Class Notes for

Physics 1

(including AP® Physics 1)

in Plain English

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This is a set of class notes that can be used for an algebra-based, first-year high school Physics 1 course at the college preparatory (CP1), honors, or AP® level. This hardcopy is provided so that you can fully participate in class discussions without having to worry about writing everything down.

While a significant amount of detail is included in these notes, they are intended as a supplement to textbooks, classroom discussions, experiments and activities. These class notes and any textbook discussion of the same topics are intended to be complementary. In some cases, the notes and the textbook differ in method or presentation, but the physics is the same. There may be errors and/or omissions in any textbook. There are almost certainly errors and omissions in these notes, despite my best efforts to make them clear, correct, and complete.

Topics

The AP® curriculum is, of course, set by the College Board. The choice of topics for the CP1 *vs.* honors course is arbitrary. Choices I have made are that the honors course and the AP® course are similarly challenging, but the honors course has more flexibility with regard to pacing, difficulty, and topics. The CP1 course does not require trigonometry or solving problems symbolically before substituting numbers. However, all physics students should take algebra and geometry courses before taking physics, and should be very comfortable solving problems that involve algebra.

Topics that are part of the curriculum for some courses but not others are marked in the left margin as follows:

CP1 & honors (not AP®) *honors only (not AP®)* *honors & AP®* *AP® (only)*

Topics that are not otherwise marked should be assumed to apply to all courses at all levels.

About the Homework Problems

The homework problems include a mixture of easy and challenging problems. *The process of making yourself smarter involves challenging yourself, even if you are not sure how to proceed.* By spending at least 10 minutes attempting each problem, you build neural connections between what you have learned and what you are trying to do. Even if you are not able to get the answer, when we go over those problems in class, you will reinforce the neural connections that led in the correct direction.

Answers to most problems are provided so you can check your work and see if you are on the right track. Do not simply write those answers down in order to receive credit for work you did not do. This will give you a false sense of confidence, and will actively prevent you from using the problems to make yourself smarter. *You have been warned.*

Using These Notes

As we discuss topics in class, you will want to add your own notes to these. If you have purchased this copy, you are encouraged to write directly in it, just as you would write in your own notebook. If this copy was issued to you by the school and you intend to return it at the end of the year, you will need to write your supplemental notes on separate paper. If you do this, be sure to write down page numbers in your notes, to make cross-referencing easier.

You should bring these notes to class every day, because lectures and discussions will follow these notes, which will also be projected onto the SMART board.

Features

These notes, and the course they accompany, are designed to follow both the 2016 Massachusetts Curriculum Frameworks, which are based on the Next Generation Science Standards (NGSS), and the AP® Physics 1 curriculum. (Note that the AP® learning objectives are the ones from 2014.) The notes also utilize strategies from the following popular teaching methods:

- Each topic includes Mastery Objectives and Success Criteria. These are based on the *Studying Skillful Teaching* course, from Research for Better Teaching (RBT), and are in "Students will be able to…" language.
- AP® topics include Learning Objectives from the College Board.
- Each topic includes Tier 2 vocabulary words and language objectives for English Learners, based on the Rethinking Equity and Teaching for English Language Learners (RETELL) course.
- Notes are organized in Cornell notes format as recommended by Keys To Literacy (KtL).
- Problems in problem sets are designated "Must Do" (M), "Should Do" (S) and "Aspire to Do" (A), as recommended by the Modern Classrooms Project (MCP).

Conventions

Some of the conventions in these notes are different from conventions in some physics textbooks. Although some of these are controversial and may incur the ire of other physics teachers, here is an explanation of my reasoning:

- When working sample problems, the units are left out of the algebra until the end. While I agree that there are good reasons for keeping the units to show the dimensional analysis, many students confuse units for variables, *e.g.,* confusing the unit "m" (meters) with the variable "*m*" (mass).
- Problems are worked using $g = 10\,\frac{\text{m}}{\text{s}^2}$. This is because many students are not adept with algebra, and have trouble seeing where a problem is going once they take out their calculators. With simpler numbers, students have an easier time following the physics.
- Vector quantities are denoted with arrows as well as boldface, *e.g.,* $\vec{\boldsymbol{v}}, \vec{\boldsymbol{d}}, \vec{\boldsymbol{F}}_g$. This is to help students keep track of which quantities are vectors and which are scalars.
- Forces are denoted the variable $\vec{\boldsymbol{F}}$ with a subscript, e.g., $\vec{\boldsymbol{F}}_g, \vec{\boldsymbol{F}}_f, \vec{\boldsymbol{F}}_N, \vec{\boldsymbol{F}}_T$, *etc.* instead of $m\vec{\boldsymbol{g}}, \vec{\boldsymbol{f}}, \vec{\boldsymbol{N}}, \vec{\boldsymbol{T}}$, *etc.* This is to reinforce the connection between a quantity (force), a single variable $(\vec{\boldsymbol{F}})$, and a unit.
- Average velocity is denoted $\vec{\boldsymbol{v}}_{ave.}$ instead of $\bar{\vec{\boldsymbol{v}}}$. I have found that using the subscript "ave." helps students remember that average velocity is different from initial and final velocity.
- The variable V is used for electric potential. Voltage (potential difference) is denoted by ΔV. Although $\Delta V = IR$ is different from how the equation looks in most physics texts, it is useful to teach circuits starting with electric potential, and it is useful to maintain the distinction between absolute electric potential (V) and potential difference (ΔV). (This is also how the College Board represents electric potential *vs.* voltage on AP® Physics exams.)
- Equations are typeset on one line when practical. While there are very good reasons for teaching $\vec{\boldsymbol{a}} = \dfrac{\vec{\boldsymbol{F}}_{net}}{m}$ rather than $\vec{\boldsymbol{F}}_{net} = m\vec{\boldsymbol{a}}$ and $I = \dfrac{\Delta V}{R}$ rather than $\Delta V = IR$, students' difficulty in solving for a variable in the denominator often causes more problems than does their lack of understanding of which are the manipulated and responding variables.

Learning Progression

There are several categories of understandings and skills that simultaneously build on themselves throughout this course:

Content

The sequence of topics starts with preliminaries—laboratory and then mathematical skills. After these topics, most of the rest of the course is spent on mechanics: kinematics (motion), then forces (which cause changes to motion), then energy (which makes it possible to apply a force), and momentum (what happens when moving objects interact and transfer some of their energy to each other. After mechanics, the course touches on other topics that are required in the Massachusetts state frameworks: electricity & magnetism, waves, heat, and atomic & particle physics. (Note that topics other than mechanics are not part of the AP® Physics 1 curriculum.)

Problem-Solving

This course teaches problem-solving skills. The problems students will be asked to solve represent real-life situations. You will need to determine which equations and which assumptions apply in order to solve them. The problems start fairly simple and straightforward, requiring only one equation and basic algebra. As the topics progress, some of the problems require multiple steps and multiple equations, often requiring students to use equations from earlier in the course in conjunction with later ones.

Laboratory

This course teaches experimental design. The intent is never to give a student a laboratory procedure, but instead to teach the student to determine which measurements are needed and which equipment to use. (This does, however, require teaching students to use complicated equipment and giving them sufficient time to practice with it, such as probes and the software that collects data from them.)

Early topics, with their one-step equations, are used to teach the basic skills of determining which measurements are needed for a single calculation and how to take them. As later topics connect equations to earlier ones, the experiments become more complex, and students are required to stretch their ability to connect the quantities that they want to relate with ones they can measure.

Scientific Discourse

As topics progress, the causal relationships between quantities become more complex, and students' explanations need to become more complex as a result. Students need to be given opportunities to explain these relationships throughout the course, both orally and in writing.

Acknowledgements

These notes would not have been possible without the assistance of many people. It would be impossible to include everyone, but I would particularly like to thank:

- Every student I have ever taught, for helping me learn how to teach, and how to explain and convey challenging concepts.
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- Every teacher I have worked with, for their kind words, sympathetic listening, helpful advice and suggestions, and other contributions great and small that have helped me to enjoy and become competent at the profession of teaching.
- The department heads, principals and curriculum directors I have worked with, for mentoring me, encouraging me, allowing me to develop my own teaching style, and putting up with my experiments, activities and apparatus that place students physically at the center of a physics concept. In particular: Mark Greenman, Marilyn Hurwitz, Scott Gordon, Barbara Osterfield, Wendell Cerne, Maura Walsh, Lauren Mezzetti, Jill Joyce and Anastasia Mower.
- Everyone else who has shared their insights, stories, and experiences in physics, many of which are reflected in some way in these notes.

I am reminded of Sir Isaac Newton's famous quote, *"If I have seen further it is because I have stood on the shoulders of giants."*

About the Author

Jeff Bigler is a physics teacher at Lynn English High School in Lynn, Massachusetts. He has degrees from MIT in chemical engineering and biology, and is a National Board certified teacher in Science–Adolescence and Young Adulthood. He worked in biotech and IT prior to starting his teaching career in 2003. He has taught both physics and chemistry at all levels from conceptual to AP®.

He is married and has two adult daughters. His hobbies are music and Morris dancing.

Errata

As is the case in just about any large publication, these notes undoubtedly contain errors despite my efforts to find and correct them all.

Known errata for these notes are listed at:
https://www.mrbigler.com/Physics-1/Notes-Physics-1-errata.shtml

Contents

MA Curriculum Frameworks for Physics

Standard	Topics	Chapters
HS-PS1-8	fission, fusion & radioactive decay: α, β & γ; energy released/absorbed	17
HS-PS2-1	Newton's 2nd ($F_{net} = ma$), motion graphs; ramps, friction, normal force, gravity, magnetic force	3, 5, 8, 14
HS-PS2-2	conservation of momentum	10
HS-PS2-3	lab: reduce impulse in a collision	10
HS-PS2-4	gravitation & coulomb's law including relative changes	8, 12
HS-PS2-5	electromagnetism: current produces magnetic field & vice-versa, including examples	14
HS-PS2-9(MA)	Ohm's Law, circuit diagrams, evaluate series & parallel circuits for V, I or R.	13
HS-PS2-10(MA)	free-body diagrams, algebraic expressions & Newton's laws to predict acceleration for 1-D motion, including motion graphs	3, 5
HS-PS3-1	conservation of energy including thermal, kinetic, gravitational, magnetic or electrical including gravitational & electric fields	9, 12, 16
HS-PS3-2	energy can be motion of particles or stored in fields. kinetic $\rightarrow$ thermal, evaporation/condensation, gravitational potential energy, electric fields	5, 12, 16
HS-PS3-3	lab: build a device that converts energy from one form to another.	9
HS-PS3-4a	zero law of thermodynamics (heat flow & thermal equilibrium)	16
HS-PS3-5	behavior of charges or magnets attracting & repelling	12, 14
HS-PS4-1	waves: $v = f\lambda$ & T = 1/f, EM waves traveling through space or a medium vs. mechanical waves in a medium	15
HS-PS4-3	EM radiation is both wave & particle. Qualitative behavior of resonance, interference, diffraction, refraction, photoelectric effect and wave vs. particle model for both	15, 17
HS-PS4-5	Devices use waves and wave interactions with matter, such as solar cells, medical imaging, cell phones, wi-fi	15, 17

MA Science Practices

Practice	Description
SP1	Asking questions.
SP2	Developing & using models.
SP3	Planning & carrying out investigations.
SP4	Analyzing & interpreting data.
SP5	Using mathematics & computational thinking.
SP6	Constructing explanations.
SP7	Engaging in argument from evidence.
SP8	Obtaining, evaluating and communicating information.

AP® Physics 1 Big Ideas & Learning Objectives

These are current as of 2015, except that topics that were removed in 2021–22 have been omitted.

KINEMATICS

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.A.1.1: The student is able to express the motion of an object using narrative, mathematical, and graphical representations. [**SP 1.5**, **2.1**, **2.2**]
3.A.1.2: The student is able to design an experimental investigation of the motion of an object. [**SP 4.2**]
3.A.1.3: The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. [**SP 5.1**]

FORCES

BIG IDEA 1: Objects and systems have properties such as mass and charge. Systems may have internal structure.
1.C.1.1: The student is able to design an experiment for collecting data to determine the relationship between the net force exerted on an object its inertial mass and its acceleration. [**SP 4.2**]
1.C.3.1: The student is able to design a plan for collecting data to measure gravitational mass and to measure inertial mass and to distinguish between the two experiments. [S**P 4.2**]

BIG IDEA 2: Fields existing in space can be used to explain interactions.
2.B.1.1: The student is able to apply $\vec{F}_g = m\vec{g}$ to calculate the gravitational force on an object with mass *m* in a gravitational field of strength *g* in the context of the effects of a net force on objects and systems. [**SP 2.2**, **7.2**]

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.A.2.1: The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. [**SP 1.1**]
3.A.3.1: The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. [**SP 6.4**, **7.2**]
3.A.3.2: The student is able to challenge a claim that an object can exert a force on itself. [**SP 6.1**]
3.A.3.3: The student is able to describe a force as an interaction between two objects and identify both objects for any force. [**SP 1.4**]
3.A.4.1: The student is able to construct explanations of physical situations involving the interaction of bodies using Newton's third law and the representation of action-reaction pairs of forces. [**SP 1.4**, **6.2**]
3.A.4.2: The student is able to use Newton's third law to make claims and predictions about the action-reaction pairs of forces when two objects interact. [**SP 6.4**, **7.2**]

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.A.4.3: The student is able to analyze situations involving interactions among several objects by using free-body diagrams that include the application of Newton's third law to identify forces. [**SP 1.4**]
3.B.1.1: The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations with acceleration in one dimension. [**SP 6.4**, **7.2**]
3.B.1.2: The student is able to design a plan to collect and analyze data for motion (static, constant, or accelerating) from force measurements and carry out an analysis to determine the relationship between the net force and the vector sum of the individual forces. [**SP 4.2**, **5.1**]
3.B.1.3: The student is able to re-express a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. [**SP 1.5**, **2.2**]
3.B.2.1: The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. [**SP 1.1**, **1.4**, **2.2**]
3.C.4.1: The student is able to make claims about various contact forces between objects based on the microscopic cause of those forces. [**SP 6.1**]
3.C.4.2: The student is able to explain contact forces (tension, friction, normal, buoyant, spring) as arising from interatomic electric forces and that they therefore have certain directions. [**SP 6.2**]

BIG IDEA 4: Interactions between systems can result in changes in those systems.
4.A.1.1 The student is able to use representations of the center of mass of an isolated two-object system to analyze the motion of the system qualitatively and semi-quantitatively. [**SP 1.2**, **1.4**, **2.3**, **6.4**]
4.A.2.1: The student is able to make predictions about the motion of a system based on the fact that acceleration is equal to the change in velocity per unit time, and velocity is equal to the change in position per unit time. [**SP 6.4**]
4.A.2.2: The student is able to evaluate using given data whether all the forces on a system or whether all the parts of a system have been identified. [**SP 5.3**]
4.A.2.3: The student is able to create mathematical models and analyze graphical relationships for acceleration, velocity, and position of the center of mass of a system and use them to calculate properties of the motion of the center of mass of a system. [**SP 1.4**, **2.2**]
4.A.3.1: The student is able to apply Newton's second law to systems to calculate the change in the center-of-mass velocity when an external force is exerted on the system. [**SP 2.2**]
4.A.3.2: The student is able to use visual or mathematical representations of the forces between objects in a system to predict whether or not there will be a change in the center-of-mass velocity of that system. [**SP 1.4**]

CIRCULAR MOTION AND GRAVITATION

BIG IDEA 1: Objects and systems have properties such as mass and charge. Systems may have internal structure.
1.C.3.1: The student is able to design a plan for collecting data to measure gravitational mass and to measure inertial mass and to distinguish between the two experiments. [**SP 4.2**]

BIG IDEA 2: Fields existing in space can be used to explain interactions.
2.B.1.1: The student is able to apply $\vec{F}_g = m\vec{g}$ to calculate the gravitational force on an object with mass *m* in a gravitational field of strength *g* in the context of the effects of a net force on objects and systems. [**SP 2.2**, **7.2**]
2.B.2.1: The student is able to apply $g = \frac{GM}{r^2}$ to calculate the gravitational field due to an object with mass *M*, where the field is a vector directed toward the center of the object of mass *M*. [**SP 2.2**]
2.B.2.2: The student is able to approximate a numerical value of the gravitational field (*g*) near the surface of an object from its radius and mass relative to those of the Earth or other reference objects. [**SP 2.2**]

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.A.2.1: The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. [**SP 1.1**]
3.A.3.1: The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. [**SP 6.4**, **7.2**]
3.A.3.3: The student is able to describe a force as an interaction between two objects and identify both objects for any force. [**SP 1.4**]
3.A.4.1: The student is able to construct explanations of physical situations involving the interaction of bodies using Newton's third law and the representation of action-reaction pairs of forces. [**SP 1.4**, **6.2**]
3.A.4.2: The student is able to use Newton's third law to make claims and predictions about the action-reaction pairs of forces when two objects interact. [**SP 6.4**, **7.2**]
3.A.4.3: The student is able to analyze situations involving interactions among several objects by using free-body diagrams that include the application of Newton's third law to identify forces. [**SP 1.4**]
3.B.1.2: The student is able to design a plan to collect and analyze data for motion (static, constant, or accelerating) from force measurements and carry out an analysis to determine the relationship between the net force and the vector sum of the individual forces. [**SP 4.2**, **5.1**]
3.B.1.3: The student is able to re-express a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. [**SP 1.5**, **2.2**]
3.B.2.1: The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. [**SP 1.1**, **1.4**, **2.2**]
3.C.1.1: The student is able to use Newton's law of gravitation to calculate the gravitational force the two objects exert on each other and use that force in contexts other than orbital motion. [**SP 2.2**]
3.C.1.2: The student is able to use Newton's law of gravitation to calculate the gravitational force between two objects and use that force in contexts involving orbital motion [**SP 2.2**]
3.C.2.2: The student is able to connect the concepts of gravitational force and electric force to compare similarities and differences between the forces. [**SP 7.2**]

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.G.1.1: The student is able to articulate situations when the gravitational force is the dominant force and when the electromagnetic, weak, and strong forces can be ignored. [**SP 7.1**]

BIG IDEA 4: Interactions between systems can result in changes in those systems.
4.A.2.2: The student is able to evaluate using given data whether all the forces on a system or whether all the parts of a system have been identified. [**SP 5.3**]

ENERGY

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.E.1.1: The student is able to make predictions about the changes in kinetic energy of an object based on considerations of the direction of the net force on the object as the object moves. [**SP 6.4**, **7.2**]
3.E.1.2: The student is able to use net force and velocity vectors to determine qualitatively whether kinetic energy of an object would increase, decrease, or remain unchanged. [**SP 1.4**]
3.E.1.3: The student is able to use force and velocity vectors to determine qualitatively or quantitatively the net force exerted on an object and qualitatively whether kinetic energy of that object would increase, decrease, or remain unchanged. [**SP 1.4**, **2.2**]
3.E.1.4: The student is able to apply mathematical routines to determine the change in kinetic energy of an object given the forces on the object and the displacement of the object. [**SP 2.2**]

BIG IDEA 4: Interactions between systems can result in changes in those systems.
4.C.1.1: The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy. [**SP 1.4**, **2.1**, **2.2**]
4.C.1.2: The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system. [**SP 6.4**]
4.C.2.1: The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass. [**SP 6.4**]
4.C.2.2: The student is able to apply the concepts of Conservation of Energy and the Work-Energy theorem to determine qualitatively and/or quantitatively that work done on a two-object system in linear motion will change the kinetic energy of the center of mass of the system, the potential energy of the systems, and/or the internal energy of the system. [**SP 1.4**, **2.2**, **7.2**]

BIG IDEA 5: Changes that occur as a result of interactions are constrained by conservation laws.
5.A.2.1: The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. [**SP 6.4**, **7.2**]
5.B.1.1: The student is able to set up a representation or model showing that a single object can only have kinetic energy and use information about that object to calculate its kinetic energy. [**SP 1.4**, **2.2**]
5.B.1.2: The student is able to translate between a representation of a single object, which can only have kinetic energy, and a system that includes the object, which may have both kinetic and potential energies. [**SP 1.5**]

BIG IDEA 5: Changes that occur as a result of interactions are constrained by conservation laws.
5.B.2.1: The student is able to calculate the expected behavior of a system using the object model (i.e., by ignoring changes in internal structure) to analyze a situation. Then, when the model fails, the student can justify the use of conservation of energy principles to calculate the change in internal energy due to changes in internal structure because the object is actually a system. [**SP 1.4**, **2.1**]
5.B.3.1: The student is able to describe and make qualitative and/or quantitative predictions about everyday examples of systems with internal potential energy. [**SP 2.2**, **6.4**, **7.2**]
5.B.3.2: The student is able to make quantitative calculations of the internal potential energy of a system from a description or diagram of that system. [**SP 1.4**, **2.2**]
5.B.3.3: The student is able to apply mathematical reasoning to create a description of the internal potential energy of a system from a description or diagram of the objects and interactions in that system. [**SP 1.4**, **2.2**]
5.B.4.1: The student is able to describe and make predictions about the internal energy of systems. [**SP 6.4**, **7.2**]
5.B.4.2: The student is able to calculate changes in kinetic energy and potential energy of a system, using information from representations of that system. [**SP 1.4**, **2.1**, **2.2**]
5.B.5.1: The student is able to design an experiment and analyze data to examine how a force exerted on an object or system does work on the object or system as it moves through a distance. [**SP 4.2**, **5.1**]
5.B.5.2: The student is able to design an experiment and analyze graphical data in which interpretations of the area under a force-distance curve are needed to determine the work done on or by the object or system. [**SP 4.2**, **5.1**]
5.B.5.3: The student is able to predict and calculate from graphical data the energy transfer to or work done on an object or system from information about a force exerted on the object or system through a distance. [**SP 1.4**, **2.2**, **6.4**]
5.B.5.4: The student is able to make claims about the interaction between a system and its environment in which the environment exerts a force on the system, thus doing work on the system and changing the energy of the system (kinetic energy plus potential energy). [**SP 6.4**, **7.2**]
5.B.5.5: The student is able to predict and calculate the energy transfer to (i.e., the work done on) an object or system from information about a force exerted on the object or system through a distance. [**SP 2.2**, **6.4**]
5.D.1.1: The student is able to make qualitative predictions about natural phenomena based on conservation of linear momentum and restoration of kinetic energy in elastic collisions. [**SP 6.4**, **7.2**]
5.D.1.2: The student is able to apply the principles of conservation of momentum and restoration of kinetic energy to reconcile a situation that appears to be isolated and elastic, but in which data indicate that linear momentum and kinetic energy are not the same after the interaction, by refining a scientific question to identify interactions that have not been considered. Students will be expected to solve qualitatively and/or quantitatively for one-dimensional situations and only qualitatively in two-dimensional situations. [**SP 2.2**, **3.2**, **5.1**, **5.3**]
5.D.1.3: The student is able to apply mathematical routines appropriately to problems involving elastic collisions in one dimension and justify the selection of those mathematical routines based on conservation of momentum and restoration of kinetic energy. [**SP 2.1**, **2.2**]

BIG IDEA 5: Changes that occur as a result of interactions are constrained by conservation laws.
5.D.1.4: The student is able to design an experimental test of an application of the principle of the conservation of linear momentum, predict an outcome of the experiment using the principle, analyze data generated by that experiment whose uncertainties are expressed numerically, and evaluate the match between the prediction and the outcome. [**SP 4.2**, **5.1**, **5.3**, **6.4**]
5.D.1.5: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values. [**SP 2.1**, **2.2**]
5.D.2.1: The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic. [**SP 6.4**, **7.2**]
5.D.2.3: The student is able to apply the conservation of linear momentum to a closed system of objects involved in an inelastic collision to predict the change in kinetic energy. [**SP 6.4**, **7.2**]

MOMENTUM

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.D.1.1: The student is able to justify the selection of data needed to determine the relationship between the direction of the force acting on an object and the change in momentum caused by that force. [**SP 4.1**]
3.D.2.1: The student is able to justify the selection of routines for the calculation of the relationships between changes in momentum of an object, average force, impulse, and time of interaction. [**SP 2.1**]
3.D.2.2: The student is able to predict the change in momentum of an object from the average force exerted on the object and the interval of time during which the force is exerted. [**SP 6.4**]
3.D.2.3: The student is able to analyze data to characterize the change in momentum of an object from the average force exerted on the object and the interval of time during which the force is exerted. [**SP 5.1**]
3.D.2.4: The student is able to design a plan for collecting data to investigate the relationship between changes in momentum and the average force exerted on an object over time. [**SP 4.2**]

BIG IDEA 4: Interactions between systems can result in changes in those systems.
4.B.1.1: The student is able to calculate the change in linear momentum of a two-object system with constant mass in linear motion from a representation of the system (data, graphs, etc.). [**SP 1.4**, **2.2**]
4.B.1.2: The student is able to analyze data to find the change in linear momentum for a constant-mass system using the product of the mass and the change in velocity of the center of mass. [**SP 5.1**]
4.B.2.1: The student is able to apply mathematical routines to calculate the change in momentum of a system by analyzing the average force exerted over a certain time on the system. [**SP 2.2**]
4.B.2.2: The student is able to perform analysis on data presented as a force-time graph and predict the change in momentum of a system. [**SP 5.1**]

BIG IDEA 5: Changes that occur as a result of interactions are constrained by conservation laws.
5.A.2.1: The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. [**SP 6.4**, **7.2**]
5.D.1.1: The student is able to make qualitative predictions about natural phenomena based on conservation of linear momentum and restoration of kinetic energy in elastic collisions. [**SP 6.4**, **7.2**]
5.D.1.2: The student is able to apply the principles of conservation of momentum and restoration of kinetic energy to reconcile a situation that appears to be isolated and elastic, but in which data indicate that linear momentum and kinetic energy are not the same after the interaction, by refining a scientific question to identify interactions that have not been considered. Students will be expected to solve qualitatively and/or quantitatively for one-dimensional situations and only qualitatively in two-dimensional situations. [**SP 2.2**, **3.2**, **5.1**, **5.3**]
5.D.1.3: The student is able to apply mathematical routines appropriately to problems involving elastic collisions in one dimension and justify the selection of those mathematical routines based on conservation of momentum and restoration of kinetic energy. [**SP 2.1**, **2.2**]
5.D.1.4: The student is able to design an experimental test of an application of the principle of the conservation of linear momentum, predict an outcome of the experiment using the principle, analyze data generated by that experiment whose uncertainties are expressed numerically, and evaluate the match between the prediction and the outcome. [**SP 4.2**, **5.1**, **5.3**, **6.4**]
5.D.1.5: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values. [**SP 2.1**, **2.2**]
5.D.2.1: The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic. [**SP 6.4**, **7.2**]
5.D.2.2: The student is able to plan data collection strategies to test the law of conservation of momentum in a two-object collision that is elastic or inelastic and analyze the resulting data graphically. [**SP 4.1**, **4.2**, **5.1**]
5.D.2.3: The student is able to apply the conservation of linear momentum to a closed system of objects involved in an inelastic collision to predict the change in kinetic energy. [**SP 6.4**, **7.2**]
5.D.2.4: The student is able to analyze data that verify conservation of momentum in collisions with and without an external friction force. [**SP 4.1**, **4.2**, **4.4**, **5.1**, **5.3**]
5.D.2.5: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum as the appropriate solution method for an inelastic collision, recognize that there is a common final velocity for the colliding objects in the totally inelastic case, solve for missing variables, and calculate their values. [**SP 2.1**, **2.2**]
5.D.3.1: The student is able to predict the velocity of the center of mass of a system when there is no interaction outside of the system but there is an interaction within the system (i.e., the student simply recognizes that interactions within a system do not affect the center of mass motion of the system and is able to determine that there is no external force). [**SP 6.4**]

SIMPLE HARMONIC MOTION

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.B.3.1: The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. [**SP 6.4**, **7.2**]
3.B.3.2: The student is able to design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force. [**SP 4.2**]
3.B.3.3: The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown. [**SP 2.2**, **5.1**]
3.B.3.4: The student is able to construct a qualitative and/or a quantitative explanation of oscillatory behavior given evidence of a restoring force. [**SP 2.2**, **6.2**]

BIG IDEA 5: Changes that occur as a result of interactions are constrained by conservation laws.
5.B.2.1: The student is able to calculate the expected behavior of a system using the object model (i.e., by ignoring changes in internal structure) to analyze a situation. Then, when the model fails, the student can justify the use of conservation of energy principles to calculate the change in internal energy due to changes in internal structure because the object is actually a system. [**SP 1.4**, **2.1**]
5.B.3.1: The student is able to describe and make qualitative and/or quantitative predictions about everyday examples of systems with internal potential energy. [**SP 2.2**, **6.4**, **7.2**]
5.B.3.2: The student is able to make quantitative calculations of the internal potential energy of a system from a description or diagram of that system. [**SP 1.4**, **2.2**]
5.B.3.3: The student is able to apply mathematical reasoning to create a description of the internal potential energy of a system from a description or diagram of the objects and interactions in that system. [**SP 1.4**, **2.2**]
5.B.4.1: The student is able to describe and make predictions about the internal energy of systems. [**SP 6.4**, **7.2**]
5.B.4.2: The student is able to calculate changes in kinetic energy and potential energy of a system, using information from representations of that system. [**SP 1.4**, **2.1**, **2.2**]

TORQUE AND ROTATIONAL MOTION

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.F.1.1: The student is able to use representations of the relationship between force and torque. [**SP 1.4**]
3.F.1.2: The student is able to compare the torques on an object caused by various forces. [**SP 1.4**]
3.F.1.3: The student is able to estimate the torque on an object caused by various forces in comparison to other situations. [**SP 2.3**]
3.F.1.4: The student is able to design an experiment and analyze data testing a question about torques in a balanced rigid system. [**SP 4.1**, **4.2**, **5.1**]

BIG IDEA 3: The interactions of an object with other objects can be described by forces.
3.F.1.5: The student is able to calculate torques on a two-dimensional system in static equilibrium, by examining a representation or model (such as a diagram or physical construction). [**SP 1.4**, **2.2**]
3.F.2.1: The student is able to make predictions about the change in the angular velocity about an axis for an object when forces exerted on the object cause a torque about that axis. [**SP 6.4**]:
3.F.2.2: The student is able to plan data collection and analysis strategies designed to test the relationship between a torque exerted on an object and the change in angular velocity of that object about an axis. [**SP 4.1**, **4.2**, **5.1**]
3.F.3.1: The student is able to predict the behavior of rotational collision situations by the same processes that are used to analyze linear collision situations using an analogy between impulse and change of linear momentum and angular impulse and change of angular momentum. [**SP 6.4**, **7.2**]
3.F.3.2: In an unfamiliar context or using representations beyond equations, the student is able to justify the selection of a mathematical routine to solve for the change in angular momentum of an object caused by torques exerted on the object. [**SP 2.1**]
3.F.3.3: The student is able to plan data collection and analysis strategies designed to test the relationship between torques exerted on an object and the change in angular momentum of that object. [**SP 4.1**, **4.2**, **5.1**, **5.3**]

BIG IDEA 4: Interactions between systems can result in changes in those systems.
4.A.1.1 The student is able to use representations of the center of mass of an isolated two-object system to analyze the motion of the system qualitatively and semi-quantitatively. [**SP 1.2**, **1.4**, **2.3**, **6.4**]
4.D.1.1: The student is able to describe a representation and use it to analyze a situation in which several forces exerted on a rotating system of rigidly connected objects change the angular velocity and angular momentum of the system. [**SP 1.2**, **1.4**]
4.D.1.2: The student is able to plan data collection strategies designed to establish that torque, angular velocity, angular acceleration, and angular momentum can be predicted accurately when the variables are treated as being clockwise or counterclockwise with respect to a well-defined axis of rotation, and refine the research question based on the examination of data. [**SP 3.2**, **4.1**, **4.2**, **5.1**, **5.3**]
4.D.2.1: The student is able to describe a model of a rotational system and use that model to analyze a situation in which angular momentum changes due to interaction with other objects or systems. [**SP 1.2**, **1.4**]
4.D.2.2: The student is able to plan a data collection and analysis strategy to determine the change in angular momentum of a system and relate it to interactions with other objects and systems. [**SP 4.2**]
4.D.3.1: The student is able to use appropriate mathematical routines to calculate values for initial or final angular momentum, or change in angular momentum of a system, or average torque or time during which the torque is exerted in analyzing a situation involving torque and angular momentum. [**SP 2.2**]
4.D.3.2: The student is able to plan a data collection strategy designed to test the relationship between the change in angular momentum of a system and the product of the average torque applied to the system and the time interval during which the torque is exerted. [**SP 4.1**, **4.2**]

BIG IDEA 5: Changes that occur as a result of interactions are constrained by conservation laws.
5.E.1.1: The student is able to make qualitative predictions about the angular momentum of a system for a situation in which there is no net external torque. [**SP 6.4**, **7.2**]
5.E.1.2: The student is able to make calculations of quantities related to the angular momentum of a system when the net external torque on the system is zero. [**SP 2.1**, **2.2**]
5.E.2.1: The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses. [**SP 2.2**]

Introduction: Study Skills

Unit: Study Skills

Topics covered in this chapter:

The purpose of this chapter is to help you develop study skills that will help you to be successful, not just in this physics class, but in all of your classes throughout high school and college.

- *Cornell (Two-Column) Notes* describes a method of setting up and using a note-taking page in order to make it easy to find information later.
- *Reading & Taking Notes from a Textbook* discusses a strategy for using note-taking as a way to organize information in your brain and actually learn from it.
- *Taking Notes in Class* discusses strategies for taking effective class notes that build on your textbook notes and help you study for tests and get the most out of what you are learning.
- *Taking Notes on Math Problems* discusses strategies for taking effective notes on *how* to solve a math problem instead of just writing down the solution.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

This chapter does not specifically address any of the Massachusetts curriculum frameworks or science practices.

AP® Physics 1 Learning Objectives & Science Practices:

This chapter does not specifically address any of the AP® Physics 1 learning objectives or science practices.

Use this space for summary and/or additional notes:

Cornell (Two-Column) Notes

Unit: Study Skills

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Use the Cornell note-taking system to take effective notes, or add to existing notes.

Success Criteria:

- Notes are in two columns with appropriate main ideas on the left and details on the right.
- Bottom section includes summary and/or other important points.

Language Objectives:

- Understand and describe how Cornell notes are different from other forms of note-taking.

Tier 2 Vocabulary: N/A

Notes:

The Cornell note-taking system was developed in the 1950s by Walter Pauk, an education professor at Cornell University. Besides being a useful system for note-taking in general, it is an especially useful system for interacting with someone else's notes (such as these) in order to get more out of them.

The main features of Cornell notes are:

1. The main section of the page is for the details of what actually gets covered in class.
2. The left section (Cornell notes call for 2½ inches, though I have shrunk it to 2 inches) is for "big ideas"—the organizational headings that help you organize these notes and find details that you are looking for. These have been left blank for you to add throughout the year, because the process of deciding what is important is a key element of understanding and remembering.
3. Cornell notes call for the bottom section (2 inches) to be used for a 1–2 sentence summary of the page in your own words. This is always a good idea, but you may also choose to use that space for other things you want to remember that aren't in these notes.

Use this space for summary and/or additional notes:

How to Get Nothing Worthwhile Out Of These Notes

If you are using these notes as a combination of your textbook and a set of notes, you may be tempted to sleep through class because "it's all in the notes," and then use these notes look up how to do the homework problems when you get confused. If you do this, you will learn very little physics, and you will find this class to be both frustrating and boring.

How to Get the Most Out Of These Notes

These notes are provided so you can preview topics before you learn about them in class. This way, you can pay attention and participate in class without having to worry about writing everything down. However, because active listening, participation and note-taking improve your ability to understand and remember, it is important that you interact with these notes and the discussion.

The "Big Ideas" column on the left of each page has been deliberately left blank. This is to give you the opportunity to go through your notes and categorize each section according to the big ideas it contains. Doing this throughout the year will help you keep the information organized in your brain—it's a lot easier to remember things when your brain has a place to put them!

If we discuss something in class that you want to remember, *mark or highlight it in the notes*! If we discuss an alternative way to think about something that works well for you, *write it in*! You paid for these notes—don't be afraid to use them!

There is a summary section at the bottom of each page. Utilize it! If you can summarize something, you understand it; if you understand something, it is much easier to remember.

Use this space for summary and/or additional notes:

Reading & Taking Notes from a Textbook

Unit: Study Skills

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Use information from the organization of a textbook to take well-organized notes.

Success Criteria:

- Section headings from text are represented as main ideas.
- All information in section summary is represented in notes.
- Notes include page numbers.

Language Objectives:

- Understand and be able to describe the strategies presented in this section.

Tier 2 Vocabulary: N/A

Notes:

If you read a textbook the way you would read a novel, you probably won't remember much of what you read. Before you can understand anything, your brain needs enough context to know how to file the information. This is what Albert Einstein was talking about when he said, "It is the theory which decides what we are able to observe."

When you read a section of a textbook, you need to create some context in your brain, and then add a few observations to solidify the context before reading in detail.

René Descartes described this process in one (very long) sentence in 1644, in the preface to his *Principles of Philosophy*:

> "I should also have added a word of advice regarding the manner of reading this work, which is, that I should wish the reader at first go over the whole of it, as he would a romance, without greatly straining his attention, or tarrying at the difficulties he may perhaps meet with, and that afterwards, if they seem to him to merit a more careful examination, and he feels a desire to know their causes, he may read it a second time, in order to observe the connection of my reasonings; but that he must not then give it up in despair, although he may not everywhere sufficiently discover the connection of the proof, or understand all the reasonings—it being only necessary to mark with a pen the places where the difficulties occur, and continue reading without interruption to the end; then, if he does not grudge to take up the book a third time, I am confident that he will find in a fresh perusal the solution of most of the difficulties he will have marked before; and that, if any remain, their solution will in the end be found in another reading."

Use this space for summary and/or additional notes:

Descartes is advocating reading the text four times. However, it is not necessary to do a thorough reading each time. It is indeed useful to make four passes over the text, but each one should add a new level of understanding, and three of those four passes are quick and require minimal effort.

The following 4-step system takes approximately the same amount of time that you're probably used to spending on reading and taking notes, but it will likely make a tremendous difference in how much you understand and how much you remember.

1. Make a Cornell notes template. **Copy the title/heading of each section** as a big idea in the left column. (If the author has taken the trouble to organize the textbook, you should take advantage of it!) *Write the page numbers next to the headings so you will know where to go if you need to look up details in the textbook.* For each big idea, only give yourself about ¼ to ½ page of space for the details. (Don't do anything else yet.) This process should take only about 1–2 minutes.

   Assuming you are going to be taking notes from the textbook *before* discussing the same topic in class (which is ideal), *start a new sheet of paper for each section* (which means everything below your ¼ to ½ page of notes will be blank), so you can *use the same paper to add notes from class to your notes from the textbook*.

2. Do not write anything else yet! **Look through the section for pictures, graphs, and tables.** Take a moment to look at each one of these—if the author gave them space in the textbook, they must be important. Also read over (but don't try to answer) the homework questions/problems at the end of the section—these illustrate what the author thinks you should be able to do once you know the content. This process should take about 10–15 minutes.

3. **Actually read the text**, one section at a time. For each section, jot down key terms and sentence fragments that remind you of the key ideas about the text, and the pictures and questions/problems from step 1 above. Remember that you shouldn't write more than the ¼ to ½ page allotted. (You don't need to write out the details—those are in the book, which you already have!) This is the time-consuming step, though it is probably less time-consuming than what you're used to doing.

4. **Read the summary at the end** of the chapter or section—this is what the author thinks you should know now that you've finished the reading. If there's anything you don't recognize, go back, look it up, and add it to your notes. This process should take about 5–10 minutes.

For a high school textbook, you shouldn't need to use more than about one side of a sheet of paper of actual writing* per 5 pages of reading!

* However, you will use more sheets of paper than that because you will use a separate sheet of paper for each topic.

Use this space for summary and/or additional notes:

Helpful Hints

- When you write key terms/vocabulary words in your notes, highlight them and define them in your own words, in a way that makes sense to you. (Formal academic language is only useful when you understand it.)
- When you write equations in your notes, highlight them and/or leave space around them to make them easier to see. (Taking notes in multiple colors or using highlighters is helpful for this.)
- Indicate which concepts, equations or words are related to each other (and how they are related), ideally in a different color from the notes themselves. (If relationships have their own separate color, they are easier to follow.) These relationships are likely to be the most important parts of each concept.

Use this space for summary and/or additional notes:

Taking Notes in Class

Unit: Study Skills

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Take useful notes during a lecture/discussion.

Success Criteria:

- Notes contain key information.
- Notes indicate context/hierarchy.

Language Objectives:

- Highlight any words that are new to you.
- Highlight any words that sometimes have a different meaning from the scientific meaning.

Tier 2 Vocabulary: N/A

Notes:

Taking good notes during a lecture or discussion can be challenging. Unlike a textbook, which you can skim first to get an idea of the content, you can't pre-listen to a live lecture or discussion.

Preview the Content

Whenever possible, take notes from the textbook (as described in the section *Reading & Taking Notes from a Textbook*, starting on page 24) before discussing the same topic in class.*

Combine your Textbook Notes with your Class Notes

During the lecture/discussion, get out the textbook notes you already took. Take your class notes for each topic on the same sheet of paper as your ¼ to ½ page of textbook notes, starting below your horizontal line. This way, your notes will be organized by topic, and your class notes will be correlated with your textbook notes and the corresponding sections of the textbook.

* If your teacher doesn't assign reading before teaching about a topic, ask the teacher at the end of each class, "What will we be learning next time?" This way you can proactively take notes from the textbook in advance, to prepare your brain for the class discussion.

Use this space for summary and/or additional notes:

What to Write Down

You can't write every word the teacher says. And you can't rely on only writing what the teacher writes on the board, because the teacher might say important things without writing them down, and the teacher might use the board to give examples.

As with textbook notes, when a teacher introduces a topic, write down the name of the topic at the beginning, and treat it the same way you would treat a section heading in a textbook.

As with textbook notes, highlight vocabulary words/key terms and equations so you can find them easily.

Focus on relationships. Write arrows connecting things that are related, ideally in a different color from the notes themselves.

If the teacher writes down instructions or a procedure for doing something, that's one of the few times when you really want to write down everything.

If the teacher allows you to take a picture of notes on the board, remember that ***the picture is not a substitute for taking effective notes***! The process of writing things down and organizing them is a large part of what helps you understand and remember them. If you take a picture, it is important that you transcribe the information in the picture into your notes (by hand) as soon afterwards as is practical, before you forget everything.

Review Your Notes at the Beginning of the Next Class

Each topic in class follows from the previous topic. While your classmates are still arriving and the teacher is getting ready to teach, get out your class notes from the previous day and your textbook notes on the new topic. Quickly skim both to refresh your memory. This will help your brain connect the new lecture/discussion to the previous one.

Keep a Binder

A binder can be helpful for keeping your notes organized. If you do this, it's usually easiest to organize everything by topic.

- Try to put everything in the binder immediately. Put assignments right after your notes on the same topic. This is useful when doing the assignments, because your notes will already be with them. It's useful when studying for a test, because the notes show you the information and the assignments show what kinds of questions your teacher asked about them.
- If your teacher hands back quizzes or tests, put those right after the last thing that is covered.
- At the end of each unit, put in a divider so you can find where one unit ends and the next one begins.

Use this space for summary and/or additional notes:

Studying for Tests

When studying for tests:

- Review your notes to make sure you remember everything, focusing on key terms/vocabulary, key equations, concepts and relationships.
- If your teacher didn't give you practice problems, re-do some of the homework problems. *Don't just look at the problems and think, "Yes, I remember doing that."* Instead, cover up your solutions and try to solve the problem without looking at your work or the answer.
- Make a study sheet for the test, even if you're not allowed to use it during the test. The process of organizing everything onto one sheet of paper will help you remember what is important and organize it in your brain.
- If the class has a mid-term and/or final exam, keep your study sheets for each test, and use them to study for the mid-term or final. This will save you a lot of time!
- If your teacher handed back quizzes and tests, keep those to study for the mid-term or final. Anything your teacher asked before is highly likely to show up again!
- Make sure you are familiar with the calculator that you will be using during the test. If you always use the calculator app on your phone, the calculator you use during the test may require you to put in the values and operations in a different order, which may confuse you.

Use this space for summary and/or additional notes:

Taking Notes on Math Problems

Unit: Study Skills

MA Curriculum Frameworks (2016): SP5

AP® Physics 1 Learning Objectives: SP5

Mastery Objective(s): (Students will be able to…)

- Take notes on math problems that both show and explain the steps.

Success Criteria:

- Notes show the order of the steps, from start to finish.
- A reason or explanation is indicated for each step.

Language Objectives:

- Be able to describe and explain the process of taking notes on a math problem.

Tier 2 Vocabulary: N/A

Notes:

If you were to copy down a math problem and look at it a few days or weeks later, chances are you'll recognize the problem, but you won't remember how you solved it.

