Solving Linear & Rotational Motion Problems

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.2

AP Physics 1 Learning Objectives: 3.A.1.3

Skills:

- solve problems involving motion in one or two dimensions

Language Objectives:

- Set up and solve word problems relating to linear or angular motion.

Notes:

The following is a summary of the variables used for motion problems:

Linear			Angular		
Var.	**Unit**	**Description**	**Var.**	**Unit**	**Description**
x	m	position	θ	— (rad)	angle; angular position
$\vec{d}, \Delta x$	m	displacement	$\Delta\theta$	— (rad)	angular displacement
$\vec{v}$	$\frac{m}{s}$	velocity	$\vec{\omega}$	$\frac{1}{s}$ $\left(\frac{rad}{s}\right)$	angular velocity
$\vec{a}$	$\frac{m}{s^2}$	acceleration	$\vec{\alpha}$	$\frac{1}{s^2}$ $\left(\frac{rad}{s^2}\right)$	angular acceleration
t	s	time	t	s	time

Notice that each of the linear variables has an angular counterpart.

Note that "radian" is not a unit. A radian is a ratio that describes an angle as the ratio of the arc length to the radius. This ratio is dimensionless (has no unit), because the units cancel. This means that an angle described in radians has no unit, and therefore never needs to be converted from one unit to another. However, we often write "rad" after an angle measured in radians to remind ourselves that the quantity describes an angle.

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We have learned the following equations for solving motion problems:

Linear Equation	Angular Equation	Relation	Comments
$\vec{d} = \Delta\vec{x} = \vec{x} - \vec{x}_o$	$\Delta\vec{\theta} = \vec{\theta} - \vec{\theta}_o$	$\vec{s} = r\Delta\vec{\theta}$	Definition of displacement.
$\bar{\vec{v}} = \frac{\vec{d}}{t} = \frac{\Delta\vec{x}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$	$\bar{\vec{\omega}} = \frac{\Delta\vec{\theta}}{t} = \frac{\vec{\omega}_o + \vec{\omega}}{2}$	$\vec{v}_T = r\vec{\omega}$	Definition of *average* velocity. Note that you can't use $\bar{v}$ or $\bar{\omega}$ if there is acceleration.
$\vec{a} = \frac{\Delta\vec{v}}{t} = \frac{\vec{v} - \vec{v}_o}{t}$	$\vec{\alpha} = \frac{\Delta\vec{\omega}}{t} = \frac{\vec{\omega} - \vec{\omega}_o}{t}$	$\vec{a}_T = r\vec{\alpha}$	Definition of acceleration.
$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2}\vec{a}t^2$	$\vec{\theta} = \vec{\theta}_o + \vec{\omega}_o t + \frac{1}{2}\vec{\alpha}t^2$		Position formula.
$\vec{v}^2 = \vec{v}_o^2 + 2\vec{a}\Delta\vec{x}$	$\vec{\omega}^2 = \vec{\omega}_o^2 + 2\vec{\alpha}\Delta\vec{\theta}$		Relates velocities, acceleration and distance. Useful if time is not known.
$\vec{a}_c = \frac{\vec{v}^2}{r}$		$\vec{a}_c = r\vec{\omega}^2$	Centripetal acceleration (acceleration toward the center of a circle.)

Note that vector quantities (shown in bold) can be positive or negative, depending on direction.

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Selecting the Right Equation

When you are faced with a problem, choose an equation based on the following criteria:

- The equation must contain the variable you are looking for.
- All other quantities in the equation must be either given in the problem or assumed from the description of the problem.
  - If an object starts at rest (not moving), then $\vec{v}_o = 0$ or $\vec{\omega}_o = 0$.
  - If an object comes to a stop, then $\vec{v} = 0$ or $\vec{\omega} = 0$.
  - If gravity is involved (*e.g.,* the object is falling), $\vec{a} = \vec{g} = 10\,\frac{\text{m}}{\text{s}^2}$.

    (Applies to linear acceleration problems only.)

This means you can choose the appropriate equation by making a list of what you are looking for and what you know. The equation in which you know everything except what you are looking for is the one to use.

