear **angular**

As before, be careful to distinguish between the lower case Greek letter "α" and the lower case Roman letter "a".

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As with linear acceleration, if the object has angular velocity and then accelerates, the position equation looks like this:

$$\vec{x} = \vec{x}_o + \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2 \qquad \vec{\theta} = \vec{\theta}_o + \vec{\omega}_o t + \tfrac{1}{2}\vec{\alpha}t^2$$

linear **angular**

tangential acceleration: the linear acceleration of a point on a rigid, rotating body. The term tangential acceleration is used because the instantaneous direction of the acceleration is tangential to the direction of rotation.

The tangential acceleration of a point on a rigid, rotating body is:

$$\vec{a}_T = r\vec{\alpha}$$

Sample Problem:

Q: A bicyclist is riding at an initial (linear) velocity of $7.5\,\frac{\text{m}}{\text{s}}$, and accelerates to a velocity of $10.0\,\frac{\text{m}}{\text{s}}$ over a duration of 5.0 s. If the wheels on the bicycle have a radius of 0.343 m, what is the angular acceleration of the bicycle wheels?

A: First we need to find the initial and final angular velocities of the bike wheel. We can do this from the tangential velocity, which equals the velocity of the bicycle.

$$\vec{v}_{o,T} = r\vec{\omega}_o$$
$$7.5 = (0.343)\vec{\omega}_o$$
$$\vec{\omega}_o = \frac{7.5}{0.343} = 21.87\,\frac{\text{rad}}{\text{s}}$$

$$\vec{v}_T = r\vec{\omega}$$
$$10.0 = (0.343)\vec{\omega}$$
$$\vec{\omega} = \frac{10.0}{0.343} = 29.15\,\frac{\text{rad}}{\text{s}}$$

Then we can use the equation:

$$\vec{\omega} - \vec{\omega}_o = \vec{\alpha}t$$
$$29.15 - 21.87 = \vec{\alpha}(5.0)$$
$$7.28 = 5.0\vec{\alpha}$$
$$\vec{\alpha} = \frac{7.28}{5.0} = 1.46\,\frac{\text{rad}}{\text{s}^2}$$

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An alternative method is to solve the equation by finding the linear acceleration first:

$$\vec{\boldsymbol{v}} - \vec{\boldsymbol{v}}_o = \vec{\boldsymbol{a}}t$$
$$10.0 - 7.5 = \vec{\boldsymbol{a}}(5)$$
$$2.5 = 5\vec{\boldsymbol{a}}$$
$$\vec{\boldsymbol{a}} = \frac{2.5}{5} = 0.5\,\tfrac{\text{m}}{\text{s}^2}$$

Then we can use the relationship between tangential acceleration and angular acceleration:

$$\vec{\boldsymbol{a}}_T = r\vec{\boldsymbol{\alpha}}$$
$$0.5 = (0.343)\vec{\boldsymbol{\alpha}}$$
$$\vec{\boldsymbol{\alpha}} = \frac{a}{0.343} = 1.46\,\tfrac{\text{rad}}{\text{s}^2}$$

Homework Problems

1. A turntable rotating at 33⅓ RPM is shut off. It slows down at a constant rate and coasts to a stop in 26 s. What is its angular acceleration?

   Answer: $-0.135\ \frac{\text{rad}}{\text{s}^2}$

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2. A turntable rotating with an angular velocity of ω_o is shut off. It slows down at a constant rate and coasts to a stop in time *t*. What is its angular acceleration, α? (You may use your work from problem #1 above to guide you through the algebra.)

   Answer: $\alpha = \frac{-\omega_o}{t}$

3. One of the demonstrations we saw in class was swinging a bucket of water in a vertical circle without spilling any of the water.

   a. Explain why the water stayed in the bucket.

   b. If the combined length of your arm and the bucket is 0.90 m, what is the minimum tangential velocity that the bucket must have in order to not spill any water?

   Answer: $3.33\ \frac{\text{m}}{\text{s}}$

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