Solving a math problem is a process. For notes to be useful, ***your notes need to capture the process as it happens, not just the final result***.

If you want to take good notes on how to solve a problem, you need your notes to show what you did at each step

Use this space for summary and/or additional notes:

For example, consider the following physics problem:

> A 25 kg cart is accelerated from rest to a velocity of $3.5\,\frac{\text{m}}{\text{s}}$ over an interval of 1.5 s. Find the net force applied to the cart.

The solved problem looks like this:

> A <u>25 kg</u> (m) cart is accelerated <u>from rest</u> ($v_o = 0$) to a velocity of <u>$3.5\,\frac{\text{m}}{\text{s}}$</u> ($v$) over an interval of <u>1.5 s</u> (t). Find the <u>net force</u> ($\boxed{F_{net}}$) applied to the cart.

$$
\begin{aligned}
F_{net} &= ma & \qquad v - v_o &= at \\
F_{net} &= 25a & 3.5 - 0 &= (a)(1.5) \\
F_{net} &= (25)(5.5) & 3.5 &= 1.5a \\
F_{net} &= 138.\overline{8}\,\text{N} & a &= 5.5\,\tfrac{\text{m}}{\text{s}^2}
\end{aligned}
$$

This looks nice, and it's the right answer. But if you look at it now (or look back at it in a month), you won't know what you did.

The quickest and easiest way to fix this is to number the steps and add a couple of words of description for each step:

1. <u>Label quantities</u> (<u>Given</u> & <u>Unknown</u>)

> A <u>25 kg</u> (m) cart is accelerated <u>from rest</u> ($v_o = 0$) to a velocity of <u>$3.5\,\frac{\text{m}}{\text{s}}$</u> ($v$) over an interval of <u>1.5 s</u> (t). Find the <u>net force</u> ($\boxed{F_{net}}$) applied to the cart.

2. Find <u>Equation</u> that has desired quantity

$$F_{net} = ma$$
$$F_{net} = 25a$$

3. Need another equation to find *a*

$$v - v_o = at$$
$$3.5 - 0 = (a)(1.5)$$

4. <u>Solve</u> for *a*

$$a = 5.5\,\tfrac{\text{m}}{\text{s}^2}$$

5. <u>Substitute</u> *a* into 1st equation

$$F_{net} = (25)(5.5)$$
$$F_{net} = 138.\overline{8}\,\text{N}$$

6. Remember the unit!

The math is exactly the same as above, but notice that the annotated problem includes two features:

- Steps are numbered, so you can see what order the steps were in.
- Each step has a short description so you know exactly what was done and why.

Annotating problems this way allows you to ***study the process***, not just the answer!

Use this space for summary and/or additional notes:

Introduction: Laboratory & Measurement

Unit: Laboratory & Measurement

Topics covered in this chapter:

The purpose of this chapter is to teach skills necessary for designing and carrying out laboratory experiments, recording data, and writing summaries of the experiment in different formats.

- *The Scientific Method* describes scientific thinking and how it applies to physics and to this course.
- *The AP® Physics Science Practices* lists & describes the scientific practices that are required by the College Board for an AP® Physics course.
- *Designing & Performing Experiments* discusses strategies for coming up with your own experiments and carrying them out.
- *Random vs. Systematic Error, Uncertainty & Error Analysis,* and *Significant Figures* discuss techniques for estimating how closely measured data can quantitatively predict an outcome.
- *Graphical Solutions (Linearization)* discusses strategies for turning a relationship into a linear equation and using the slope of a best-fit line to represent the quantity of interest.
- *Keeping a Laboratory Notebook, Internal Laboratory Reports,* and *Formal Laboratory Reports* discuss ways in which you might record and communicate (write up) your laboratory experiments.

Calculating uncertainty (instead of relying on significant figures) is a new and challenging skill that will be used in lab write-ups throughout the year.

Use this space for summary and/or additional notes:

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

This chapter addresses the following MA science and engineering practices:

Practice 1: Asking Questions and Defining Problems

Practice 2: Developing and Using Models

Practice 3: Planning and Carrying Out Investigations

Practice 4: Analyzing and Interpreting Data

Practice 6: Constructing Explanations and Designing Solutions

Practice 7: Engaging in Argument from Evidence

Practice 8: Obtaining, Evaluating, and Communicating Information

AP®

AP® Physics 1 Learning Objectives & Science Practices:

This chapter addresses the following AP Physics 1 science practices:

SP 4.1 The student can justify the selection of the kind of data needed to answer a particular scientific question.

SP 4.2 The student can design a plan for collecting data to answer a particular scientific question.

SP 4.3 The student can collect data to answer a particular scientific question.

SP 4.4 The student can evaluate sources of data to answer a particular scientific question.

SP 5.1 The student can analyze data to identify patterns or relationships.

SP 5.2 The student can refine observations and measurements based on data analysis.

SP 5.3 The student can evaluate the evidence provided by data sets in relation to a particular scientific question.

Skills learned & applied in this chapter:

- Designing laboratory experiments
- Estimating uncertainty in measurements
- Propagating uncertainty through calculations
- Writing up lab experiments

Use this space for summary and/or additional notes:

The Scientific Method

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP1, SP2, SP6, SP7

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain how the scientific method can be applied to a problem or question.

Success Criteria:

- Steps in a specific process are connected in consistent and logical ways.
- Explanation correctly uses appropriate vocabulary.

Language Objectives:

- Understand and correctly use terms relating to the scientific method, such as "peer review".

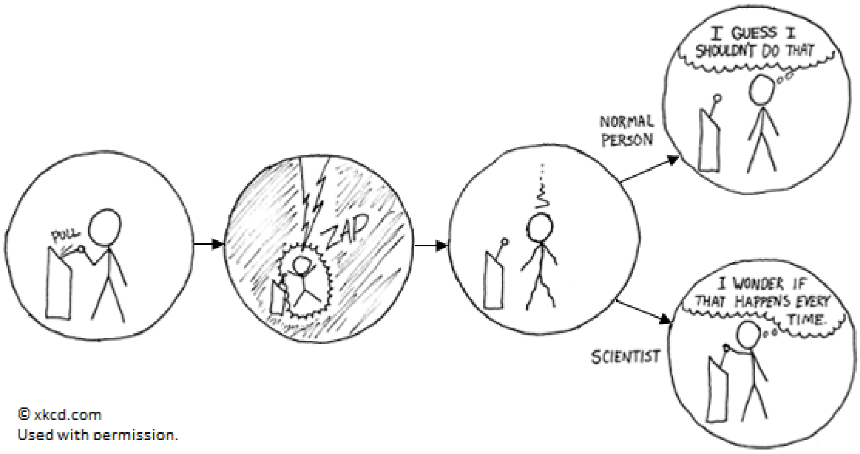
Tier 2 Vocabulary: theory, model, claim, law, peer

Notes:

The scientific method is a fancy name for "figure out what happens by trying it."

In the middle ages, "scientists" were called "philosophers." These were church scholars who decided what was "correct" by a combination of observing the world around them and then arguing and debating with each other about the mechanisms and causes.

During the Renaissance, scientists like Galileo Galilei and Leonardo da Vinci started using experiments instead of argument to decide what really happens in the world.

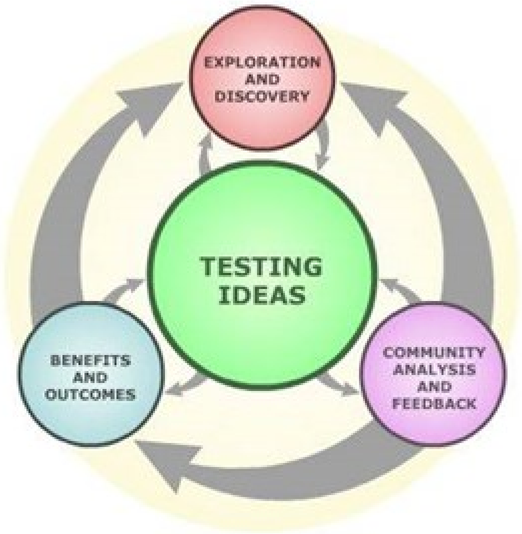


Use this space for summary and/or additional notes:

A Mindset, Not a Recipe

The scientific method is a mindset, which basically amounts to "let nature speak". Despite what you may have been taught elsewhere, the scientific method does not have specific "steps," and does not necessarily require a hypothesis.

The scientific method looks more like a map, with testing ideas (experimentation) at the center:

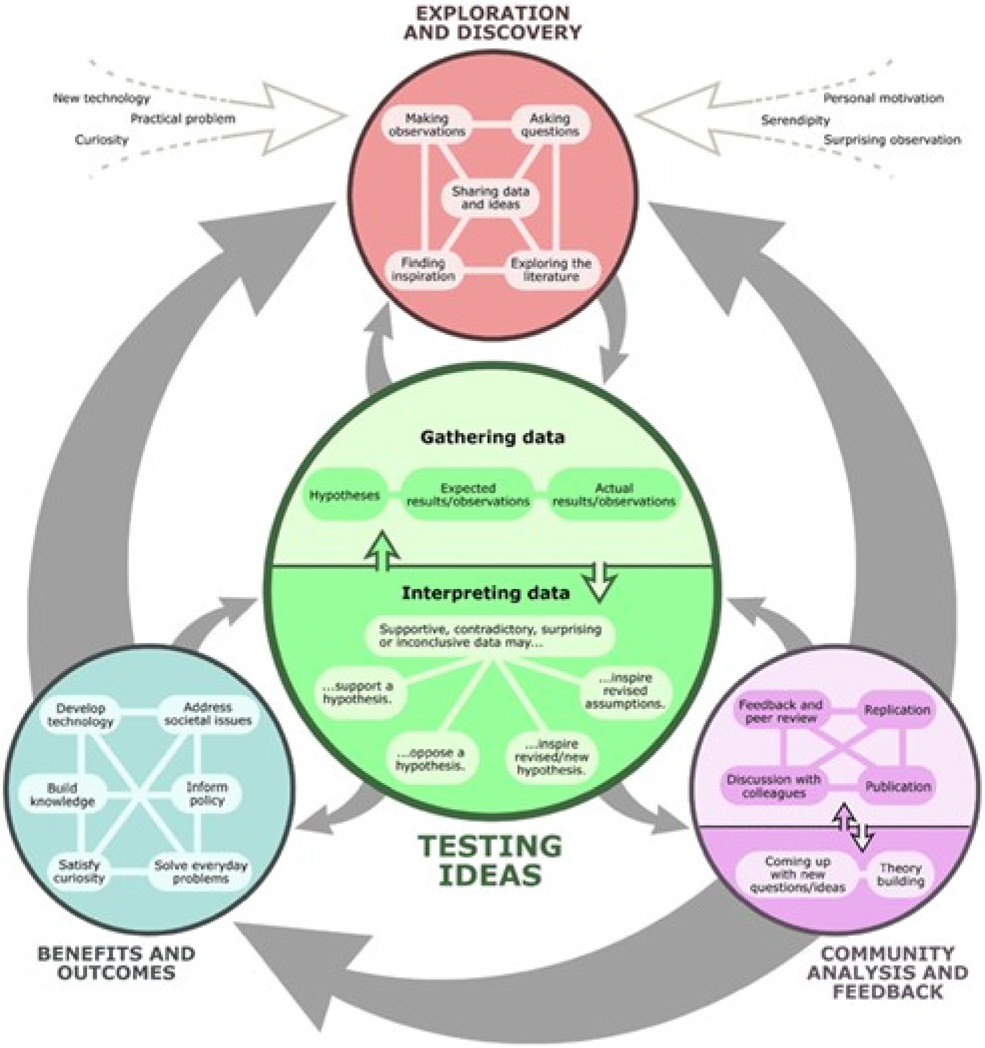


from the *Understanding Science* website*

* Understanding Science. 2018. University of California Museum of Paleontology. 1 July 2018 http://www.understandingscience.org. Used with permission.

Use this space for summary and/or additional notes:

Each of the circles in the above diagram is a broad area that contains many processes:



from the *Understanding Science* website

Use this space for summary and/or additional notes:

When scientists conclude something interesting that they think is important and want to share, they state it in the form of a ***claim***, which states that something happens, under what conditions it happens, and in some cases gives a possible explanation.

Before a claim is taken seriously, the original scientist and any others who are interested try everything they can think of to disprove the claim. If the claim holds up despite many attempts to disprove it, the claim gains support.

peer review: the process by which scientists scrutinize, evaluate and attempt to disprove each other's claims.

If a claim has gained widespread support among the scientific community and can be used to predict the outcomes of experiments (and it has *never* been disproven), it might eventually become a theory or a law.

theory: a claim that has never been disproven, that gives an explanation for a set of observations, and that can be used to predict the outcomes of experiments.

model: a way of viewing a set of concepts and their relationships to one another. A model is one type of theory.

law: a claim that has never been disproven and that can be used to predict the outcomes of experiments, but that does not attempt to model or explain the observations.

Note that the word "theory" in science has a different meaning from the word "theory" in everyday language. In science, a theory is a model that:

- *has never failed* to explain a collection of related observations
- *has never failed* to successfully predict the outcomes of related experiments

For example, the theory of evolution *has never failed* to explain the process of changes in organisms caused by factors that affect the survivability of the species.

If a repeatable experiment contradicts a theory, and the experiment passes the peer review process, the theory is deemed to be wrong. If the theory is wrong, it must either be modified to explain the new results, or discarded completely.

Use this space for summary and/or additional notes:

Theories *vs.* Natural Laws

The terms "theory" and "law" developed organically over many centuries, so any definition of either term must acknowledge that common usage, both within and outside of the scientific community, will not always be consistent with the definitions.

Nevertheless, the following rules of thumb may be useful:

A *theory* is a model that attempts to explain *why* or *how* something happens. A *law* simply describes or quantifies what happens without attempting to provide an explanation. Theories and laws can both be used to predict the outcomes of related experiments.

> For example, the *Law of Gravity* states that objects attract other objects based on their masses and distances from each other. It is a law and not a theory because the Law of Gravity does not explain *why* masses attract each other.
>
> *Atomic Theory* states that matter is made of atoms, and that those atoms are themselves made up of smaller particles. The interactions between these particles are used to explain certain properties of the substances. This is a theory because we cannot see atoms or prove that they exist. However, the model gives an explanation for *why* substances have the properties that they do.

A theory cannot become a law for the same reasons that a definition cannot become a measurement, and a postulate cannot become a theorem.

Use this space for summary and/or additional notes:

The Language of Science

Because science is concerned with defining the limits of what we know and how confident we are that we know it, there are several words that have different meanings in science than they do in the vernacular*.

Term	Science	Vernacular
opinion	Judgments, insights and interpretations that are grounded in expertise and based on evidence.	Subjective preferences, tastes, viewpoints.
skepticism	Judgment of a claim based solely on the strength and quality of the evidence.	Cynicism, negativity, contrarianism, denial.
consensus	Broad agreement based on an extensive body of evidence.	A popular opinion or belief within a group of people.
fact	A claim that has been extensively confirmed and is widely accepted by the scientific community. Acceptance is provisional; new evidence can disprove something previously thought to be fact.	Immutable truth.
law	An observation that something always happens and can be predicted, but does not necessarily offer an explanation.	A requirement that something happens, with the threat of a penalty or punishment if the law is contradicted (broken).
theory	An explanation of a phenomenon that fits all of the evidence that has ever been observed and has high predictive power.	Speculation, hunch, guess.
model	A representation of something that helps envision or understand it.	An exact duplicate of something at a smaller scale.
uncertainty	Measured or calculated range of confidence in findings.	Ignorance.

* Everyday language.

Use this space for summary and/or additional notes:

AP®

The AP® Physics Science Practices

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP1, SP2, SP3, SP4, SP5, SP6, SP7

AP® Physics 1 Learning Objectives: SP1, SP2, SP3, SP4, SP5, SP6, SP7

Mastery Objective(s): (Students will be able to…)

- Describe what the College Board and the State of Massachusetts want you to know about how science is done.

Language Objectives:

- Explain what the student is expected to do for each of the AP® Science Practices.

Tier 2 Vocabulary: data, claim, justify

Notes:

The College Board has described the scientific method in practical terms, dividing them into seven Science Practices that students are expected to learn in AP Physics 1.

Science Practice 1: The student can use representations and models to communicate scientific phenomena and solve scientific problems.

A model is any mental concept that can explain and predict how something looks, works, is organized, or behaves. Atomic theory is an example of a model: matter is made of atoms, which are made of protons, neutrons, and electrons. The number, location, behavior and interactions of these sub-atomic particles explains and predicts how different types of matter behave.

1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.

1.2 The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.

1.3 The student can *refine representations and models* of natural or man-made phenomena and systems in the domain.

1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

1.5 The student can express key elements of natural phenomena across multiple representations in the domain.

Use this space for summary and/or additional notes:

AP®

Science Practice 2: The student can use mathematics appropriately.

Physics is the representation of mathematics in nature. It is impossible to understand physics without a solid understand of mathematics and how it relates to physics. For AP Physics 1, this means having an intuitive feel for how algebra works, and how it can be used to relate quantities or functions to each other. If you are the type of student who solves algebra problems via memorized procedures, you may struggle to develop the kind of mathematical understanding that is necessary in AP Physics 1.

2.1 The student can justify the selection of a mathematical routine to solve problems.

2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

2.3 The student *can estimate numerically quantities* that describe natural phenomena.

Science Practice 3: The student can engage in scientific questioning to extend thinking or to guide investigations within the context of the AP course.

Ultimately, the answer to almost any scientific question is "maybe" or "it depends". Scientists pose questions to understand not just what happens, but the extent to which it happens, the causes, and the limits beyond which outside factors become dominant.

3.1 The student can pose scientific questions.

3.2 The student can refine scientific questions.

3.3 The student can evaluate scientific questions.

Science Practice 4: The student can plan and implement data collection strategies in relation to a particular scientific question.

Scientists do not "prove" things. Mathematicians and lawyers prove that something must be true. Scientists collect data in order to evaluate what happens under specific conditions, in order to determine what is likely true, based on the information available. Data collection is important, because the more and better the data, the more scientists can determine from it.

4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.

4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

4.3 The student can *collect data* to answer a particular scientific question.

4.4 The student can *evaluate sources of data* to answer a particular scientific question.

Use this space for summary and/or additional notes:

AP®

Science Practice 5: The student can perform data analysis and evaluation of evidence.

Just as data collection is important, analyzing data and being able to draw meaningful conclusions is the other crucial step to understanding natural phenomena. Scientists need to be able to recognize patterns that actually exist within the data, and to be free from the bias that comes from expecting a particular result beforehand.

5.1 The student can *analyze data* to identify patterns or relationships.

5.2 The student can *refine observations and measurements* based on data analysis.

5.3 The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.

Science Practice 6: The student can work with scientific explanations and theories.

In science, there are no "correct" answers, only claims and explanations. A scientific claim is any statement that is believed to be true. In order to be accepted, a claim must be verifiable based on evidence, and any claim or explanation must be able to make successful predictions, which are also testable. Science does not prove claims to be universally true or false; science provides supporting evidence. Other scientists will accept or believe a claim provided that there is sufficient evidence to support it, and no evidence that directly contradicts it.

6.1 The student can justify claims with evidence.

6.2 The student can *construct explanations of phenomena based on evidence* produced through scientific practices.

6.3 The student can articulate the reasons that scientific explanations and theories are refined or replaced.

6.4 The student can make *claims and predictions about natural phenomena* based on scientific theories and models.

6.5 The student can evaluate alternative scientific explanations.

Use this space for summary and/or additional notes:

AP®

Science Practice 7: The student is able to connect and relate knowledge across various scales, concepts, and representations in and across domains.

If a scientific principle is true in one domain, scientists must be able to consider that principle in other domains and apply their understanding from the one domain to the other. For example, conservation of momentum is believed by physicists to be universally true on every scale and in every domain, and it has implications in the contexts of laboratory-scale experiments, quantum mechanical behaviors at the atomic and sub-atomic levels, and special relativity.

7.1 The student can *connect phenomena and models* across spatial and temporal scales.

7.2 The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

Use this space for summary and/or additional notes:

Designing & Performing Experiments

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP1, SP3, SP8

AP Physics 1 Learning Objectives: SP 1, SP2, SP3, SP4, SP5, SP6, SP7

MA Curriculum Frameworks (2006): N/A

Mastery Objective(s): (Students will be able to…)

- Create a plan and procedure to answer a question through experimentation.

Success Criteria:

- Experimental Design utilizes backward design.
- Experimental Design uses logical steps to connect the desired answer or quantity to quantities that can be observed or measured.
- Procedure gives enough detail to set up experiment.
- Procedure establishes values of control and manipulated variables.
- Procedure explains how to measure responding variables.

Language Objectives:

- Understand and correctly use the terms "responding variable" and "manipulated variable."
- Understand and be able to describe the strategies presented in this section.

Tier 2 Vocabulary: inquiry, independent, dependent, control

Notes:

If your experience in science classes is like that of most high school students, you have always done "experiments" that were devised, planned down to the finest detail, painstakingly written out, and debugged before you ever saw them. You learned to faithfully follow the directions, and as long as everything that happened matched the instructions, you knew that the "experiment" must have come out right.

If someone asked you immediately after the "experiment" what you just did or what its significance was, you had no answers for them. When it was time to do the analysis, you followed the steps in the handout. When it was time to write the lab report, you had to frantically read and re-read the procedure in the hope of understanding enough of what the "experiment" was about to write something intelligible.

This is not how science is supposed to work.

Use this space for summary and/or additional notes:

In an actual scientific experiment, you would start with an objective, purpose or goal. You would figure out what you needed to know, do, and/or measure in order to achieve that objective. Then you would set up your experiment, observing, doing and measuring the things that you decided upon. Once you had your results, you would figure out what those results told you about what you needed to know. At that point, you would draw some conclusions about how well the experiment worked, and what to do next.

That is precisely how experiments work in this course. You and your lab group will design every experiment that you perform. You will be given an objective or goal and a general idea of how to go about achieving it. You and your lab group (with help) will decide the specifics of what to do, what to measure (and how to measure it), and how to make sure you are getting good results. The education "buzzword" for this is ***inquiry-based experiments***.

Types of Experiments

There are many ways to categorize experiments. For the purpose of this discussion, we will categorize them as either qualitative experiments or quantitative experiments.

Qualitative Experiments

If you are trying to cause something to happen, observe whether or not something happens, or determine the conditions under which something happens, you are performing a qualitative experiment. Your experimental design section needs to address:

- What it is that you are trying to observe or measure.
- If something needs to happen, what you will do to try to make it happen.
- How you will observe it.
- How you will determine whether or not the thing you were looking for actually happened.

Often, determining whether or not the thing happened is the most challenging part. For example, in atomic & particle physics (as was also the case in chemistry), what "happens" involves atoms and sub-atomic particles that are too small to see. For example, you might detect radioactive decay by using a Geiger counter to detect charged particles that are emitted.

Use this space for summary and/or additional notes:

Quantitative Experiments

If you are trying to determine the extent to which something happens, your experiment almost certainly involves measurements and calculations. Your experimental design section needs to address:

- What it is that you are trying to measure.
- If something needs to happen, what you will do to try to make it happen.
- What you can actually measure, and how to connect it to the quantities of interest.
- How to set up your experimental conditions so the quantities that you will measure are within measurable limits.
- How to calculate and interpret the quantities of interest based on your results.

What to Control and What to Measure

In every experiment, there are some quantities that you need to keep constant, some that you need to change, and some that you need to observe. These are called ***control variables***, ***manipulated (independent) variables***, and ***responding (dependent) variables***.

<u>control variables</u>: conditions that are being kept constant. These are usually parameters that could be manipulated variables in a different experiment, but are being kept constant so they do not affect the relationship between the variables that you are testing in this experiment. For example, if you are dropping a ball from different heights to find out how long it takes to hit the ground, you want to make sure the wind is the same speed and direction for each trial, so wind does not affect the outcome of the experiment. This means wind speed and direction are *control* variables.

<u>manipulated variables</u> (also known as independent variables): the conditions you are setting up. These are the parameters that you specify when you set up the experiment. They are called *independent variables* because you are choosing the values for these variables, which means they are *independent* of what happens in the experiment. For example, if you are dropping a ball from different heights to find out how long it takes to hit the ground, you are choosing the heights before the experiment begins, so height is the *manipulated (independent)* variable.

<u>responding variables</u> (also known as dependent variables): the things that happen during the experiment. These are the quantities that you won't know the values for until you measure them. They are called *dependent variables* because they are *dependent* on what happens in the experiment. For example, if you are dropping a ball from different heights to find out how long it takes to hit the ground, the times depend on what happens after you let go of the ball. This means time is the *responding (dependent)* variable.

Use this space for summary and/or additional notes:

If someone asks what your manipulated, dependent and control variables are, the question simply means:

- "What did you vary on purpose (manipulated variables)?"
- "What did you measure (responding variables)?"
- "What did you keep the same for each trial (control variables)?"

Variables in Qualitative Experiments

If the goal of your experiment is to find out ***whether or not*** something happens at all, you need to set up a situation in which the phenomenon you want to observe can either happen or not, and then observe whether or not it does. The only hard part is making sure the conditions of your experiment don't bias whether the phenomenon happens or not.

If you want to find out ***under what conditions*** something happens, what you're really testing is whether or not it happens under different sets of conditions that you can test. In this case, you need to test three situations:

1. A situation in which you are sure the thing will happen, to make sure you can observe it. This is your **positive control**.
2. A situation in which you sure the thing cannot happen, to make sure your experiment can produce a situation in which it doesn't happen and you can observe its absence. This is your **negative control**.
3. A condition or situation that you want to test to see whether or not the thing happens. The condition is your **manipulated variable**, and whether or not the thing happens is your **responding variable**.

Variables in Quantitative Experiments

If the goal of your experiment is to quantify (find a numerical relationship for) the extent to which something happens (the responding variable), you need to figure out a set of conditions that enable you to measure the thing that happens. Once you know that, you need to figure out how much you can change the parameter you want to test (the manipulated variable) and still be able to measure the result. This gives you the highest and lowest values of your manipulated variable. Then perform the experiment using a range of values for the manipulated value that cover the range from the lowest to the highest (or *vice-versa*).

For quantitative experiments, a good rule of thumb is the **8 & 10 rule**: you should have at least 8 data points, and the range from the highest to the lowest values of your manipulated variables should span at least a factor of 10.

Use this space for summary and/or additional notes:

Letting the Equations Design the Experiment

Most high school physics experiments are relatively simple to understand, set up and execute—much more so than in chemistry or biology. This makes physics well-suited for teaching you how to design experiments.

Determining what to measure usually means determining what you need to know and then figuring out how to get there starting from *quantities that you can measure*.

For a quantitative experiment, if you have a mathematical formula that includes the quantity you want to measure, you need to find the values of the other quantities in the equation.

For example, suppose you need to calculate the force of friction that brings a sliding object to a stop. If we design the experiment so that there are no other horizontal forces, friction will be the net force. We can then calculate force from the equation for Newton's Second Law:

$$F_f = F_{net} = \underline{\boldsymbol{m}}a$$

In order to use this equation to calculate force, we need to know:

- **<u>mass</u>**: we can measure this directly, using a balance. (*Note that **<u>m</u>** is underlined because we can measure it directly, which means we don't need to pursue another equation to calculate it.*)
- **<u>acceleration</u>**: we could measure this with an accelerometer, but we do not have one in the lab. This means we will need to find the acceleration some other way.

Because we need to *calculate* acceleration rather than measuring it, that means we need to expand our experiment in order to get the necessary data to do so. Instead of just measuring force and acceleration, we now need to:

1. Measure the mass.
2. *Perform an experiment* in which we apply the force and collect enough information to *determine the acceleration*.
3. Calculate the force on the object, using the mass and the acceleration.

Use this space for summary and/or additional notes:

In order to determine the acceleration, we need another equation. We can use:

$$\underline{\boldsymbol{v}} = v_o + a\underline{\boldsymbol{t}}$$

This means in order to calculate acceleration, we need to know:

- **<u>final velocity</u> (*v*)**: the force is being applied until the object is at rest (stopped), so the final velocity $v = 0$. (*Underlined because we have designed the experiment in a way that we know its value.*)
- **<u>initial velocity</u> (v_o)**: not known; we need to either measure or calculate this.
- **<u>time</u> (*t*)**: we can measure this directly with a stopwatch. (*Underlined because we can measure it directly.*)

Now we need to expand our experiment further, in order to calculate v_o. We can calculate the initial velocity from the equation:

$$\boldsymbol{v}_{ave.} = \frac{\underline{\boldsymbol{d}}}{\underline{\boldsymbol{t}}} = \frac{v_o + \cancelto{0}{\underline{\boldsymbol{v}}}}{2}$$

We have already figured out how to measure $\underline{\boldsymbol{t}}$, and we set up the experiment so that $\underline{\boldsymbol{v}} = 0$ at the end. This means that to calculate v_o, the only quantities we need to measure are:

- **<u>time</u> (*t*)**: as noted above, we can measure this directly with a stopwatch. (*Underlined because we can measure it directly.*)
- **<u>displacement</u> (*d*)**: the change in the object's position. We can measure this with a meter stick or tape measure. (*Underlined because we can measure it.*)

Notice that every quantity is now expressed in terms of quantities that we know or can measure, or quantities we can calculate, so we're all set. We simply need to set up an experiment to measure the underlined quantities.

Use this space for summary and/or additional notes:

To facilitate this approach, it is often helpful to use a table. For the above experiment, such a table might look like the following:

Desired Quantity	Equation	Description/ Explanation	Known Quantities	Measured Quantities	Quantities to be Calculated *(still needed)*
$\vec{F}_f$	$\vec{F}_f = \vec{F}_{net}$	Set up experiment so other forces cancel	—	—	$\vec{F}_{net}$
$\vec{F}_{net}$	$\vec{F}_{net} = m\vec{a}$	Newton's 2nd Law	—	m	$\vec{a}$
$\vec{a}$	$\vec{v} - \vec{v}_o = \vec{a}t$	Kinematic equation #2	$\vec{v} = 0$	t	$\vec{v}_o$
$\vec{v}_o$	$\dfrac{\vec{d}}{t} = \dfrac{\vec{v}_o + \vec{v}}{2}$	Kinematic equation #1	$\vec{v} = 0$	$\vec{d}$, t	—

In this table, we started with the quantity we want to determine ($\vec{F}_f$). We found an equation that contains it ($\vec{F}_f = \vec{F}_{net}$). In that equation, $\vec{F}_{net}$ is neither a fixed control variable nor a constant and we cannot measure it, so it is "still needed" and becomes the start of a new row in the table.

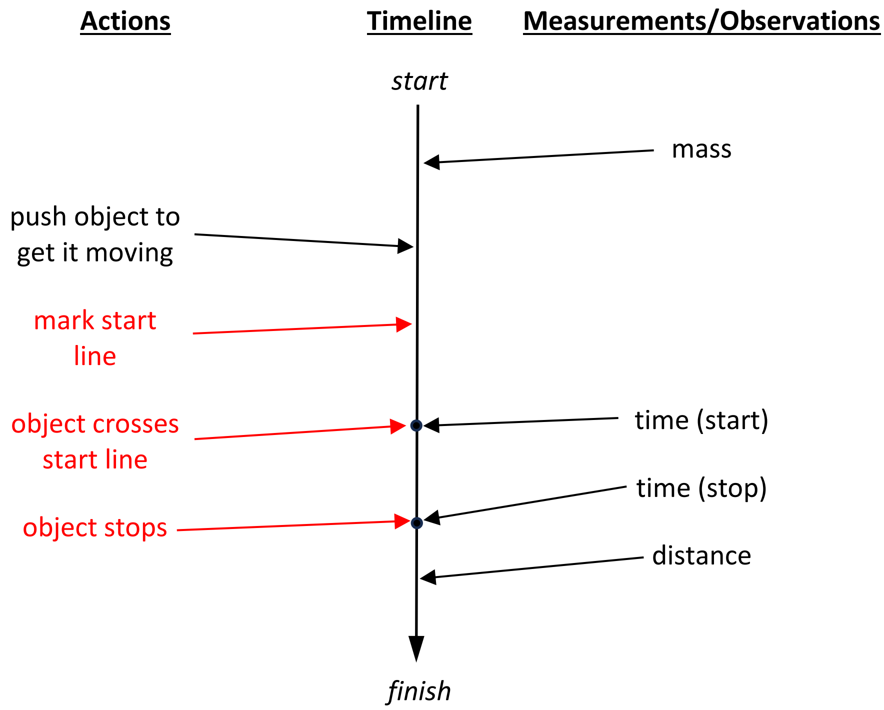
This process continues until every quantity that is needed is either a Fixed quantity (control variable or constant) or can be measured, and there are no quantities that are still needed.

- Notice that every variable in the equation is either the desired variable, or it appears in one of the three columns on the right.
- Notice that when we get to the third row, the equation contains a control variable that is designed into the experiment ($\vec{v} = 0$ because the object stops at the end), a quantity that can be measured (*t*, using a stopwatch), and a quantity that is still needed ($\vec{v}_o$). This quantity is known without having to be measured because of how we set up the experiment.
- Notice that every quantity that you need to measure appears in the "Measured Quantities" column.
- Notice that your experimental conditions need to account for the control variables in the "Known Quantities" column.
- Notice that your calculations are simply the entire "Equation" column, starting at the bottom and working your way back to the top.

Use this space for summary and/or additional notes:

Flow Chart

In the flow chart, note that actions are on one side and measurements (which appear in the "Measured Quantities" column of the table) are on the other. ***Do not include anything else in the flow chart.***



When we realized that measuring time must involve both starting and stopping the stopwatch, we needed to add actions so we can determine when to start and stop the stopwatch.

Note that a dot on the timeline indicates that the action on the left and the measurement on the right need to happen at exactly the same time.

The purpose of this flow chart is to show the procedure in a visual, easy-to-follow manner. The procedure starts at the top ("*start*" on the timeline) and ends at the bottom ("*finish*" on the timeline). As you move down the timeline, perform each action and/or measurement in order from top to bottom.

The flow chart makes it easy to perform the experiment and later on when writing the procedure into a lab report, because it shows everything that is happening in chronological order.

Use this space for summary and/or additional notes:

Procedure

The procedure follows directly from the flow chart. If we start at the top of the timeline ("start") on the flow chart and proceed downward, the first thing we encounter is "mass," on the "Measurements/Observations" side. This means the first thing we need to do is measure the mass.

Next, we encounter "push object to get it moving," on the "actions" side, so that is the second step.

After that, we encounter "object crosses start line" and "time (start)" that must happen at the same time (as indicated by the dot on the timeline arrow). The third step needs to therefore include both.

Continue down the flow chart in the same manner until we reach "finish" at the bottom. The resulting procedure looks like this:

1. Measure the mass of the object.
2. Mark a start line.
3. Get the object moving.
4. Start a stopwatch when the object crosses the start line.
5. Stop the stopwatch when the object stops.
6. Measure the distance the object traveled.
7. Repeat the experiment, using different masses based on the **8 & 10 rule**—take at least **8 data points**, varying the mass over at least a **factor of 10**.

Data

We need to make sure we have recorded the measurements (including uncertainties, which are addressed in the Uncertainty & Error Analysis topic, starting on page 57) of every quantity we need in order to calculate our result. In this experiment, we need measurements for **mass**, **displacement** and **time**.

Use this space for summary and/or additional notes:

Analysis

Most of our analysis is our calculations. Start from the bottom of the experimental design table and work upward.

In this experiment that means start with:

$$\frac{d}{t} = \frac{v_o + \cancel{v}^{\,0}}{2}$$

The reason we needed this equation was to find v_o, so we need to rearrange it to:

$$v_o = \frac{2d}{t}$$

(We are allowed to use *d* and *t* in the equation because we measured them.)

Now we go to the equation above it in our experimental design and substitute our expression for v_o into it:

$$\cancel{v}^{\,0} = v_o + at$$

$$0 = \frac{2d}{t} + at$$

The purpose of this equation was to find acceleration, so we need to rearrange it to:

$$a = \frac{-2d}{t^2}$$

(We can drop the negative sign because we are only interested in the magnitude of the acceleration.)

Our last equation is $F_f = F_{net} = ma$. If we are interested only in finding one value of F_f, we can just substitute and solve:

$$F_{net} = ma = m\left(-\frac{2d}{t^2}\right) = \frac{-2md}{t^2}$$

However, we will get a much better answer if we plot a graph relating each of our values of mass (remember the 8 & 10 rule) to the resulting acceleration and calculate the force using the graph. This process is described in detail in the "Graphical Solutions & Linearization" section, starting on page 83.

Use this space for summary and/or additional notes:

Generalized Approach

The generalized approach to experimental design is therefore:

Experimental Design

1. Find an equation that contains the quantity you want to find.
2. Using a table to organize your information, work your way from that equation through related equations until every quantity in every equation is either something you can calculate or something you can measure.

Procedure

3. Determine the actions and measurements that are needed.
4. Create a flow chart that shows the order of events.
5. Turn the flow chart into a written procedure. (You should take notes on the detailed procedure while performing the experiment and write it out afterwards, because you will almost certainly make decisions while performing the experiment that you need to capture in your procedure.)

Data & Observations

6. Set up your experiment and do a test run. *This means **you need to perform the calculations for your test run before doing the rest of the experiment,*** in case you need to modify your procedure. You will be extremely frustrated if you finish your experiment and go home, only to find out at 2:00 am the night before the write-up is due that it didn't work.
7. Record your measurements and other data.
8. Remember to record the uncertainty for *every* quantity that you measure. (See the "Uncertainty & Error Analysis" section, starting on page 57.)

Analysis

9. Calculate the results. Whenever possible, apply the **8 & 10 rule** and calculate your answer graphically.

AP®

If you are taking one of the AP® Physics exams, you should answer the experimental design question by writing the Experimental Design, Procedure, and Analysis sections above.

Use this space for summary and/or additional notes:

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Random *vs.* Systematic Error

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP3

AP® Physics 1 Learning Objectives: SP4

Mastery Objective(s): (Students will be able to…)

- Correctly use the terms “random error” and “systematic error” in a scientific context.
- Explain the difference between random and systematic errors.

Success Criteria:

- Be able to recognize situations as accurate/inaccurate and/or precise/imprecise.

Language Objectives:

- Be able to describe the difference between random errors and systematic errors.

Tier 2 Vocabulary: random, systematic, accurate, precise

Notes:

Science relies on making and interpreting measurements, and the accuracy and precision of these measurements affect what you can conclude from them.

Random vs. Systematic Errors

<u>random errors</u>: are natural uncertainties in measurements because of the limits of precision of the equipment used. Random errors are assumed to be distributed around the actual value, without bias in either direction.

<u>systematic errors</u>: occur from specific problems in your equipment or your procedure. Systematic errors are often biased in one direction more than another, and can be difficult to identify.

“Accuracy” *vs.* “Precision”

The words “accuracy” and “precision” are not used in science because these words are often used as synonyms in everyday English. However, because some high school science teachers insist on using the terms, their usual meanings are:

<u>accuracy:</u> the amount of systematic error in a measurement. A measurement is said to be accurate if it has low systematic error.

<u>precision</u>: either how finely a measurement was made or the amount of random error in a set of measurements. A single measurement is said to be precise if it was measured within a small fraction of its total value. A group of measurements is said to be precise if the amount of random error is small (the measurements are close to each other).

Use this space for summary and/or additional notes:

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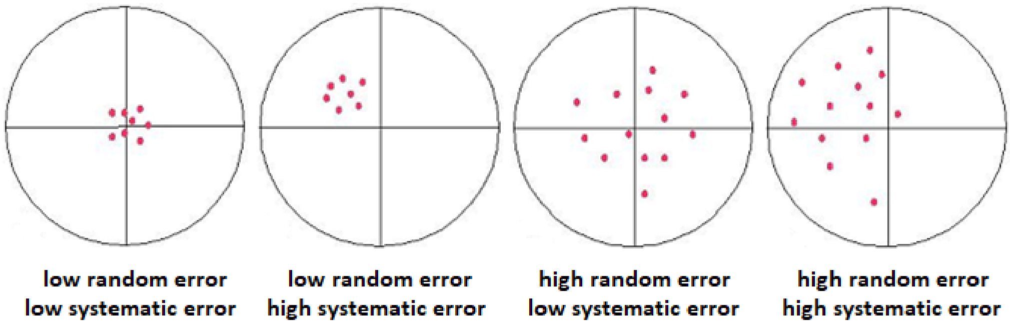
Examples:

Suppose the following drawings represent arrows shot at a target.



The first set has *low random error* because the points are close to each other. It has *low systematic error* because the points are approximately equally distributed about the expected value.

The second set has *low random error* because the points are close to each other. However, it has *high systematic error* because the points are centered on a point that is noticeably far from the expected value.

The third set has *low systematic error* because the points are approximately equally distributed around the expected value. However, it has *high random error* because the points are not close to each other.

The fourth set has *high random error* because the points are not close to each other. It has *high systematic error* because the points are centered on a point that is noticeably far from the expected value.

Use this space for summary and/or additional notes:

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For another example, suppose a teacher is 55 years old, and two of their classes estimate their age.

High Systematic Error

The first class's estimates are 72, 73, 77, and 78 years old. These measurements have low random error because they are close together, but high systematic error (because the average is 75, which is far from the expected value of 55).

When there is a significant amount of systematic error, it often means there is some problem with the way the experiment was set up or performed (or a problem with the equipment) that caused all of the numbers to be off in the same direction.

In this example, the teacher may have gray hair and very wrinkled skin, and may appear much older than they actually are.

High Random Error

The second class's estimates are 10, 31, 77 and 98. This *set* of data has low systematic error (because the average is 54, which is close to the expected value), but high random error because the individual values are not close to each other.

When there is a significant amount of random error, it can also mean a problem with the way the experiment was set up or performed (or a problem with the equipment). However, it can also mean that the experiment is not actually measuring what the scientist thinks it is measuring.

If there is a lot of random error, it can look like there is no relationship between the manipulated variables and the responding variables. If there is no relationship between the manipulated variables and the responding variables, it can look like there is a lot of random error. Scientists must consider both possibilities.

In this example, the class may have not cared about providing valid numbers, or they may not have realized that the numbers they were guessing were supposed to be the age of a person.

Use this space for summary and/or additional notes:

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Uncertainty & Error Analysis

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP4

AP® Physics 1 Learning Objectives: SP5

Mastery Objective(s): (Students will be able to…)

- Determine the uncertainty of a measured or calculated value.

Success Criteria:

- Take analog measurements to one extra digit of precision.
- Correctly estimate measurement uncertainty.
- Correctly read and interpret stated uncertainty values.
- Correctly propagate uncertainty through calculations involving addition/subtraction and multiplication/division.

Language Objectives:

- Understand and correctly use the terms "uncertainty" and "relative error."
- Correctly explain the process of estimating and propagating uncertainty.

Tier 2 Vocabulary: uncertainty, error

Notes:

In science, unlike mathematics, there is no such thing as an exact answer. Ultimately, every quantity is limited by the precision and accuracy of the measurements that it came from. If you can only measure a quantity to within 10 %, that means any calculation that is derived from that measurement can't be any better than ±10 %.

Error analysis is the practice of determining and communicating the causes and extents of uncertainty in your results. Error analysis involves understanding and following the uncertainty in your data, from the initial measurements to the final calculated and reported results.

Note that the word "error" in science has a different meaning from the word "error" in everyday language. In science, **"error" means "uncertainty."** If you report that you drive (2.4 ± 0.1) miles to school every day, you would say that this distance has an error of ±0.1 mile. This does not mean your car's odometer is wrong; it means that the actual distance *could be* 0.1 mile more or 0.1 mile less—*i.e.,* somewhere between 2.3 and 2.5 miles. ***When you are analyzing your results, <u>never</u> use the word "error" to mean mistakes that you might have made!***

Use this space for summary and/or additional notes:

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Uncertainty

The uncertainty or error of a measurement describes how close the actual value is likely to be to the measured value. For example, if a length was measured to be 22.3 cm, and the uncertainty was 0.3 cm (meaning that the measurement is only known to within ± 0.3 cm), we could represent this measurement in either of two ways:

22.3 ± 0.3 cm* 22.3(3) cm

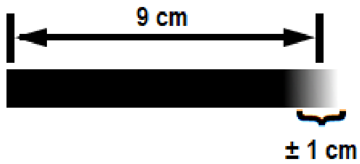
The first of these states the variation (±) explicitly in cm (the actual unit). The second is shows the variation in the last digits shown.

What it means is that the true length is approximately 22.3 cm, and is statistically likely† to be somewhere between 22.0 cm and 22.6 cm.

Absolute Uncertainty (Absolute Error)

Absolute uncertainty (or absolute error) refers to the uncertainty in the actual measurement.

For example, consider the rectangle below (not to scale):



The length of this rectangle is approximately 9 cm, but the exact length is uncertain because we can't determine exactly where the right edge is.

We would express the measurement as 9 ± 1 cm. The ± 1 cm of uncertainty is called the *absolute error*.

Every measurement has a limit to its precision, based on the method used to measure it. This means that ***every measurement has uncertainty***.