What an AP Motion Graph Problem Looks Like

AP motion problems almost always involve either graphs or projectiles. Free-response problems will often ask you to compare two graphs, such as a position-time graph *vs.* a velocity-time graph, or a velocity-time graph *vs.* an acceleration-time graph.

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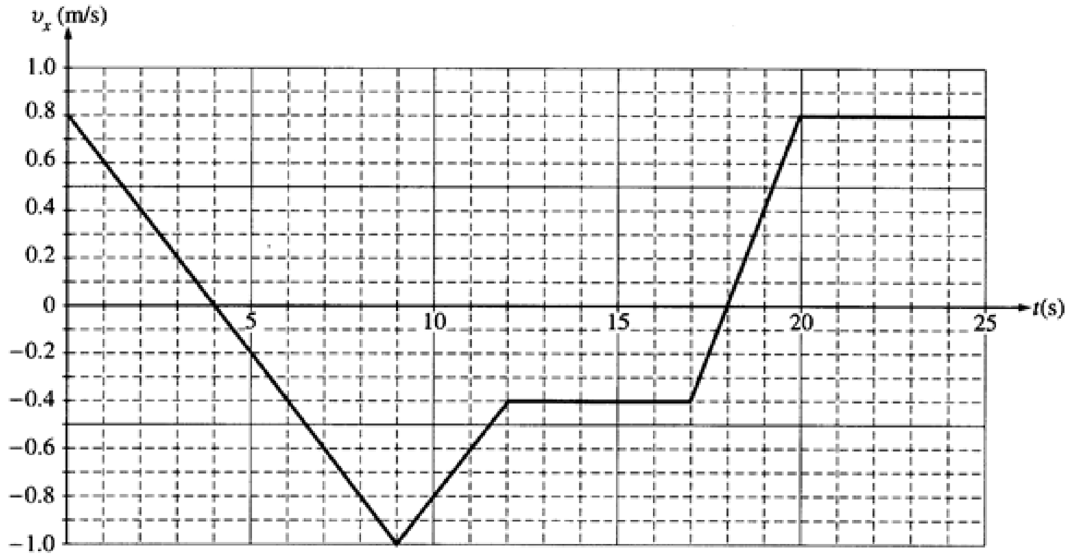
Here is an example of a free-response question involving motion graphs:

Q: A 0.50 kg cart moves on a straight horizontal track. The graph of velocity *v* versus time *t* for the cart is given below.



a. Indicate every time *t* for which the cart is at rest.

The cart is at rest whenever the velocity is zero. Velocity is the y-axis, so we simply need to find the places where y = 0. These are at $\boxed{t = 4\text{ s}}$ *and* $\boxed{t = 18\text{ s}}$.

b. Indicate every time interval for which the speed (magnitude of velocity) of the cart is increasing.

For the velocity vector, we use positive and negative to indicate direction. Therefore, the magnitude is the absolute value. The magnitude of the velocity is increasing whenever the graph is moving away from the x-axis, which happens in the intervals $\boxed{4\text{–}9\text{ s}}$ *and* $\boxed{18\text{–}20\text{ s}}$.

The most likely mistake would be to give the times when the acceleration is positive. Positive acceleration can mean that the speed is increasing in the positive direction, but it can also mean that it is decreasing in the negative direction.

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c. Determine the horizontal position x of the cart at t = 9.0 s if the cart is located at x = 2.0 m at t = 0.

Position is the area under a velocity-time graph. Therefore, if we add the positive and subtract the negative areas from t = 0 *to t* = 9.0 s*, the result is the position at t* = 9.0 s.

The area of the triangular region from 0–4 s *is* $(\frac{1}{2})(4)(0.8) = 1.6$ m.