* The unit is assumed to apply to both the value and the uncertainty. It would be more pedantically correct to write (9 ± 1) cm, but this is rarely done. The unit for the value and uncertainty should be the same. For example, a value of 10.63 m ± 2 cm should be rewritten as 10.63 ± 0.02 m

† Statistically, the standard uncertainty is one standard deviation, which is discussed on page 61.

Use this space for summary and/or additional notes:

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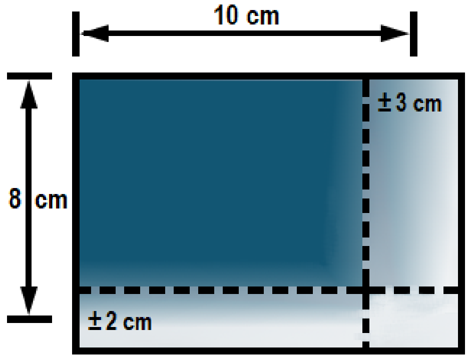
Relative Uncertainty (Relative Error)

Relative uncertainty (usually called relative error) shows the error or uncertainty as a fraction of the measurement.

The formula for relative error is $\text{R.E.} = \dfrac{\text{uncertainty}}{\text{measured value}}$

For example, consider the following rectangle. (Note that the black solid lines are not part of the rectangle. They were added to show the boundaries.) Note that it is deliberately uncertain exactly where the edges of the rectangle are.



The base (length) of the rectangle is 10 ± 3 cm, and the height (width) is 8 ± 2 cm. This means that the area is approximately 80 cm^2. The area of the uncertainty of the base is 2 × 10 = 20 cm^2. The area of the uncertainty of the height is 3 × 8 = 24 cm^2. The total uncertainty is 20 + 24 = 44 cm^2. (In this case we double-count the overlap, because it's uncertain ***both*** because of the uncertainty in the base ***and*** because of the uncertainty of the height.)

The fraction of the length that is uncertain (the relative error of the length) is $\dfrac{3\,\cancel{\text{cm}}}{10\,\cancel{\text{cm}}} = 0.3$. The fraction of the width that is uncertain (the relative error of the width) is $\dfrac{2\,\cancel{\text{cm}}}{8\,\cancel{\text{cm}}} = 0.25$.

Note that relative error is dimensionless (does not have any units), because the numerator and denominator have the same units, which means the units cancel.

If we add these relative errors together, we get 0.3 + 0.25 = 0.55, which is the total relative error.

If we multiply the total relative error by the area of the rectangle, we get the uncertainty for the area: (0.55)(80 cm^2) = ± 44 cm^2.

Percent Error

Percent error is simply the relative error expressed as a percentage. You can turn relative error into percent error by multiplying by 100.

In the example above, the relative error of 0.55 would be 55 % error.

Use this space for summary and/or additional notes:

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Uncertainty of A Single Measurement

If you have the ability to measure a quantity that is not changing (such as the mass or length of an object), you will get the same value every time you measure it. This means you have only one data point.

When you have only one data point, the uncertainty is the limit of how well you can measure it. This will be your best educated guess, based on how closely you think you actually measured the quantity. This means you need to take measurements as carefully and precisely as possible, because *every careless measurement needlessly increases the uncertainty of the result*.

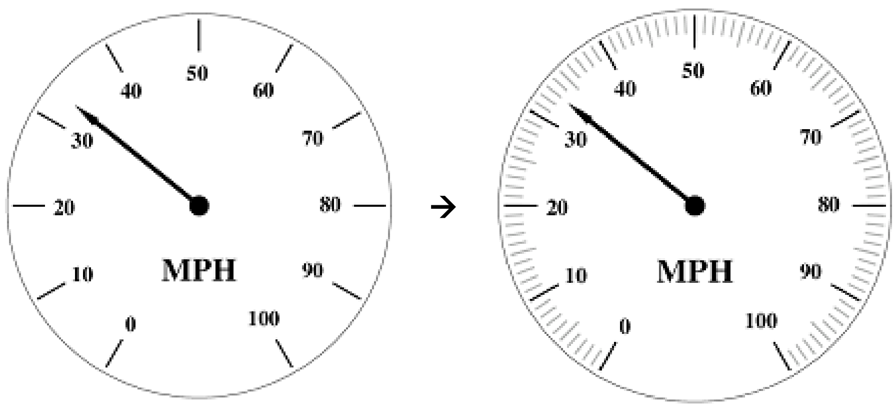
Digital Measurements

For digital equipment, if the reading is *stable* (not changing), look up the published precision of the instrument in its user's manual. (For example, many balances used in high schools have a readability of 0.01 g but are only precise to within ± 0.02 g.) If there is no published value (or the manual is not available), assume the uncertainty is ± 1 in the last digit.

If the reading is *unstable* (changing), state the reading as the average of the highest and lowest values, and the uncertainty as half of the range: (highest – lowest)/2, which is the amount that you would need to add to or subtract from the average to obtain either of the extremes. (However, the uncertainty can never be less than the published uncertainty of the equipment).

Analog Measurements

When making analog measurements, *always* estimate one extra digit beyond the finest markings on the equipment. For example, if you saw the speedometer on the left, you would imagine that each tick mark was divided into ten smaller tick marks like the one on the right.



what you see: between 30 & 40 MPH

what you visualize: 33 ± 1 MPH

Use this space for summary and/or additional notes:

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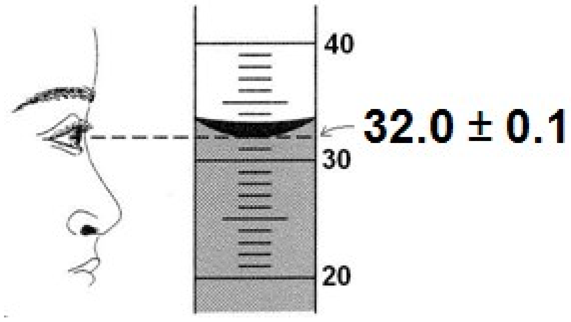
Note that the ***measurement and uncertainty must be expressed to the same decimal place***.

For example, in the diagram below, the graduated cylinder is marked in 1 mL increments. When measuring volume in this graduated cylinder*, you would estimate and write down the volume to the nearest 0.1 mL, as shown:



In the above experiment, you *must* record the volume as:

32.0 ± 0.1 mL ← correct

32 ± 0.1 mL ← wrong

32 ± 1 mL ← inadequate

In other words, the zero at the end of 32.0 mL is *required*. It is necessary to show that *you measured the volume to the nearest tenth, not to the nearest one.*

When estimating, the uncertainty depends on how well you can see the markings, but you can usually assume that the estimated digit has an uncertainty of $\pm\frac{1}{10}$ of the finest markings on the equipment. Here are some examples:

Equipment	Typical Markings	Estimate To	Assumed Uncertainty
ruler	1 mm	0.1 mm	± 0.1 mm
25 mL graduated cylinder	0.2 mL	0.02 mL	± 0.02 mL
thermometer	1 °C	0.1 °C	± 0.1 °C

* Remember that for most liquids, which have a downward meniscus, volume is measured at the *bottom* of the meniscus.

Use this space for summary and/or additional notes:

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Calculating the Uncertainty of a *Set* of Measurements

When you have measurements of multiple separately-generated data points, the uncertainty is calculated using statistics, so that some specific percentage of the measurements will fall within the average, plus or minus the uncertainty.

Note that statistical calculations are beyond the scope of this course. This information is provided for students who have taken (or are taking) a statistics course and are interested in how statistics are applied to uncertainty.

Ten or More Independent Measurements: Standard Deviation

If you have a large enough set of independent measurements (at least 10), then the uncertainty is the standard deviation of the mean. (Independent measurements means you set up the situation that generated the data point on separate occasions. *E.g.,* if you were measuring the length of the dashes that separate lanes on a highway, independent measurements would mean measuring different lines that were likely generated by different line painting apparatus. Measuring the same line ten times would not be considered independent measurements.)

standard deviation (σ): the average of how far each data point is from its expected value.

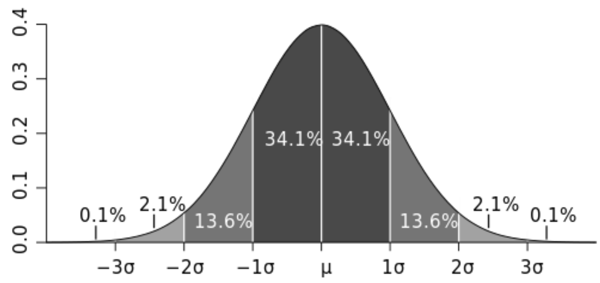
The standard deviation is calculated mathematically as the average difference between each data point and the value predicted by the best-fit line.

A small standard deviation means that most or all of the data points lie close to the best-fit line. A larger standard deviation means that on average, the data points lie farther from the line.

Unless otherwise stated, ***the standard deviation is the uncertainty (the "plus or minus") of a calculated quantity***. *E.g.,* a measurement of 25.0 cm with a standard deviation of 0.5 cm would be expressed as (25.0 ± 0.5) cm.

The expected distribution of values relative to the mean is called the Gaussian distribution (named after the German mathematician Carl Friedrich Gauss.) It looks like a bell, and is often called a "bell curve".

Statistically, approximately two-thirds (actually 68.2 %) of the measurements are expected to fall within one standard deviation of the mean, *i.e.,* within the standard uncertainty.



Use this space for summary and/or additional notes:

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There is an equation for the standard deviation, though most people don't use the equation because they calculate the standard deviation using the statistics functions on a calculator or computer program.

However, note that most calculators and statistics programs calculate the *sample* standard deviation (σ_s), whereas the uncertainty should be the standard deviation *of the mean* (σ_m). This means:

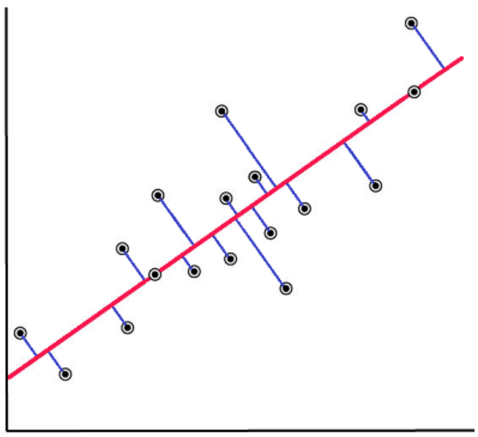
$$u = \sigma_m = \frac{\sigma_s}{\sqrt{n}}$$

and:

$$\text{reported value} = \bar{x} \pm u = \bar{x} \pm \sigma_m = \bar{x} \pm \frac{\sigma_s}{\sqrt{n}}$$

Best-Fit Lines & Standard Deviation

If the quantity of interest is calculated using the slope of a best-fit line (see Graphical Solutions & Linearization on page 83), the standard deviation is the average distance from each data point to the best-fit line.



best-fit line: a line that represents the expected value of your responding variable for values of your manipulated variable. The best-fit line minimizes the total accumulated error (difference between each actual data point and the line).

correlation coëfficient (R or R^2 value): a measure of how linear the data are—how well they approximate a straight line. In general, an R^2 value of less than 0.9 means that the data are not linear, there was a problem with one or more data points, or there was a problem with the entire experiment.

Use this space for summary and/or additional notes:

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Fewer than Ten Independent Measurements

While the standard deviation of the mean is the correct approach when we have a sufficient number of data points, often we have too few data points (small values of *n*), which causes the calculated standard deviation to predict a much larger uncertainty than we probably actually have.

If you have only a few independent measurements (fewer than 10), then you have too few data points to for the standard deviation to represent the uncertainty. In this case, we can estimate the standard uncertainty by finding the range and dividing by two.*

Example:

Suppose you measured a mass on a balance and the reading drifted between 3.46 g and 3.58 g:

$$\bar{x} = \frac{3.46 + 3.58}{2} = 3.52$$
$$range = 3.58 - 3.46 = 0.12$$
$$u \approx \frac{range}{2} \approx \frac{0.12}{2} \approx 0.06$$

You would record the balance reading as 3.52 ± 0.06 g.

* Some texts suggest dividing by $\sqrt{3}$ instead of dividing by 2. For so few data points, the distinction is not important enough to add another source of confusion for students.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Propagating Uncertainty in Calculations

When you perform calculations using numbers that have uncertainty, you need to propagate the uncertainty through the calculation.

Crank Three Times

The simplest way to calculate uncertainty is the "crank three times" method. The "crank three times" method involves:

1. Perform the calculation using the actual numbers. This gives the result (the part before the ± symbol).

2. Perform the calculation a second time, using the end of the range for each value that would give the *smallest* result. (Note that with fractions, this means you need to subtract the uncertainty for values in the numerator and add the uncertainty for values in the denominator.) This gives the lower limit of the range.

3. Perform the calculation a third time using the end of the range for each value that would give the *largest* result. This gives the upper limit of the range.

4. Assuming you have fewer than ten data points, use the approximation that the uncertainty = $u \approx \frac{range}{2}$.

The advantage to "crank three times" is that it's easy to understand and you are therefore less likely to make a mistake. The disadvantage is that it can become unwieldy when you have multi-step calculations.

Use this space for summary and/or additional notes:

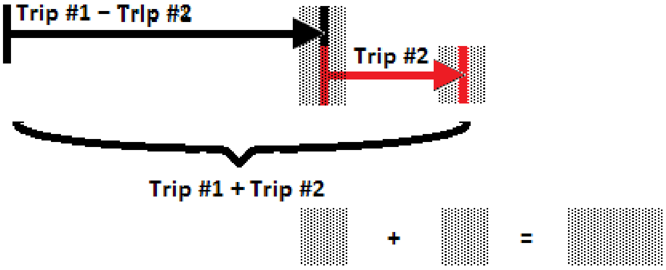
CP1 & honors (not AP®)

Addition & Subtraction: Add the *Absolute* Errors

When quantities with uncertainties are added or subtracted, the uncertainties have the same units, which means we can add the quantities to get the answer, and then just add the uncertainties to get the total uncertainty.

If the calculation involves ***addition*** or ***subtraction***, add the *absolute* errors.

Imagine you walked for a distance and measured it. That measurement has some uncertainty. Then imagine that you started from where you stopped and walked a second distance and measured it. The second measurement also has uncertainty. The total distance is the distance for Trip #1 + Trip #2.



Because there is uncertainty in the distance of Trip #1 and *also* uncertainty in the distance of Trip #2, it is easy to see that the total uncertainty when the two trips are added together is the sum of the two uncertainties.

Use this space for summary and/or additional notes:

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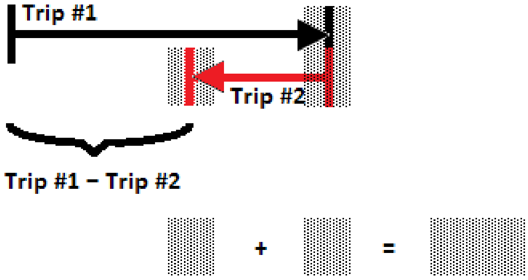
Now imagine that you walked for a distance and measured it, but then you turned around and walked back toward your starting point for a second distance and measured that. Again, both measurements have uncertainty.



Notice that, even though the distances are subtracted to get the answer, the *uncertainties still accumulate*. As before, the uncertainty in where Trip #1 ended becomes the uncertainty in where Trip #2 started. There is also uncertainty in where Trip #2 ended, so again, the total uncertainty is the *sum* of the two uncertainties.

For a numeric example, consider the problem:

$$(8.45 \pm 0.15\text{ cm}) - (5.43 \pm 0.12\text{ cm})$$

Rewriting in column format:

$$\begin{array}{rl} & 8.45 \pm 0.15\text{ cm} \\ - & \underline{5.43 \pm 0.12\text{ cm}} \\ & \boxed{3.02 \pm 0.27\text{ cm}} \end{array}$$

Notice that even though we had to subtract to find the answer, we had to *add the uncertainties*.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Multiplication & Division: Add the *Relative* Errors

If the calculation involves ***multiplication or division***, we can't just add the uncertainties (absolute errors), because the units do not match. Therefore we need to add the *relative* errors to get the total relative error, and then convert the relative error back to absolute error afterwards.

Note: ***Most of the calculations that you will perform in physics involve multiplication and/or division, which means almost all of your uncertainty calculations throughout the course will use relative error.***

For example, if we have the problem (2.50 ± 0.15 kg) × $(0.30 \pm 0.06\ \frac{\text{m}}{\text{s}^2})$, we would do the following:

1. **Calculate the result** using the equation.

   $F_{net} = ma$

   $F_{net} = (2.50)(0.30) = 0.75\ \text{N}$ ← Result

2. **Calculate the relative error for each of the measurements**:

   The relative error of (2.50 ± 0.15) kg is $\frac{0.15\,\cancel{\text{kg}}}{2.50\,\cancel{\text{kg}}} = 0.06$

   The relative error of $(0.30 \pm 0.06)\ \frac{\text{m}}{\text{s}^2}$ is $\frac{0.06\,\cancel{\frac{\text{m}}{\text{s}^2}}}{0.30\,\cancel{\frac{\text{m}}{\text{s}^2}}} = 0.20$

   (Notice that the units cancel.)

3. **Add the relative errors** to find the total relative error:

   0.06 + 0.20 = 0.26 ← Total Relative Error

4. **Multiply the total relative error** (step 3) **by the result** (from step 1 above) to convert the uncertainty back to the correct units.

   (0.26)(0.75 N) = 0.195 N

   (Notice that the units come from the result.)

5. **Combine the result with its uncertainty** and round appropriately:

   $F_{net} = 0.75 \pm 0.195\ \text{N}$

   Because the uncertainty is specified, the answer is technically correct without rounding, but it is good form to round uncertainties to the appropriate number of significant figures, and *round the result to the same decimal place*:

   $F_{net} = 0.75 \pm 0.20\ \text{N}$

For the kinds of experiments you will do in physics class, it is usually sufficient to show the uncertainty to one or two significant figures, and then round the answer to the same place value.

Use this space for summary and/or additional notes:

honors (not AP®)

Exponents

Calculations that involve ***exponents*** use the same rule as for multiplication and division. If you think of exponents as multiplying a number by itself the indicated number of times, it means you would need to add the relative error of that number that many times.

In other words, when a value is raised to an exponent, multiply its relative error by the exponent.

Note that this applies even when the exponent is a fraction (meaning roots). For example:

A ball is dropped from a height of 1.8 ± 0.2 m and falls with an acceleration of $9.81 \pm 0.02\,\frac{\text{m}}{\text{s}^2}$. You want to find the time it takes to fall, using the equation $t=\sqrt{\frac{2a}{d}}$. Because $\sqrt{x}$ can be written as $x^{1/2}$, the equation can be rewritten as $t=\frac{\sqrt{2a}}{\sqrt{d}}=\frac{(2a)^{1/2}}{d^{1/2}}$

Using the steps on the previous page:

1. The result is $t=\sqrt{\frac{2a}{d}}=\sqrt{\frac{2(9.81)}{1.8}}=\sqrt{10.9}=3.30\text{ s}$

2. The relative errors are:

   distance: $\frac{0.2\text{ m}}{1.8\text{ m}}=0.111$

   acceleration: $\frac{0.02\,\frac{\text{m}}{\text{s}^2}}{9.81\,\frac{\text{m}}{\text{s}^2}}=0.0020$

3. Because of the square roots in the equation, the total relative error is: $\frac{1}{2}(0.111)+\frac{1}{2}(0.002)=0.057$

4. The absolute uncertainty for the time is therefore (3.30)(0.057) = ± 0.19 s.

5. The answer is therefore 3.30 ± 0.19 s. However, we have only one significant figure of uncertainty for the height, so it would be better to round to 3.3 ± 0.2 s.

Use this space for summary and/or additional notes:

Summary of Uncertainty Calculations

Uncertainty of a Single Quantity

Measured Once

Make your best educated guess of the uncertainty based on how precisely you were able to measure the quantity and the uncertainty of the instrument(s) that you used.

Measured Multiple Times (Independently)

- If you have a lot of data points, the uncertainty is the standard deviation of the mean, which you can get from a calculator that has statistics functions.
- If you have few data points, use the approximation $u \approx \frac{r}{2}$.

Uncertainty of a Calculated Value

Calculations Use Only Addition & Subtraction

The uncertainties all have the same units, so just add the uncertainties of each of the measurements. The total is the uncertainty of the result.

Calculations Use Multiplication & Division (and possibly Exponents)

The uncertainties don't all have the same units, so you need to use relative error.:

1. Perform the desired calculation. (Answer the question without worrying about the uncertainty.)
2. Find the relative error of each measurement. $\text{R.E.} = \frac{\text{uncertainty } (\pm)}{\text{measured value}}$
3. If the equation includes an exponent (including roots, which are fractional exponents), multiply each relative error by its exponent in the equation.
4. Add the relative errors to find the total relative error.
5. Multiply the total relative error from step 4 by the answer from step 1 to get the absolute uncertainty (±) in the correct units.
6. If desired, round the uncertainty to the appropriate number of significant digits and round the answer to the same place value.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Homework Problems

Because the answers are provided, you must show sufficient work in order to receive credit.

1. **(M = Must Do)** In a 4 × 100 m relay race, the four runners' times were: (10.52 ± 0.02) s, (10.61 ± 0.01) s, (10.44 ± 0.03) s, and (10.21 ± 0.02) s. What was the team's (total) time for the event, including the uncertainty?

   Answer: 41.78 ± 0.08 s

2. **(S = Should Do)** After school, you drove a friend home and then went back to your house. According to your car's odometer, you drove 3.4 miles to your friend's house (going past your house on the way). Then you drove 1.2 miles back to your house. If the uncertainty in your car's odometer reading is 0.1 mile, how far is it from school directly to your house (including the uncertainty)?

   Answer: 2.2 ± 0.2 mi.

3. **(M = Must Do)** A baseball pitcher threw a baseball for a distance of (18.44 ± 0.05) m in (0.52 ± 0.02) s.

   a. What was the velocity of the baseball in meters per second? (*Divide the distance in meters by the time in seconds.*)

      Answer: $35.46\,\frac{\text{m}}{\text{s}}$

   b. What are the relative errors of the distance and time? What is the total relative error?

      Answer: distance: 0.0027; time: 0.0385; total R.E.: 0.0412

   c. Calculate the uncertainty of the velocity of the baseball and express your answer as the velocity (from part a above) plus or minus the uncertainty that you just calculated, with correct rounding.

      Answer: $35.46 \pm 1.46\,\frac{\text{m}}{\text{s}}$ which rounds to $35 \pm 1\,\frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

4. **(S)** A rock that has a mass of 8.15 ± 0.25 kg is sitting on the top of a cliff that is 27.3 ± 1.1 m high. What is the gravitational potential energy of the rock (including the uncertainty)? The equation for this problem is $U_g = mgh$. In this equation, *g* is the acceleration due to gravity on Earth, which is equal to $9.81 \pm 0.02\,\frac{\text{m}}{\text{s}^2}$, and the unit for energy is J (joules).

   Answer: 2 183 ± 159 J

5. **(S)** You drive West on the Mass Pike, from Route 128 to the New York state border, a distance of 127 miles. The EZ Pass transponder determines that your car took 1 hour and 54 minutes (1.9 hours) to complete the trip, and you received a ticket in the mail for driving $66.8\,\frac{\text{mi.}}{\text{hr.}}$ in a $65\,\frac{\text{mi.}}{\text{hr.}}$ zone. The uncertainty in the distance is ± 1 mile and the uncertainty in the time is ± 30 seconds (± 0.0083 hours). Can you use this argument to fight the ticket and win? (You can win if you prove that because of the uncertainty, your speed *could* have been less than $65\,\frac{\text{mi.}}{\text{hr.}}$.)

   Answer: No, this argument won't work. Your average speed is $66.8 \pm 0.8\,\frac{\text{mi.}}{\text{hr.}}$. Therefore, the minimum that your speed could have been is $66.8 - 0.8 = 66.0\,\frac{\text{mi.}}{\text{hr.}}$.

Use this space for summary and/or additional notes:

Significant Figures

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: SP5

Mastery Objective(s): (Students will be able to…)

- Identify the significant figures in a number.
- Perform calculations and round the answer to the appropriate number of significant figures

Success Criteria:

- Be able to identify which digits in a number are significant.
- Be able to count the number of significant figures in a number.
- Be able to determine which places values will be significant in the answer when adding or subtracting.
- Be able to determine which digits will be significant in the answer when multiplying or dividing.
- Be able to round a calculated answer to the appropriate number of significant figures.

Language Objectives:

- Explain the concepts of significant figures and rounding.

Tier 2 Vocabulary: significant, round

Notes:

Because it would be tedious to calculate the uncertainty for every calculation in physics, we can use significant figures (or significant digits) as a simple way to estimate and represent the uncertainty.

Significant figures are based on the following approximations:

- All stated values are rounded off so that the uncertainty is only in the last unrounded digit.
- Assume that the uncertainty in the last unrounded digit is ±1.
- The results of calculations are rounded so that the uncertainty of the result is only in the last unrounded digit, and is assumed to be ±1.

While these assumptions are often (though not always) the right order of magnitude, they rarely give a close enough approximation of the uncertainty to be useful. For this reason, ***significant figures are used as a convenience, and are used <u>only</u> when the uncertainty does not actually matter***.

If you need to express the uncertainty of a measured or calculated value, you must express the uncertainty separately from the measurement, as described in the previous section.

Use this space for summary and/or additional notes:

Therefore, when you take measurements and perform calculations in the laboratory, you will specifically state the measurements and their uncertainties. ***Never use significant figures in lab experiments!***

For homework problems and written tests, you will not be graded on your use of significant figures, but you may use them as a simple way to keep track of the approximate effects of uncertainty on your answers, if you wish.

The *only* reasons that significant figures are presented in these notes are:

1. If you are taking the AP® exam, you are expected to round your answers to an appropriate number of significant figures.
2. After a year of surviving the emotional trauma of significant figures in chemistry class, students expect to be required to use significant figures in physics and every science course afterwards. It is kinder to just say "[*sigh*] Yes, please do your best to round to the correct number of significant figures." than it is to say "Nobody actually uses significant figures. All that trauma was for nothing."

Every time you perform a calculation, you need to express your answer to enough digits that you're not introducing additional uncertainty. However, as long as that is true, feel free to round your answer off in order to omit digits that are one or more orders of magnitude smaller than the uncertainty.

In the example on page 69, we rounded the number 1 285.74 off to the tens place, resulting in the value of 1 290, because we couldn't show more precision than we actually had.

In the number 1 290, we would say that the first three digits are "significant", meaning that they are the part of the number that is not rounded off. The zero in the ones place is "insignificant," because the digit that was there was lost when we rounded.

<u>significant figures</u> (significant digits): the digits in a measured value or calculated result that are not rounded off. (Note that the terms "significant figures" and "significant digits" are used interchangeably.)

<u>insignificant figures</u>: the digits in a measured value or calculated result that were "lost" (became zeroes before a decimal point or were cut off after a decimal point) due to rounding.

Use this space for summary and/or additional notes:

Identifying the Significant Digits in a Number

The first significant digit is where the "measured" part of the number begins—the first digit that is not zero.

The last significant digit is the last "measured" digit—the last digit whose true value is known.

- If the number doesn't have a decimal point, the last significant digit will be the last digit that is not zero. (Anything after that has been rounded off.)

  Example: If we round the number 234 567 to the thousands place, we would get 235 000. (Note that because the digit after the "4" in the thousands place was 5 or greater, so we had to "round up".) In the rounded-off number, the first three digits (the 2, 3, and 5) are the significant digits, and the last three digits (the zeroes at the end) are the insignificant digits.

- If the number has a decimal point, the last significant digit will be the last digit shown. (Anything rounded after the decimal point gets chopped off.)

  Example: If we round the number 11.223 344 to the hundredths place, it would become 11.22. When we rounded the number off, we "chopped off" the extra digits.

- If the number is in scientific notation, it has a decimal point. Therefore, the above rules tell us (correctly) that all of the digits before the "times" sign are significant.

In the following numbers, the significant figures have been underlined:

- $\underline{13}\,000$
- $0.0\underline{275}$
- $0.0\underline{150}$
- $\underline{6\,804.305\,00}$
- $\underline{6.0} \times 10^{23}$
- $\underline{3400.}$ (note the decimal point at the end)

Digits that are not underlined are insignificant. Notice that only zeroes can ever be insignificant.

Use this space for summary and/or additional notes:

Mathematical Operations with Significant Figures

Addition & Subtraction

When adding or subtracting, calculate the total normally. Then identify the smallest place value where nothing is rounded. Round your answer to that place.

For example, consider the following problem.

problem:		“sig figs” equivalent:
123 000	± 1000	123 ???.????
0.0075	± 0.0001	0.0075
+ 1 650	± 10	+ 1 65?.????
124 650.0075	± 1010.0001	124 ???.????
		(Check this digit for rounding)

In the first number (123 000), the hundreds, tens, and ones digit are zeros, presumably because the number was rounded to the nearest 1000. The second number (0.0075) is presumably rounded to the ten-thousandths place, and the number 1650 is presumably rounded to the tens place.

The first number has the largest uncertainty, so we need to round our answer to the thousands place to match, giving 125 000 ± 1 000.

A silly (but pedantically correct) example of addition with significant digits is:

$$100 + 37 = 100$$

Use this space for summary and/or additional notes:

Multiplication and Division

When multiplying or dividing, calculate the result normally. Then count the total *number* of significant digits in the values that you used in the calculation. Round your answer so that it has the same number of significant digits as the value that had the *fewest*.

Consider the problem:

$$34.52 \times 1.4$$

The answer (without taking significant digits into account) is $34.52 \times 1.4 = 48.328$

The number 1.4 has the fewest significant digits (2). Remember that 1.4 really means 1.4 ± 0.1, which means the actual value, if we had more precision, could be anything between 1.3 and 1.5. Using "crank three times," the actual answer could therefore be anything between 34.52 × 1.3 = 44.876 and 34.52 × 1.5 = 51.780.

To get from the answer of 48.328 to the largest and smallest answers we would get from "crank three times," we would have to add or subtract approximately 3.5. (Notice that this agrees with the number we found previously for this same problem by propagating the relative error.) If the uncertainty is in the ones digit (greater than or equal to 1, but less than 10), this means that the ones digit is approximate, and everything beyond it is unknown. Therefore, using the rules of significant figures, we would report the number as 48.

In this problem, notice that the least significant term in the problem (1.4) had 2 significant digits, and the answer (48) also has 2 significant digits. This is where the rule comes from.

A silly (but pedantically correct) example of multiplication with significant digits is:

$$141 \times 1 = 100$$

Use this space for summary and/or additional notes:

Mixed Operations

For mixed operations, keep all of the digits until you're finished (so round-off errors don't accumulate), but keep track of the last significant digit in each step by putting a line over it (even if it's not a zero). Once you have your final answer, round it to the correct number of significant digits. Don't forget to use the correct order of operations (PEMDAS)!

For example:

$$137.4 \times 52 + 120 \times 1.77$$
$$(137.4 \times 52) + (120 \times 1.77)$$
$$7\,\overline{1}44.8 + 2\overline{1}2.4 = 7\,\overline{3}57.2 = 7\,400$$

Note that in the above example, ***we kept all of the digits and didn't round until the end***. This is to avoid introducing small rounding errors at each step, which can add up to enough to change the final answer. Notice how, if we had rounded off the numbers at each step, we would have gotten the wrong answer:

$$137.4 \times 52 + 120 \times 1.77$$
$$(137.4 \times 52) + (120 \times 1.77)$$
$$7\,\overline{1}00 + 2\overline{1}0 = 7\,\overline{3}10 = 7\,300 \longleftarrow \text{☹}$$

However, if we had done actual error propagation (remembering to add absolute errors for addition/subtraction and relative errors for multiplication/division), we would get the following:

$$137.4 \times 52 = 7144.8;\ \text{ R.E. } = \frac{0.1}{137.4} + \frac{1}{52} = 0.01996$$
$$\text{partial answer} = 7144.8 \pm 142.6$$
$$120 \times 1.77 = 212.4;\ \text{ R.E. } = \frac{1}{120} + \frac{0.01}{1.77} = 0.01398$$
$$\text{partial answer} = 212.4 \pm 2.97$$

The total absolute error is therefore 142.6 + 2.97 = 145.6

The best answer is therefore 7357.2 ± 145.6. *I.e.,* the actual value lies between approximately 7200 and 7500.

Use this space for summary and/or additional notes:

What to Do When Rounding Doesn't Give the Correct Number of Significant Figures

If you have a different number of significant digits from what the rounding shows, you can place a line over the last significant digit, or you can place the whole number in scientific notation. Both of the following have four significant digits, and both are equivalent to writing 13 000 ± 10

- $13\,\bar{0}00$
- 1.300×10^4

When Not to Use Significant Figures

Significant figure rules only apply in situations where the numbers you are working with have a limited precision. This is usually the case when the numbers represent measurements. Exact numbers have infinite precision, and therefore have an infinite number of significant figures. Some examples of exact numbers are:

- Pure numbers, such as the ones you encounter in math class.
- Anything you can count. (*E.g.,* there are 24 people in the room. That means exactly 24 people, not 24.0 ± 0.1 people.)
- Whole-number exponents in formulas. (*E.g.,* the area of a circle is πr^2. The exponent "2" is a pure number.)
- Exact values. (*E.g.,* in the International System of Units, the speed of light is defined to be *exactly* $2.997\,924\,58 \times 10^8\ \frac{\text{m}}{\text{s}}$.)

You should also avoid significant figures any time the uncertainty is likely to be substantially different from what would be implied by the rules for significant figures, or any time you need to quantify the uncertainty more exactly.

Summary

Significant figures are a source of ongoing stress among physics students. To make matters simple, realize that few formulas in physics involve addition or subtraction, so you can usually just apply the rules for multiplication and division: look at each of the numbers you were given in the problem. Find the one that has the fewest significant figures, and round your final answer to the same number of significant figures.

If you have absolutely no clue what else to do, ***round to three significant figures and stop worrying***. You would have to measure quite carefully to have more than three significant figures in your original data, and three is usually enough significant figures to avoid unintended loss of precision, at least in a high school physics course. ☺

Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** For each of the following, Underline the significant figures in the number and Write the assumed uncertainty as ± the appropriate quantity.

   57 300 ± 100 ← Sample problem with correct answer.

   a. 13 500
   b. 26.0012
   c. 01902
   d. 0.000 000 025
   e. 320.
   f. 6.0×10^{-7}
   g. 150.00
   h. 10
   i. 0.005 310 0

2. **(M)** Round off each of the following numbers as indicated and indicate the last significant digit if necessary.

   a. 13 500 to the nearest 1000
   b. 26.0012 to the nearest 0.1
   c. 1902 to the nearest 10
   d. 0.000 025 to the nearest 0.000 01
   e. 320. to the nearest 10
   f. 6.0×10^{-7} to the nearest 10^{-6}
   g. 150.00 to the nearest 100
   h. 10 to the nearest 100

Use this space for summary and/or additional notes:

3. Solve the following math problems and round your answer to the appropriate number of significant figures.

   a. **(M)** $3\,521 \times 220$

   b. **(S)** $13\,580.160 \div 113$

   c. **(M)** $2.71828 + 22.4 - 8.31 - 62.4$

   d. **(A)** $23.5 + 0.87 \times 6.02 - 105$ (Remember PEMDAS!)

Use this space for summary and/or additional notes:

honors & AP®

Graphical Solutions & Linearization

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP4, SP5

AP® Physics 1 Learning Objectives: SP2, SP5

Mastery Objective(s): (Students will be able to…)

- Use a graph to calculate the relationship between two variables.

Success Criteria:

- Graph has the manipulated variable on the *x*-axis and the responding variable on the *y*-axis.
- Graph includes best-fit line that appears to minimize the total accumulated distance between the points and the line.
- Axes and best-fit line drawn with straightedge.
- Divisions on axes are evenly spaced.
- Slope of line determined correctly (rise/run).
- Slope used correctly in calculation of desired result.

Language Objectives:

- Explain why a best-fit line gives a better answer than calculating an average.
- Explain how the slope of the line relates to the desired quantity.

Tier 2 Vocabulary: plot, axes

Notes:

Most experiments in a high-school physics class involve finding a mathematical relationship between two quantities. While it is possible to simply measure each quantity once and calculate, or to measure several times and calculate the average, an approach that measures the relationship across a range of values will provide a better result.

A common way to achieve this is to manipulate equations and plot data such that the expected result is a straight line, and the experimental quantity is calculated by plotting a best-fit line and determining the slope. This means that you need to plot graphs ***accurately***, either on graph paper or using a computer or calculator. If you use graph paper:

- The data points need to be as close to their actual locations as you are capable of drawing.
- The best-fit line needs to be as close as you can practically get to its mathematically correct location.
- The best-fit line must be drawn with a straightedge.
- The slope needs to be calculated using the actual rise and run of points on the best-fit line.

Use this space for summary and/or additional notes:

honors & AP®

As mentioned in the previous section, a good rule of thumb for quantitative experiments is the **8 & 10 rule**: you should have at least 8 data points, and the range from the highest to the lowest values tested should span at least a factor of 10.

Once you have your data points, arrange the equation into $y = mx + b$ form, such that the slope (or $^1/_{slope}$) is the quantity of interest. Then accurately plot your data and draw a best-fit line. The slope of this line will be the quantity of interest (or its reciprocal).

For example, suppose you wanted to calculate the spring constant of a spring by measuring the displacement caused by an applied force. You are given the following data:

Applied Force (N)	0.0	1.0	2.0	3.0	5.0
Displacement (m)	−0.01	0.05	0.16	0.20	0.34
Uncertainty (m)	± 0.06	± 0.06	± 0.06	± 0.06	± 0.06

The equation is $F_s = kx$, which is already in $y = mx + b$ form. However, we varied the force and measured the displacement, which means force is the manipulated variable (x-axis), and displacement is the responding variable (y-axis). Therefore, we need to rearrange the equation to:

$$\begin{array}{ccccc} y & = & m & x & +b \\ \downarrow & & \downarrow & \downarrow & \downarrow \\ x & = & \left(\frac{1}{k}\right) & F_s & +0 \end{array}$$

This means that if we plot a graph of all of our data points, a graph of F_s *vs.* x will have a slope of $\frac{1}{k}$.

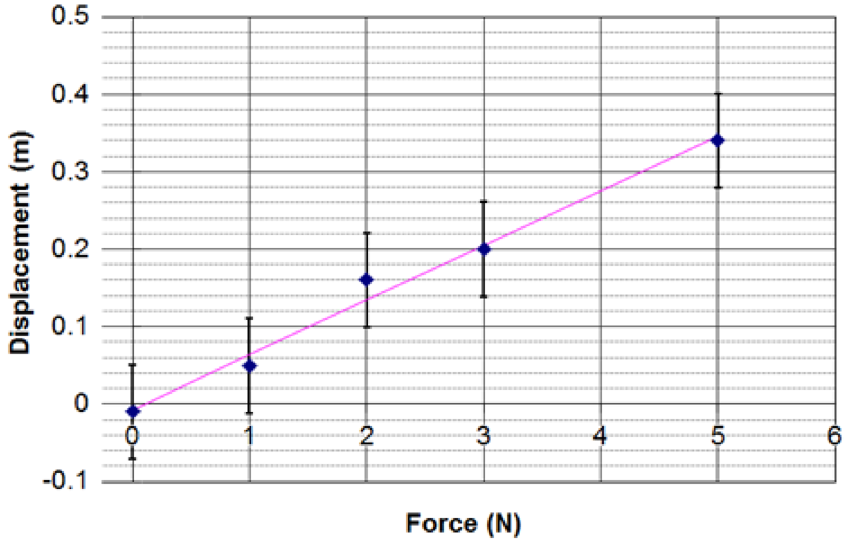
You therefore need to:

1. Plot the data points, expressing the uncertainties as error bars.
2. Draw a best-fit line that passes through each error bar and minimizes the total accumulated distance away from each data point. (You can use linear regression, provided that the regression line actually passes through each error bar. If the line cannot pass through all of the error bars, you need to determine what the problem was with the outlier(s).) You may disregard a data point in your determination of the best-fit line *only* if you know *and can explain* the problem that caused it to be an outlier.

Use this space for summary and/or additional notes:

honors & AP®

The plot looks like the following*:



We calculate the slope using the actual rise (Δy) and run (Δx) from the graph. The best-fit line goes through the points (0, 0) and (3.0, 0.21). From these points, we would calculate the slope as:

$$m = \frac{\Delta y}{\Delta x} = \frac{0.21-0}{3.0-0} = 0.07$$

Because the slope is $\frac{1}{k}$, the spring constant is the reciprocal of the slope of the above graph. $\frac{1}{0.07} = 14\,\frac{\text{N}}{\text{m}}$ (rounded to two significant figures).

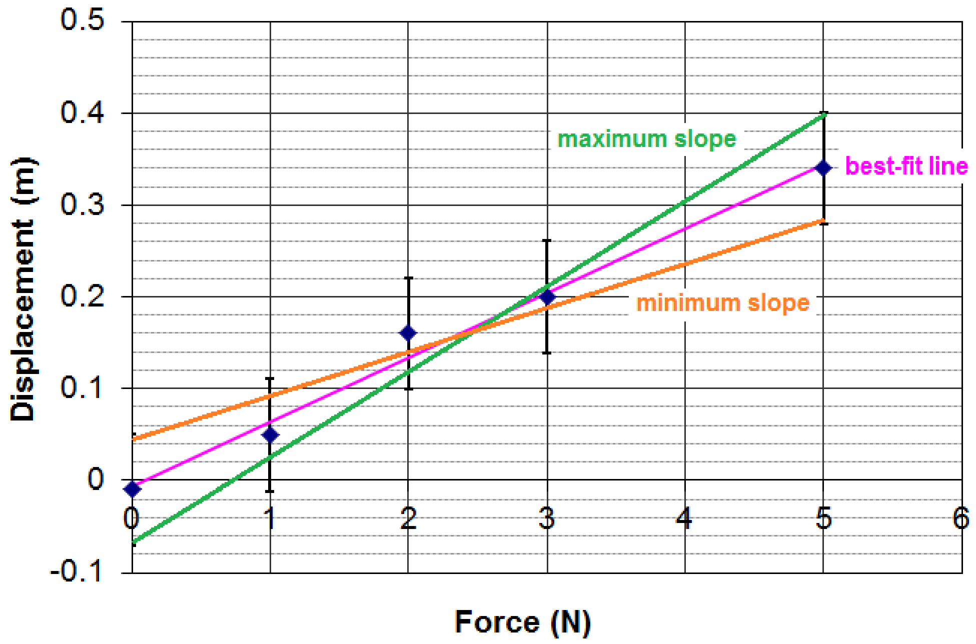
* Note that graphs of Hooke's Law are almost always drawn with displacement on the abscissa (*x*-axis) and force on the ordinate (*y*-axis). The axes were reversed intentionally in this text for three reasons:

1. In most cases it is better to plot the responding variable on the ordinate, to show mathematically that the responding variable is a function of the manipulated variable, $y = f(x)$.
2. Plotting this way creates an example that shows what to do when the slope of the graph is the reciprocal of the quantity of interest.
3. To illustrate the fact that the graph is a valid representation of the data (and may therefore be used for calculations) regardless of which quantity is plotted on which axis.

Use this space for summary and/or additional notes:

Uncertainty and Linearization

If you need to determine the uncertainty for a quantity that was calculated using the slope of a graph, you can draw "lines of worst fit" (which still need to go through the error bars) and then "crank three times" to find the maximum and minimum slope.

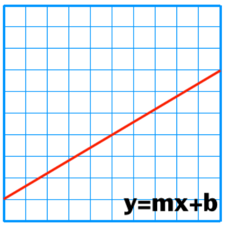
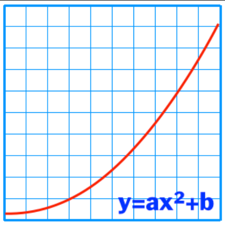
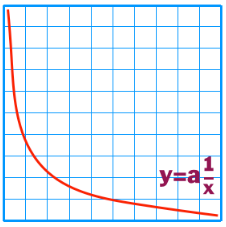
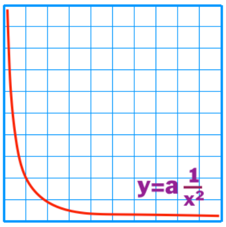
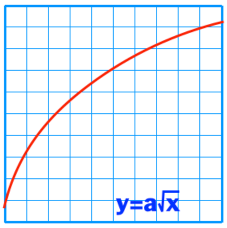


Note that calculating the uncertainty for quantities that are determined graphically is beyond the scope of this course.

Use this space for summary and/or additional notes:

Recognizing Shapes of Graphs

Of course, not all graphs are linear. When you know the equation in advance, it is easy to rearrange the equation in order to linearize it. However, if you do not know the equation before looking at the data, it is useful to memorize the general shapes of these graphs so you can predict the type of relationship and the form of the equation.*

Plot of *y vs. x*	Equation	Linear Plot
	**Linear** $y = mx + b$ b = y-intercept	y vs. x slope = m
	**Power** $y = ax^2$ or $y = ax^2 + b$ b = minimum y-value	y vs. x^2 slope = a
	**Inverse** $y = \frac{a}{x}$ or $y = a \cdot \frac{1}{x}$ undefined (∞) at $x = 0$	y vs. $\frac{1}{x}$ slope = a
	**Inverse Square** $y = \frac{a}{x^2}$ or $y = a \cdot \frac{1}{x^2}$ undefined (∞) at $x = 0$	y vs. $\frac{1}{x^2}$ slope = a
	**Square Root** $y = a\sqrt{x}$	y vs. $\sqrt{x}$ slope = a

*Graphs by Tony Wayne. Used with permission.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Keeping a Laboratory Notebook

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP3, SP8

AP® Physics 1 Learning Objectives: SP4

Mastery Objective(s): (Students will be able to…)

- Determine which information to record in a laboratory notebook.
- Record information in a laboratory notebook according to practices used in industry.

Success Criteria:

- Record data accurately and correctly, with units and including estimated digits.
- Use the correct protocol for correcting mistakes.

Language Objectives:

- Understand and be able to describe the process for recording lab procedures and data.

Tier 2 Vocabulary: N/A

Notes:

A laboratory notebook serves two important purposes:

1. It is a legal record of what you did and when you did it.
2. It is a diary of exactly what you did, so you can look up the details later.

Your Notebook as an Official Record

Laboratory notebooks are kept by scientists in research laboratories and high tech companies. If a company or research institution needs to prove (perhaps in a court case) that you did a particular experiment on a particular date and got a particular set of results, your lab notebook is the primary evidence. While there is no right or wrong way for something to exist as a piece of evidence, the goal is for you to maintain a lab notebook that gives the best chance that it can be used to prove beyond a reasonable doubt exactly what you did, exactly when you did it, and exactly what happened.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

For companies that use laboratory notebooks in this way, there are a set of guidelines that exist to prevent mistakes that could compromise the integrity of the notebook. Details may vary somewhat from one company to another, but are probably similar to these, and the spirit of the rules is the same.

- All entries in a lab notebook must be hand-written in ink. (*This proves that you did not erase information.*)
- Your actual procedure and all data must be recorded directly into the notebook, not recorded elsewhere and copied in. (*This proves that you could not have made copy errors.*)
- All pages must be numbered consecutively, to show that no pages have been removed. If your notebook did not come with pre-numbered pages, you need to write the page number on each page before using it. (*This proves that no pages were removed.*) ***Never remove pages from a lab notebook for any reason.*** If you need to cross out an entire page, you may do so with a single large "X". If you do this, write a brief explanation of why you crossed out the page, and sign and date the cross-out.
- Start each experiment on a new page. (*This way, if you have to submit an experiment as evidence, you don't end up submitting parts of other experiments.*)
- Sign and date the bottom of the each page when you finish recording information on it. Make sure your supervisor witnesses each page within a few days of when you sign it. (*The legal date of an entry is the date it was witnessed. The date is important in patent claims.*)
- When crossing out an incorrect entry in a lab notebook, never obliterate it. Always cross it out with a single line through it, so that it is still possible to read the original mistake. (*This is to prove that it was a mistake, and you didn't change your data or observations. Erased or covered-up data is considered the same as faked or changed data in the scientific community.*) ***Never use "white-out" in a laboratory notebook.*** Any time you cross something out, write your initials and the date next to the change.
- ***Never, ever change data after the experiment is completed.*** Your data, right or wrong, is what you actually observed. Changing your data constitutes fraud, which is a form of cheating that is worse than plagiarism.
- ***Never change <u>anything</u> on a page you have already signed and dated.*** If you realize that an experiment was flawed, leave the bad data where it is and add a note that says "See page ____." with your initials and date next to the addendum. On the new page, refer back to the page number of the bad data and describe briefly what was wrong with it. Then, give the correct information and sign and date it as you would an experiment.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Recording Your Procedure

Recording a procedure in a laboratory notebook is a challenging problem, because on the one hand, you need to have a legal record of what you did that is specific enough to be able to stand as evidence in court. On the other hand, you also need to be able to perform the experiment quickly and efficiently without stopping to write down every detail.

If your experiment is complicated and you need to plan your procedure ahead of time, you can record your intended procedure in your notebook before performing the experiment. Then all you need to do during the experiment is to note any differences between the intended procedure and what you actually did.

If the experiment is quick and simple, or if you suddenly think of something that you want to do immediately, without taking time to plan a procedure beforehand, you can jot down brief notes during the experiment for anything you may not remember, such as instrument settings and other information that is specific to the values of your manipulated variables. Then, as soon as possible after finishing the experiment, write down *all* of the details of the experiment. Include absolutely *everything*, including the make and model number of any major equipment that you used. Don't worry about presentation or whether the procedure is written in a way that would be easy for someone else to duplicate; concentrate on making sure the specifics are accurate and complete. The other niceties matter in reports, but not in a notebook.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Recording Data

Here are some general rules for working with data. (Most of these are courtesy of Dr. John Denker, at http://www.av8n.com/physics/uncertainty.htm):

- Write something about what you did on the same page as the data, even if it is a very rough outline. Your procedure notes should not get in the way of actually performing the experiment, but there should be enough information to corroborate the detailed summary of the procedure that you will write afterwards. (Also, for evidence's sake, the sooner after the experiment that you write the detailed summary, the more weight it will carry in court.)
- Keep *<u>all</u>* of the raw data, whether you will use it or not.
- Don't discard a measurement, even if you think it is wrong. Record it anyway and put a "?" next to it. You can always choose not to use the data point in your calculations (as long as you give an explanation).
- Never erase or delete a measurement. The only time you should ever cross out recorded data is if you accidentally wrote down the wrong number.
- Record all digits. Never round off original data measurements. If the last digit is a zero, you must record it anyway!
- For analog readings (*e.g.,* ruler, graduated cylinder, thermometer), always estimate and record one extra digit.
- Always write down the units with each measurement!
- Record *<u>every</u>* quantity that will be used in a calculation, whether it is changing or not.
- Don't convert in your head before writing down a measurement. Record the original data in the units you actually measured it in, and convert in a separate step.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Calculations

In general, calculations only need to be included in a laboratory notebook when they lead directly to another data point or another experiment. When this is the case, the calculation should be accompanied by a short statement of the conclusion drawn from it and the action taken. Calculations in support of the after-the-fact analysis of an experiment or set of experiments may be recorded in a laboratory notebook if you wish, or they may appear elsewhere.

Regardless of where calculations appear, you must:

- Use enough digits to avoid unintended loss of significance. (Don't introduce round-off errors in the middle of a calculation.) This usually means use at least two more "guard" digits beyond the number of "significant figures" you expect your answer to have.
- You may round for convenience only to the extent that you do not lose significance.
- Always calculate and express uncertainty separately from the measurement. Never rely on "sig figs" to express uncertainty. (In fact, you should never rely on "sig figs" at all!)
- Leave digits in the calculator between steps. (Don't round until the end.)
- When in doubt, keep plenty of "guard digits" (digits after the place where you think you will end up rounding).

Integrity of Data

Your data are your data. In classroom settings, people often get the idea that the goal is to report an uncertainty that reflects the difference between the measured value and the "correct" value. That idea certainly doesn't work in real life—if you knew the "correct" value you wouldn't need to make measurements!

In all cases—in the classroom and in real life—you need to determine the uncertainty of your own measurement by scrutinizing your own measurement procedures and your own analysis. Then you judge how well they agree. For example, we would say that the quantities 10 ± 2 and 11 ± 2 agree reasonably well, because there is considerable overlap between their probability distributions. However, 10 ± 0.2 does not agree with 11 ± 0.2, because there is no overlap.

If you get an impossible result or if your results disagree with well-established results, you should look for and comment on possible problems with your procedure and/or measurements that could have caused the differences you observed. You must *never* fudge your data to improve the agreement.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Your Laboratory Notebook is *Not* a Report

Many high school students are taught that a laboratory notebook should be a journal-style book in which they must write perfect after-the-fact reports, but they are not allowed to change anything if they make a mistake. ***If you have been taught this, you need to unlearn it right now, because it's is false and damaging!***

A laboratory notebook was never meant to communicate your experiments to anyone else. A laboratory notebook is only your official signed and dated record of your procedure (what you did) and your data (what happened) at the exact instant that you took it and wrote it down. If anyone asks to see your laboratory notebook, they should not necessarily expect to understand anything in it without an explanation.

Of course, because it is your journal, your laboratory notebook *may* contain anything that you think is relevant. You may choose to include an explanation of the motivations for one or more experiments, the reasons you chose the procedure that you used, alternative procedures or experiments you may have considered, ideas for future experiments, *etc.* Or you may choose to record these things separately and cross-reference them to specific pages in your lab notebook.

Use this space for summary and/or additional notes:

Internal Laboratory Reports

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP3, SP8

AP® Physics 1 Learning Objectives: SP5

Success Criteria:

- The report has the correct sections in the correct order.
- Each section contains the appropriate information.

Language Objectives:

- Understand and be able to describe the sections of an internal laboratory report, and which information goes in each section.
- Write an internal laboratory report with the correct information in each section.

Tier 2 Vocabulary: N/A

Notes:

An internal laboratory report is written for co-workers, your boss, and other people in the company or research facility that you work for. It is usually a company confidential document that is shared internally, but not shared outside the company or facility.

Every lab you work in, whether in high school, college, research, or industry, will have its own internal report format. It is much more important to understand what *kinds* of information you need to report and what you will use it for than it is to get attached to any one format.

Most of the write-ups you will be required to do this year will be internal write-ups, as described in this section. The format we will use is based on the outline of the actual experiment.

AP®

Although lab reports are not specifically required for AP® Physics, each section of the internal laboratory report format described here is presented in a way that can be used directly in the "design an experiment" question.

Title & Date

Each experiment should have the title and date the experiment was performed written at the top. The title should be a descriptive sentence fragment (usually without a verb) that gives some information about the purpose of the experiment.

Objective

This should be a one or two-sentence description of what you are trying to determine or calculate by performing the experiment.

Use this space for summary and/or additional notes:

Experimental Design

This is the most important section in your report. This section needs to explain:

- What you were trying to observe or measure.
- If something needed to happen, how you made it happen. A flow chart can be useful for this
- Which aspects of the outcome you needed to observe or measure. (Note that you do not need to include the details of how to make the observations or measurements. That information will be included in your procedure.)

Qualitative Experiments

If you are trying to cause something to happen, observe whether or not something happens, or determine the conditions under which something happens, you are performing a qualitative experiment. Your experimental design section needs to explain:

- What you are trying to observe or measure.
- If something needs to happen, what you will do to try make it happen.
- How you will determine whether or not it has happened.
- How you will interpret your results.

Interpreting results is usually the challenging part. For example, in atomic & particle physics (as well as in chemistry), what "happens" involves atoms and electrons that are too small to see. You might detect radioactive decay by using a Geiger counter to detect the charged particles that are emitted.

As you define your experiment, you will need to pay attention to:

- Which conditions you need to keep constant (control variables)
- Which conditions you are changing intentionally (manipulated variables)
- Which outcomes you are observing or measuring (responding variables)

Quantitative Experiments

If you are trying to determine the extent to which something happens, your experiment almost certainly involves measurements and calculations. Your experimental design section needs to explain:

- Your approach to solving the problem and/or gathering the data that you need.
- The specific quantities that you are going to vary (your manipulated variables).
- The specific quantities that you are going to keep constant (your control variables).
- The specific quantities that you are going to measure or observe (your responding variables) .
- How you are going to calculate or interpret your results.

Use this space for summary and/or additional notes:

A good way to record this is to use a table like the following. For example, if you were writing up the experiment described in the section, "Designing & Performing Experiments" (starting on page 44), your experimental design table looked like the following:

Desired Quantity	Equation	Description/ Explanation	Known Quantities	Quantities that Can be Measured	Quantities that Need to be Calculated
$\vec{F}_f$	$\vec{F}_f = \vec{F}_{net}$	Set up experiment so other forces cancel	—	—	$\vec{F}_{net}$
$\vec{F}_{net}$	$\vec{F}_{net} = m\vec{a}$	Newton's 2nd Law	—	m	$\vec{a}$
$\vec{a}$	$\vec{v} - \vec{v}_o = \vec{a}t$	Kinematic equation #2	$\vec{v}$	t	$\vec{v}_o$
$\vec{v}_o$	$\frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$	Kinematic equation #1	$\vec{v}$	$\vec{d}$, t	—

You can include this table directly in the write-up, along with the following information:

Actions (what needs to happen in the experiment):

The object needed to slide from a starting point until it stops on its own due to friction.

Known Quantities:

- constants: none
- control variables that do not need to be measured: final velocity $\vec{v} = 0$

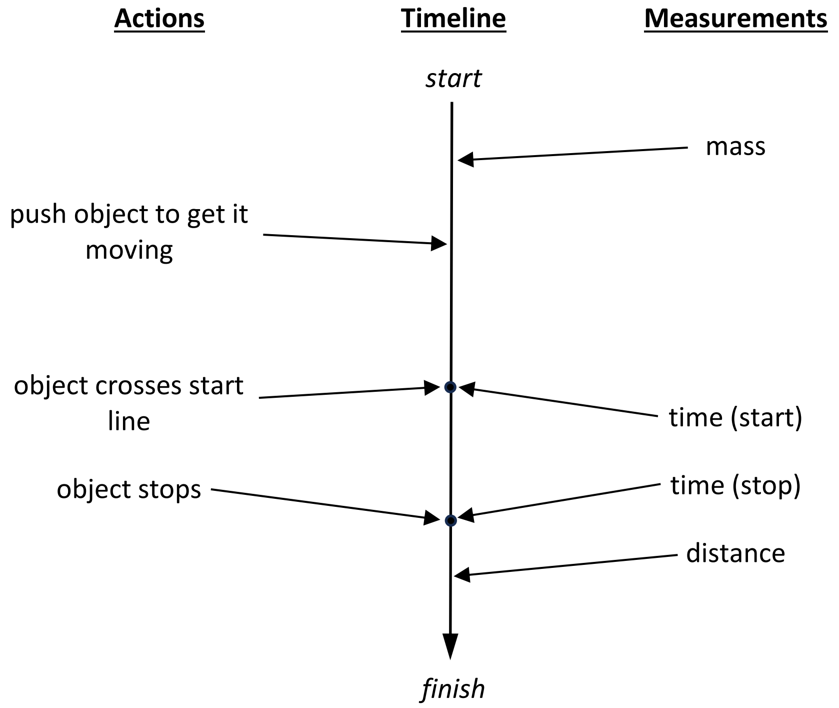
Measured Quantities:

- control variables that need to be measured: mass (*m*) using a balance
- manipulated (independent) variables: none
- responding (dependent) variables: time (*t*) using a stopwatch; distance (*d*) using a meter stick or tape measure

Use this space for summary and/or additional notes:

Flow Chart:

In the flow chart, note that actions are on one side and measurements are on the other. Do not include anything else in the flow chart.



The purpose of this flow chart when you designed the experiment was to show you what you needed to do in a visual, easy-to-follow manner. This also makes it easy to write up the experiment. The procedure starts at the top ("*start*" on the timeline) and ends at the bottom ("*finish*" on the timeline), which means you write the procedure starting at the top and moving down the timeline, describing each action and/or measurement in order from top to bottom.

Use this space for summary and/or additional notes:

Procedure

Your procedure is a detailed description of exactly what you did in order to take your measurements. You already identified what you needed to measure in your experimental design section. Your procedure will therefore be fairly brief and much easier to write. This section is where you give a detailed description of everything you need to do in order to take those data.

You need to include:

- A photograph or sketch of your apparatus, with *each component labeled* (with *both* dimensions *and* specifications), and details about how the components were connected. You need to do this even if the experiment is simple. The picture will serve to answer many questions about how you set up the experiment and most of the key equipment you used.
- A list of any significant equipment that is not labeled in your sketch or photograph. (You do not need to mention generic items like pencils and paper.)
- A narrative description of how you set up the experiment, referring to your sketch or photograph. Generic lab safety procedures and protective equipment may be assumed, but mention any unusual precautions that you needed to take.
- A descriptive list of your *control variables*, including their *values* and how you ensured that they remain constant.
- A descriptive list of your *manipulated variables*, including their *values* and how you set them.
- A descriptive list of your *responding variables* and a step-by-step description of everything you did to measure their values. (Do not include the *values* of the responding variables here—you will present those in your Data & Observations section.)
- Any significant things you did as part of the experiment besides the ones mentioned above.
- Never say "Gather the materials." This is assumed.

Use this space for summary and/or additional notes:

Data & Observations

This is a section in which you present all of your data. Be sure to record every quantity specified in your procedure, including quantities that are not changing (your control variables), quantities that are changing (your manipulated variables), and quantities you measured (your responding variables). ***Remember to include the units!***

For a high school lab write-up, it is usually sufficient to present one or more data tables that include your measurements for each trial and the quantities that you calculated from them. However, if you have other data or observations that you recorded during the lab, they must be listed here.

You must also include estimates of the *uncertainty for every quantity that you measured*. You will also need to state the calculated uncertainty for the final quantity that your experiment is intended to determine.

Although calculated values are actually part of your analysis, it is often more convenient (and easier for the reader) to include them in your data table, even though the calculations will be presented in the next section.

Analysis

The analysis section is where you interpret your data. Your analysis should mirror your Experimental Design section (possibly in the same order, but more likely in reverse), with the goal of guiding the reader from your data to the quantity that you ultimately want to calculate or determine.

Your analysis needs to include:

- A narrative description (one or more paragraphs) of the outcome of the experiment, which guides the reader from your data through your calculations to the quantity you set out to determine.
- One (and only one) sample calculation for each separate equation that you used. For example, if you calculated acceleration for each of five data points, you would write down the formula, and then choose one set of data to plug in and show how you got the answer.
- Any calculated values that did not appear in the data table in your Data & Observations section

Use this space for summary and/or additional notes:

- If you need to do a graphical analysis, include a carefully-plotted graph showing the data points you took for your dependent *vs.* manipulated variables. Often, the quantity you are calculating will be the slope of this graph (or its reciprocal). The graph needs to show the region in which the slope is linear, because this is the range over which your experiment is valid. Note that ***any graphs you include in your write-up must be drawn accurately to scale, using graph paper, and using a ruler/straightedge wherever a straight line is needed***. (When an accurate graph is required, you will lose points if you include a freehand sketch instead.)

  It is acceptable to use a linear regression program to determine the slope. If you do this, you need to say so and give the correlation coëfficient. However, you still need to plot an accurate graph.

- Quantitative error analysis. In general, most quantities in a high school physics class are calculated from equations that use multiplication and division. Therefore, you need to:

  1. Determine the uncertainty of each your measurements.
  2. Calculate the relative error for each measurement.
  3. Combine your relative errors to get the total relative error for your calculated value(s).
  4. Multiply the total relative error by your calculated values to get the absolute uncertainty (±) for each one.

- Sources of uncertainty: this is a list of factors ***inherent in your procedure*** that limit how precise your answer can be. In general, you need a source of uncertainty for each measured quantity.

  Never include mistakes, especially mistakes you aren't sure whether or not you made! A statement like "We might have written down the wrong number." or "We might have done the calculations incorrectly." is really saying, "We might be stupid and you shouldn't believe anything else in this report." (Any "we might be stupid" statements will not count toward your required number of sources of uncertainty.)

  However, if a problem *actually occurred*, and if you *used that data point in your calculations anyway*, you need to explain what happened and why you were unable to fix the problem during the experiment, and you also need to calculate an estimate of the effects on your results.

Use this space for summary and/or additional notes:

Conclusion

Your conclusion should be worded similarly to your objective, but this time including your final calculated result(s) and uncertainty. You do not need to restate sources of uncertainty in your conclusions unless you believe they were significant enough to create some doubt about your results.

Your conclusion should also include 1–2 sentences describing ways the experiment could be improved. These should specifically address the sources of uncertainty that you listed in the analysis section above.

Summary

You can think of the sections of the report in pairs. For each pair, the first part describes the intent of the experiment, and the corresponding second part describes the result.

- **Objective**: describes the purpose of the experiment
    - **Experimental Design**: explains how the details of the experiment were determined
        - **Procedure**: describes in detail how the data were acquired
        - **Data & Observations**: lists the data acquired via the procedure
    - **Analysis**: describes in detail what was learned from the experiment, including calculations and uncertainty.
- **Conclusions**: addresses how well the objective was achieved

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Formal Laboratory Reports

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP3, SP8

AP® Physics 1 Learning Objectives: SP5

Mastery Objective(s): (Students will be able to…)

- Write a formal (journal article-style) laboratory report that appropriately communicates all of the necessary information.

Success Criteria:

- The report has the correct sections in the correct order.
- Each section contains the appropriate information.
- The report contains an abstract that conveys the appropriate amount of information.

Language Objectives:

- Understand and be able to describe the sections of a formal laboratory report, and which information goes in each section.
- Write a formal laboratory report with the correct information in each section.

Tier 2 Vocabulary: abstract

Notes:

A formal laboratory report serves the purpose of communicating the results of your experiment to other scientists outside of your laboratory or institution.

A formal report is a significant undertaking. In a research laboratory, you might submit as many as one or two articles to a scientific journal in a year. Some college professors require students to write their lab reports in journal article format.

The details of what to include are similar to the Internal Report format described in the previous section, except as noted below. The format of a formal journal article-style report is as follows:

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Abstract

This is the most important part of your report. It is a (maximum) 200-word executive summary of *everything* about your experiment—the procedure, results, analysis, and conclusions. In most scientific journals, the abstracts are searchable via the internet, so it needs to contain enough information to enable someone to find your abstract, and after reading it, to know enough about your experiment to determine whether or not to purchase a copy of the full article (which can sometimes cost $100 or more). It also needs to be short enough that the person doing the search won't just think "TL; DR" ("Too Long; Didn't Read") and move on to the next abstract.

Because the abstract is a complete summary, it is always best to wait to write it until you have already written the rest of your report.

Introduction

Your introduction is actually a mini research paper on its own, including citations. (For a high school lab report, it should be 1–3 pages; for scientific journals, 5–10 pages is not uncommon.) Your introduction needs to describe background information that another scientist might not know, plus all of the background information that specifically led to your experiment. Assume that your reader has a similar knowledge of physics as you, but does not know anything about this experiment. The introduction is usually the most time-consuming part of the report to write.

Materials and Methods

This section combines both the experimental design and procedure sections of an informal lab write-up. Unlike an informal write-up, the Materials and Methods section of a formal report is written in paragraph form, in the past tense, using the passive voice, and avoiding pronouns. As with the informal write-up, a labeled photograph or drawing of your apparatus is a necessary part of this section, but you need to *also* describe the set-up in the text.

Also unlike the informal write-up, your Materials and Methods section needs to give some *explanation* of your choices of the values used for your control and manipulated variables.

Data and Observations

This section is similar to the same section in the lab notebook write-up, except that:

1. You should present only data you actually recorded/measured in this section. (Calculated values are presented in the Discussion section.)
2. You need to *introduce* the data table. (This means you need to describe the important things that someone should notice in the table first, and then say something like "Data are shown in Table 1.")

Note that all figures and tables in the report need to be numbered separately and consecutively.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Discussion

This section is similar to the Analysis section in the lab notebook write-up, but with some important differences.

As with the rest of the formal report, your discussion must be in paragraph form. Your discussion is essentially a long essay discussing your results and what they mean. You need to introduce and present a table with your calculated values and your uncertainty. After presenting the table, you should discuss the results, uncertainties, and sources of uncertainty in detail. If your results relate to other experiments, you need to discuss the relationship and include citations for those other experiments.

Your discussion needs to include each of the formulas that you used as part of your discussion and give the results of the calculations, but you do not need to show the intermediate step of substituting the numbers into the equation.

Conclusions

Your conclusions are written much like in the internal write-up. You need at least two paragraphs. In the first, restate your findings and summarize the significant sources of uncertainty. In the second paragraph, list and explain improvements and/or follow-up experiments that you suggest.

Works Cited

As with a research paper, you need to include a complete list of bibliography entries for the references you cited in your introduction and/or discussion sections.

Your ELA teachers probably require MLA-style citations; scientific papers typically use APA style. However, in a high school physics class, while it is important that you know which information needs to be cited and *what* information needs to go into each citation, you may use any format you like as long as you use it correctly and consistently.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Typesetting Superscripts and Subscripts

Because formal laboratory reports need to be typed, and because physics uses superscripts and subscripts extensively, it is important to know how to typeset superscripts and subscripts.

You can make use of the following shortcuts:

superscript: text that is raised above the line, such as the exponent "2" in $A=\pi r^2$.

In Google Docs, select the text, then hold down "Ctrl" and press the "." (period) key.

In Microsoft programs (such as Word) running on Windows, select the text, then hold down "Ctrl" and "Shift" and press the "+" key.

On a Macintosh, select the text, then hold down "Command" and "Control" and press the "+" key.

subscript: text that is lowered below the line, such as the "o" in $x=x_o+v_o t$.

In Google Docs, select the text, then hold down "Ctrl" and press the "," (comma) key.

In Microsoft programs (such as Word) running on Windows, select the text, then hold down "Ctrl" and press the "–" key.

On a Macintosh, select the text, then hold down "Command" and "Control" and press the "–" key.

Note that you will lose credit in laboratory reports if you don't use superscripts and subscripts correctly. For example, you will lose credit if you type *d* = *vot* + 1/2*at*^2 instead of $d = v_o t + \frac{1}{2} a t^2$.

Use this space for summary and/or additional notes:

Introduction: Mathematics

Unit: Mathematics

Topics covered in this chapter:

The purpose of this chapter is to familiarize you with mathematical concepts and skills that will be needed in physics.

- *Standard Assumptions in Physics* discusses what you can and cannot assume to be true in order to be able to solve the problems you will encounter in this class.
- *The International System of Units* and *Scientific Notation* briefly review skills that you are expected to remember from your middle school math and science classes.
- *Solving Problems Symbolically* discusses rearranging equations to solve for a particular variable before (or without) substituting values.
- *Solving Word Problems Systematically* discusses how to solve word problems, including determining which quantity and which variable apply to a number given in a problem based on the units, choosing an equation that applies to a problem, and substituting numbers from the problem into the equation.
- *Right-Angle Trigonometry* is a review of sine, cosine and tangent (SOH CAH TOA), and an explanation of how these functions are used in physics.
- *Vectors*, *Vectors* vs. *Scalars in Physics*, and *Vector Multiplication* discuss the use and manipulation of vectors (quantities that have a direction) to represent quantities in physics.

Use this space for summary and/or additional notes:

- *Degrees, Radians & Revolutions* and *Polar, Cylindrical & Spherical Coördinates* explain how to work with angles and coördinate systems that are needed for the rotational problems encountered in AP® Physics.

Depending on your math background, some of the topics, such as trigonometry and vectors, may be unfamiliar. These topics may be taught, reviewed or skipped, depending on the needs of the students in the class.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

This chapter addresses the following MA science and engineering practices:

Practice 4: Analyzing and Interpreting Data

Practice 5: Using Mathematics and Computational Thinking

Practice 8: Obtaining, Evaluating, and Communicating Information

AP®

AP® Physics 1 Learning Objectives & Science Practices:

SP 2.1: The student can *justify the selection of a mathematical routine* to solve problems.

SP 2.2: The student can *apply mathematical routines* to quantities that describe natural phenomena.

SP 2.3: The student can *estimate numerically quantities* that describe natural phenomena.

Skills learned & applied in this chapter:

- Identifying quantities in word problems and assigning them to variables
- Choosing a formula based on the quantities represented in a problem
- Using trigonometry to calculate the lengths of sides and angles of triangles
- Representing quantities as vectors
- Adding and subtracting vectors
- Multiplying vectors using the dot product and cross product

Use this space for summary and/or additional notes:

Prerequisite Skills:

These are the mathematical understandings that are necessary for Physics 1 that are taught in the MA Curriculum Frameworks for Mathematics.

- Construct and use tables and graphs to interpret data sets.
- Solve algebraic expressions.
- Perform basic statistical procedures to analyze the center and spread of data.
- Measure with accuracy and precision (*e.g.,* length, volume, mass, temperature, time)
- Convert within a unit (*e.g.,* centimeters to meters).
- Use common prefixes such as milli-, centi-, and kilo-.
- Use scientific notation, where appropriate.
- Use ratio and proportion to solve problems.

Fluency in all of these understandings is a prerequisite for this course. Students who lack this fluency may have difficulty passing the course.

Use this space for summary and/or additional notes:

Standard Assumptions in Physics

Unit: Mathematics

MA Curriculum Frameworks (2016): SP1, SP2

AP® Physics 1 Learning Objectives: SP 1.1, 1.2, 1.3, 1.4

Mastery Objective(s): (Students will be able to…)

- Make reasonable assumptions in order to be able to solve problems using the information given.

Success Criteria:

- Assumptions account for quantities that might affect the situation, but whose effects are either negligible.

Language Objectives:

- Explain why we need to make assumptions in our everyday life.

Tier 2 Vocabulary: assumption

Notes:

Many of us have been told not to make assumptions. There is a popular expression that states that "when you assume, you make an ass of you and me":

ass|u|me

In science, particularly in physics, this adage is crippling. Assumptions are part of everyday life. When you cross the street, you assume that the speed of cars far away is slow enough for you to walk across without getting hit. When you eat your lunch, you assume that the food won't cause an allergic reaction. When you run down the hall and slide across the floor, you assume that the friction between your shoes and the floor will be enough to stop you before you crash into your friend.

assumption: something that is unstated but considered to be fact for the purpose of making a decision or solving a problem. Because it is impossible to measure and/or calculate everything that is going on in a typical physics or engineering problem, it is almost always necessary to make assumptions.

Use this space for summary and/or additional notes:

In a first-year physics course, in order to make problems and equations easier to understand and solve, we will often assume that certain quantities have a minimal effect on the problem, even in cases where this would not actually be true. The term used for these kinds of assumptions is “ideal”. Some of the ideal physics assumptions we will use include the following. Over the course of the year, you can make each of these assumptions unless you are explicitly told otherwise.

- Constants are constant and variables vary as described. This means that constants (such as acceleration due to gravity) have the same value in all parts of the problem, and variables change in the manner described by the relevant equation(s).
- Ideal machines and other objects that are not directly considered in the problem have negligible mass, inertia, and friction. (Note that these idealizations may change from problem-to-problem. A pulley may have negligible mass in one problem, but another pulley in another problem may have significant mass that needs to be considered as part of the problem.)
- If a problem does not give enough information to determine the effects of friction, you may assume that sliding (kinetic) friction between surfaces is negligible. In physics problems, ice is assumed to be frictionless unless you are explicitly told otherwise.
- If a problem does not mention air resistance and air resistance is not a central part of the problem, you may assume that friction due to air resistance is negligible.
- The mass of an object can often be assumed to exist at a single point in 3-dimensional space. (This assumption does not hold for problems where you need to calculate the center of mass, or torque problems where the way the mass is spread out is part of the problem.)
- All energy can be accounted for when energy is converted from one form to another. (This is always true, but in an ideal collision, energy lost to heat is usually assumed to be negligible.)
- Collisions between objects are assumed to be either perfectly elastic or perfectly inelastic, unless the problem states otherwise.
- The amount that solids and liquids expand or contract due to changes in temperature or pressure is negligible. (This will not be the case in problems involving thermal expansion.)
- Gas molecules do not interact when they collide or are forced together from pressure. (Real gases can form liquids and solids or participate in chemical reactions.)
- Electrical wires have negligible resistance.
- Physics students always do all of their homework. ☺

Use this space for summary and/or additional notes:

In some topics, a particular assumption may apply to some problems but not others. In these cases, the problem needs to make it clear whether or not you can make the relevant assumption. (For example, in the "forces" topic, some problems involve friction and others do not. A problem that does not involve friction might state that "a block slides across a frictionless surface.")

If you are not sure whether you can make a particular assumption, you should ask the teacher. If this is not practical (such as an open response problem on a standardized test), you should decide for yourself whether or not to make the assumption, and ***explicitly state what you are assuming as part of your answer***.

Use this space for summary and/or additional notes:

The International System of Units

Unit: Mathematics

MA Curriculum Frameworks (2016): SP5

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Use and convert between metric prefixes attached to units.

Success Criteria:

- Conversions between prefixes move the decimal point the correct number of places.
- Conversions between prefixes move the decimal point in the correct direction.
- The results of conversions have the correct answers with the correct units, including the prefixes.

Language Objectives:

- Set up and solve problems relating to the concepts described in this section.

Tier 2 Vocabulary: unit, prefix

Notes:

This section is intended to be a brief review. You learned to use the metric system and its prefixes in elementary school. Although you will learn many new S.I. units this year, ***you are expected to be able to fluently apply any metric prefix to any unit and be able to convert between prefixes in any problem you might encounter throughout the year****.*

A unit is a specifically defined measurement. Units describe both the type of measurement, and a base amount.

For example, 1 cm and 1 inch are both lengths. They are used to measure the same dimension, but the specific amounts are different. (In fact, 1 inch is exactly 2.54 cm.)

Every measurement is a number multiplied by its units. In algebra, the term “3x” means “3 times x”. Similarly, the distance “75 m” means “75 times the distance 1 meter”.

The number and the units are <u>*both*</u> *necessary to describe any measurement.* You *always* need to write the units. Saying that “12 is the same as 12 g” would be as ridiculous as saying “12 is the same as 12 × 3”.

Use this space for summary and/or additional notes:

The International System (often called the metric system) is a set of units of measurement that is based on natural quantities (on Earth) and powers of 10.

The metric system has 7 fundamental "base" units:

Unit	**Quantity**
meter (m)	length
kilogram (kg)	mass
second (s)	time
Kelvin (K)	temperature
mole (mol)	amount of substance
ampere (A)	electric current
candela (cd)	intensity of light

All other S.I. units are combinations of one or more of these seven base units.

For example:

Velocity (speed) is a change in distance over a period of time, which would have units of distance/time (m/s).

Force is a mass subjected to an acceleration. Acceleration has units of distance/time^2 (m/s^2), and force has units of mass × acceleration. In the metric system this combination of units (kg·m/s^2) is called a Newton, which means:
$1\text{ N} \equiv 1\text{ kg·m/s}^2$

(The symbol "≡" means "is identical to," whereas the symbol "=" means "is equivalent to".)

The S.I. base units are calculated from these seven definitions, after converting the derived units (joule, coulomb, hertz, lumen and watt) into the seven base units (second, meter, kilogram, ampere, kelvin, mole and candela).

Use this space for summary and/or additional notes:

Prefixes

The metric system uses prefixes to indicate multiplying a unit by a power of ten. Prefixes are defined for powers of ten from 10^{-30} to 10^{30}:

Factor		Prefix	Symbol
1 000 000 000 000 000 000 000 000 000 000	10^{30}	quetta	Q
1 000 000 000 000 000 000 000 000 000	10^{27}	ronna	R
1 000 000 000 000 000 000 000 000	10^{24}	yotta	Y
1 000 000 000 000 000 000 000	10^{21}	zeta	Z
1 000 000 000 000 000 000	10^{18}	exa	E
1 000 000 000 000 000	10^{15}	peta	P
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto	h
10	10^{1}	deca	da
1	10^{0}	—	—
0.1	10^{-1}	deci	d
0.01	10^{-2}	centi	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	µ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p
0.000 000 000 000 001	10^{-15}	femto	f
0.000 000 000 000 000 001	10^{-18}	atto	a
0.000 000 000 000 000 000 001	10^{-21}	zepto	z
0.000 000 000 000 000 000 000 001	10^{-24}	yocto	y
0.000 000 000 000 000 000 000 000 001	10^{-27}	ronto	r
0.000 000 000 000 000 000 000 000 000 001	10^{-30}	quecto	q

↑ Move Decimal Point to the Left

↓ Move Decimal Point to the Right

Note that some of the prefixes skip by a factor of 10 and others skip by a factor of 10^3. This means ***you can't just count the steps in the table—you have to actually look at the exponents***.

The most commonly used prefixes are:

- mega (M) = 10^6 = 1 000 000
- kilo (k) = 10^3 = 1000
- centi (c) = $10^{-2} = \frac{1}{100}$ = 0.01
- milli (m) = $10^{-3} = \frac{1}{1\,000}$ = 0.001
- micro (µ) = $10^{-6} = \frac{1}{1\,000\,000}$ = 0.000 001

Any metric prefix is allowed with any metric unit. For example, "35 cm" means "35 × c × m" or "(35)($\frac{1}{100}$)(m)". If you multiply this out, you get 0.35 m.

Note that some units have two-letter abbreviations. *E.g.,* the unit symbol for pascal (a unit of pressure) is (Pa). Standard atmospheric pressure is 101 325 Pa. This same number could be written as 101.325 kPa or 0.101 325 MPa.

Use this space for summary and/or additional notes:

There is a popular geek joke based on the ancient Greek heroine Helen of Troy. She was said to have been the most beautiful woman in the world, and she was an inspiration to the entire Trojan fleet. She was described as having "the face that launched a thousand ships." Therefore a milliHelen must be the amount of beauty required to launch one ship.

Conversions

If you need to convert from one prefix to another, simply move the decimal point.

- Use the starting and ending powers of ten to determine the number of places to move the decimal point.
- When you convert, the actual measurement needs to stay the same. This means that if the prefix gets larger, the number needs to get smaller (move the decimal point to the left), and if the prefix gets smaller, the number needs to get larger (move the decimal point to the right).

Definitions

In order to have measurements be the same everywhere in the universe, any system of measurement needs to be based on some defined values. As of May 2019, instead of basing units on physical objects or laboratory measurements, all S.I. units are defined by specifying exact values for certain fundamental constants:

- The Planck constant, *h,* is exactly $6.626\,070\,15 \times 10^{-34}$ J·s
- The elementary charge, *e,* is exactly $1.602\,176\,634 \times 10^{-19}$ C
- The Boltzmann constant, *k,* is exactly $1.380\,649 \times 10^{-23}$ $\text{J}\cdot\text{K}^{-1}$
- The Avogadro constant, N_A, is exactly $6.022\,140\,76 \times 10^{23}$ mol^{-1}
- The speed of light, *c,* is exactly 299 792 458 $\text{m}\cdot\text{s}^{-1}$
- The ground state hyperfine splitting frequency of the caesium-133 atom, $\Delta\nu(^{133}\text{Cs})_{\text{hfs}}$, is exactly 9 192 631 770 Hz
- The luminous efficacy, K_{cd}, of monochromatic radiation of frequency 540×10^{12} Hz is exactly 683 $\text{lm}\cdot\text{W}^{-1}$

The exact value of each of the base units is calculated from combinations of these fundamental constants, and every derived unit is calculated from combinations of base units.

Use this space for summary and/or additional notes:

The MKS vs. cgs Systems

Because physics heavily involves units that are derived from other units, it is important to make sure that all quantities are expressed in the appropriate units before applying formulas. (This is how we get around having to do factor-label unit-cancelling conversions—like you learned in chemistry—for every single physics problem.)

There are two measurement systems commonly used in physics. In the MKS, or "meter-kilogram-second" system, units are derived from the S.I. units of meters, kilograms, seconds, moles, Kelvins, amperes, and candelas. In the cgs, or "centimeter-gram-second" system, units are derived from the units of centimeters, grams, seconds, moles, Kelvins, amperes, and candelas. The following table shows some examples:

Quantity	**MKS Unit**	**Base Units Equivalent**	**cgs Unit**	**Base Units Equivalent**
force	newton (N)	$\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$	dyne (dyn)	$\frac{\text{g}\cdot\text{cm}}{\text{s}^2}$
energy	joule (J)	$\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$	erg	$\frac{\text{g}\cdot\text{cm}^2}{\text{s}^2}$
magnetic flux density	tesla (T)	$\frac{\text{N}}{\text{A}}$, $\frac{\text{kg}\cdot\text{m}}{\text{A}\cdot\text{s}^2}$	gauss (G)	$\frac{0.1\,\text{dyn}}{\text{A}}$, $\frac{0.1\,\text{g}\cdot\text{cm}}{\text{A}\cdot\text{s}^2}$

In general, because 1 kg = 1 000 g and 1 m = 100 cm, each MKS unit is 100 000 times the value of its corresponding cgs unit.

In this class, we will use exclusively MKS units. This means you have to learn only one set of derived units. However, you can see the importance, when you solve physics problems, of making sure all of the quantities are in MKS units before you plug them into a formula!

Use this space for summary and/or additional notes:

Formatting Rules for S.I. Units

- The value of a quantity is written as a number followed by a non-breaking space (representing multiplication) and a unit symbol; *e.g.,* 2.21 kg, 7.3×10^2 m^2, or 22 K. This rule explicitly includes the percent sign (*e.g.,* 10 %, not 10%) and the symbol for degrees of temperature (*e.g.,* 37 °C, not 37°C). (However, note that angle measurements in degrees are written next to the number without a space.)
- Units do not have a period at the end, except at the end of a sentence.
- A prefix is part of the unit and is attached to the beginning of a unit symbol without a space. Compound prefixes are not allowed.
- Symbols for derived units formed by multiplication are joined with a center dot (·) or a non-breaking space; *e.g.,* N·m or N m.
- Symbols for derived units formed by division are joined with a solidus (fraction line), or given as a negative exponent. *E.g.,* "meter per second" can be written $\frac{\text{m}}{\text{s}}$, m/s, m s^{-1}, or m·s^{-1}.
- The first letter of symbols for units derived from the name of a person is written in upper case; otherwise, they are written in lower case. *E.g.,* the unit of pressure is the pascal, which is named after Blaise Pascal, so its symbol is written "Pa" (note that "Pa" is a two-letter symbol). Conversely, the mole is not named after anyone, so the symbol for mole is written "mol". Note, however, that the symbol for liter is "L" rather than "l", because a lower case "l" is too easy to confuse with the number "1".
- A plural of a symbol must not be used; *e.g.,* 25 kg, not 25 kgs.
- Units and prefixes are case-sensitive. *E.g.,* the quantities 1 mW and 1 MW represent two different quantities (milliwatt and megawatt, respectively).
- The symbol for the decimal marker is either a point or comma on the line. In practice, the decimal point is used in most English-speaking countries and most of Asia, and the comma is used in most of Latin America and in continental European countries.
- Spaces should be used as a thousands separator (1 000 000) instead of commas (1,000,000) or periods (1.000.000), to reduce confusion resulting from the variation between these forms in different countries.
- Any line break inside a number, inside a compound unit, or between a number and its unit should be avoided.

Use this space for summary and/or additional notes:

Homework Problems

Perform the following conversions.

1. **(M)** 2.5 m = __________ cm

2. **(M)** 18mL = __________ L

3. **(M)** 68 kJ = __________ J

4. **(M)** 6 500 mg = __________ kg

5. **(M)** 101 kPa = __________ Pa

6. **(M)** 325 ms = __________ s

Use this space for summary and/or additional notes:

Scientific Notation

Unit: Mathematics

MA Curriculum Frameworks (2016): SP5

AP® Physics 1 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Correctly use numbers in scientific notation in mathematical problems.

Success Criteria:

- Numbers are converted correctly to and from scientific notation.
- Numbers in scientific notation are correctly entered into a calculator.
- Math problems that include numbers in scientific notation are set up and solved correctly.

Language Objectives:

- Explain how numbers are represented in scientific notation, and what each part of the number represents.

Tier 2 Vocabulary: N/A

Notes:

This section is intended to be a brief review. You learned to use the scientific notation in elementary or middle school. ***You are expected to be able to fluently perform calculations that involve numbers in scientific notation, and to express the answer correctly in scientific notation when appropriate.***

Scientific notation is a way of writing a very large or very small number in compact form. The value is always written as a number between 1 and 10, multiplied by a power of ten.

For example, the number 1 000 would be written as 1×10^3. The number 0.000 075 would be written as 7.5×10^{-5}. The number 602 000 000 000 000 000 000 000 would be written as 6.02×10^{23}. The number 0.000 000 000 000 000 000 000 000 000 000 000 663 would be written as 6.63×10^{-34}.

Scientific notation is really just math with exponents, as shown by the following examples:

$$5.6 \times 10^3 = 5.6 \times 1\,000 = 5\,600$$

$$2.17 \times 10^{-2} = 2.17 \times \frac{1}{10^2} = 2.17 \times \frac{1}{100} = \frac{2.17}{100} = 0.0217$$

Notice that if 10 is raised to a positive exponent means you're multiplying by a power of 10. This makes the number larger, which means the decimal point moves to the right. If 10 is raised to a negative exponent, you're actually dividing by a power of 10. This makes the number smaller, which means the decimal point moves to the left.

Use this space for summary and/or additional notes:

Significant figures are easy to use with scientific notation: all of the digits before the "×" sign are significant. The power of ten after the "×" sign represents the (insignificant) zeroes, which would be the rounded-off portion of the number. In fact, the mathematical term for the part of the number before the "×" sign is the *significand*.

Math with Scientific Notation

Because scientific notation is just a way of rewriting a number as a mathematical expression, all of the rules about how exponents work apply to scientific notation.