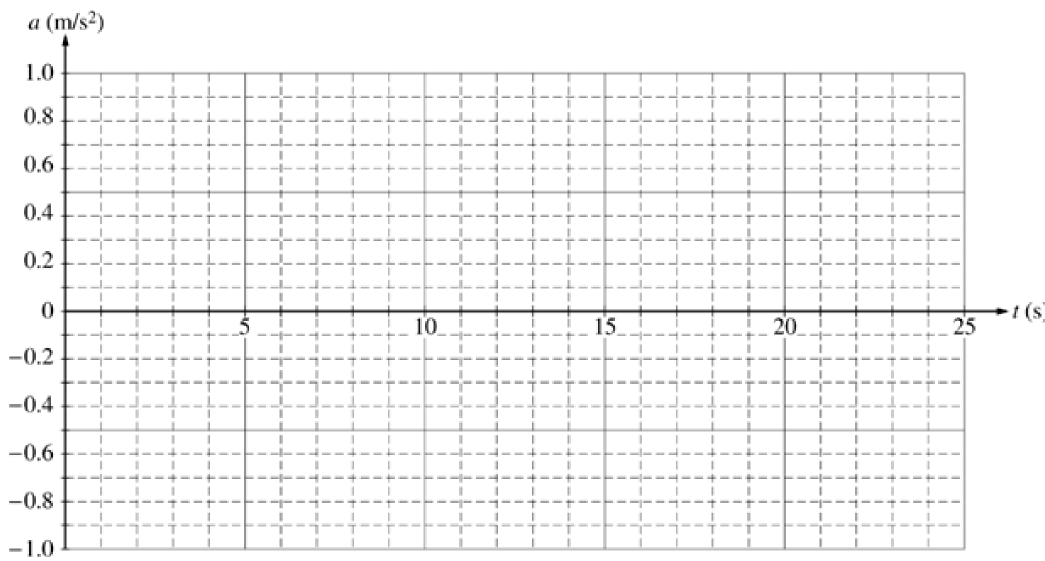
The area of the triangular region from 4–9 s *is* $(\frac{1}{2})(5)(-1.0) = -2.5$ m.

The total displacement is therefore $\Delta x = 1.6 + (-2.5) = -0.9\,\text{m}$.

Because the cart's initial position was +2.0 m*, its final position is* 2.0 + (−0.9) = **+1.1 m**.

The most likely mistakes would be to add the areas regardless of whether they are negative or positive, and to forget to add the initial position after you have found the displacement.

d. On the axes below, sketch the acceleration a versus time t graph for the motion of the cart from t = 0 to t = 25 s.



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Acceleration is the slope of a velocity-time graph. Because the graph is discontininuous, we need to split it at each point where the slope suddenly changes. Each of the regions is a straight line (constant slope), which means all of the accelerations are constant (horizontal lines on the graph).

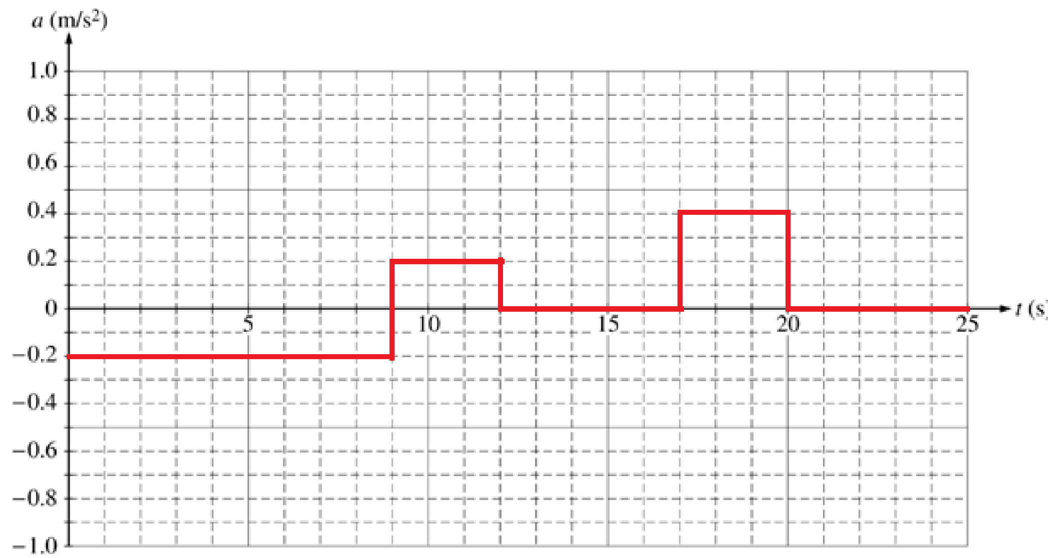
From 0–9 s*, the slope is* $\frac{\Delta y}{\Delta x} = \frac{-1.8}{9} = -0.2\,\frac{\text{m}}{\text{s}^2}$.