Adding & Subtracting: adjust one or both numbers so that the power of ten is the same, then add or subtract the significands.

$$\begin{aligned}&\left(3.50\times10^{-6}\right)+\left(2.7\times10^{-7}\right)=\left(3.50\times10^{-6}\right)+\left(0.27\times10^{-6}\right)\\&=(3.50+0.27)\times10^{-6}=3.77\times10^{-6}\end{aligned}$$

Multiplying & dividing: multiply or divide the significands. If multiplying, add the exponents. If dividing, subtract the exponents.

$$\frac{6.2\times10^{8}}{3.1\times10^{10}}=\frac{6.2}{3.1}\times10^{8-10}=2.0\times10^{-2}$$

Exponents: raise the significand to the exponent. Multiply the exponent of the power of ten by the exponent to which the number is raised.

$$\left(3.00\times10^{8}\right)^{2}=\left(3.00\right)^{2}\times\left(10^{8}\right)^{2}=9.00\times10^{(8\times2)}=9.00\times10^{16}$$

Use this space for summary and/or additional notes:

Using Scientific Notation on Your Calculator

Scientific calculators are designed to work with numbers in scientific notation. It's possible to can enter the number as a math problem (always use parentheses if you do this!) but math operations can introduce mistakes that are hard to catch.

Scientific calculators all have some kind of scientific notation button. The purpose of this button is to enter numbers directly into scientific notation and make sure the calculator stores them as a single number instead of a math equation. (This prevents you from making PEMDAS errors when working with numbers in scientific notation on your calculator.) On most Texas Instruments calculators, such as the TI-30 or TI-83, you would do the following:

What you type	What the calculator shows	What you would write
6.6 $\boxed{\text{EE}}$ −34	6.6E−34	6.6×10^{-34}
1.52 $\boxed{\text{EE}}$ 12	1.52E12	1.52×10^{12}
−4.81 $\boxed{\text{EE}}$ −7	−4.81E−7	-4.81×10^{-7}

On some calculators, the scientific notation button is labeled $\boxed{\text{EXP}}$ or $\boxed{\times 10^x}$ instead of $\boxed{\text{EE}}$.

***Important notes**:*

- *Many high school students are afraid of the $\boxed{EE}$ button because it is unfamiliar. If you are afraid of your $\boxed{EE}$ button, you need to get over it and start using it anyway. However, if you insist on clinging to your phobia, you need to at least use parentheses around all numbers in scientific notation, in order to minimize the likelihood of PEMDAS errors in your calculations.*
- *Regardless of how you enter numbers in scientific notation into your calculator, always place parentheses around the denominator of fractions.*

$$\frac{2.75\times10^{3}}{5.00\times10^{-2}} \quad \textit{becomes} \quad \frac{2.75\times10^{3}}{(5.00\times10^{-2})}$$

- *You need to **write** answers using correct scientific notation. For example, if your calculator displays the number* 1.52E12, *you need to write* 1.52×10^{12} *(plus the appropriate unit, of course) in order to receive credit.*

Use this space for summary and/or additional notes:

Homework Problems

Convert each of the following between scientific and algebraic notation.

1. **(M)** $2.65 \times 10^{9} =$

2. **(M)** 387 000 000 =

3. **(M)** $1.06 \times 10^{-7} =$

4. **(M)** 0.000 000 065 =

Solve each of the following on a calculator that can do scientific notation.

5. **(M)** $(2.8\times 10^{6})(1.4\times 10^{-2}) =$

   Answer: 3.9×10^{4}

6. **(S)** $\dfrac{3.75\times 10^{8}}{1.25\times 10^{4}} =$

   Answer: 3.00×10^{4}

7. **(M)** $\dfrac{1.2\times 10^{-3}}{5.0\times 10^{-1}} =$

   Answer: 2.4×10^{-3}

Use this space for summary and/or additional notes:

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Solving Equations Symbolically

Unit: Mathematics

MA Curriculum Frameworks (2016): SP5

AP® Physics 1 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Rearrange algebraic expressions to solve for any variable in the expression.

Success Criteria:

- Rearrangements are algebraically correct.

Language Objectives:

- Describe how the rules of algebra are applied to expressions that contain only variables.

Tier 2 Vocabulary: equation, variable

Notes:

In solving physics problems, we are more often interested in the relationship between the quantities in the problem than we are in the numerical answer.

For example, suppose we are given a problem in which a person with a mass of 65 kg accelerates on a bicycle from rest $(0\,\frac{\text{m}}{\text{s}})$ to a velocity of $10\,\frac{\text{m}}{\text{s}}$ over a duration of 12 s and we wanted to know the force that was applied.

We could calculate acceleration as follows:

$$v - v_o = at$$
$$10 - 0 = a(12)$$
$$a = \frac{10}{12} = 0.8\overline{3}\,\frac{\text{m}}{\text{s}^2}$$

Then we could use Newton's second law:

$$F = ma$$
$$F = (65)(0.8\overline{3}) = 54.2\,\text{N}$$

We have succeeded in answering the question. However, the question and the answer are of no consequence. Obtaining the correct answer shows that we can manipulate two related equations and come out with the correct number.

Use this space for summary and/or additional notes:

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However, if instead we decided that we wanted to come up with an expression for force in terms of the quantities given (mass, initial and final velocities and time), we would need to rearrange the relevant equations to give an expression for force in terms of those quantities.

Just like algebra with numbers, rearranging an equation to solve for a variable is simply "undoing PEMDAS:"

1. "Undo" addition and subtraction by doing the inverse (opposing) operation. If a variable is added, subtract it from both sides; if the variable is subtracted, then add it to both sides.

$$\begin{aligned} a+c&=b \\ -c&=-c \\ a&=b-c \end{aligned}$$

2. "Undo" multiplication and division by doing the inverse operation. If a variable is multiplied, divide both sides by it; if the variable is in the denominator, multiply both sides by it. *Note: whenever you have variables in the denominator that are on the same side of the equation as the variable you are solving for, always multiply both sides by it to clear the fraction.*

$$\frac{x\cancel{y}}{\cancel{y}}=\frac{z}{y}$$

$$x=\frac{z}{y}$$

$$\frac{n}{r}=s$$

$$\cancel{r}\cdot\frac{n}{\cancel{r}}=s\cdot r$$

$$\frac{n}{s}=\frac{\cancel{s}r}{\cancel{s}}$$

$$\frac{n}{s}=r$$

3. "Undo" exponents by the inverse operation, which is taking the appropriate root of both sides. (Most often, the exponent will be 2, which means take the square root.) Similarly, you can "undo" roots by raising both sides to the appropriate power.

$$\begin{aligned} t^2&=4ab \\ \sqrt{t^2}&=\sqrt{4ab} \\ t&=\sqrt{4}\cdot\sqrt{ab}=2\sqrt{ab} \end{aligned}$$

4. When you are left with only parentheses and nothing outside of them, you can drop the parentheses, and then repeat steps 1–3 above until you have nothing left but the variable of interest.

Use this space for summary and/or additional notes:

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Returning to the previous problem:

We know that $F = ma$. We are given *m*, but not *a*, which means we need to replace *a* with an expression that includes only the quantities given.

First, we find an expression that contains *a*:

$$v - v_o = at$$

We recognize that $v_o = 0$, and we use algebra to rearrange the rest of the equation so that *a* is on one side, and everything else is on the other side.

$$v - v_o = \underline{a}t$$
$$v - 0 = \underline{a}t$$
$$v = \underline{a}t$$
$$\underline{a} = \frac{v}{t}$$

Finally, we replace *a* in the first equation with $\frac{v}{t}$ from the second:

$$F = ma$$
$$F = (m)(\frac{v}{t})$$
$$F = \frac{mv}{t}$$

If the only thing we want to know is the value of *F* in one specific situation, we can substitute numbers at this point. However, we can also see from our final equation that increasing the mass or velocity will increase the numerator, which will increase the value of the fraction, which means the force would increase. We can also see that increasing the time would increase the denominator, which would decrease the value of the fraction, which means the force would decrease.

Solving the problem symbolically gives a relationship that holds true for all problems of this type in the natural world, instead of merely giving a number that answers a single pointless question. This is why the College Board and many college professors insist on symbolic solutions to equations.

Use this space for summary and/or additional notes:

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Homework Problems

1. **(S)** Given $a = 2bc$ and $e = c^2 d$, write an expression for *e* in terms of *a*, *b*, and *d*.

2. **(M)** Given $w = \frac{3}{2}xy^2$ and $z = \frac{q}{y}$:

   a. **(M)** Write an expression for *z* in terms of *q*, *w*, and *x*.

   b. **(M)** If you wanted to maximize the value of the variable *z* in question #2 above, what adjustments could you make to the values of *q*, *w*, and *x*?

   c. **(M)** Changing which of the variables *q*, *w*, or *x* would give the largest change in the value of *z*?

Use this space for summary and/or additional notes:

Solving Word Problems Systematically

Unit: Mathematics

MA Curriculum Frameworks (2016): SP1, SP5

AP® Physics 1 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Assign (declare) variables in a word problem according to the conventions used in physics.
- Substitute values for variables in an equation.

Success Criteria:

- Variables match the quantities given and match the units.
- Quantities are substituted for the correct variables in the equation.

Language Objectives:

- Describe the quantities used in physics, list their variables, and explain why that particular variable might have been chosen for the quantity.

Tier 2 Vocabulary: equation, variable

Notes:

Math is a language. Like other languages, it has nouns (numbers), pronouns (variables), verbs (operations), and sentences (equations), all of which must follow certain rules of syntax and grammar.

This means that turning a word problem into an equation is translation from English to math.

Mathematical Operations

You have probably been taught translations for most of the common math operations:

word	meaning	word	meaning	word	meaning
and, more than (but not "is more than")	+	percent ("per" + "cent")	÷ 100	**is** at least	≥
less than (but not "is less than")	−	change in *x*, difference in *x*	Δx	**is** more than	>
of	×	is	=	**is** at most	≤
per, out of	÷			**is** less than	<

Use this space for summary and/or additional notes:

Identifying Variables

In science, almost every measurement must have a unit. These units are your key to what kind of quantity the numbers describe. Some common quantities in physics and their units are:

quantity	S.I. unit	variable	quantity	S.I. unit	variable
mass	kg	*m*	work	J	*W*
distance, length	m	*d* , *L*	power	W	*P**
height	m	*h*	pressure	Pa	*P**
area	m^2	*A*	momentum	N·s	*p**
acceleration	m/s^2	*a*	density	kg/m^3	*ρ**
volume	m^3	*V*	moles	mol	*n*
velocity (speed)	m/s	*v*	temperature	K	*T*
time	s	*t*	heat	J	*Q*
force	N	*F*	electric charge	C	*q* , *Q*

*Note the subtle differences between uppercase "*P*", lowercase "*p*", and the Greek letter *ρ* ("rho").

Any time you see a number in a word problem that has a unit that you recognize (such as one listed in this table), notice which quantity the unit is measuring, and label the quantity with the appropriate variable.

Be especially careful with uppercase and lowercase letters. In physics, the same uppercase and lowercase letter may be used for completely different quantities.

Use this space for summary and/or additional notes:

Variable Substitution

Variable substitution simply means taking the numbers you have from the problem and substituting those numbers for the corresponding variable in an equation. A simple version of this is a density problem:

If you have the formula:

$$\rho^{*} = \frac{m}{V} \quad \text{and you're given:} \quad m = 12.3\text{ g} \quad \text{and} \quad V = 2.8\text{ cm}^3$$

simply substitute 12.3 g for m, and 2.8 cm^3 for V, giving:

$$\rho = \frac{12.3\text{ g}}{2.8\text{ cm}^3} = 4.4\,\tfrac{\text{g}}{\text{cm}^3}$$

Because variables and units both use letters, it is often safer to leave the units out when you substitute numbers for variables and then add them back in at the end:[†]

$$\rho = \frac{12.3}{2.8} = 4.4\,\tfrac{\text{g}}{\text{cm}^3}$$

[*] Physicists use the Greek letter ρ ("rho") for density. Note that the Greek letter ρ is different from the Roman letter "p".

[†] Many physics teachers disagree with this approach and insist on having students include the units with the number throughout the calculation. However, this can lead to confusion about which symbols are variables and which are units. For example, if a device applies a power of 150 W for a duration of 30 s and we wanted to find out the amount of work done, we would have:

$$P = \frac{W}{t}$$

$$150\text{ W} = \frac{W}{30\text{ s}} \quad vs. \quad 150 = \frac{W}{30}$$

In the left equation, the student would need to realize that the W on the left side is the unit "watts", and the W on the right side of the equation is the variable W, which stands for "work".

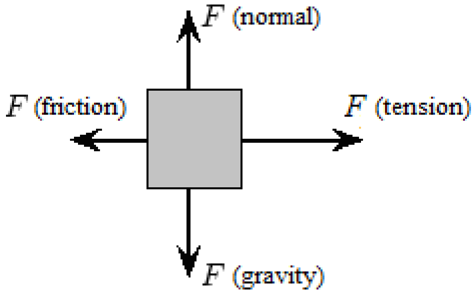
Use this space for summary and/or additional notes:

Subscripts

In physics, one problem can often have several instances of the same quantity. For example, consider a box with four forces on it:

1. The force of gravity, pulling downward.
2. The "normal" force of the table resisting gravity and holding the box up.
3. The tension force in the rope, pulling the box to the right.
4. The force of friction, resisting the motion of the box and pulling to the left.

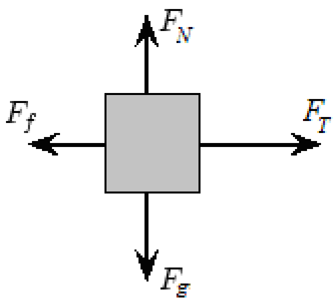
The variable for force is "*F*". There are four types of forces, which means "*F*" means four different things in this problem:



To make the diagram easier to read, we add subscripts to the variable "*F*". Note that in most cases, the subscript is the first letter of the word that describes the particular instance of the variable:

1. F_g is the force of gravity.
2. F_N is the normal force.
3. F_T is the tension in the rope.
4. F_f is friction.

This results in the following free-body diagram:



We use these same subscripts in the equations that relate to the problem. For example:

$$F_g = mg \quad \text{and} \quad F_f = \mu F_N$$

Use this space for summary and/or additional notes:

When writing variables with subscripts, be especially careful that the subscript looks like a subscript—***it needs to be smaller than the other letters and lowered slightly***. For example, when we write F_g, we add the subscript $_g$ (which stands for "gravity") to the variable F (force). ***Note that the subscript is part of the variable***; the variable is no longer F, but F_g.

An example is the following equation:

$$F_g = mg \qquad \leftarrow \qquad \text{right ☺}$$

It is important that the subscript $_g$ on the left does not get confused with the variable g on the right. Otherwise, the following error might occur:

$$Fg = mg \qquad \leftarrow \qquad \text{wrong! ☹}$$
$$F\cancel{g} = m\cancel{g}$$
$$F = m$$

A common use of subscripts is the subscript "o" to mean "initial". (Imagine that the word problem or "story problem" is shown as a video. When the slider is at the beginning of the video, the time is 0, and the values of all of the variables at that time are shown with a subscript of o.)

For example, if an object is moving slowly at the beginning of a problem and then it speeds up, we need subscripts to distinguish between the initial velocity and the final velocity. Physicists do this by calling the initial velocity "v_o*" where the subscript "o" means "at time zero", *i.e.,* at the beginning of the problem. The final velocity is simply "v" without the zero.

* pronounced "v-sub-zero", "v-zero" or "v-naught"

Use this space for summary and/or additional notes:

The Problem-Solving Process: "*GUESS*"

The following is an overview of the problem-solving process. The acronym "GUESS" may be helpful to remember it.

1. **Given**: Identify the given quantities in the problem, based on the units and any other information in the problem.
   - Assign the appropriate variables to those quantities.
2. **Unknown**: Identify the quantity that the question is asking for.
   - Assign the appropriate variable.
3. **Equation**: Find an equation that contains the Unknown and one or more of the Given quantities.
   - The best choice where *every quantity in the equation* is either the Unknown or one of the Givens.
   - If there is no equation in which every quantity is the unknown or one of the givens, choose the one that comes closest. However, *the equation must contain the unknown* or you won't be able to solve for it!
4. **Solve**: Use algebra to rearrange the equation to Solve it for the variable you're looking for. (Move all of the other quantities to the other side by "undoing PEMDAS.")* This process is explained in more detail in the next section, *Solving Word Problems Systematically*, starting on page 127.
5. **Substitute**: Replace the Given variables with their values and calculate the answer.
   - If you can't calculate the answer because you still need a variable, go back to step 2 above. The variable you need is your new unknown. Complete steps 2–5 above to find the value of that variable, then continue with the original equation.
6. Apply the appropriate unit(s) to the result.

* For CP1 physics, if students do not have strong algebra skills you may need to switch the order of steps 4 & 5, having students first substitute values into the equation, and then rearrange the equation when there is only one variable.

Use this space for summary and/or additional notes:

Sample Problem

A net force of 30 N acts on an object with a mass of 1.5 kg. What is the acceleration of the object? (*mechanics/forces*)

1. **Given**: Identify the Given quantities in the problem and assign variables to them. We can use *Table C. Quantities, Variables and Units* on page 683 of your Physics Reference Tables:

   - 30 N uses the unit N (newtons). Newtons are used for force, and the variable for force is $\vec{F}$.
   - 1.5 kg uses the unit kg (kilograms). Kilograms are used for mass, and the variable for mass is *m*.

   A net force of $\underbrace{\text{30 N}}_{\vec{F}}$ acts on an object with a mass of $\underbrace{\text{1.5 kg}}_{m}$. What is the acceleration of the object?

2. **Unknown**: Identify the quantity that the question is asking for and assign a variable to it.

   - The unknown quantity is acceleration. From *Table C. Quantities, Variables and Units* on page 683, acceleration uses the variable $\vec{a}$, and the units $\frac{m}{s^2}$ (which we will need later for the answer).

   A net force of $\underbrace{\text{30 N}}_{\vec{F}}$ acts on an object with a mass of $\underbrace{\text{1.5 kg}}_{m}$. What is the $\underbrace{\text{acceleration}}_{\vec{a}}$ of the object?

3. **Equation**: Find an equation that includes the Unknown and one or more of the Given quantities:

   $$\vec{F}_{net} = m\vec{a}$$

4. **Solve**: Use algebra ("undo PEMDAS") to rearrange the equation.

   We need to get $\vec{a}$ by itself. In the equation, *m* is attached to $\vec{a}$ by multiplication, so we need to get rid of *m* by ***undoing multiplication***, which means we ***divide*** by *m* on both sides.

   $$\frac{\vec{F}_{net}}{m} = \frac{\cancel{m}\vec{a}}{\cancel{m}}$$

   $$\frac{\vec{F}_{net}}{m} = \vec{a}$$

5. **Substitute**: Replace the Given quantities with their values and calculate the answer. (*Remember to add the units!*)

   $$\frac{\vec{F}_{net}}{m} = \vec{a} \rightarrow \frac{30}{1.5} = \vec{a} \rightarrow \boxed{20\,\frac{m}{s^2}} = \vec{a}$$

Use this space for summary and/or additional notes:

Homework Problems

To solve these problems, refer to your Physics Reference Tables starting on page 679. To make the equations easier to find, *the table and section of the table in your Physics Reference Tables where the equation can be found is given in parentheses*.

Note that this is probably one of the most frustrating assignments in this course. The process is unfamiliar, the problem set feels more like a scavenger hunt than a problem set, and the problems intentionally contain pesky details that you will encounter throughout the year that you will learn about here by struggling with them. Please be advised that this is meant to be a *productive* struggle!

For problems #1–3 below, ***identify the variables*** that correspond with the Given and Unknown quantities in the following problems. (*You do not need to find the equation or solve the problem.*)

1. **(M = Must Do)** What is the average velocity of a car that travels 90. m in 4.5 s? (*mechanics*)

2. **(M = Must Do)** If a net force of 100. N acts on a mass of 5.0 kg, what is its acceleration? (*mechanics*)

3. **(S = Should Do)** A 25 Ω resistor is placed in an electrical circuit with a voltage of 110 V. How much current flows through the resistor? (*electricity*)

For problems #4–6 below, ***identify the variables*** (as above) and ***find the equation*** that relates those variables. (*You do not need to rearrange the equation or solve the problem.*)

4. **(M)** What is the potential energy due to gravity of a 95 kg anvil that is about to fall off a 150 m cliff onto Wile E. Coyote's head?
Note: "fall" means gravity is involved and will appear in the equation.
(*mechanics/energy, work & power*)

Use this space for summary and/or additional notes:

5. **(S – honors & AP®; M – CP1)** If the momentum of a block is 18 N·s and its velocity is $3\,\frac{\text{m}}{\text{s}}$, what is the mass of the block?
(*mechanics/momentum*)

honors & AP®

6. **(M – honors & AP®; A – CP1)** If the momentum of a block is *p* and its velocity is *v*, derive an expression for the mass of the block.
(*mechanics/momentum*) (If you're not sure how to solve this, #5 is the same problem, but with numbers.

For the remaining problems (#7–20 below), use the *GUESS* method to ***identify the variables***, ***find the equation***, and ***solve*** the problems. (Answers are given so you can check your work; <u>*credit will be given only if all steps of GUESS are shown*</u>.)

7. **(M)** What is the <u>frequency</u> of a wave that is traveling at a velocity of <u>$300.\,\frac{\text{m}}{\text{s}}$</u> and has a wavelength of <u>10. m</u>?
(*waves/waves*)

Answer: 30. Hz

8. **(S)** What is the <u>energy of a photon</u> that has a frequency of <u>6×10^{15} Hz</u>?
Note: the equation includes Planck's constant, which you need to look up.
(*atomic, particle, and nuclear physics/energy*)

Answer: 3.96×10^{-18} J

9. **(S)** A piston with an area of $2.0\,\text{m}^2$ is compressed by a force of 10 000 N. What is the pressure applied by the piston?
(*fluids/pressure*)

Answer: 5 000 Pa

Use this space for summary and/or additional notes:

honors & AP®

10. **(M – honors & AP®; A – CP1)** Derive an expression for the acceleration (a) of a car whose velocity changes from v_o to v in time t.

    (*If you are not sure how to do this problem, do #11 below and use the steps to guide your algebra.*)

    (*mechanics/kinematics*)

    Answer: $a = \frac{v - v_o}{t}$

11. **(M)** What is the acceleration of a car whose velocity changes from $60.\,\frac{m}{s}$ to $80.\,\frac{m}{s}$ over a period of 5.0 s?

    Hint: v_o is the initial velocity and v is the final velocity.

    (*<u>You must start with the equations in your</u>* <u>Physics Reference Tables</u> *<u>and show all of the steps of GUESS.</u> You may only use the answer to question #10 above as a starting point if you have already solved that problem.*)

    (*mechanics/kinematics*)

    Answer: $4.0\,\frac{m}{s^2}$

12. **(S)** If the normal force on an object is 100. N and the coëfficient of kinetic friction between the object and the surface it is sliding on is 0.35, what is the force of friction on the object as it slides along the surface?
    Note: the coefficient of kinetic friction is a material-specific constant whose value is given in the problem.
    (*mechanics/forces*)

    Answer: 35 N

13. **(M)** A 1200 W hair dryer is plugged into a electrical circuit with a voltage of 110 V. How much electric current flows through the hair dryer?
    (*electricity/circuits*)

    Answer: 10.9 A

Use this space for summary and/or additional notes:

honors & AP®

14. **(S – honors & AP®; A – CP1)** A car has mass *m* and kinetic energy *K*. Derive an expression for its velocity (*v*).

    (*If you are not sure how to do this problem, do #15 below and use the steps to guide your algebra.*)

    (*mechanics/energy*)

    Answer: $v=\sqrt{\dfrac{2K}{m}}$

15. **(S)** A car has a mass of 1 200 kg and kinetic energy of 240 000 J. What is its velocity?

    (*You must start with the equations in your* Physics Reference Tables and show all of the steps of GUESS. *You may only use the answer to question #14 above as a starting point if you have already solved that problem.*)

    (*mechanics/energy*)

    Answer: $20.\ \frac{\text{m}}{\text{s}}$

16. **(S)** What is the velocity of a photon (wave of light) as it passes through a block of clear plastic that has an index of refraction of 1.40?

    Hint: The index of refraction is a material-specific constant whose value is given in the problem.

    (*waves/reflection & refraction*)

    Answer: $2.14\times10^{8}\ \frac{\text{m}}{\text{s}}$

17. **(M)** If a pressure of 100 000 Pa is applied to a gas and the volume decreases by $0.05\,\text{m}^3$, how much work was done on the gas?

    Note: ΔV is two symbols, but it is a single variable that represents the change in volume. Pay attention to whether ΔV is positive or negative.

    (*heat/thermodynamics*)

    Answer: 5 000 J

Use this space for summary and/or additional notes:

honors & AP®

18. **(S – honors & AP®; A – CP1)** If the distance from a mirror to an object is s_o and the distance from the mirror to the image is s_i, derive an expression for the distance from the lens to the focus (f).

    (*If you are not sure how to do this problem, do #19 below and use the steps to guide your algebra.*)

    (*waves/mirrors & lenses*)

    Answer: $f = \frac{s_i s_o}{s_i + s_o}$

19. **(S)** If the distance from a mirror to an object is 0.8 m and the distance from the mirror to the image is 0.6 m, what is the distance from the mirror to the focus?

    (*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #18 above as a starting point if you have already solved that problem.*)

    (*waves/mirrors & lenses*)

    Answer: 0.343 m

20. **(S)** What is the momentum of a photon that has a wavelength of 400 nm? *Hint: you will need to convert nanometers to meters.*
    (*atomic, Particle, and Nuclear physics/energy*)

    Answer: $1.65\times10^{-27}\ \text{N}\cdot\text{s}$

Use this space for summary and/or additional notes:

honors & AP®

Right-Angle Trigonometry

Unit: Mathematics

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Use the Pythagorean theorem to find one side of a right triangle, given the other two sides.
- Use the trigonometry functions sine, cosine and tangent to find one side of a right triangle, given one of the non-right angles and one other side.
- Use the inverse trigonometry functions arcsine ($\sin^{-1}$), arccosine ($\cos^{-1}$), and arctangent ($\tan^{-1}$) to find one of the non-right angles of a right triangle, given any two sides.

Success Criteria:

- Sides and angles are correctly identified (opposite, adjacent, hypotenuse).
  - Correct function/equation is chosen based on the relationship between the sides and angles.

Language Objectives:

- Describe the relationships between the sides and angles of a right triangle.

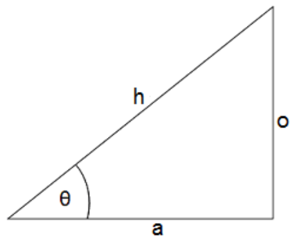
Tier 2 Vocabulary: opposite, adjacent

Notes:

The word trigonometry comes from "trigon*" = "triangle" and "ometry" = "measurement", and is the study of relationships among the sides and angles of triangles.

If we have a right triangle, such as the one shown to the right:

- side "h" (the longest side, opposite the right angle) is the *__hypotenuse__*.
- side "o" is the side of the triangle that is *__opposite__* (across from) angle θ.
- side "a" is the side of the triangle that is *__adjacent__* to (connected to) angle θ (and is not the hypotenuse).

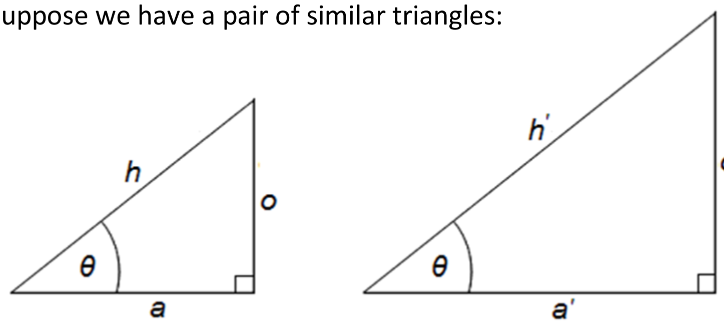


*"trigon" is another word for a 3-sided polygon (triangle), just as "octagon" is an 8-sided polygon.

Use this space for summary and/or additional notes:

honors & AP®

Now, suppose we have a pair of similar triangles:



Because the triangles are similar, the corresponding angles and the ratios of corresponding pairs of sides must be equal. For example, the ratio of the opposite side to the hypotenuse would be $\frac{o}{h}=\frac{o'}{h'}$. ***This ratio must be the same for every triangle that is similar to the ones above***, *i.e.*, ***for every right triangle that has an angle equal to θ***. This means that if we know the angle, θ, then we know the ratio of the opposite side to the hypotenuse.

We define this ratio as the sine of the angle, *i.e.,* $\text{sine}\,(\theta)=\sin\theta=\frac{o}{h}=\frac{o'}{h'}$.

We define similar quantities for ratios of other sides:

$$\text{cosine}\,(\theta) \;=\; \cos\theta \;=\; \frac{a}{h} \;=\; \frac{a'}{h'}$$

$$\text{tangent}\,(\theta) \;=\; \tan\theta \;=\; \frac{o}{a} \;=\; \frac{o'}{a'} \;=\; \frac{\sin\theta}{\cos\theta}$$

We can create a table of these ratios (sines, cosines and tangents) for different values of the angle θ, as shown in *Table II. Values of Trigonometric Functions* on page 698 of your Physics Reference Tables. The **sin**, **cos** and **tan** buttons on your calculator calculate this ratio for any value of θ. (Just make sure your calculator is correctly set for degrees or radians, depending on how θ is expressed.)*

There are a lot of stupid mnemonics for remembering which sides are involved in which functions. (You may been taught SOH CAH TOA.) My favorite of these is "**O**h **h**eck, **a**nother **h**our **o**f **a**lgebra!"

* In physics, problems that use Cartesian coördinates use degrees, and problems involving rotation (which is studied in AP® Physics 1, but not the CP1 or honors course) use polar coördinates and radians. This means that if you are taking CP1 or honors Physics 1, angles will always be expressed in degrees. If you are taking AP® Physics 1, you will need to use degrees for some problems and radians for others.

Use this space for summary and/or additional notes:

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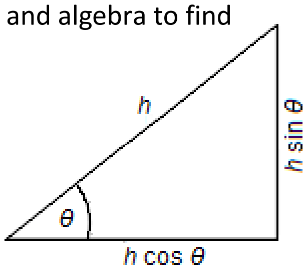
The most common use of trigonometry functions in physics is to decompose a vector into its components in the x- and y-directions. If we know the angle of inclination of the vector quantity, we can use trigonometry and algebra to find the components of the vector in the x- and y-directions:

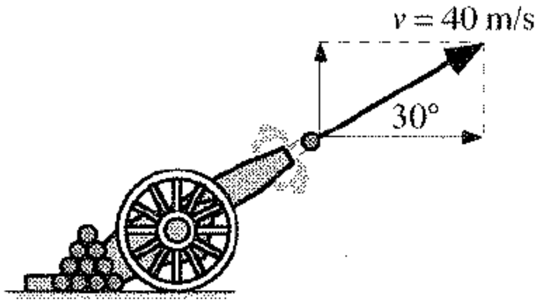
$$\cos\theta = \frac{a}{h} \rightarrow h\cdot\cos\theta = \frac{a}{\cancel{h}}\cdot\cancel{h} \rightarrow \boxed{a = h\cos\theta}$$

$$\sin\theta = \frac{o}{h} \rightarrow h\cdot\sin\theta = \frac{o}{\cancel{h}}\cdot\cancel{h} \rightarrow \boxed{o = h\sin\theta}$$



Memorize these relationships! It will save you a lot of time throughout the rest of the year.

For example, consider the following situation:



The horizontal velocity of the cannon ball is:

$$v_x = h\cos\theta = (40\,\tfrac{m}{s})\cos(30°) = (40)(0.866) = 34.6\,\tfrac{m}{s}$$

The initial vertical velocity of the cannon ball is:

$$v_{o,y} = h\sin\theta = (40\,\tfrac{m}{s})\sin(30°) = (40)(0.5) = 20\,\tfrac{m}{s}$$

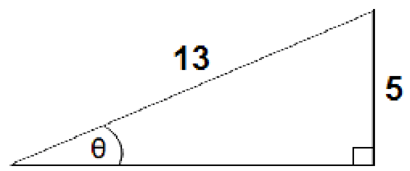
You can look up the sine and cosine of 30° on a trigonometry table similar to the one in *Table II. Values of Trigonometric Functions* on page 698 of your Physics Reference Tables. You can, of course, also use the sin, cos and tan functions on your calculator.

Use this space for summary and/or additional notes:

honors & AP®

Finding Angles (Inverse Trigonometry Functions)

If you know the sides of a triangle and you need to find an angle, you can use the inverse of a trigonometric function. For example, suppose we had the following triangle:



We don't know what angle θ is, but we know that $\sin\theta = \frac{5}{13} = 0.385$.

If we look for a number that is close to 0.385 in the sine column of *Table II. Values of Trigonometric Functions* on page 698 of your Physics Reference Tables, we see that 0.385 would be somewhere between 22° and 23°, a little closer to 23°. (By inspection, we might guess 22.6° or 22.7°.)*

To perform the same function on a calculator, we use the inverse of the sine function (which means to go from the sine of an angle to the angle itself, instead of the other way around). The inverse sine (the proper name of the function is actually "arcsine") is usually labeled $\sin^{-1}$ on calculators. Doing this, we see that:

$$\theta = \sin^{-1}(0.385) = 22.64°$$

which is between 22.6° and 22.7°, as expected.

Summary

- If you know two sides of a right triangle, *a* and *b*, you can find the third side from the Pythagorean theorem: $c^2 = a^2 + b^2$
- If you know one of the acute angles, θ, of a right triangle, the other acute angle is $90° - \theta$.
- If you know one side of a right triangle and one acute angle (*e.g.,* a problem involving a force or velocity at an angle), you can find the remaining sides using sine, cosine, or tangent. (Especially remember $x = h\cos\theta$ and $y = h\sin\theta$.)
- If you know two sides of a right triangle and you need an angle, use one of the inverse trigonometric functions, *i.e.,* $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$.

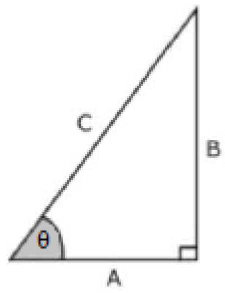
* Although it may be tempting to just teach students to use the inverse trig functions on their calculators, they will gain a more intuitive understanding of what an inverse trig function is if they start with trig tables.

Use this space for summary and/or additional notes:

honors & AP®

Homework Problems

Questions 1–5 are based on the following right triangle, with sides *A*, *B*, and *C*, and angle θ between *A* and *C*.



Note that the drawing is not to scale, and that angle *x* and the lengths of *A*, *B* and *C* will be different for each problem.

Some problems may also require use of the fact that the angles of a triangle add up to 180◦.

1. **(S)** If $A = 5$ and $C = 13$, what is *B*?

2. **(M)** If $A = 5$ and $C = 13$, what is $\sin\theta$?

3. **(M)** If $C = 20$ and $\theta = 50°$, what are *A* and *B*?

4. **(M)** If $A = 100$ and $C = 150$, what is θ?

5. **(S)** If $B = 100$ and $C = 150$, what is θ?

Use this space for summary and/or additional notes:

honors & AP®

6. **(M)** You are a golfer, and your ball is in a sand trap with a hill next to it. You need to hit your ball so that it goes over the hill to the green. If your ball is 10. m away from the side of the hill and the hill is 2.5 m high, what is the minimum angle above the horizontal that you need to hit the ball in order to just get it over the hill? (*Hint: draw a sketch.*)

7. **(M)** If a force of 80 N is applied at an angle of 40° above the horizontal, how much of that force is applied in the horizontal direction?

Use this space for summary and/or additional notes:

honors (not AP®)

The Laws of Sines & Cosines

Unit: Mathematics

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Use the law of sines to find the missing side or angle in a non-right triangle.
- Use the law of cosines to find the missing side in a non-right triangle.

Success Criteria:

- Sides and angles are correctly identified (opposite, adjacent, hypotenuse).
- Correct equation/law is chosen based on the relationship between the sides and angles.

Language Objectives:

- Describe the relationships between the sides and angles of a right triangle.

Tier 2 Vocabulary: opposite, adjacent

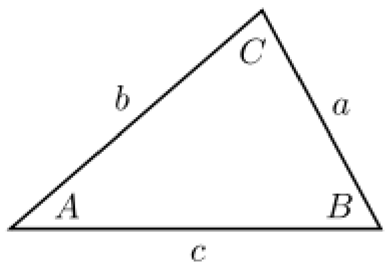
Notes:

The Law of Sines and the Law of Cosines are often needed to calculate distances or angles in physics problems that involve non-right triangles. ***Trigonometry involving non-right triangles is beyond the scope of this course.***

Any triangle has three degrees of freedom, which means it is necessary to specify a minimum of three pieces of information in order to describe the triangle fully.

The law of sines and the law of cosines each relate four quantities, meaning that if three of the quantities are specified, the fourth can be calculated.

Consider the following triangle *ABC*, with sides *a*, *b*, and *c*, and angles *A*, *B*, and *C*. Angle *A* has its vertex at point *A*, and side *a* is opposite vertex *A* (and hence is also opposite angle *A*).



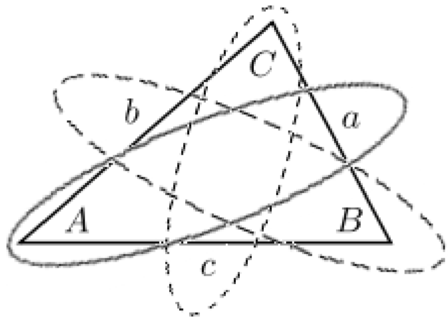
Use this space for summary and/or additional notes:

honors (not AP®)

The Law of Sines

The law of sines states that, for any triangle:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



The four quantities related by the law of sines are two sides and their opposite angles. This means that in order to the law of sines, you need to know one angle and the length of the opposite side, plus any other side or any other angle. From this information, you can find the unknown side or angle, and from there you can work your way around the triangle and calculate every side and every angle.

Use this space for summary and/or additional notes:

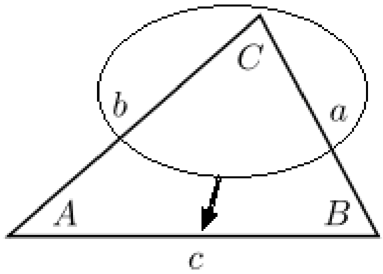
honors (not AP®)

The Law of Cosines

The law of cosines states that, for any triangle:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

You can use the law of cosines to find any angle or the length of the third side of a triangle as long as you know any two sides and the included angle:



You can also use the law of cosines to find one of the angles if you know the lengths of all three sides.

Remember that which sides and angles you choose to be *a*, *b* and *c*, and *A*, *B* and *C* are arbitrary. This means you can switch the labels around to fit your situation, as long as angle C is opposite side c and so on. Thus the law of cosines can also be written:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$

Notice that the Pythagorean Theorem is simply the law of cosines in the special case where $C = 90°$ (because $\cos 90° = 0$).

The law of cosines is algebraically less convenient than the law of sines, so a good strategy would be to use the law of sines whenever possible, reserving the law of cosines for situations when it is not possible to use the law of sines.

Use this space for summary and/or additional notes:

Vectors

Unit: Mathematics

MA Curriculum Frameworks (2016): SP5

AP® Physics 1 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Identify the magnitude and direction of a vector.
- Combine vectors graphically and calculate the magnitude and direction.

Success Criteria:

- Magnitude is calculated correctly (Pythagorean theorem).
- Direction is correct: angle (using trigonometry) or direction (*e.g.,* "south", "to the right", "in the negative direction", *etc.*)

Language Objectives:

- Explain what a vector is and what its parts are.

Tier 2 Vocabulary: magnitude, direction

Notes:

vector: a quantity that has a direction as well as a magnitude (value/quantity).

E.g., if you are walking $1\frac{\text{m}}{\text{s}}$ to the north, the magnitude is $1\frac{\text{m}}{\text{s}}$ and the direction is north.

scalar: a quantity that has a value/quantity but does not have a direction. (A scalar is what you think of as a "regular" number, including its unit.)

magnitude: the part of a vector that is not the direction (*i.e.,* the value including its units). If you have a force of 25 N to the east, the magnitude of the force is 25 N.

The mathematical operation of taking the magnitude of a vector is represented by two double vertical bars (like double absolute value bars) around the vector. For example, if $\vec{\boldsymbol{F}}$ is 25 N to the east, then $\|\vec{\boldsymbol{F}}\| = 25\,\text{N}$. However, to make typesetting easier, it is common to use regular absolute value bars instead, *e.g.,* $|\vec{F}| = 25\,\text{N}$.

resultant: a vector that is the result of a mathematical operation (such as the addition of two vectors).

Use this space for summary and/or additional notes:

Variables that represent vectors are traditionally typeset in ***bold Italics***. Vector variables may also optionally have an arrow above the letter:

$$\boldsymbol{J}, \vec{\boldsymbol{F}}, \boldsymbol{v}$$

Variables that represent scalars are traditionally typeset in *plain Italics:*

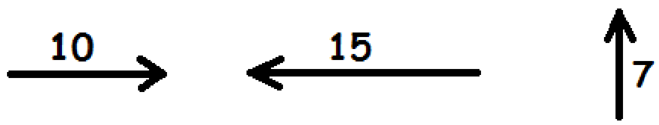
$$V,\ t,\ \lambda$$

Variable that represent only the magnitude of a vector (*e.g.,* in equations where the direction is not relevant) are typeset as if they were scalars:

For example, suppose $\vec{\boldsymbol{F}}$ is a vector representing a force of 25 N to the east. (Notice that the vector includes the magnitude or amount ***and*** the direction.)

If we needed a variable to represent only the magnitude of 25 N, we would use the variable *F*.

Vectors are represented graphically using arrows. The length of the arrow represents the magnitude of the vector, and the direction of the arrow represents the direction of the vector:



magnitude 10	magnitude 15	magnitude 7
direction: "to the right", 0°	direction: "to the left", +180°	direction: "up", +90°

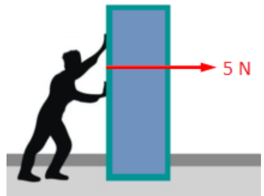
The negative of a vector is a vector with the same magnitude in the opposite direction:

Use this space for summary and/or additional notes:

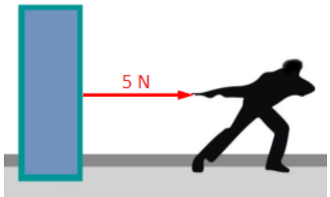
Translating Vectors

Vectors have a magnitude and direction but not a location. This means we can translate a vector (in the geometry sense, which means to move it without changing its size or orientation), and it's still the same vector quantity.

For example, consider a person pushing against a box with a force of +5 N (shown in this diagram as 5 N to the right):

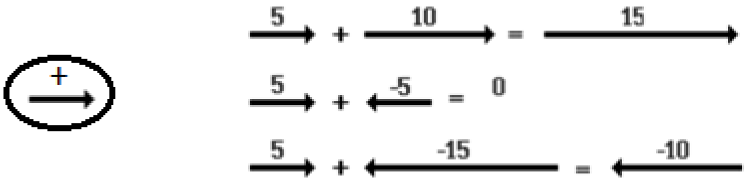


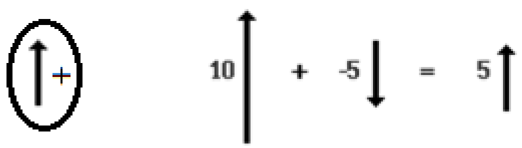
If the force is moved to the other side of the box, it's still +5 N, which means it's still the same vector.



Adding Vectors in One Dimension

If you are combining vectors in one dimension (*e.g,* horizontal), adding vectors is just adding positive and/or negative numbers:



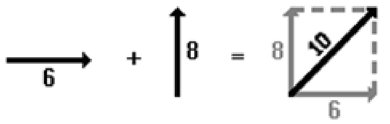


Use this space for summary and/or additional notes:

Adding vectors in Two Dimensions

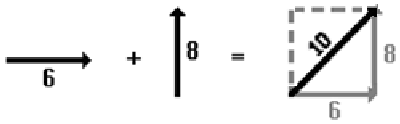
If the vectors are not in the same direction, we translate (slide) them until they meet, either tail-to-tail or tip-to-tail, and complete the parallelogram.

If the vectors are at right angles to one another, the parallelogram is a rectangle and we can use the Pythagorean theorem to find the magnitude of the resultant:



$(6^2 + 8^2 = 10^2)$

tail-to-tail



$(6^2 + 8^2 = 10^2)$

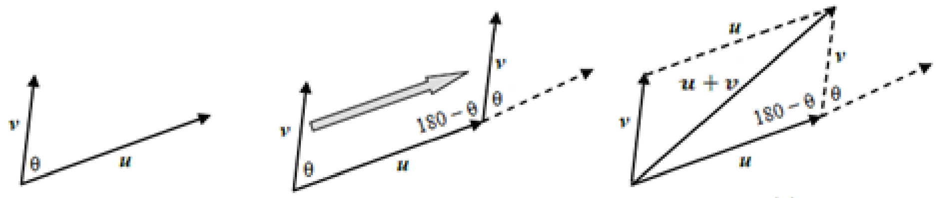
tip-to-tail

Note that the sum of these two vectors has a magnitude (length) of 10, not 14; "Adding" vectors means combining them using geometry.

The vector sum comes out the same whether you combine the vectors tail-to-tail or tip-to-tail. The decision of how to represent the vectors depends on the situation that you are modeling with them:

- Two forces pulling on the same object (think of two ropes connected to the same point) is best represented by drawing the vectors tail-to-tail.
- The displacement* of a walking path that starts in one direction and then turns is best represented by drawing the vectors tip-to-tail.

We would use exactly the same process to add vectors that are not perpendicular:

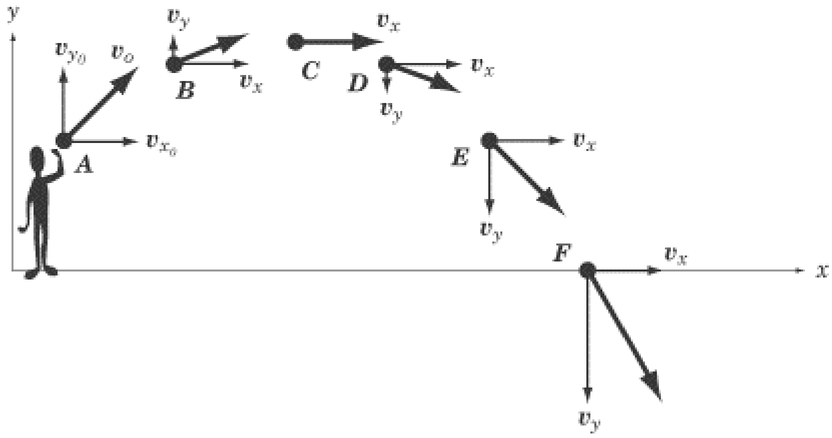


However, the trigonometry needed for these calculations is beyond the scope of this course.

* Displacement is a vector quantity that represents the straight-line distance from one point to another. Displacement is covered in the next unit, Kinematics in One Dimension.

Use this space for summary and/or additional notes:

One type of physics problem that commonly uses vectors is two-dimensional projectile motion. If the motion of the projectile is represented by a vector, $\vec{\boldsymbol{v}}$, at angle θ, the vector can be represented as the sum of a horizontal vector $\vec{\boldsymbol{v}}_x$ and a vertical vector $\vec{\boldsymbol{v}}_y$. This is useful because the horizontal vector $\vec{\boldsymbol{v}}_x$ gives us the component (portion) of the vector in the *x*-direction, and the vertical vector $\vec{\boldsymbol{v}}_y$ gives us the component of the vector in the *y*-direction.



Notice that $\vec{\boldsymbol{v}}_x$ remains constant, but $\vec{\boldsymbol{v}}_y$ changes (because of the effects of gravity).

Because perpendicular vectors do not affect each other, we can apply equations to the two directions separately.

For example, in projectile motion (which you will learn about in detail in the Projectile Motion topic starting on page 218), we usually use the equation $\vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}_o t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$, applying it separately in the *x*- and *y*-directions. This gives us two equations.

In the horizontal (*x*)-direction:

$$\vec{\boldsymbol{d}}_x = \vec{\boldsymbol{v}}_{o,x} t + \tfrac{1}{2}\cancelto{0}{\vec{\boldsymbol{a}}_x} t^2$$
$$\vec{\boldsymbol{d}}_x = \vec{\boldsymbol{v}}_x t$$

In the vertical (*y*)-direction:

$$\vec{\boldsymbol{d}}_y = \vec{\boldsymbol{v}}_{o,y} t + \tfrac{1}{2}\vec{\boldsymbol{a}}_y t^2$$
$$\vec{\boldsymbol{d}}_y = \vec{\boldsymbol{v}}_{o,y} t + \tfrac{1}{2}\vec{\boldsymbol{g}} t^2$$

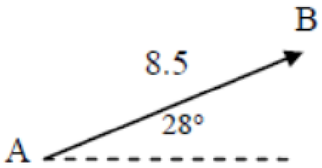
Note that each of the vector quantities ($\vec{\boldsymbol{d}}, \vec{\boldsymbol{v}}_o$ and $\vec{\boldsymbol{a}}$) has independent *x*- and *y*-components. For example, $\vec{\boldsymbol{v}}_{o,x}$ (the component of the initial velocity in the *x*-direction) is independent of $\vec{\boldsymbol{v}}_{o,y}$ (the component of the initial velocity in the *x*-direction). This means *we treat them as completely separate variables*, and we can solve for one without affecting the other.

Use this space for summary and/or additional notes:

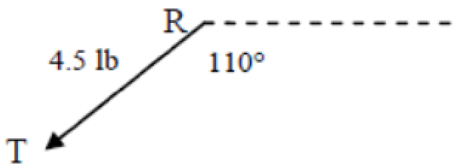
Homework Problems

Label the magnitude and direction (relative to horizontal) of each of the following:

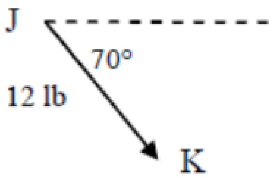
1. **(M)**



2. **(M)**

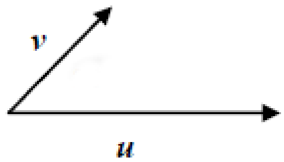


3. **(S)**

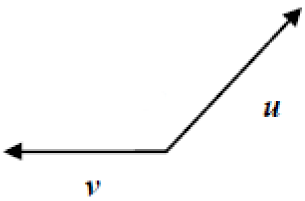


Sketch the resultant of each of the following.

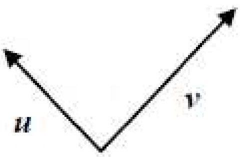
4. **(M)**



5. **(M)**

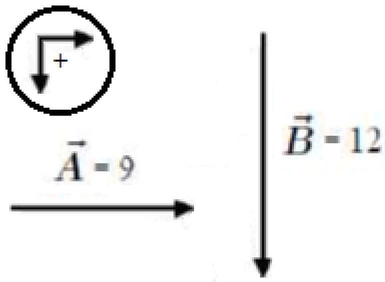


6. **(S)**



Use this space for summary and/or additional notes:

Consider the following vectors $\vec{A} \& \vec{B}$.



Vector $\vec{A}$ has a magnitude of 9 and its direction is the positive horizontal direction (to the right).

Vector $\vec{B}$ has a magnitude of 12 and its direction is the positive vertical direction (down).

7. **(M)** Sketch the resultant of $\vec{A}+\vec{B}$, and determine its magnitude and direction*.

8. **(S)** Sketch the resultant of $\vec{A}-\vec{B}$ (which is the same as $\vec{A}+(-\vec{B})$, and determine its magnitude and direction*.

* Finding the direction requires trigonometry. If your teacher skipped the right-angle trigonometry section, you should only find the magnitude.

Use this space for summary and/or additional notes:

Vectors *vs.* Scalars in Physics

Unit: Mathematics

MA Curriculum Frameworks (2016): SP5

AP® Physics 1 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Identify vector *vs.* scalar quantities in physics.

Success Criteria:

- Quantity is correctly identified as a vector or a scalar.

Language Objectives:

- Explain why some quantities have a direction and others do not.

Tier 2 Vocabulary: magnitude, direction

Notes:

In physics, most numbers represent quantities that can be measured or calculated from measurements. Most of the time, there is no concept of a "deficit" of a measured quantity. For example, quantities like mass, energy, and power can only be nonnegative, because in classical mechanics there is no such thing as "anti-mass," "anti-energy," or "anti-power."

However, vector quantities have a direction as well as a magnitude, and direction can be positive or negative.

A rule of thumb that works *most* of the time in a high school physics class is:

Scalar quantities. These are usually positive, with a few notable exceptions (*e.g.,* work and electric charge).

Vector quantities. Vectors have a direction associated with them. For one-dimensional vectors, the direction is conveyed by defining a direction to be "positive". Vectors in the positive direction are expressed as positive numbers, and vectors in the opposite (negative) direction are expressed as negative numbers.

In some cases, you will need to split a vector in two component vectors, one vector in the x-direction, and a separate vector in the y-direction. In these cases, you will need to choose which direction is positive and which direction is negative for *both* the x- and y-axes. Once you have done this, every vector quantity must be assigned a positive or negative value, according to the directions you have chosen.

Differences. The difference or change in a variable is indicated by the Greek letter Δ in front of the variable. Any difference can be positive or negative. However, note that a difference can either be a vector, indicating a change relative to the positive direction (*e.g.,* $\Delta\boldsymbol{x}$, which indicates a change in position), or scalar, indicating an increase or decrease (*e.g.,* ΔV, which indicates a change in volume).

Use this space for summary and/or additional notes:

Example:

Suppose you have a problem that involves throwing a ball straight upwards with a velocity of $15\,\frac{\text{m}}{\text{s}}$. Gravity is slowing the ball down with a downward acceleration of $10\,\frac{\text{m}}{\text{s}^2}$. You want to know how far the ball has traveled in 0.5 s.

Displacement, velocity, and acceleration are all vectors. The motion is happening in the y-direction, so we need to choose whether "up" or "down" is the positive direction. Suppose we choose "up" to be the positive direction. This means:

- When the ball is first thrown, it is moving upwards. This means its velocity is in the *positive* direction, so we would represent the initial velocity as $\vec{\mathbf{v}}_o = +15\,\frac{\text{m}}{\text{s}}$.
- Gravity is accelerating the ball downwards, which is the *negative* direction. We would therefore represent the acceleration as $\vec{\boldsymbol{a}} = -10\,\frac{\text{m}}{\text{s}^2}$.
- Time is a scalar quantity, so its value is +0.5 s.

If we had to substitute the numbers into the formula:

$$\vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}_o t + \tfrac{1}{2}\vec{\boldsymbol{a}}t^2$$

we would do so as follows:

$$\vec{\boldsymbol{d}} = (+15)(0.5) + (\tfrac{1}{2})(-10)(0.5)^2$$

and we would find out that $\vec{\boldsymbol{d}} = +6.25\,\text{m}$.

The answer is *positive*. Earlier, we defined positive as "up", so the answer tells us that the displacement is upwards from the starting point.

Use this space for summary and/or additional notes:

What if, instead, we had chosen "down" to be the positive direction?

- When the ball is first thrown, it is moving upwards. This means its velocity is now in the *negative* direction, so we would represent the initial velocity as $\vec{v}_o = -15\,\tfrac{\text{m}}{\text{s}}$.
- Gravity is accelerating the ball downwards, which is the *positive* direction. We would therefore represent the acceleration as $\vec{a} = +10\,\tfrac{\text{m}}{\text{s}^2}$.
- Time is a scalar quantity, so its value is +0.5 s.

If we had to substitute the numbers into the formula:

$$\vec{d} = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2$$

we would do so as follows:

$$\vec{d} = (-15)(0.5) + (\tfrac{1}{2})(10)(0.5)^2$$

and we would find out that $\vec{d} = -6.25\,\text{m}$.

The answer is *negative*. However, remember that we defined "down" to be positive, which means "up" is the negative direction. This means the displacement is *upwards* from the starting point, as before.

In any problem you solve, the choice of which direction is positive *vs.* negative is arbitrary. The only requirement is that *every vector quantity in the problem* needs to be consistent with your choice.

Use this space for summary and/or additional notes:

Vector Multiplication

Unit: Mathematics

MA Curriculum Frameworks (2016): SP5

AP® Physics 1 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Correctly use and interpret the symbols "•" and "×" when multiplying vectors.
- Finding the dot product & cross product of two vectors.

Success Criteria:

- Magnitudes and directions are correct.

Language Objectives:

- Explain how to interpret the symbols "•" and "×" when multiplying vectors.

Tier 2 Vocabulary: magnitude, direction, dot, cross

Notes:

With scalar (ordinary) numbers, there is only one way to multiply them, which you learned in elementary school. Vectors, however, can be multiplied in three different ways.

dot product: multiplication of two vectors that results in a scalar.

$$\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}} = C$$

cross product: multiplication of two vectors that results in a new vector.

$$\vec{\boldsymbol{I}} \times \vec{\boldsymbol{J}} = \vec{\boldsymbol{K}}$$

tensor product: multiplication of two vectors that results in a tensor. $\vec{\boldsymbol{A}} \otimes \vec{\boldsymbol{B}}$ is a matrix of vectors that results from multiplying the respective components of each of the two vectors. It describes the effect of each component of the vector on each component of every other vector in the array. Tensors are beyond the scope of a high school physics course.

Use this space for summary and/or additional notes:

Multiplying a Vector by a Scalar

Multiplying a vector by a scalar is like multiplying a variable by a number. The magnitude changes, but the direction does not. For example, in physics, displacement equals velocity times time:

$$\vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}t$$

Velocity is a vector; time is a scalar. The magnitude is the velocity times the time, and the direction of the displacement is the same as the direction of the velocity.

If the two vectors have opposite directions, the equation needs a negative sign. For example, the force applied by a spring equals the spring constant (a scalar quantity) times the displacement:

$$\vec{\boldsymbol{F}}_s = -k\vec{\boldsymbol{x}}$$

The negative sign in the equation signifies that the force applied by the spring is in the opposite direction from the displacement.

The Dot (Scalar) Product of Two Vectors

The scalar product of two vectors is called the "dot product". Dot product multiplication of vectors is represented with a dot:

$$\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}}^{*}$$

The dot product of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ is:

$$\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}} = AB\cos\theta$$

where *A* is the magnitude of $\vec{\boldsymbol{A}}$, *B* is the magnitude of $\vec{\boldsymbol{B}}$, and θ is the angle between the two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$.

For example, in physics, *work* (a scalar quantity) is the dot product of the vectors *force* and *displacement* (distance):

$$W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}} = Fd\cos\theta$$

* pronounced "A dot B"

Use this space for summary and/or additional notes:

The Cross (Vector) Product of Two Vectors

The vector product of two vectors is called the cross product. Cross product multiplication of vectors is represented with a multiplication sign:

$$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$$ *

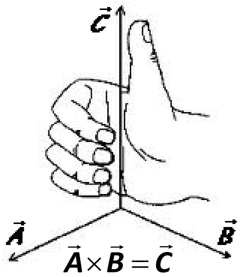
The magnitude of the cross product of vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ that have an angle of θ between them is given by the formula:

$$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = AB\sin\theta$$

The direction of the cross product is a little difficult to make sense out of. You can figure it out using the "right hand rule":

Position your right hand so that your fingers curl from the first vector to the second. Your thumb points in the direction of the resultant vector.

Note that this means that the resultant vectors for $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$ and $\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}}$ point in *opposite* directions, *i.e.,* the cross product of two vectors is *not* commutative!



$$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = \vec{\boldsymbol{C}}$$

$$\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}} = -\vec{\boldsymbol{C}}$$

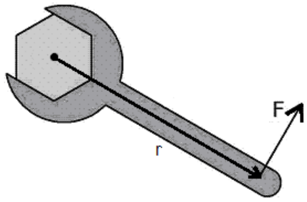
On a two-dimensional piece of paper, a vector coming toward you (out of the page) is denoted by a set of $\odot \quad \odot \quad \odot \quad \odot \quad \odot$ symbols, and a vector going away from you (into the page) is denoted by a set of $\otimes \quad \otimes \quad \otimes \quad \otimes \quad \otimes$ symbols.

Think of these symbols as representing an arrow inside a tube or pipe. The dot represents the tip of the arrow coming toward you, and the "X" represents the fletches (feathers) on the tail of the arrow going away from you.)

* pronounced "A cross B"

Use this space for summary and/or additional notes:

In physics, torque is a vector quantity that is derived by a cross product.



The torque produced by a force $\vec{\boldsymbol{F}}$ acting at a radius $\vec{\boldsymbol{r}}$ is given by the equation:

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} = rF\sin\theta$$

Because the direction of the force is usually perpendicular to the displacement, it is usually true that $\sin\theta = \sin 90° = 1$. This means the magnitude $rF\ \sin\theta = rF(1) = rF.$ Using the right-hand rule, we determine that the *direction* of the resultant torque vector is coming out of the page.

(The force generated by the interaction between charges and magnetic fields, a topic covered in AP® Physics 2, is also a cross product.)

Thus, if you are tightening or loosening a nut or bolt that has right-handed (standard) thread, the torque vector will be in the direction that the nut or bolt moves.

Vector Jokes

Now that you understand vectors, here are some bad vector jokes:

Q: What do you get when you cross an elephant with a bunch of grapes?

A:   $\sin\theta$

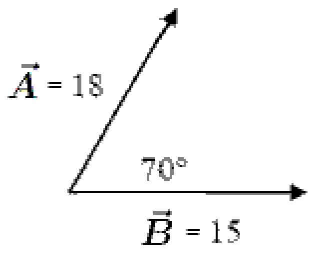
Q: What do you get when you cross an elephant with a mountain climber?

A: You can't do that! A mountain climber is a scalar ("scaler," meaning someone who scales a mountain).

Use this space for summary and/or additional notes:

Homework Problems

For the following vectors $\vec{A} \& \vec{B}$:



1. **(M)** Determine $\vec{A} \bullet \vec{B}$

2. **(M)** Determine $\vec{A} \times \vec{B}$ (both magnitude and direction)

Use this space for summary and/or additional notes:

AP®

Degrees, Radians and Revolutions

Unit: Mathematics

MA Curriculum Frameworks (2016): S&E P5

AP® Physics 1 Learning Objectives: SP 2.2

Knowledge/Understanding:

- Express angles and arc length in degrees, radians, and full revolutions.

Skills:

- Convert between degrees, radians and revolutions.

Language Objectives:

- Understand and correctly use the terms "degree," "radian," and "revolution".

Tier 2 Vocabulary: degree, revolution

Notes:

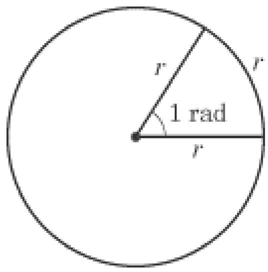
degree: an angle equal to $\frac{1}{360}$ of a full circle. A full circle is therefore 360°.

revolution: a rotation of exactly one full circle (360°) by an object.

radian: the angle that results in an arc length equal to the radius of a circle. *I.e.*, one "radius" of the way around the circle. Because the distance all the way around the circle is 2π times the radius, a full circle (or one rotation) is therefore 2π radians.

This means that $1\,\text{radian} = \frac{1}{2\pi}$ of a circle $= \left(\frac{1}{2\pi}\right)(360°) \approx 57.3°$

We are used to measuring angles in degrees. However, trigonometry functions are often more convenient if we express the angle in radians:

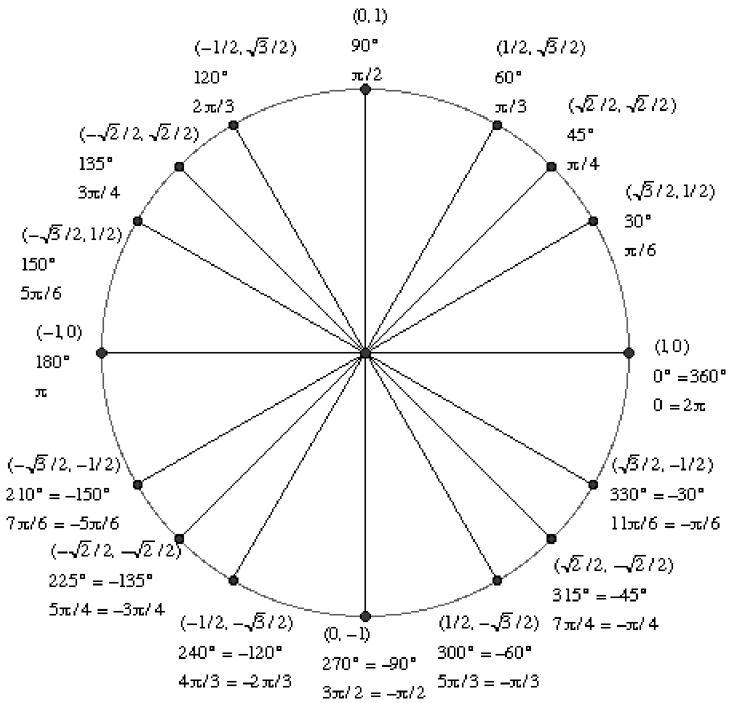


This is often convenient because if we express the angle in radians, the angle is equal to the arc length (distance traveled around the circle) times the radius, which makes much easier to switch back and forth between the two quantities.

Use this space for summary and/or additional notes:

AP®

On the following unit circle (a circle with a radius of 1), several of the key angles around the circle are marked in radians, degrees, and the (*x,y*) coördinates of the corresponding point around the circle.



In each case, the angle in radians is equal to the distance traveled around the circle, starting from the point (1,0).

It is particularly useful to memorize the following:

Degrees	0°	90°	180°	270°	360°
Rotations	0	¼	½	¾	1
Radians	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

In general, you can convert between degrees and radians using the conversion factor $360^\circ = 2\pi \text{ rad}$. For example, to convert 225° to radians, we would do:

$$\frac{225^\circ}{1} \times \frac{2\pi}{360^\circ} = 1.25\pi \text{ radians}$$

Note that because a radian is arc length divided by radius (distance divided by distance), it is a dimensionless quantity, *i.e.,* a quantity that has no unit. This is convenient because it means you never have to convert radians from one unit to another.

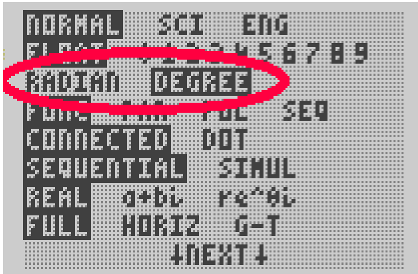
Use this space for summary and/or additional notes:

AP®

Precalculus classes often emphasize learning to convert between degrees and radians. However, in practice, these conversions are rarely, if ever necessary. Expressing angles in radians is useful in rotational problems in physics because it combines all of the quantities that depend on radius into a single variable, and avoids the need to use degrees at all. If a conversion is necessary,

In physics, you will usually use degrees for linear (Cartesian) problems, and radians for rotational problems. For this reason, when using trigonometry functions it is important to make sure your calculator mode is set correctly for degrees or radians, as appropriate to each problem:

TI-30 scientific calculator



TI-83 or later graphing calculator

If you switch your calculator between degrees and radians, don't forget that this will affect math class as well as physics!

Use this space for summary and/or additional notes:

AP®

Polar, Cylindrical & Spherical Coördinates

Unit: Mathematics

MA Curriculum Frameworks (2016): S&E P5

AP® Physics 1 Learning Objectives: SP 2.2

Knowledge/Understanding:

- Express a position in Cartesian, polar, cylindrical, or spherical coördinates.

Skills:

- Convert between Cartesian coördinates and polar, cylindrical and/or spherical coördinates.

Language Objectives:

- Accurately describe and apply the concepts described in this section using appropriate academic language.

Tier 2 Vocabulary: polar

Notes:

In your math classes so far, you have expressed the location of a point using Cartesian coördinates—either (*x, y*) in two dimensions or (*x, y, z*) in three dimensions.

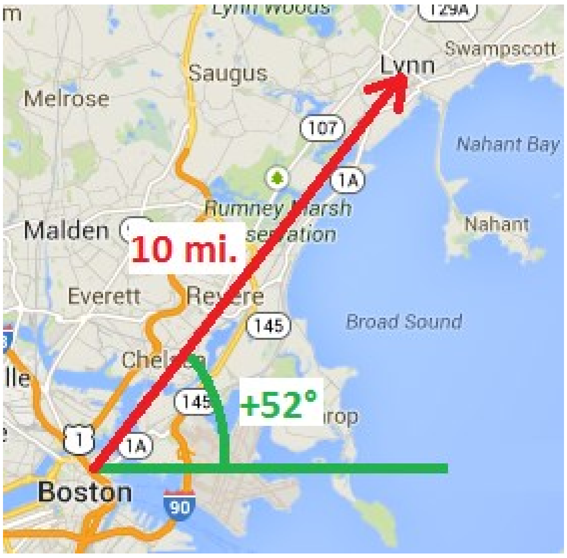
Cartesian coördinates: (or rectangular coördinates): a two- or three-dimensional coordinate system that specifies locations by separate distances from each of two or three axes (lines). These axes are labeled *x*, *y*, and *z*, and a point is specified using its distance from each axis, in the form (*x, y*) or (*x, y, z*).

Use this space for summary and/or additional notes:

AP®

<u>polar coördinates</u>: a two-dimensional coördinate system that specifies locations by their distance from the origin (radius) and angle from some reference direction. The radius is *r*, and the angle is θ (the Greek letter "theta"). A point is specified using the distance and angle, in the form (r, θ).

For example, if we say that the city of Lynn, Massachusetts is 10 miles from Boston, at heading of 52° north of due east, we are using polar coördinates:



(Note: cardinal or "compass" direction is traditionally specified with North at 0° and 360°, and clockwise as the positive direction, meaning that East is 90°, South is 180°, West is 270°. This means that the compass heading from Boston to Lynn would be 38° to the East of true North. However, in this class we will specify angles as mathematicians do, with 0° indicating the direction of the positive *x*-axis.)

<u>cylindrical coördinates</u>: a three-dimensional coördinate system that specifies locations by distance from the origin (radius), angle from some reference direction, and height above the origin. The radius is *r*, the angle is θ, and the height is *z*. A point is specified using the distance and angle, and height in the form (r, θ, z).

Use this space for summary and/or additional notes:

AP®

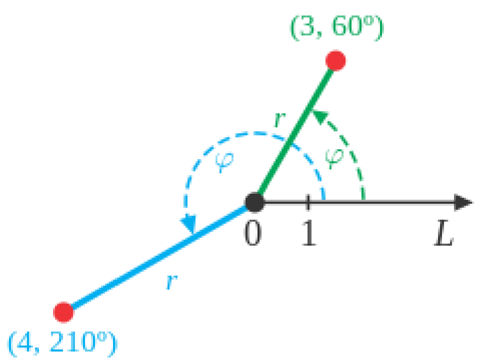
spherical coördinates: a three-dimensional coördinate system that specifies locations by distance from the origin (radius), and two separate angles, one from some horizontal reference direction and the other from some vertical reference direction. The radius is labeled *r*, the horizontal angle is θ, and the vertical angle is ϕ (the Greek letter "phi"). A point is specified using the distance and angle, and height in the form (r, θ, ϕ).

When we specify a point on the Earth using longitude and latitude, we are using spherical coördinates. The distance is assumed to be the radius of the Earth (because the interesting points are on the surface), the longitude is θ, and the latitude is ϕ. (Note, however, that latitude on the Earth is measured up from the equator. In physics, we generally use the convention that $\phi = 0°$ is straight upward, meaning ϕ will indicate the angle *downward* from the "North pole".)

In AP® Physics 1, the problems we will see are one- or two-dimensional. For each problem, we will use the simplest coördinate system that applies to the problem: Cartesian (x, y) coördinates for linear problems and polar (r, θ) coördinates for problems that involve rotation.

Note that while mathematicians almost always prefer to express angles in radians, physicists typically usually use degrees for linear problems and radians for rotational problems.

The following example shows the locations of the points (3, 60°) and (4, 210°) using polar coördinates:



Use this space for summary and/or additional notes:

AP®

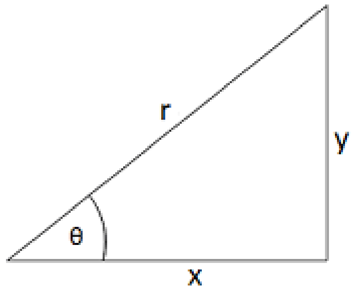
Converting Between Cartesian and Polar Coördinates

If vectors make sense to you, you can simply think of polar coördinates as the magnitude (r) and direction (θ) of a vector.

Converting from Cartesian to Polar Coördinates

If you know the *x*- and *y*-coördinates of a point, the radius (r) is simply the distance from the origin to the point. You can calculate r from x and y using the distance formula:

$$r = \sqrt{x^2 + y^2}$$



The angle comes from trigonometry:

$\tan\theta = \frac{y}{x}$, which means $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Sample Problem:

Q: Convert the point (5,12) to polar coördinates.

A: $r = \sqrt{x^2 + y^2}$

$r = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{12}{5}\right) = \tan^{-1}(2.4) = 67.4° = 1.18\,\text{rad}$

$\boxed{(13, 67.4°)}$ or $\boxed{(13, 1.18\text{ rad})}$

Converting from Polar to Cartesian Coördinates

As we saw in our review of trigonometry, if you know r and θ, then $x = r\cos\theta$ and $y = r\sin\theta$.

Sample Problem:

Q: Convert the point (8, 25°) to Cartesian coördinates.

A: $x = 8\cos(25°) = (8)(0.906) = 7.25$

$y = 8\sin(25°) = (8)(0.423) = 3.38$

$\boxed{(7.25, 3.38)}$

In practice, you will rarely need to convert between the two coördinate systems. The reason for using polar coördinates in a rotating system is because the quantities of interest are based on the rotational angle and the distance from the center of rotation. Using polar coördinates for these problems *avoids* the need to use trigonometry to convert between systems.

Use this space for summary and/or additional notes:

Introduction: Kinematics (Motion) in One Dimension

Unit: Kinematics (Motion) in One Dimension

Topics covered in this chapter:

In this chapter, you will study how things move and how the relevant quantities are related.

- *Linear Motion, Speed & Velocity* and *Acceleration* deal with understanding and calculating the velocity (change in position) and acceleration (change in velocity) of an object, and with representing and interpreting graphs involving these quantities.
- *Dot Diagrams* deals with a representation of motion using a series of dots that show the location of an object at equal time intervals.
- *Newton's Equations of Motion* deals with solving motion problems algebraically, using equations.
- *Motion Graphs* deals with creating and interpreting graphs of position *vs.* time and velocity *vs.* time.

Some of the challenging tasks include identifying quantities from their units, choosing the equation that relates the quantities of interest, and keeping track of positive and negative directions when working with vector quantities.

Note to Teachers

In most physics textbooks, Motion Graphs are presented before Newton's Equations of Motion because the graphs are visual, and the intuitive understanding derived from graphs can then be applied to the equations. However, in recent years, many students have a weak understanding of graphs. I have found that reversing the usual order enables students to use their understanding of algebra to better understand the graphs. This is especially true in this text because students have already learned most of the relevant concepts in the Word Problems topic in the Mathematics chapter.

Use this space for summary and/or additional notes:

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-10(MA). Use ~~free-body force diagrams~~, algebraic expressions, and Newton's laws of motion to predict changes to velocity and acceleration for an object moving in one dimension in various situations.

AP®

AP® Physics 1 Learning Objectives:

3.A.1.1: The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **[SP 1.5, 2.1, 2.2]**

3.A.1.2: The student is able to design an experimental investigation of the motion of an object. **[SP 4.2]**

3.A.1.3: The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **[SP 5.1]**

Topics from this chapter assessed on the SAT Physics Subject Test:

Kinematics, such as velocity, acceleration, motion in one dimension, and motion of projectiles

1. Displacement
2. Speed, velocity and acceleration
3. One-dimensional motion with uniform acceleration
4. Kinematics with graphs

Skills learned & applied in this chapter:

- Choosing from a set of equations based on the quantities present.
- Working with vector quantities.
- Relating the slope of a graph and the area under a graph to equations.
- Using graphs to represent and calculate quantities.

Use this space for summary and/or additional notes:

Linear Motion, Speed & Velocity

Unit: Kinematics (Motion) in One Dimension

MA Curriculum Frameworks (2016): HS-PS2-10(MA)

AP® Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3

Mastery Objective(s): (Students will be able to…)

- Correctly describe the position, speed, velocity, and acceleration of an object based on a description of its motion (or lack thereof).

Success Criteria:

- Description of vector quantities (position, velocity & acceleration) indicates both magnitude (amount) and direction.
- Description of scalar quantities does not include direction.

Language Objectives:

- Explain the Tier 2 words "position," "distance," "displacement," "speed," "velocity," and "acceleration" and how their usage in physics is different from the vernacular.
- Explain why we do not use the word "deceleration" in physics.

Tier 2 Vocabulary: position, speed, velocity, acceleration, direction

Labs, Activities & Demonstrations:

- Walk in the positive and negative directions (with positive or negative velocity).
- Walk and change direction to show distance *vs.* displacement.

Notes:

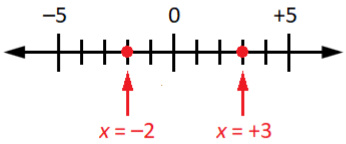
coördinate system: a framework for describing an object's ***position*** (location), based on its distance (in one or more directions) from a specifically-defined point (the ***origin***). (You should remember these terms from math.)

direction: which way an object is oriented or moving within its coördinate system. Note that direction can be positive or negative.

Use this space for summary and/or additional notes:

position ($\vec{x}$): [vector*] the location of an object relative to the origin (zero point) of its coördinate system. We will consider position to be a zero-dimensional vector.

If we are representing position in only one dimension (*e.g.,* along the *x*-axis), we represent it as a positive or negative number, which means position can be positive or negative.

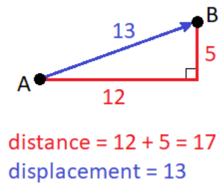


If we are representing position in two dimensions, we can use either cartesian coördinates—the position in each direction (*x*,*y*)—or polar coordinates—the distance from the origin and an angle (*r*,*θ*). See Polar, Cylindrical & Spherical Coördinates starting on page 166 for more information.

distance (*d*): [scalar] the length of the path that an object took when it moved. Distance does not depend on direction and is always positive or zero.

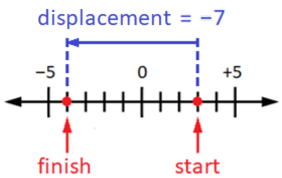
displacement ($\vec{d}$ or $\Delta\vec{x}$): [vector] how far an object's current position is from its starting position ("initial position"). If an object travels in a straight line and does not reverse direction, distance and the magnitude of the displacement will be the same. However, if the object changes direction, then the distance traveled will be greater than the displacement.

For example, suppose an object travelled a distance of 12 units to the east, and then 5 units to the north. The object has travelled a total *distance* of 17 units, However, by the Pythagorean theorem, the object's *displacement* is 13 units from where it started.



Note that because physics problems are usually described in the vernacular, problems will often use the word "distance" when what is actually meant is displacement.

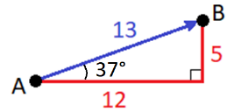
As with position, if motion is in only one dimension (*e.g.,* only along the x-axis), we define one direction along that dimension to be the positive direction and the opposite direction to be negative. Thus displacement will be positive, negative, or zero. For example, the object at the right's displacement is −7 units (7 units in the negative direction) from where it started.



* Position is a zero-dimensional vector. An object's position is a location that, like other vector quantities, can be positive or negative and is dependent on the coördinate system chosen.

Use this space for summary and/or additional notes:

If motion is in two dimensions, the displacement is usually described in polar coördinates, using the straight-line distance from start to finish and the angle from some reference direction (*e.g.,* the *x*-axis). Using the above example, we would describe the object's displacement as 13 at an angle of 37° above horizontal.



rate: the change in any quantity over a specific period of time.

motion: when an object's *position* is changing over time.

speed: [scalar] the rate at which an object is moving at an instant in time. Speed does not depend on direction, and is always positive or zero.

An object's (instantaneous) speed is the distance that it would travel in a given amount of time, divided by that amount of time.

If the object's speed is constant, then its $\text{average speed} = \dfrac{\text{distance}}{\text{time}}$

velocity: ($\vec{\boldsymbol{v}}$) [vector] the rate of change of an object's position (its displacement) over a given period of time. Because velocity is a *vector*, it has a *direction* as well as a *magnitude*; think of velocity as the vector equivalent of speed.

An object's instantaneous velocity is the same as its instantaneous speed, with the addition of some way to indicate the direction it is moving.

We use $\vec{\boldsymbol{v}}$ without a subscript to indicate an object's instantaneous velocity. If the object's velocity is changing, we use $\vec{\boldsymbol{v}}_o$ for the initial velocity (the subscript "0" means "at time zero"), and $\vec{\boldsymbol{v}}$ for the final velocity.

If an object is moving in one dimension and does not change direction, then (the magnitude of) its average velocity will be the same as its average speed.

As with average speed, an object's $\text{average velocity} = \dfrac{\text{displacement}}{\text{time}}$

$$\vec{\boldsymbol{v}}_{ave.} = \frac{\vec{\boldsymbol{d}}}{t} = \frac{\Delta \boldsymbol{x}}{t}$$

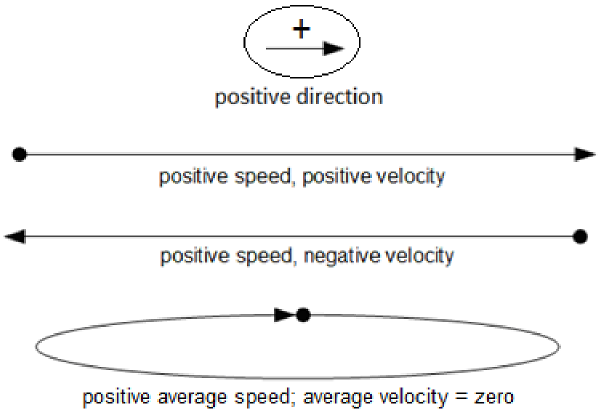
Note that because the average velocity is neither the initial nor final velocity, we need to use a descriptive subscript to indicate what sort of velocity it is.

Note that if the direction changes, the object's average speed will be greater than its average velocity, because the distance traveled is greater than the object's displacement.

Use this space for summary and/or additional notes:

As with position and displacement, if velocity is in one dimension (e.g., along the x-axis), we use positive and negative numbers to indicate the direction. A positive instantaneous velocity means the object is moving in the positive direction; a negative instantaneous velocity means the object is moving in the negative direction; an instantaneous velocity of zero means the object is "at rest" (not moving).

If an object returns to its starting point, its average velocity is zero, because its displacement is zero.



In the MKS system, speed and velocity are measured in meters per second.

$$1\,\frac{\text{m}}{\text{s}} \approx 2.24\,\frac{\text{mi.}}{\text{hr.}}$$

<u>uniform motion</u>: motion at a constant velocity (*i.e.,* constant speed and direction)

Use this space for summary and/or additional notes:

Variables Used to Describe Linear Motion

Variable	Quantity	Unit	Variable	Quantity	Unit
$\vec{x}$	(final) position	m	$\vec{v}$	(final) velocity	$\frac{m}{s}$
$\vec{x}_o$	initial (starting) position	m	$\vec{v}_o$	initial (starting) velocity	$\frac{m}{s}$
d	distance	m	$\vec{v}_{ave.}$	average velocity	$\frac{m}{s}$
$\vec{d}$, $\Delta\vec{x}$	displacement	m	$\vec{a}$	acceleration	$\frac{m}{s^2}$
$\vec{h}$	height	m	$\vec{g}$	acceleration due to gravity	$\frac{m}{s^2}$
			t	time	s

By convention, physicists use the variable $\vec{g}$ to mean acceleration due to gravity of an object in free fall, and $\vec{a}$ to mean acceleration under any other conditions.

The average velocity of an object is its displacement (change in position) divided by the elapsed time.

$$\vec{v}_{ave} = \frac{\vec{d}}{t}$$

The acceleration of an object is its change in velocity divided by the elapsed time. (Acceleration will be covered in detail in the next section.)

$$\vec{a} = \frac{\Delta\vec{v}}{t}$$

Signs of Vector Quantities

As described above, for motion in one dimension, the sign of a vector (positive or negative) is used to indicate its direction.

- Displacement is positive if the change in position of object in question is toward the positive direction, and negative if the change in position is toward the negative direction.
- Velocity is positive if the object is moving in the positive direction, and negative if the object is moving in the negative direction.
- Acceleration is positive if the *change* in velocity is positive (*i.e.,* if the velocity is becoming more positive or less negative). Acceleration is negative if the *change* in velocity is negative (*i.e.,* if the velocity is becoming less positive or more negative).

Use this space for summary and/or additional notes:

Sample Problems

Q: A car travels 1200 m in 60 seconds. What is its average velocity?

A: $v_{ave.} = \dfrac{d}{t} = \dfrac{1200\text{ m}}{60\text{ s}} = 20\ \frac{\text{m}}{\text{s}}$

Q: A person walks 320 m at an average velocity of $1.25\ \frac{\text{m}}{\text{s}}$. How long did it take?

A: "How long" means what length of *__time__*.

$$\vec{\boldsymbol{v}}_{ave.} = \frac{\vec{\boldsymbol{d}}}{t} \qquad (\vec{\boldsymbol{v}}_{ave.})t = \vec{\boldsymbol{d}} \qquad t = \frac{\vec{\boldsymbol{d}}}{\vec{\boldsymbol{v}}_{ave.}} = \frac{320}{1.25} = 256\ s$$

Notice that when solving for a variable in the denominator, it is safest to do it in two steps—first multiply both sides by the denominator and then divide to isolate the variable in a second step. Many students attempt to rearrange the variables in one step, often with little success.

Use this space for summary and/or additional notes:

Linear Acceleration

Unit: Kinematics (Motion) in One Dimension

MA Curriculum Frameworks (2016): HS-PS2-10(MA)

AP® Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3

Mastery Objective(s): (Students will be able to…)

- Calculate acceleration given initial & final velocity and time.
- Describe the motion of an object that is accelerating.

Success Criteria:

- Calculations for acceleration have the correct value, correct direction (sign), and correct units.
- Descriptions of motion account for the starting and final velocity and any changes of direction.

Language Objectives:

- Correctly use the term "acceleration" the way it is used in physics. Translate the vernacular term "deceleration" into a physics-appropriate description.

Tier 2 Vocabulary: velocity, acceleration, direction

Lab Activities & Demonstrations:

- Walk with different combinations of positive/negative velocity and positive/negative acceleration.
- Fan cart, especially to show the cart moving in one direction but accelerating in the opposite direction.
- Have students make two strings of beads, one spaced at equal distances and the other spaced so they land at equal time intervals.

Notes:

<u>acceleration</u> ($\vec{a}$): [vector] a change in velocity; the rate of change of velocity.

$$\vec{a} = \frac{\Delta\vec{v}}{t} = \frac{\vec{v} - \vec{v}_o}{t}$$

The MKS unit for acceleration is $\frac{\text{m}}{\text{s}^2}$. This is because $\Delta\vec{v}$ has units $\frac{\text{m}}{\text{s}}$, which means $\vec{a} = \frac{\Delta\vec{v}}{t}$ has units $\frac{\text{m}/\text{s}}{\text{s}} = \frac{\text{m}}{\text{s}} \cdot \frac{1}{\text{s}} = \frac{\text{m}}{\text{s}^2}$*.

<u>uniform acceleration</u>: constant acceleration; a constant rate of change of velocity.

Because this is an algebra-based course, acceleration will be assumed to be uniform in all of the problems in this course that involve acceleration.

* The unit for acceleration is sometimes described as "meters per second per second".

Use this space for summary and/or additional notes:

In the vernacular, we use the term "acceleration" to mean "speeding up," and "deceleration" to mean "slowing down." In physics, we always use the term "acceleration". If an object is moving (in one dimension) in the positive direction, then ***positive acceleration*** means "speeding up" and ***negative acceleration*** means "slowing down".

Note that acceleration is a vector quantity, which means it has a direction. This means that acceleration is _any_ change in velocity, including a change in speed or a change in direction. There is a popular joke in which a physics student is taking a driving lesson. The instructor says, "Apply the accelerator." The physics student replies, "Which one? I've got three!"

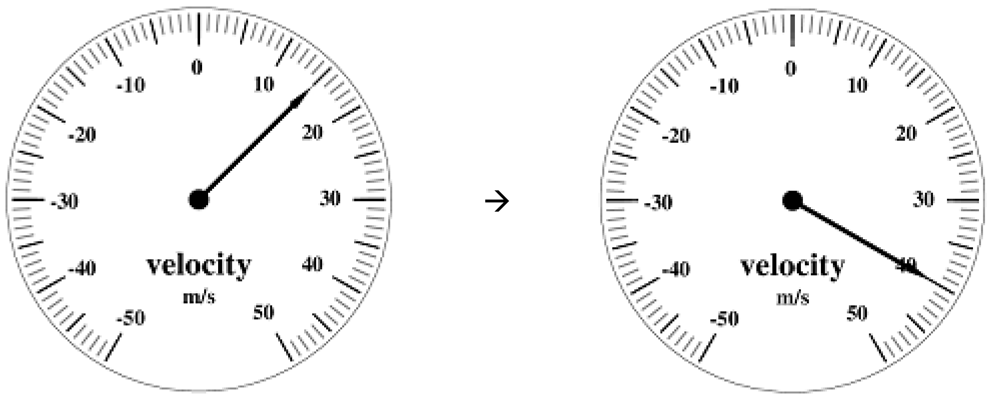
Note that if an object is moving in the negative direction, then the sign of acceleration is reversed. Positive acceleration for an object moving in the negative direction would mean that the object is actually slowing down, and negative acceleration for an object moving in the negative direction would mean that the object is actually speeding up.