From 9–12 s*, the slope is* $\frac{\Delta y}{\Delta x} = \frac{+0.6}{3} = +0.2\,\frac{\text{m}}{\text{s}^2}$.

From 12–17 s *and from* 20–25 s*, the slope is zero.*

From 17–20 s*, the slope is* $\frac{\Delta y}{\Delta x} = \frac{+1.2}{3} = +0.4\,\frac{\text{m}}{\text{s}^2}$.

The graph therefore looks like the following:



e. The original problem also included a part (E), which was a simple projectile problem (discussed later).

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Strategies for Linear Motion Problems Involving Gravity

Linear motion problems in physics often involve gravity. These problems usually fall into one of two categories:

1. If you have an object in free fall, the problem will probably give you either the distance it fell ($x - x_o$), or the time it fell (t). Use the position equation $\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2}\vec{a}t^2$ to calculate whichever one you don't know. (If the object starts from rest, that means $\vec{v}_o = 0$.)

2. If an object is thrown upwards, it will decelerate at a rate of $-10\,\frac{\text{m}}{\text{s}^2}$ (assuming "up" is the positive direction) until it stops moving ($\vec{v} = 0$). Then it will fall. This means you need to split the problem into two parts:

   a. When the object is moving upward, the initial velocity, $\vec{v}_o$, is usually given and $\vec{v}$ (at the top) = zero. From these, you can use $\vec{v}^2 = \vec{v}_o^2 - 2\vec{a}\Delta\vec{x}$ to figure out the maximum height.

   b. Once you know the maximum height, you know the distance to the ground, $\vec{v}_o = 0$, and you can use the position equation (this time with $\vec{a} = +10\,\frac{\text{m}}{\text{s}^2}$) to find the time it spends falling. The total time (up + down) will be twice as much.

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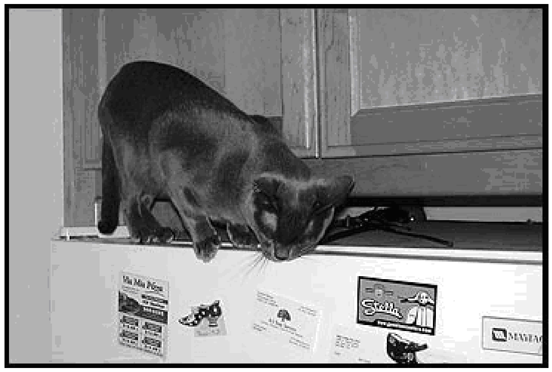
Sample Problems:

Q: If a cat jumps off a 1.8 m tall refrigerator, how long does it take to hit the ground?



A: The problem gives us d = 1.8 m. The cat is starting from rest (v_a = 0), and gravity is accelerating the cat at a rate of $a = g = 10\,\frac{\text{m}}{\text{s}^2}$. We need to find t.

Looking at the equations, the one that has what we need (t) and only quantities we know is:

$$d = v_o t + \tfrac{1}{2}at^2$$

v_o = 0, so this reduces to:

$$d = \tfrac{1}{2}at^2$$

$$1.8 = (\tfrac{1}{2})(10)\,t^2$$

$$\frac{1.8}{5} = 0.36 = t^2$$

$$t = \sqrt{0.36} = 0.6s$$

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Q: An apple falls from a tree branch at a height of 5 m and lands on Isaac Newton's head. (Assume Isaac Newton was 1.8 m tall.)

How fast was the apple traveling at the time of impact?

A: We know $d = s - s_o = 5 - 1.8 = 3.2\,\text{m}$. We also know that the apple is starting from rest ($v_a = 0$), and gravity is accelerating the apple at a rate of $a = g = 10\,\frac{\text{m}}{\text{s}^2}$. We want to find *v*.

The equation that relates all of our variables is:

$$v^2 - v_o^2 = 2ad$$

Substituting, we get:

$$v^2 - 0 = (2)(10)(3.2)$$
$$v^2 = 64$$
$$v = \sqrt{64} = 8\,\frac{\text{m}}{\text{s}}$$

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