Use this space for summary and/or additional notes:

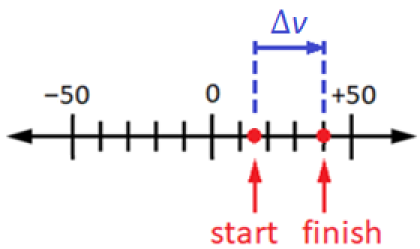
Calculating Acceleration

Suppose that instead of a speedometer, your car has a velocity meter, which displays a positive velocity when the car is going in the positive direction (forward) and a negative velocity when it is going in the negative direction (backward).

The following velocity meters show a car that starts out with a velocity of $+15\,\frac{\text{m}}{\text{s}}$ and accelerates to $+40\,\frac{\text{m}}{\text{s}}$. Suppose this acceleration happened over a time interval of 5 s.



The car's speed is faster at the end ($40\,\frac{\text{m}}{\text{s}}$ *vs.* $15\,\frac{\text{m}}{\text{s}}$), and it is traveling in the positive direction the entire time. The change in velocity ($\Delta\vec{\boldsymbol{v}}$) is $+40-(+15)=+25\,\frac{\text{m}}{\text{s}}$.



The acceleration is therefore $\vec{\boldsymbol{a}}=\frac{\Delta\vec{\boldsymbol{v}}}{t}=\frac{+40-(+15)}{5}=\frac{+25}{5}=+5\,\frac{\text{m}}{\text{s}^2}$.*

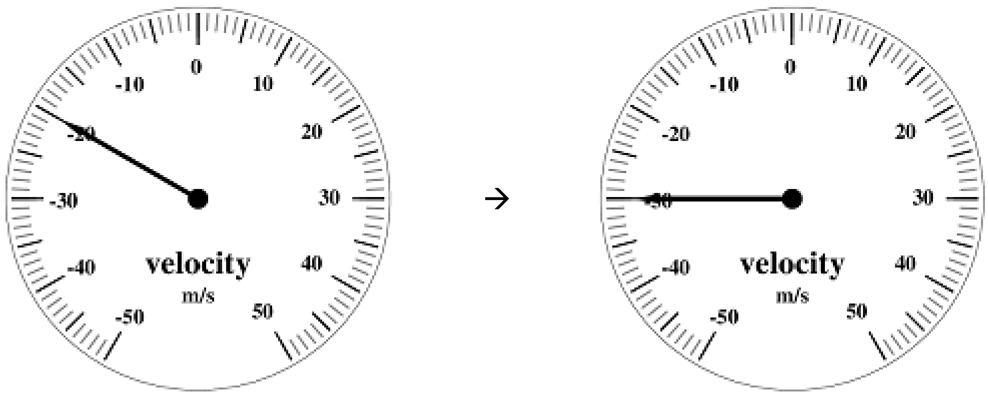
If the acceleration (rate of change of velocity) is 5 meters per second per second, then the velocity after each second would be:

time (s)	0	1	2	3	4	5
velocity $\left(\frac{\text{m}}{\text{s}}\right)$	+15	+20	+25	+30	+35	+40

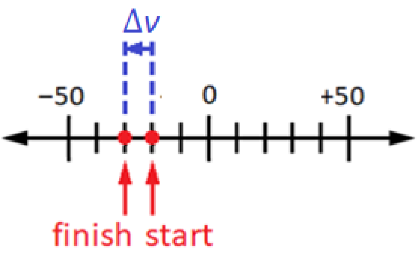
* Note that $5\,\frac{\text{m}}{\text{s}^2}$ is approximately the acceleration of a commercial jet during takeoff.

Use this space for summary and/or additional notes:

The following velocity meters show a car that starts out with a velocity of $-20\,\frac{\text{m}}{\text{s}}$ and accelerates to $-30\,\frac{\text{m}}{\text{s}}$. Suppose this acceleration also happened over a time interval of 5 s.



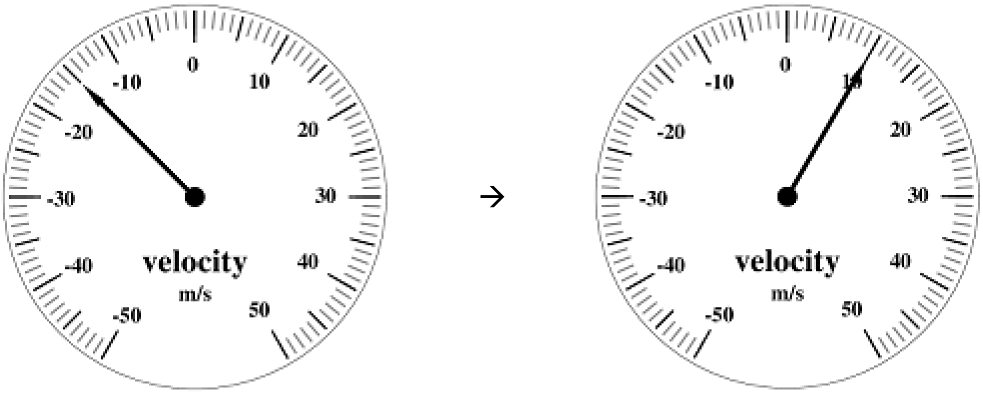
In this case, the car's speed is faster at the end ($30\,\frac{\text{m}}{\text{s}}$ *vs.* $20\,\frac{\text{m}}{\text{s}}$), but it is traveling in the negative direction the entire time. The change in velocity ($\Delta\vec{\boldsymbol{v}}$) is $-30-(-20)=-10\,\frac{\text{m}}{\text{s}}$. This means that although the car is speeding up, because it is speeding up in the negative direction, the *trend* is toward a more negative velocity.



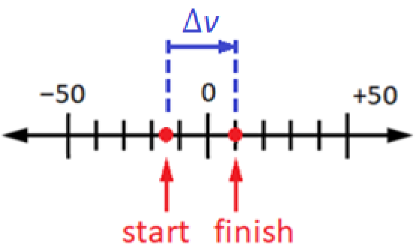
The acceleration is therefore $\vec{\boldsymbol{a}}=\dfrac{\Delta\vec{\boldsymbol{v}}}{t}=\dfrac{-30-(-20)}{5}=\dfrac{-10}{5}=-2\,\frac{\text{m}}{\text{s}^2}$

Use this space for summary and/or additional notes:

Finally, the following velocity meters show a car that starts out with a velocity of $-15\frac{\text{m}}{\text{s}}$ and accelerates to $+10\frac{\text{m}}{\text{s}}$. Suppose this acceleration also happened over a time interval of 5 s.



We calculate the change in velocity and the acceleration as before.



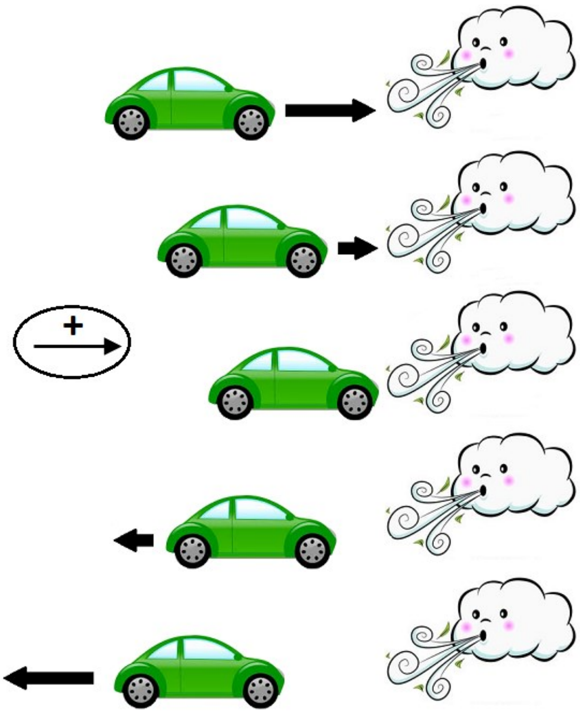
The acceleration is $\vec{a} = \frac{\Delta\vec{v}}{t} = \frac{+10-(-15)}{5} = \frac{+25}{5} = +5\frac{\text{m}}{\text{s}^2}$. This makes sense, because the velocity is continuously *trending* toward the positive direction.

However, a description of the car's motion is more complicated. The car starts out going $15\frac{\text{m}}{\text{s}}$ in the negative direction. During the first 3 s, the car slows down from $15\frac{\text{m}}{\text{s}}$ until it stops ($\vec{v} = 0$). Then, during the final 2 s it speeds up in the positive direction from rest ($\vec{v} = 0$) to a speed of $10\frac{\text{m}}{\text{s}}$.

Use this space for summary and/or additional notes:

Another Way to Visualize Acceleration

As we will study in detail later in this course, acceleration is caused by a (net) force on an object. A helpful visualization is to imagine that acceleration is caused by a strong wind exerting a force on an object.



In the above picture, the car starts out moving in the positive direction (to the right). Acceleration (represented by the wind) is in the negative direction (to the left). The negative acceleration causes the car to slow down and stop, and then to start moving and speed up in the negative direction (to the left).

Check for Understanding

A car starts out with a velocity of $+30\,\frac{\text{m}}{\text{s}}$. After 10 s, its velocity is $-10\,\frac{\text{m}}{\text{s}}$.

1. Calculate the car's acceleration.

2. Describe the motion of the car.

Use this space for summary and/or additional notes:

Free Fall (Acceleration Caused by Gravity)

The gravitational force is an attraction between objects that have mass.

free fall: when an object is freely accelerating toward the center of the Earth (or some other object with a very large mass) because of the effects of gravity, and the effects of other forces are negligible.

Objects in free fall on Earth accelerate downward at a rate of approximately $10\,\frac{\text{m}}{\text{s}^2} \approx 32\,\frac{\text{ft.}}{\text{s}^2}$. (The actual number is approximately $9.807\,\frac{\text{m}}{\text{s}^2}$ at sea level near the surface of the Earth. In this course we will usually round it to $10\,\frac{\text{m}}{\text{s}^2}$ so the calculations don't get in the way of understanding the physics.)

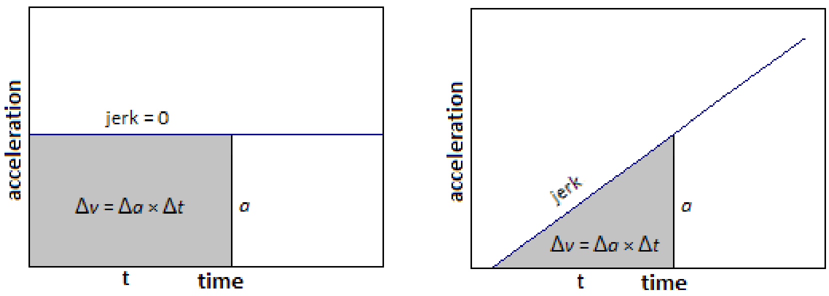
Note that an object going down a ramp is not in free fall even though gravity is the force that caused the object to accelerate. The object's motion is constrained by the ramp and it is not free to fall straight down.

Acceleration Notes

- Whether acceleration is positive or negative is based on the *trend* of the velocity (changing *toward* positive *vs.* changing *toward* negative).
- An object can have a positive velocity and a negative acceleration at the same time, or *vice versa*.
- The sign (positive or negative) of an object's velocity is the direction the object is moving. If the sign of the velocity changes (from positive to negative or negative to positive), the change indicates that the object's motion has changed directions.
- ***An object can be accelerating even when it has a velocity of zero.*** For example, if you throw a ball upward, it goes up to its maximum height and then falls back to the ground. At the instant when the ball is at its maximum height, its velocity is zero, but gravity is still causing it to accelerate toward the Earth at a rate of $10\,\frac{\text{m}}{\text{s}^2}$.

Extension

Just as a change in velocity is called acceleration, a change in acceleration with respect to time is called "jerk": $\vec{j} = \frac{\Delta \vec{a}}{\Delta t}$.



Use this space for summary and/or additional notes:

Dot Diagrams

Unit: Kinematics (Motion) in One Dimension

MA Curriculum Frameworks (2016): HS-PS2-10(MA)

AP® Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3

Mastery Objective(s): (Students will be able to…)

- Represent the motion of an object using dot diagrams.
- Describe the motion of an object based on its dot diagram.

Success Criteria:

- The dot diagram correctly shows the position of the object at each time interval.
- The description of the object's motion is correct.

Language Objectives:

- Describe the motion of an object as a sequence of events from beginning to end.

Tier 2 Vocabulary: position, velocity, acceleration

Lab Activities & Demonstrations:

- Record the motion of objects using a paper tape counter.

Notes:

The following is a famous picture called "Bob Running", taken by Harold ("Doc") Edgerton, inventor of the strobe light.

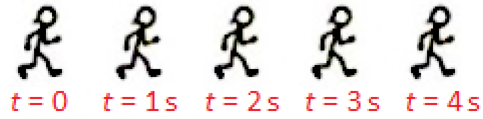
To create this picture, Edgerton opened the shutter of a camera in a dark room. A strobe light flashed at regular intervals while a child named Bob ran past. Each flash captured an image of Bob as he was running past the camera.

The images show that Bob was running at a constant velocity, because in each image he had travelled approximately the same distance relative to the previous image.

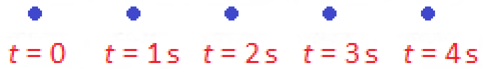
Use this space for summary and/or additional notes:

If we simplify the picture and replace the images of Bob with stick figures, they might look like this:

If the time between flashes of the strobe light was exactly one second, we would know where the stick figure was at every second:



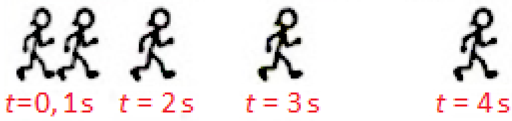
Notice that our stick figure travels the same amount of distance from one second to the next, because its velocity is constant. If we replaced the stick figures with dots, our diagram would look like this:



This is called a “dot diagram”. As with the stick figures, if the velocity is constant, the space between each dot and the next will also be constant.

<u>dot diagram</u>: a diagram that represents motion as a series of dots with a constant interval of time between each dot.

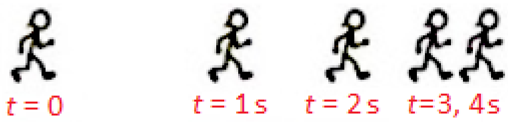
If our stick figure were accelerating, the diagram might look like this:



which would give the following dot diagram:

As our stick figure speeds up, it travels farther from one second to the next, which is why the dots get farther apart.

Similarly, if our stick figure were slowing down (negative acceleration), the diagram might look like this:



which would give the following dot diagram:

As our stick figure slows down, it travels less distance from each second to the next, which is why the dots get closer together.

Use this space for summary and/or additional notes:

Equations of Motion

Unit: Kinematics (Motion) in One Dimension

MA Curriculum Frameworks (2016): HS-PS2-10(MA)

AP® Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3

Mastery Objective(s): (Students will be able to…)

- Use the equations of motion to calculate position, velocity and acceleration for problems that involve motion in one dimension.

Success Criteria:

- Vector quantities position, velocity, and acceleration are identified and substituted correctly, including sign (direction).
- Time (scalar) is correct and positive.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Correctly identify quantities and assign variables in word problems.

Tier 2 Vocabulary: position, displacement, velocity, acceleration, direction

Notes:

As previously noted, average velocity is the displacement (change in position) with respect to time. (*E.g.,* if your displacement is 10 m over a period of 2 s, then your average velocity is $\vec{v}_{ave.} = \frac{\vec{d}}{t} = \frac{10}{2} = 5\,\frac{\text{m}}{\text{s}}$.)

Derivations of Equations

We can rearrange the formula for average velocity to show that displacement is average velocity times time:

$$\vec{v}_{ave.}(t) = \frac{\vec{d}}{\cancel{t}}\cancel{(t)} \quad \rightarrow \quad \vec{d} = (\vec{v}_{ave.})(t)$$

Note that when an object's velocity is changing, the initial velocity $\vec{v}_o$, the final velocity, $\vec{v}$, and the average velocity, $\vec{v}_{ave.}$ are *different quantities* with *different values*. (This is a common mistake that first-year physics students make.) Assuming acceleration is constant*, the average velocity is just the average of the initial and final velocities. This gives the following equation:

$$\vec{v}_{ave.} = \frac{\vec{v}_o + \vec{v}}{2} = \frac{\vec{d}}{t}$$

* In an algebra-based physics course, we will limit ourselves to problems in which acceleration is constant.

Use this space for summary and/or additional notes:

Acceleration is a change in velocity over a period of time. This means that formula for acceleration is:

$$\vec{a}_{ave.} = \frac{\vec{v} - \vec{v}_o}{t} = \frac{\Delta\vec{v}}{t} = \frac{\Delta\vec{v}}{\Delta t}$$

We can rearrange this formula to show that the change in velocity is acceleration times time:

$$\Delta\vec{v} = \vec{v} - \vec{v}_o = \vec{a}t$$

We can combine the formula for average velocity with the formula for acceleration in order to get a formula for the position of an object that is accelerating.

$$\boldsymbol{d} = (\boldsymbol{v}_{ave.})(t)$$
$$\boldsymbol{v} = \boldsymbol{a}t$$

However, the problem is that $\boldsymbol{v}$ in the formula $\boldsymbol{v} = \boldsymbol{a}t$ is the velocity at the *end*, which is not the same as the *average* velocity $\boldsymbol{v}_{ave.}$.

If the velocity of an object is changing at a constant rate (*i.e.,* the object is accelerating uniformly), the average velocity, $\boldsymbol{v}_{ave.}$ is given by the formula:

$$\boldsymbol{v}_{ave.} = \frac{\boldsymbol{v}_o + \boldsymbol{v}}{2}$$

To make the math easier to follow, let's start by assuming that the object starts at rest (not moving, which means $\boldsymbol{v}_o = 0$) and it accelerates at a constant rate. The average velocity is therefore the average of the initial velocity and the final velocity:

$$\boldsymbol{v}_{ave.} = \frac{\overset{0}{\boldsymbol{v}_o} + \boldsymbol{v}}{2} = \frac{0 + \boldsymbol{v}}{2} = \frac{\boldsymbol{v}}{2} = \frac{1}{2}\boldsymbol{v}$$

Combining all of these gives the following, for an object starting from rest:

$$\boldsymbol{d} = \boldsymbol{v}_{ave.}t = \tfrac{1}{2}\boldsymbol{v}t \quad \rightarrow \quad \vec{d} = \tfrac{1}{2}\boldsymbol{v}t = \tfrac{1}{2}(\boldsymbol{a}t)\,t = \tfrac{1}{2}\boldsymbol{a}t^2$$

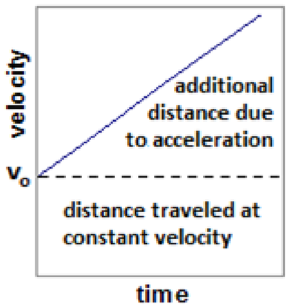
Now, recall from above that $\vec{d} = \vec{v}_{ave.}t$. Suppose that instead of starting from rest, an object's velocity is constant. The initial velocity is therefore also the final velocity and the average velocity, $(\vec{v}_o = \vec{v} = \vec{v}_{ave.})$, which means at constant velocity $\vec{d} = \vec{v}_o t$.

Therefore, if the object does not start from rest and it accelerates, we can combine these two formulas, resulting in:

$$\vec{d} = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2$$

distance the object would travel if its initial velocity were constant

additional distance the object will travel because it is accelerating



Use this space for summary and/or additional notes:

Finally, we can combine the equation $\vec{d} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2$ with the equation $\vec{v} - \vec{v}_o = \vec{a}t$ and eliminate time, giving the following equation, which relates initial and final velocity and distance:

$$\vec{v}^2 - \vec{v}_o^2 = 2\vec{a}\vec{d}\ ^{*}$$

The algebra is straightforward but tedious, and will not be presented here.

Summary of Motion Equations

Most motion problems can be calculated from Isaac Newton's equations of motion. The following is a summary of the equations presented in the previous sections:

Equation	Variables					Description
$\frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2} \left(= \vec{v}_{ave.}\right)$	$\vec{d}$	$\vec{v}_o$	$\vec{v}$		t	Average velocity is the distance per unit of time, which also equals the calculated value of average velocity.
$\vec{v} - \vec{v}_o = \vec{a}t$		$\vec{v}_o$	$\vec{v}$	$\vec{a}$	t	Acceleration is a change in velocity divided by time.
$\vec{d} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2$	$\vec{d}$	$\vec{v}_o$		$\vec{a}$	t	Total displacement is the displacement due to velocity ($\vec{v}_o t$), plus the displacement due to acceleration ($\frac{1}{2}\vec{a}t^2$).
$\vec{v}^2 - \vec{v}_o^2 = 2\vec{a}\vec{d}$	$\vec{d}$	$\vec{v}_o$	$\vec{v}$	$\vec{a}$		Velocity at the end can be calculated from velocity at the beginning, acceleration, and displacement.

* Note that this is not a proper vector expression. Vector multiplication is either a dot product, a cross product, or a tensor product; the expressions $\vec{v}^2$ and $\vec{a}\vec{d}$ are meaningless as vector expressions. The equation is presented this way to remind students that $\vec{v}, \vec{v}_o, \vec{a}$, and $\vec{d}$ are each vectors, whose signs (in one dimension) are positive or negative depending on direction.

Use this space for summary and/or additional notes:

Representing Vectors with Positive and Negative Numbers

Remember that position ($\vec{x}$), velocity ($\vec{v}$), and acceleration ($\vec{a}$) are all vectors, which means each of them can be positive or negative, depending on the direction.

- If an object is located is on the positive side of the origin (position zero), then its position, $\vec{x}$, is positive. If the object is located on the negative side of the origin, its position is negative.
- If an object is moving in the positive direction, then its velocity, $\vec{v}$, is positive. If the object is moving in the negative direction, then its velocity is negative.
- If an object's velocity is "trending positive" (increasing in the positive direction *or decreasing in the negative direction*), then its acceleration, $\vec{a}$, is positive. If the object's velocity is "trending negative" (decreasing in the positive direction *or increasing in the negative direction*), then its acceleration is negative.
- An object can have positive velocity and negative acceleration at the same time (or *vice versa*).
- An object can have a velocity of zero (for an instant) but can still be accelerating.

Selecting the Appropriate Equation

When you are faced with a problem, choose an equation based on the following criteria:

- The equation *must* contain the variable you are looking for.
- All other quantities in the equation must be either given in the problem or assumed from the description of the problem.
  - If an object *starts from rest* (not moving), that means $\vec{v}_o = 0$.
  - If an object *comes to rest* (*stops*), that means $\vec{v} = 0$. (Remember that $\vec{v}$ is the velocity at the end.)
  - If an object is moving at a constant velocity, then $\vec{v} = \text{constant} = \vec{v}_o = \vec{v}_{ave.}$ and $\vec{a} = 0$.
  - If the object is in free fall*, that means $\vec{a} = \vec{g} \approx 10\,\frac{\text{m}}{\text{s}^2}$ downward. Look for words like *drop*, *fall*, *throw*, *etc.* (Does not apply to rotation problems.)

This means you can choose the appropriate equation by making a list of what you are looking for and what you know. The equation in which you know everything except what you are looking for is the one to use.

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* See below.

Use this space for summary and/or additional notes:

Free Fall (Acceleration Caused by Gravity)

The gravitational force (or "force of gravity") is an attraction between objects that have mass.

free fall: when an object is freely accelerating toward the center of the Earth (or some other object with a very large mass) because of the effects of gravity, and the effects of other forces are negligible.

Objects in free fall on Earth accelerate downward at a rate of approximately $10\,\frac{\text{m}}{\text{s}^2} \approx 32\,\frac{\text{ft.}}{\text{s}^2}$. (The actual value is approximately $9.806\,\frac{\text{m}}{\text{s}^2}$ at sea level near the surface of the Earth. In this course we will usually round it to $10\,\frac{\text{m}}{\text{s}^2}$ so the calculations don't get in the way of understanding the physics.) When an object is in free fall, we usually replace the variable $\vec{\boldsymbol{a}}$ with the constant $\vec{\boldsymbol{g}}$.

Note that an object going down a ramp is ***not*** in free fall, even though gravity is the force that caused the object to accelerate. The object's motion is constrained by the ramp and it is not free to fall straight down.

As with any other vector quantity, acceleration due to gravity can be represented by a positive or negative number, depending on which direction you choose to be positive. For example, if we choose "up" to be the positive direction, that would mean acceleration due to gravity is in the negative direction, *i.e.,* $\vec{\boldsymbol{a}} = \vec{\boldsymbol{g}} = -10\,\frac{\text{m}}{\text{s}^2}$.

Hints for Solving Problems Involving Free Fall

1. If an object is thrown upwards, gravity will cause it to accelerate downwards. This means that if we choose the positive direction to be "up," $\vec{\boldsymbol{v}}_o$ will be positive, but $\vec{\boldsymbol{a}}$ will be $-10\,\frac{\text{m}}{\text{s}^2}$ (*i.e.,* negative because it's downwards).
2. At an object's *maximum height*, it stops moving for an instant ($\vec{\boldsymbol{v}} = 0$).
3. If an object goes up and then falls down to the *same height* it started from:
   a. There is no (vertical) displacement $(\vec{\boldsymbol{d}} = 0)$.
   b. *The time that the object spends going upwards is the same as the time it spends going downwards.* The time it takes to reach its maximum height is therefore half of the total time it takes to go up to its highest point and return to the ground.
   c. The *magnitude* of the velocity at the end will be the same as at the beginning, but the direction will be opposite. ($\vec{\boldsymbol{v}} = -\vec{\boldsymbol{v}}_o$)

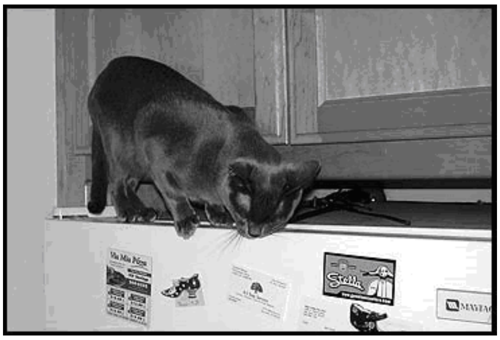
Use this space for summary and/or additional notes:

A Strategic Approach to Problem Solving

When solving motion problems, it can help to make a table of values and directions to keep track of each quantity.

Sample problems:

Q: If a cat jumps off a 1.8 m tall refrigerator, how fast is it going just before it hits the ground?



A: The cat is starting from rest $(\vec{v}_o = 0)$, and acceleration due to gravity is $\vec{a} = \vec{g} = 10\,\frac{\text{m}}{\text{s}^2}$ downwards. We need to find $\vec{v}$.

Because all of the vector quantities are in the downward direction, we will make "down" the positive direction.

var.	dir.	value
$\vec{d}$	↓	+1.8
$\vec{v}_o$	—	0
$\vec{v}$	?	?
$\vec{a}$	↓	+10
t	N/A	—

$$\frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$$

$$\vec{v} - \vec{v}_o = \vec{a}t$$

$$\vec{d} = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2$$

$$\boxed{\vec{v}^2 - \vec{v}_o^{\,2} = 2\vec{a}\vec{d}}$$

Because both of the nonzero vector quantities are downward, we will make downward the positive direction.

Using the "GUESS" method, the only equation that has the Unknown $(\vec{v})$ and the Givens ($\vec{d}, \vec{v}_o$, and $\vec{a}$) is the fourth one.

$$\vec{v}^2 - \overset{0}{\cancel{\vec{v}_o^{\,2}}} = 2\vec{a}\vec{d}$$

$$\vec{v} = \pm\sqrt{2\vec{a}\vec{d}}$$

(Note that because we introduced the square root sign, we have to consider both the positive and negative result.)

$$\vec{v} = \pm\sqrt{2\vec{a}\vec{d}} = \pm\sqrt{(2)(10)(1.8)} = \pm\sqrt{36} = \pm 6\,\frac{\text{m}}{\text{s}}$$

It is obvious from the problem that the cat is moving downward just before it hits the ground. Because downward is the positive direction, this means that the final velocity is $+6\,\frac{\text{m}}{\text{s}}$.

Use this space for summary and/or additional notes:

Q: A student throws an apple upward with a velocity of $8\,\frac{\text{m}}{\text{s}}$. The apple comes back down and hits Sir Isaac Newton in the head, at the same height as the apple was thrown.

How much time elapsed between when the apple was thrown and when it hit Newton?

A: Once again, we make a table of quantities and directions:

var.	dir.	value
$\vec{d}$	—	0
$\vec{v}_o$	↑	+8
$\cancel{\vec{v}}$	—	—
$\vec{a}$	↓	−10
t	?	N/A

$$\frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$$

$$\vec{v} - \vec{v}_o = \vec{a}t$$

$$\boxed{\vec{d} = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2}$$

$$\vec{v}^{\,2} - \vec{v}_o^{\,2} = 2\vec{a}\vec{d}$$

Note that because the apple landed at the same height as it was thrown, displacement is zero. Note also that because $\vec{v}_o$ is upward and $\vec{a}$ is downward, they need to have opposite signs. It doesn't matter which direction we choose to be positive, so for this problem let's arbitrarily choose upward to be the positive direction. This means $\vec{v}_o = +8\,\frac{\text{m}}{\text{s}}$ and $\vec{a} = -10\,\frac{\text{m}}{\text{s}^2}$.

We can now solve the problem:

$$\cancelto{0}{\vec{d}} = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2$$

$$0 = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2$$

$$0 = t(v_o + \tfrac{1}{2}at)$$

$$t = 0\,,\quad \vec{v}_o + \tfrac{1}{2}\vec{a}t = 0\vec{v}_o$$

$$t = 0\,,\quad \tfrac{1}{2}\vec{a}t = -\vec{v}_o$$

$$t = 0\,,\quad t = \frac{-2\vec{v}_o}{\vec{a}}$$

$$t = 0\,,\quad t = \frac{-2(8)}{-10} = 1.6\ \text{s}$$

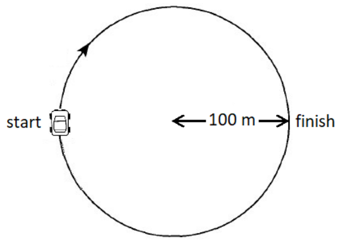
The equation helpfully tells us that the apple was at position zero twice, once at $t = 0$ when it was thrown, and again at $t = 1.6$ s when it landed on Newton's head. The problem is asking for the time when it landed, so the answer to the question that was asked is 1.6 s.

Use this space for summary and/or additional notes:

Homework Problems: Motion Equations Set #1

Try to first rearrange the equation to solve for the variable of interest and substitute numbers into the equation ***only*** after rearranging. (However, if you get stuck on a problem, feel free to solve it numerically first.)

1. **(M)** A car, traveling at constant speed, makes one lap around a circular track with a radius of 100 m. When the car has traveled halfway around the track, what *distance* did it travel? What is the magnitude of its *displacement* from the starting point?



2. **(S)** An elevator is moving upward with a speed of $11\,\frac{\text{m}}{\text{s}}$. Three seconds later, the elevator is still moving upward, but its speed has been reduced to $5.0\,\frac{\text{m}}{\text{s}}$. What is the average acceleration of the elevator during the 3.0 s interval?

   Answer: $-2\,\frac{\text{m}}{\text{s}^2}$

3. **(S – honors & AP®; M – CP1)** A car, starting from rest, accelerates in a straight-line path at a constant rate of $2.5\,\frac{\text{m}}{\text{s}^2}$. How far will the car travel in 12 seconds?

   Answer: 180 m

4. **(M – honors & AP®; S – CP1)** An object initially at rest is accelerated at a constant rate for 5.0 seconds in the positive *x* direction. If the final speed of the object is $20.0\,\frac{\text{m}}{\text{s}}$, what was the object's acceleration?

   Answer: $4\,\frac{\text{m}}{\text{s}^2}$

Use this space for summary and/or additional notes:

5. **(S – honors & AP®; M – CP1)** An object starts from rest and accelerates uniformly in a straight line in the positive *x* direction. After 10. seconds its speed is $70.\,\frac{m}{s}$.

   a. Determine the acceleration of the object.

   Answer: $7\,\frac{m}{s^2}$

   b. How far does the object travel during those first 10 seconds?

   Answer: 350 m

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6. **(M – honors & AP®; A – CP1)** A racecar has a speed of $\vec{v}_o$ when the driver releases a drag parachute. If the parachute causes a deceleration of $\vec{a}$, derive an expression for how far the car will travel before it stops.
(*If you are not sure how to do this problem, do #7 below and use the steps to guide your algebra.*)

   Answer: $\vec{d} = \dfrac{-\vec{v}_o^{\,2}}{2\vec{a}}$

   The negative sign means that $\vec{d}$ and $\vec{a}$ need to have opposite signs, which means they must be in opposite directions.

7. **(S)** A racecar has a speed of $80.\,\frac{m}{s}$ when the driver releases a drag parachute. If the parachute causes a *deceleration* of $4\,\frac{m}{s^2}$, how far will the car travel before it stops?
(*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #6 above as a starting point if you have already solved that problem.*)

   Answer: 800 m

Use this space for summary and/or additional notes:

8. **(S – honors & AP®; A – CP1)** A ball is shot straight up from the surface of the earth with an initial speed of $30.\,\frac{\text{m}}{\text{s}}$. Neglect any effects due to air resistance.

   a. What is the maximum height that the ball will reach?

   Answer: 45 m

   b. How much time elapses between the throwing of the ball and its return to the original launch point?

   Answer: 6.0 s

9. **(S – honors & AP®; M – CP1)** A brick is dropped from rest from a height of 5.0 m. How long does it take for the brick to reach the ground?

   Answer: 1 s

Use this space for summary and/or additional notes:

10. **(M – honors & AP®; A – CP1)** A ball is dropped from rest from a tower and strikes the ground 125 m below. Approximately how many seconds does it take for the ball to strike the ground after being dropped? (Neglect air resistance.)

    Answer: 5.0 s

11. **(S – honors & AP®; M – CP1)** Water drips from rest from a leaf that is 20 meters above the ground. Neglecting air resistance, what is the speed of each water drop when it hits the ground?

    Answer: $20.0\,\frac{\text{m}}{\text{s}}$

12. **(M – honors & AP®; A – CP1)** What is the maximum height that will be reached by a stone thrown straight up with an initial speed of $35\,\frac{\text{m}}{\text{s}}$?

    Answer: 61.25 m

Use this space for summary and/or additional notes:

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Homework Problems: Motion Equations Set #2

These problems are more challenging than Set #1.

1. **(S)** A car starts from rest at 50 m to the west of a road sign. It travels to the east reaching $20\,\frac{\text{m}}{\text{s}}$ after 15 s. Determine the position of the car relative to the road sign.

   Answer: 100 m east

2. **(M)** A car starts from rest at 50 m west of a road sign. It has a velocity of $20\,\frac{\text{m}}{\text{s}}$ east when it is 50 m east of the road sign. Determine the acceleration of the car.

   Answer: $2\,\frac{\text{m}}{\text{s}^2}$

3. **(S)** During a 10 s period, a car has an average velocity of $25\,\frac{\text{m}}{\text{s}}$ and an acceleration of $2\,\frac{\text{m}}{\text{s}^2}$. Determine the initial and final velocities of the car. (*Hint: this is an algebra problem with two unknowns, so it requires two equations.*)

   Answer: $v_o = 15\,\frac{\text{m}}{\text{s}}$; $v = 35\,\frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

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4. **(S)** A racing car increases its speed from an unknown initial velocity to $30\,\frac{m}{s}$ over a distance of 80 m in 4 s. Calculate the initial velocity of the car and the acceleration.

   Answer: $v_o = 10\,\frac{m}{s}$; $a = 5\,\frac{m}{s^2}$

5. **(M)** A stone is thrown vertically upward with a speed of $12.0\,\frac{m}{s}$ from the edge of a cliff that is 75.0 m high.

   a. **(M)** How much later does it reach the bottom of the cliff?

      Answer: 5.25 s

   b. **(M)** What is its velocity just before it hits the ground?

      Answer: $40.5\,\frac{m}{s}$ toward the ground ($-40.5\,\frac{m}{s}$ if "up" is positive)

   c. **(M)** What is the total distance the stone travels?

      Answer: 89.4 m

Use this space for summary and/or additional notes:

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6. **(S)** A helicopter is ascending vertically with a speed of v_o. At a height *h* above the Earth, a package is dropped from the helicopter. Derive an expression for the time, *t*, that it takes for the package to reach the ground. (*If you are not sure how to do this problem, do #7 below and use the steps to guide your algebra.*)

   Answer: $t = \frac{-v_o \pm \sqrt{v_o^2 - 2gh}}{g}$, disregarding the negative answer

7. **(M)** A helicopter is ascending vertically with a speed of $5.50\,\frac{\text{m}}{\text{s}}$. At a height of 100 m above the Earth, a package is dropped from the helicopter. How much time does it take for the package to reach the ground? (*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #6 above as a starting point if you have already solved that problem.*)

   Answer: 5.06 s

8. **(S)** A tennis ball is shot vertically upwards from the ground. It takes 3.2 s for it to return to the ground. Find the total distance the ball traveled.

   Answer: 25.6 m

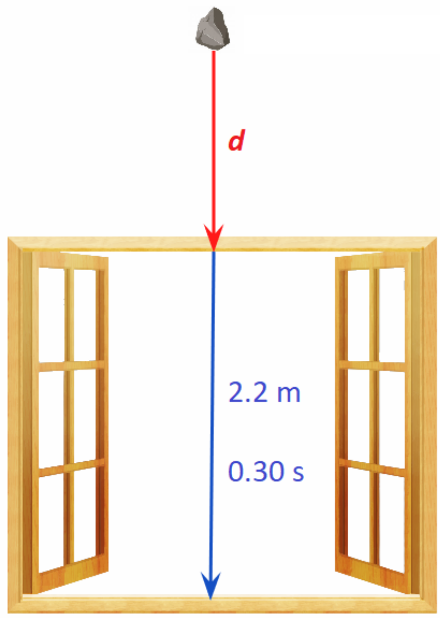
Use this space for summary and/or additional notes:

honors & AP®

9. **(S)** A kangaroo jumps vertically to a height of 2.7 m. How long will it be in the air before returning to the earth?

Answer: 1.5 s

10. **(M –AP®; S – honors)** A falling stone takes 0.30 s to travel past a window that is 2.2 m tall. From what distance above the window, *d*, did the stone fall?



Answer: 1.70 m

Use this space for summary and/or additional notes:

Motion Graphs*

Unit: Kinematics (Motion) in One Dimension

MA Curriculum Frameworks (2016): HS-PS2-10(MA)

AP® Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3

Mastery Objective(s): (Students will be able to…)

- Determine velocity, position and displacement from a position *vs.* time graph.
- Determine velocity, acceleration and displacement from a velocity *vs.* time graph.

Success Criteria:

- The correct aspect of the graph (slope or area) is used in the calculation.
- The magnitude (amount) and direction (sign, *i.e.,* + or −) is correct.

Language Objectives:

- Recall terms relating to graphs from algebra 1, such as "rise," "run," and "slope" and relate them to physics phenomena.

Tier 2 Vocabulary: position, velocity, acceleration, direction

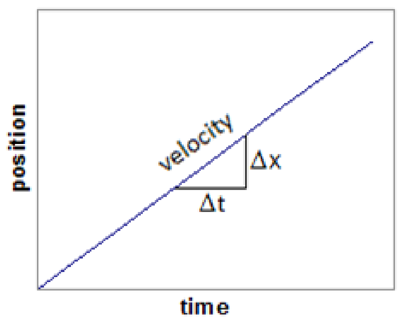
Lab Activities & Demonstrations:

- Have one student plot a position *vs.* time graph and have another student act it out.

Notes:

Position *vs.* Time Graphs

Suppose you were to plot a graph of position (x) *vs.* time (t) for an object that is moving at a constant velocity.



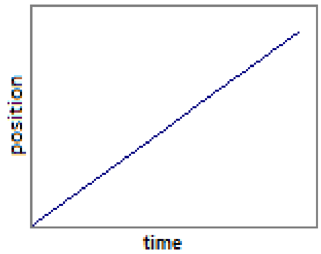
Note that $\frac{\Delta x}{\Delta t}$ is the slope of the graph. Because $\frac{\Delta x}{\Delta t} = v$, this means that the *slope* of a graph of position *vs.* time is equal to the velocity.

* Most physics texts present motion graphs before Newton's equations of motion. In this text, the order has been reversed because many students are more comfortable with equations than with graphs. This allows students to use a concept that is easier for them to help them understand one that is more challenging.

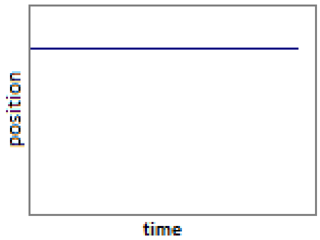
Use this space for summary and/or additional notes:

In fact, on any graph, the quantity you get when you divide the quantity on the *x*-axis by the quantity on the *y*-axis is, by definition, the slope. *I.e.,* the slope is $\frac{\Delta y}{\Delta x}$, which means the physics quantity defined by $\frac{\Delta y\text{-axis}}{\Delta x\text{-axis}}$ will always be the slope.

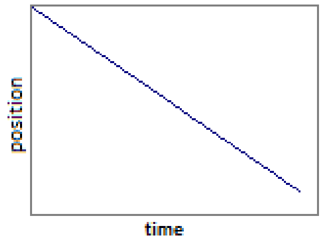
Recall that velocity is a vector, which means it can be positive, negative, or zero.



positive velocity
(moving in the positive direction)

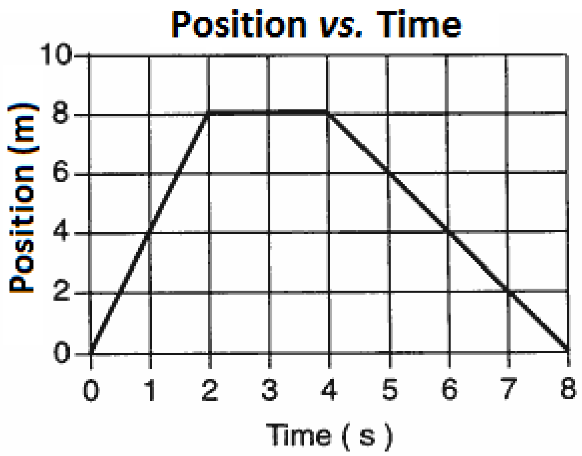


velocity = zero
(not moving)



negative velocity
(moving in the negative direction)

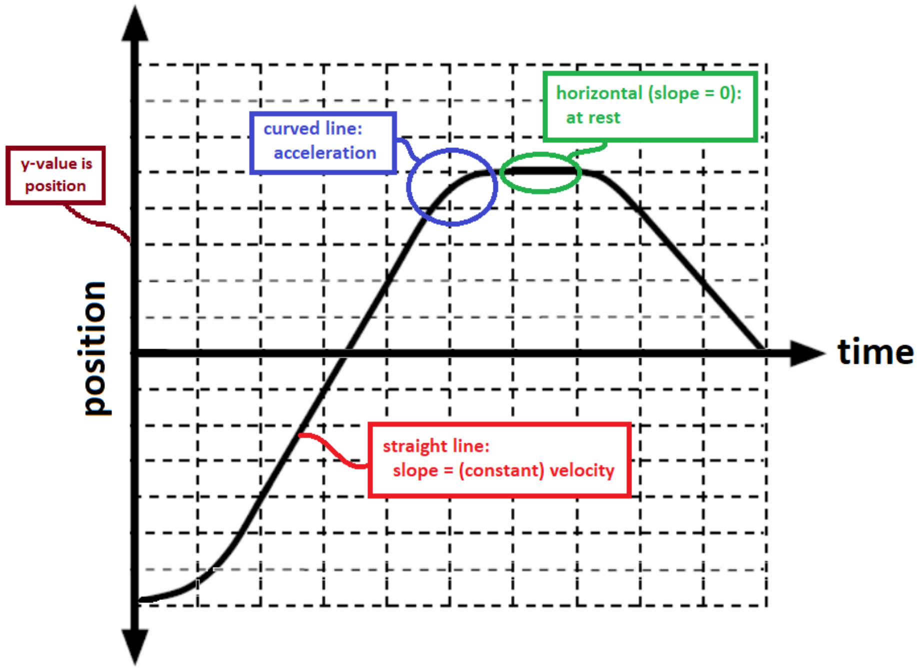
On the graph below, the velocity is $+4\,\frac{\text{m}}{\text{s}}$ from 0 s to 2 s, zero from 2 s to 4 s, and $-2\,\frac{\text{m}}{\text{s}}$ from 4 s to 8 s.



Use this space for summary and/or additional notes:

Features of Position *vs.* Time Graphs

The following diagrams show important features of position *vs.* time and velocity *vs.* time graphs.

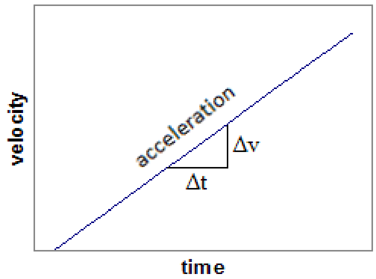


On a position *vs.* time graph, note the following:

- The *y*-value is the position (location) of the object.
- A straight line indicates constant velocity.
- A curved line indicates acceleration.
- A horizontal line indicates a velocity of zero. (The object is at rest.)
- The slope of the graph is the velocity. A positive slope indicates positive velocity (moving in the positive direction). A negative slope indicates negative velocity (moving in the negative direction).

Use this space for summary and/or additional notes:

Velocity *vs.* Time Graphs

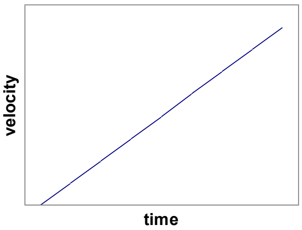


Suppose now that you were to plot a graph of *velocity* *vs.* time.

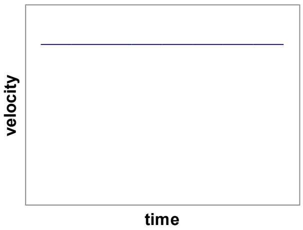
$\frac{\Delta v}{\Delta t}$ is the slope of a graph of velocity (v) *vs.* time (t). Because $\frac{\Delta v}{\Delta t} = a$, this means that acceleration is the *slope* of a graph of velocity *vs.* time.

Note the relationship between velocity-time graphs and the corresponding position-time graphs.

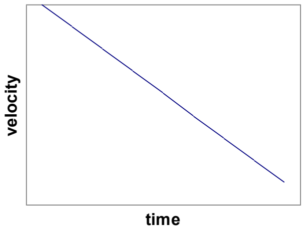
positive acceleration

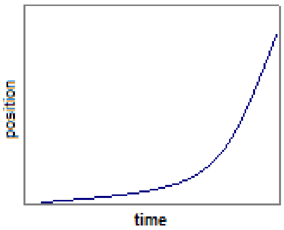


acceleration = zero

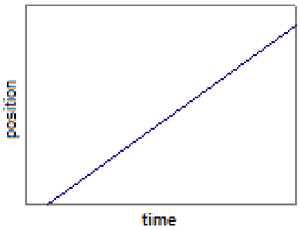


negative acceleration

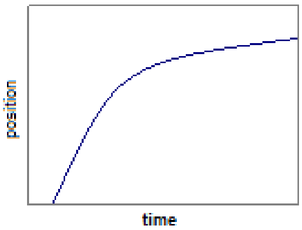




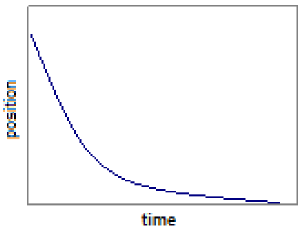
positive velocity, increasing speed



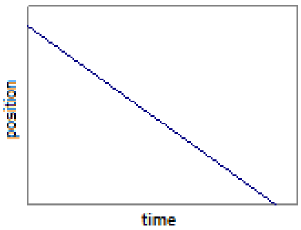
constant positive velocity



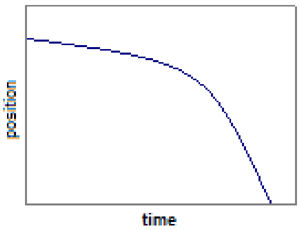
positive velocity, decreasing speed



negative velocity, decreasing speed



constant negative velocity



negative velocity, increasing speed

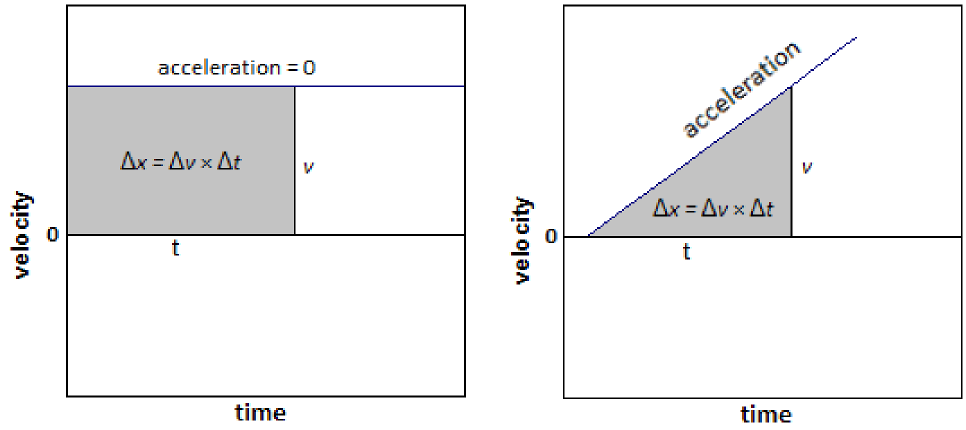
concave up

linear

concave down

Use this space for summary and/or additional notes:

Note also that $v_{ave.}t$ is the area under the graph (*i.e.,* the area between the curve and the x-axis) of velocity (v) *vs.* time (t). From the equations of motion, we know that $(v_{ave.})(t) = d$. Therefore, the *area* between a graph of velocity *vs.* time and the *x*-axis is the displacement. Note that this works both for constant velocity (the graph on the left) and changing velocity (as shown in the graph on the right).

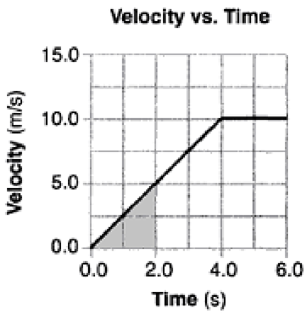


In fact, on any graph, the quantity you get when you multiply the quantities on the *x*- and *y*-axes is, by definition, the area under the graph.

Use this space for summary and/or additional notes:

In the graphs below, between 0 s and 4 s, the slope of the graph is 2.5, which means the object is accelerating at a rate of $+2.5\,\frac{m}{s^2}$.

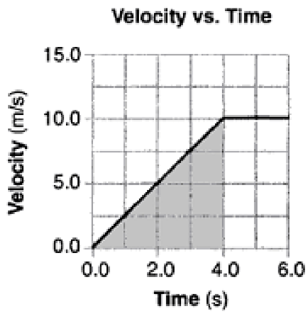
Between 4 s and 6 s the slope is zero, which indicates that object is moving at a constant velocity (of $+10\,\frac{m}{s}$) and the acceleration is zero.



Between 0 and 2 s

$a = 2.5\,\frac{m}{s^2}$

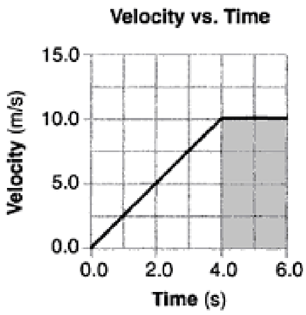
$\text{area} = \frac{1}{2}bh = \frac{1}{2}(2)(5) = 5\,\text{m}$



Between 0 and 4 s

$a = 2.5\,\frac{m}{s^2}$

$\text{area} = \frac{1}{2}bh = \frac{1}{2}(4)(10) = 20\,\text{m}$



Between 4 s and 6 s

$a = 0$

$\text{area} = bh = (2)(10) = 20\,\text{m}$

In each case, the area under the velocity-time graph equals the total distance traveled.

As we will see in the next section, the equation for displacement as a function of velocity and time is $d = v_o t + \frac{1}{2}at^2$, which becomes $d = \frac{1}{2}at^2$ for an object starting at rest. If we apply this equation to each of these situations, we would get the same numbers that we got from the area under the graph:

Between 0 and 2 s

$a = 2.5\,\frac{m}{s^2}$

$d = \frac{1}{2}(2.5)(2^2) = 5\,\text{m}$

Between 0 and 4 s

$a = 2.5\,\frac{m}{s^2}$

$d = \frac{1}{2}(2.5)(4^2) = 20\,\text{m}$

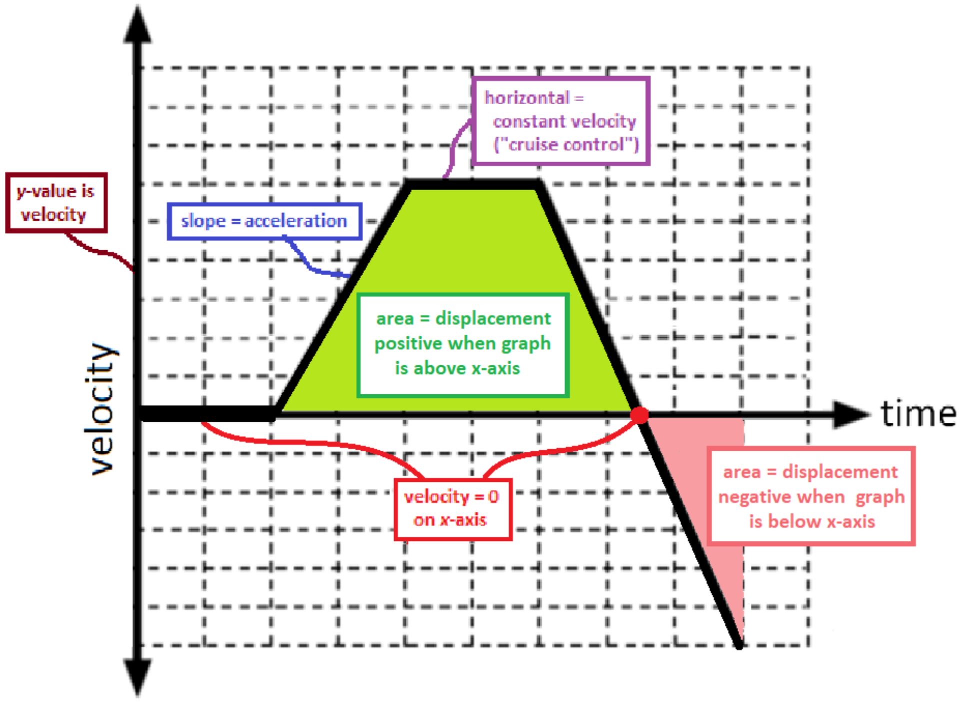
Between 4 s and 6 s

$a = 0$

$d = v_{ave.}t = (10)(2) = 20\,\text{m}$

Use this space for summary and/or additional notes:

Features of Velocity *vs.* Time Graphs



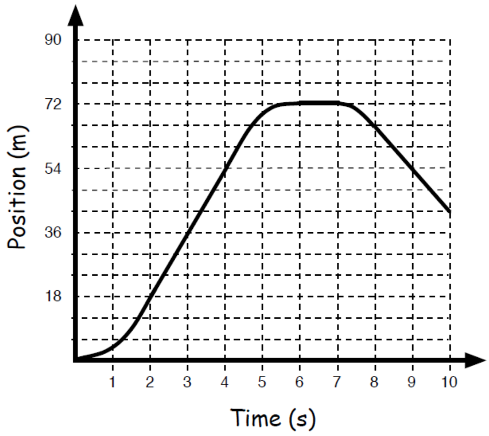
On a velocity *vs.* time graph, note the following:

- The velocity of the object is the *y*-value on the graph. When the *y*-value is positive, the object is moving in the positive direction. When the *y*-value is negative, the object is moving in the negative direction.
- A horizontal line indicates constant velocity. ***The object is at rest only when the graph is <u>on</u> the x-axis.***
- The ***slope*** of the graph is the ***acceleration***. A positive slope indicates positive acceleration, and a negative slope indicates negative acceleration.
- The ***area*** between a velocity *vs.* time graph and the *x*-axis is the ***displacement***. Areas above the x-axis indicate positive displacement, and areas below the *x*-axis indicate negative displacement.
- Note that an object cannot be moving with a nonzero velocity in the *x*- and *y*-direction at the same time. This means ***at any given time, the area can be either above the x-axis or below it, but never both***.
- The position of an object cannot be determined from a velocity *vs.* time graph.

Use this space for summary and/or additional notes:

Homework Problems: Motion Graphs

1. **(M)** An object's motion is described by the following graph of position *vs.* time:

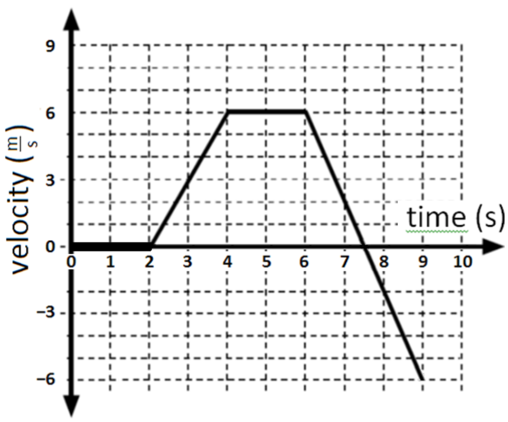


   a. What is the object doing between 2 s and 4 s? What is its velocity during that interval?

   b. What is the object doing between 6 s and 7 s? What is its velocity during that interval?

   c. During which time interval(s) is the object at rest? During which time interval(s) is it moving at a constant velocity? During which time interval(s) is it accelerating?

2. **(M)** An object's motion is described by the following graph of velocity *vs.* time:



   a. What is the object doing between 2 s and 4 s? What is its acceleration during that interval?

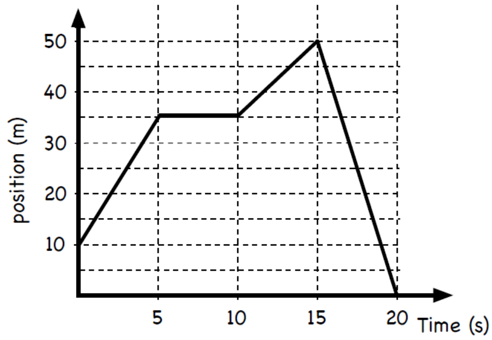
   b. What is the object doing between 4 s and 6 s? What are its velocity and acceleration during that interval?

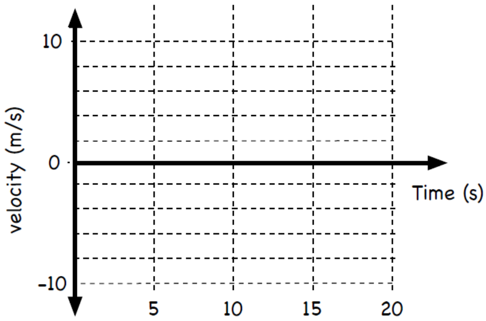
   c. What is the object's displacement between 2 s and 4 s? What is its displacement between 6 s and 9 s? (*Hint: you will need to split the areas into the area above the x-axis and the area below it.*)

Use this space for summary and/or additional notes:

3. **(M)** The graph on the left below shows the position of an object *vs.* time.

   a. Sketch a graph of velocity *vs.* time for the same object on a graph similar to the one on the right.



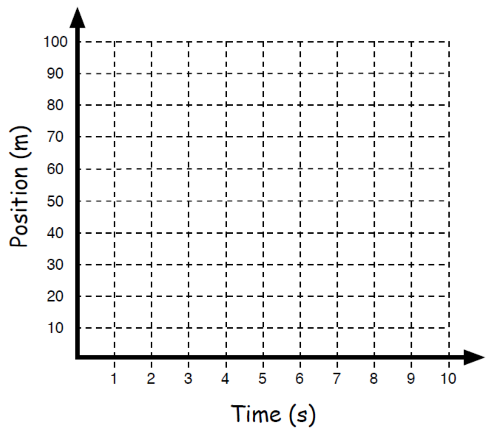


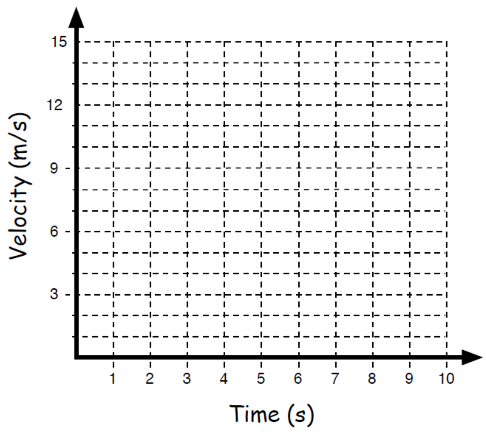
   b. Using your velocity *vs.* time graph, calculate the displacement for each 5-second segment. Use your position *vs.* time graph to check your answers.

4. **(M – AP®; S – honors & CP1)** In 1991, Carl Lewis became the first sprinter to break the 10-second barrier for the 100 m dash, completing the event in 9.86 s. The chart below shows his time for each 10 m interval.

position (m)	0	10	20	30	40	50	60	70	80	90	100
time (s)	0	1.88	2.96	3.88	4.77	5.61	6.45	7.29	8.12	8.97	9.86

Plot Lewis's position *vs.* time on the graph on the left. Draw a best-fit line (freehand). Then use the slope (rise over run) from the position *vs.* time graph to get the *y*-values for the velocity *vs.* time graph on the right.

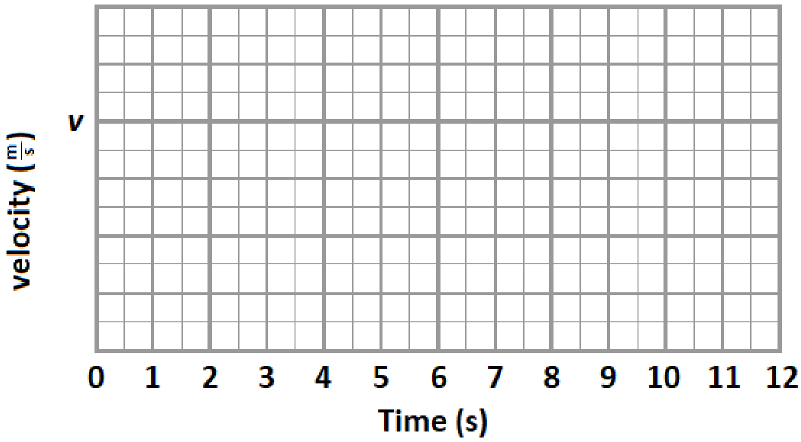




Use this space for summary and/or additional notes:

5. **(M – honors & AP®; S – CP1)** An elevator travels 4.0 m as it moves from the first floor of a building to the second floor. The elevator starts from rest and accelerates upward at a constant rate for 0.5 s, then travels at a constant (unknown) velocity *v* for 9.5 s, then decelerates at a constant rate for 0.5 s until it comes to rest on the second floor.

   a. **(M – honors & AP®; S – CP1)** Plot a graph of the elevator's motion on the graph below.



   b. **(M – honors & AP®; S – CP1)** Using your graph, determine the (constant) velocity, *v*, of the elevator during the interval from $t = 0.5$ s to $t = 10$ s.

Answer: $0.4\,\frac{m}{s}$

   c. **(M – honors & AP®; S – CP1)** Determine the acceleration of the elevator during the interval from $t = 0$ to $t = 0.5$ s.

Answer: $0.8\,\frac{m}{s^2}$

Use this space for summary and/or additional notes:

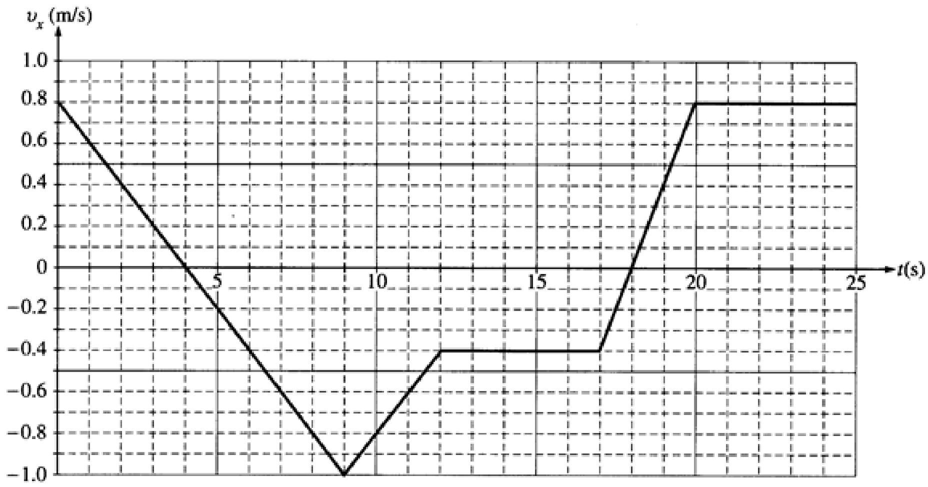
AP®

What an AP Motion Graph Problem Looks Like

AP motion problems almost always involve either graphs or projectiles. Free-response problems will often ask you to compare two graphs, such as a position-time graph *vs.* a velocity-time graph, or a velocity-time graph *vs.* an acceleration-time graph.

Here is an example of a free-response question involving motion graphs:

Q: A 0.50 kg cart moves on a straight horizontal track. The graph of velocity *v* versus time *t* for the cart is given below.



a. Indicate every time *t* for which the cart is at rest.

   The cart is at rest whenever the velocity is zero. Velocity is the *y*-axis, so we simply need to find the places where *y* = 0. These are at $\boxed{t = 4\text{ s}}$ and $\boxed{t = 18\text{ s}}$.

b. Indicate every time interval for which the speed (magnitude of velocity) of the cart is increasing.

   For the velocity vector, we use positive and negative to indicate direction. Therefore, the magnitude is the absolute value. The magnitude of the velocity is increasing whenever the graph is moving away from the x-axis, which happens in the intervals $\boxed{4\text{–}9\text{ s}}$ *and* $\boxed{18\text{–}20\text{ s}}$.

   The most likely mistake would be to give the times when the acceleration is positive. Positive acceleration can mean that the speed is increasing in the positive direction, but it can also mean that it is decreasing in the negative direction.

Use this space for summary and/or additional notes:

AP®

c. Determine the horizontal position x of the cart at $t = 9.0\,\text{s}$ if the cart is located at $x = 2.0\,\text{m}$ at $t = 0$.

Position is the area under a velocity-time graph. Therefore, if we add the positive and subtract the negative areas from $t = 0$ *to* $t = 9.0\,\text{s}$*, the result is the position at* $t = 9.0\,\text{s}$*.*

The area of the triangular region from 0–4 s *is* $(\frac{1}{2})(4)(0.8) = 1.6\,\text{m}$.

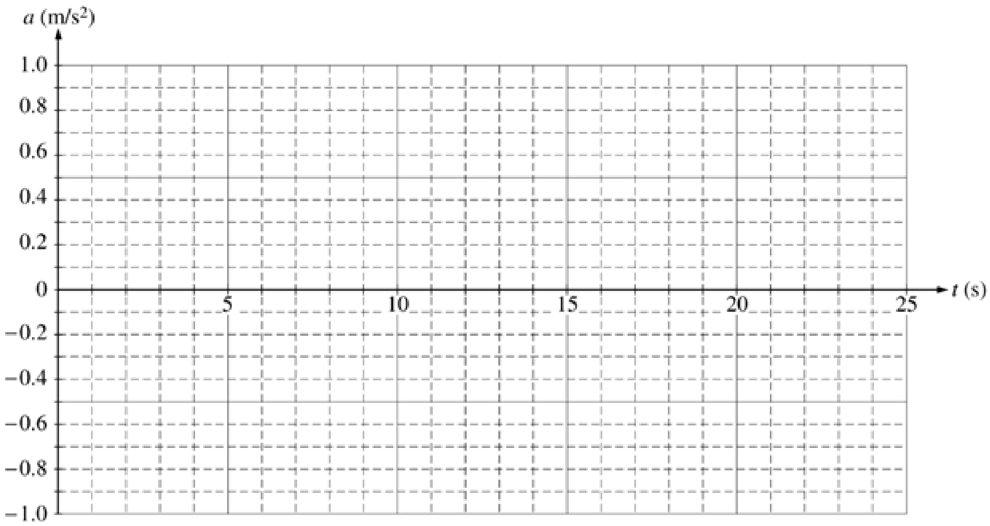
The area of the triangular region from 4–9 s *is* $(\frac{1}{2})(5)(-1.0) = -2.5\,\text{m}$.

The total displacement is therefore $\Delta x = 1.6 + (-2.5) = -0.9\,\text{m}$.

Because the cart's initial position was +2.0 m*, its final position is* 2.0 + (−0.9) = $\boxed{+1.1\text{ m}}$.

The most likely mistakes would be to add the areas regardless of whether they are negative or positive, and to forget to add the initial position after you have found the displacement.

d. On the axes below, sketch the acceleration a versus time t graph for the motion of the cart from $t = 0$ to $t = 25\,\text{s}$.



Use this space for summary and/or additional notes:

AP®

Acceleration is the slope of a velocity-time graph. Because the graph is discontininuous, we need to split it at each point where the slope suddenly changes. Each of the regions is a straight line (constant slope), which means all of the accelerations are constant (horizontal lines on the graph).

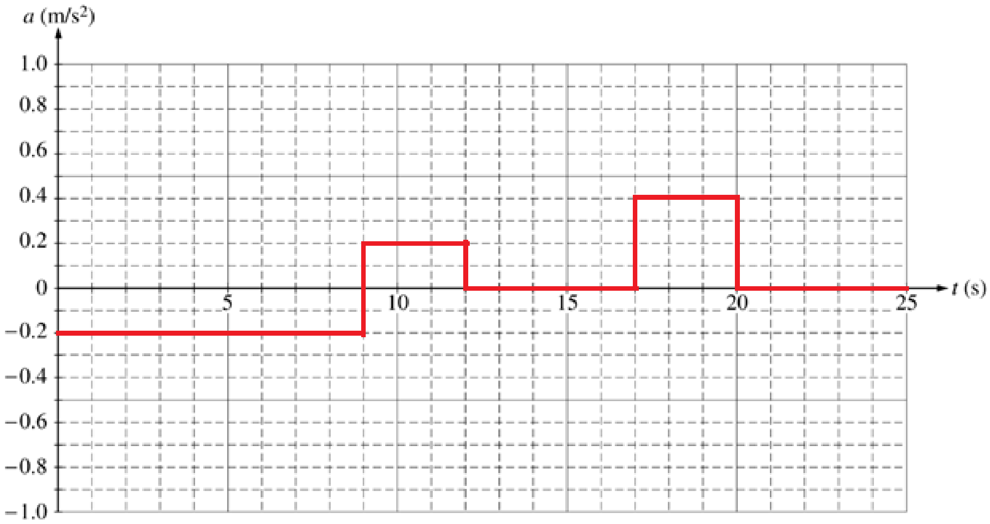
From 0–9 s*, the slope is* $\frac{\Delta y}{\Delta x} = \frac{-1.8}{9} = -0.2\,\frac{\text{m}}{\text{s}^2}$.

From 9–12 s*, the slope is* $\frac{\Delta y}{\Delta x} = \frac{+0.6}{3} = +0.2\,\frac{\text{m}}{\text{s}^2}$.

From 12–17 s *and from* 20–25 s*, the slope is zero.*

From 17–20 s*, the slope is* $\frac{\Delta y}{\Delta x} = \frac{+1.2}{3} = +0.4\,\frac{\text{m}}{\text{s}^2}$.

The graph therefore looks like the following:



e. The original problem also included a part (e), which was a simple projectile problem (discussed later).

Use this space for summary and/or additional notes:

Introduction: Kinematics in Multiple Dimensions

Unit: Kinematics (Motion) in Multiple Dimensions

Topics covered in this chapter:

In this chapter, you will study how things move and how the relevant quantities are related.

- *Projectile Motion* deals with an object that has two-dimensional motion—moving horizontally and also affected by gravity.
- *Angular Motion, Speed & Velocity* and *Angular Acceleration* deal with motion of objects that are rotating around a fixed center, using polar coördinates.
- *Centripetal Motion* deals with an object that is moving in a circle and therefore continuously accelerating toward the center.
- *Solving Linear & Rotational Motion Problems* deals with the relationships between linear and rotational kinematics problems and the types of problems that often appear on the AP® Physics exam.

Some of the challenging tasks include identifying quantities from their units, choosing the equation that relates the quantities of interest, and keeping track of positive and negative directions when working with vector quantities.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

Two-dimensional (projectile) motion and angular motion are not included in the MA Curriculum frameworks.

Use this space for summary and/or additional notes:

AP®

AP® Physics 1 Learning Objectives:

3.A.1.1: The student is able to express the motion of an object using narrative, mathematical, and graphical representations. **[SP 1.5, 2.1, 2.2]**

3.A.1.2: The student is able to design an experimental investigation of the motion of an object. **[SP 4.2]**

3.A.1.3: The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. **[SP 5.1]**

Topics from this chapter assessed on the SAT Physics Subject Test:

Kinematics, such as velocity, acceleration, and motion of projectiles

1. Two-dimensional motion with uniform acceleration

Skills learned & applied in this chapter:

- Choosing from a set of equations based on the quantities present.
- Working with vector quantities.
- Keeping track of things happening in two directions at once.

Use this space for summary and/or additional notes:

Projectile Motion

Unit: Kinematics (Motion) in Multiple Dimensions

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3

Mastery Objective(s): (Students will be able to…)

- Solve problems that involve motion in two dimensions.

Success Criteria:

- Correct quantities are chosen in each dimension (*x* & *y*).
- Positive direction is chosen for each dimension and vector quantities in each dimension have the appropriate sign (+ or −).
- Time (scalar) is correct, positive, and the same in both dimensions.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Correctly identify quantities with respect to type of quantity and direction in word problems.
- Assign variables correctly in word problems.

Tier 2 Vocabulary: projectile, dimension

Labs, Activities & Demonstrations:

- Play "catch."
- Drop one ball and punch the other at the same time.
- "Shoot the monkey."

Notes:

projectile: an object that is propelled (thrown, shot, *etc.*) horizontally and also falls due to gravity.

Because perpendicular vectors do not affect each other, the vertical and horizontal motion of the projectile are independent and can be considered separately, using a separate set of equations for each.

Use this space for summary and/or additional notes:

Assuming we can neglect friction and air resistance (which is usually the case in first-year physics problems), we make the following important assumptions:

Horizontal Motion

The horizontal motion of a projectile is not affected by anything except for air resistance. If air resistance is negligible, we can assume that there is no horizontal acceleration, and therefore the horizontal velocity of the projectile, $\vec{\boldsymbol{v}}_x$, is constant. This means the horizontal motion of a projectile can be described by the equation:

$$\vec{\boldsymbol{d}}_x = \vec{\boldsymbol{v}}_x t$$

The projectile is always moving in the same horizontal direction, so we make this the positive (horizontal, or "*x*") direction for the vector quantities of velocity and displacement.

Vertical Motion

Gravity affects projectiles the same way regardless of whether or not the projectile is also moving horizontally. All projectiles therefore have a constant downward acceleration of $\vec{\boldsymbol{g}} = 10\,\frac{\text{m}}{\text{s}^2}$ (in the vertical or "*y*" direction), due to gravity.

Therefore, the vertical motion of the particle can be described by the equations:

$$\vec{\boldsymbol{v}}_y - \vec{\boldsymbol{v}}_{o,y} = \vec{\boldsymbol{g}}t$$

$$\vec{\boldsymbol{d}}_y = \vec{\boldsymbol{v}}_{o,y}t + \frac{1}{2}gt^2$$

$$\vec{\boldsymbol{v}}_y^2 - \vec{\boldsymbol{v}}_{o,y}^2 = 2\vec{\boldsymbol{g}}\vec{\boldsymbol{d}}$$

(Notice that we have ***two*** subscripts for initial velocity, because it is ***both*** the initial velocity v_o ***and also*** the vertical velocity v_y.)

If the projectile is always moving downwards (*i.e.,* it is launched horizontally and it falls), we make down the positive vertical direction and all vector quantities (velocity, displacement and acceleration) in the *y*-direction are positive.

If the projectile is launched upwards, reaches a maximum height, and then falls, the velocity and displacement are sometimes upwards and sometimes downwards. In this case, we need to choose a direction to be positive. Usually, upward is chosen to be the positive direction, which makes $\vec{\boldsymbol{v}}_{o,y}$ positive, and makes $\vec{\boldsymbol{v}}_y$ and $\vec{\boldsymbol{g}}$ both negative. (In fact, $\vec{\boldsymbol{g}} = -10\,\frac{\text{m}}{\text{s}^2}$.)

Use this space for summary and/or additional notes:

Time

The time that the projectile spends falling must be the same as the time that the projectile spends moving horizontally. This means time (t) is the same in both equations, which means time is the variable that links the vertical problem to the horizontal problem.

The consequences of these assumptions are:

- The *time* that the object takes to fall is determined by its movement *only* in the vertical direction. (When it hits the ground, it stops moving in all directions.)
- The *horizontal distance* that the object travels is determined by the time (from the vertical equation) and by its velocity in the horizontal direction.

Therefore, the general strategy for most projectile problems is:

1. Solve the vertical problem first, to get the time.
2. Use the time from the vertical problem to solve the horizontal problem.

Use this space for summary and/or additional notes:

Sample problem:

Q: A ball is thrown horizontally at a velocity of $5\,\frac{\text{m}}{\text{s}}$ from a height of 1.5 m. How far does the ball travel (horizontally)?

A: We're looking for the horizontal distance, d_x. We know the vertical distance, $d_y = 1.5\,\text{m}$, and we know that $v_{o,y} = 0$ (there is no initial vertical velocity because the ball is thrown horizontally), and we know that $a_y = g = 10\,\frac{\text{m}}{\text{s}^2}$.

We need to separate the problem into the horizontal and vertical components.

Horizontal:

$$d_x = v_x t$$

$$d_x = 5t$$

At this point we can't get any farther, so we need to turn to the vertical problem.

Vertical:

$$d_y = v_{o,y}t + \frac{1}{2}gt^2$$

$$d_y = \frac{1}{2}gt^2$$

$$\frac{2d_y}{g} = t^2$$

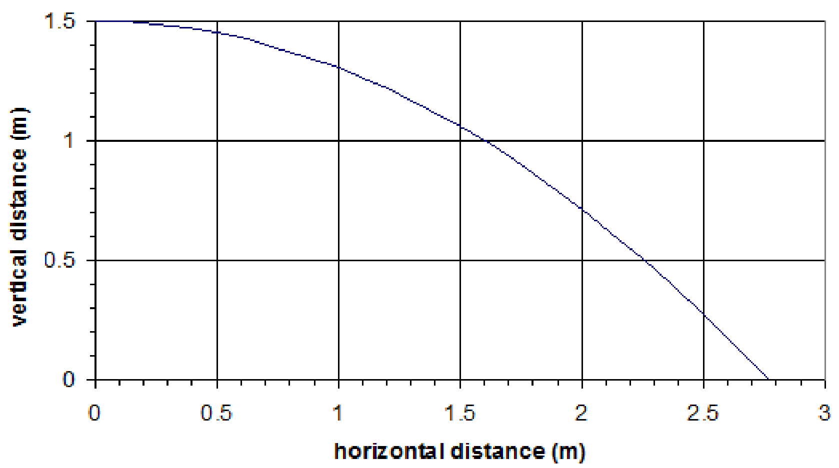
$$t = \sqrt{\frac{2d_y}{g}}$$

$$t = \sqrt{\frac{(2)(1.5)}{10}} = \sqrt{0.3} = 0.55\,\text{s}$$

Now that we know the time, we can substitute it back into the horizontal equation, giving:

$$d_x = (5)(0.55) = 2.74\,\text{m}$$

A graph of the vertical *vs.* horizontal motion of the ball looks like this:



Use this space for summary and/or additional notes:

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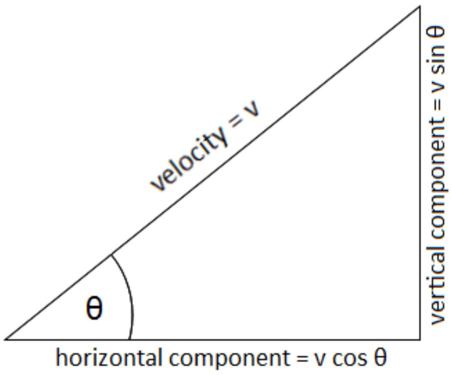
Projectiles Launched at an Angle

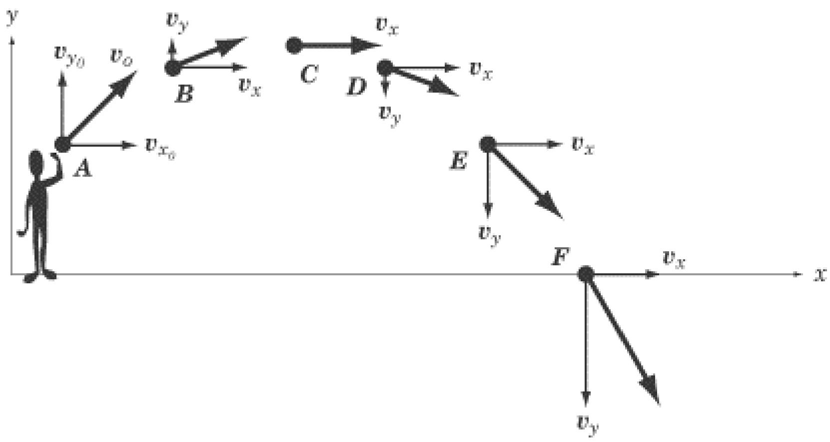
If the object is thrown/launched at an angle, you will need to use trigonometry to separate the velocity vector into its horizontal (x) and vertical (y) components:



Thus:

- horizontal velocity = $v_x = v \cos\theta$
- *initial* vertical velocity = $v_{o,y} = v \sin\theta$

Note that the vertical component of the velocity, v_y, is constantly changing because of acceleration due to gravity:



A fact worth remembering is that an angle of 45° gives the greatest horizontal displacement.

Use this space for summary and/or additional notes:

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Sample Problem:

Q: An Angry Bird[*] is launched upward from a slingshot at an angle of 40° with a velocity of $20\,\frac{\text{m}}{\text{s}}$. The bird strikes the pigs' fortress at the same height that it was launched from. How far away is the fortress?

A: We are looking for the horizontal distance, d_x .

We start with the equation:

$$d_x = v_x t$$

We need v_h and t.

We can substitute for v_x using $v_x = v\cos\theta$ to get:

$$d_x = (v\cos\theta)\,t = 20\cos(40^\circ)\,t = 15.3\,t$$

We can get t from:

$$d_y = v_{o,y}t + \tfrac{1}{2}gt^2 = v(\sin\theta)\,t + \tfrac{1}{2}gt^2 = 20(\sin 40^\circ)t + \tfrac{1}{2}(-10)t^2 = 12.9t - 5t^2$$

Because the vertical displacement is zero (the angry bird ends at the same height as it started), $d_v = 0$:

$$0 = 12.9t - 5t^2$$
$$0 = t(12.9 - 5t)$$

which has the solutions:

$$t = 0, \quad 12.9 - 5t = 0$$

The first solution ($t = 0$) is when the angry bird is launched. The second solution is the one of interest—when the angry bird lands. Solving for t gives:

$$12.9 = 5t$$
$$\frac{12.9}{5} = 2.57\,\text{s} = t$$

We can now substitute this expression into the first equation to get:

$$d_x = 15.3\,t = (15.3)(2.57) = 39.4\,\text{m}$$

[*] *Angry Birds* was a video game from 2010 in which players used slingshots to shoot birds with the necessary velocity and angle to destroy a fortress and kill the bad guys, who were green pigs.

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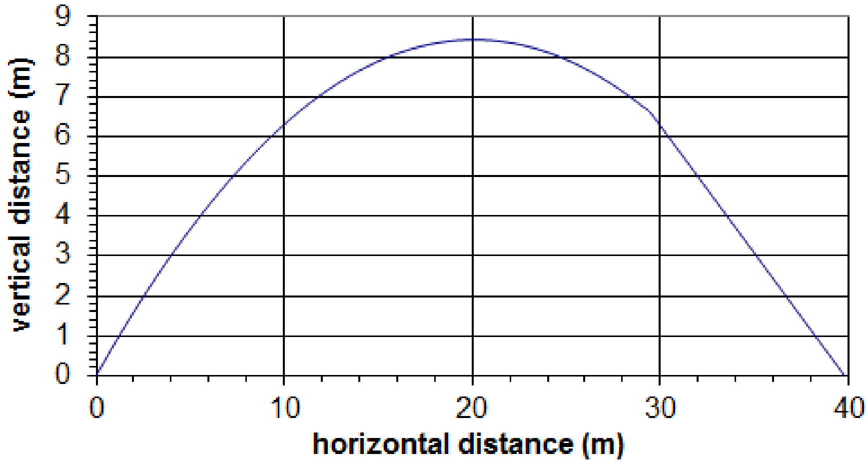
A graph of the angry bird's motion would look like the following:



If you wanted to solve this problem symbolically, you would do the following:

$$d_x = v_x t = v(\cos\theta)t$$

$$d_y = 0 = v_{o,y}t + \frac{1}{2}gt^2 = v(\sin\theta)t + \frac{1}{2}gt^2$$

$$0 = t(v\sin\theta + \frac{1}{2}gt)$$

$$v\sin\theta = -\frac{1}{2}gt$$

$$t = \frac{-2v\sin\theta}{g}$$

$$d_x = v(\cos\theta)\left(\frac{-2v\sin\theta}{g}\right) = \frac{-2v^2\sin\theta\cos\theta}{g}$$

If you have taken precalculus and you know the double angle formula, you can simplify the above expression, using $\sin 2\theta = 2\sin\theta\cos\theta$, which gives:

$$d_x = \frac{-2v^2\sin\theta\cos\theta}{g} = \frac{-v^2(2\sin\theta\cos\theta)}{g} = \frac{-v^2\sin 2\theta}{g}$$

(Of course, if you don't know the double angle formula, you can plug in the values anyway.)

Plugging in the values gives:

$$d_x = \frac{-2v\sin\theta\cos\theta}{g} = \frac{-2(20)^2(\sin 40°)(\cos 40°)}{-10} = 39.4\text{ m}$$

as before.

Use this space for summary and/or additional notes:

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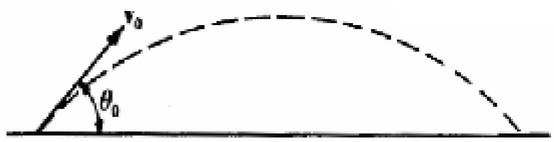
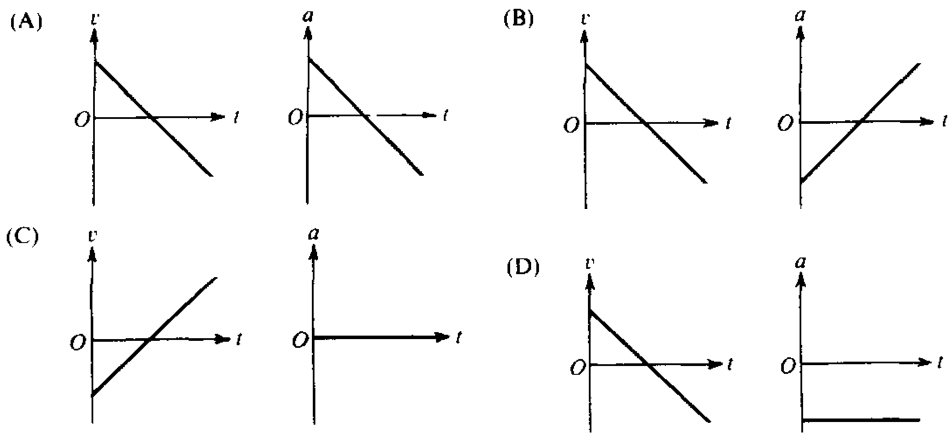
What AP® Projectile Problems Look Like

AP® motion and acceleration problems almost always involve graphs or projectiles. Here is an example that involves both:

Q:



A projectile is fired with initial velocity v_0 at an angle θ_0 with the horizontal and follows the trajectory shown above. Which of the following pairs of graphs best represents the vertical components of the velocity and acceleration, *v* and *a*, respectively, of the projectile as functions of time t?



A: *Because the object is a projectile:*

- *It can move both vertically and horizontally.*
- *It has a nonzero initial horizontal velocity. However, because the problem is asking about the vertical components, we can ignore the horizontal velocity.*
- *It has a constant acceleration of* $-g$ *(i.e., g in the downward direction) due to gravity.*

Use this space for summary and/or additional notes:

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For each pair of graphs, the first graph is velocity vs. time. The slope, $\frac{\Delta v}{\Delta t}$*, is acceleration. Because acceleration is constant, the graph has to have a constant. if we choose up to be the positive direction (which is the most common convention), correct answers would be (A), (B), and (D). If we choose down to be positive, only (C) would be correct.*

The second graph is acceleration vs. time. We know that acceleration is constant, which eliminates choices (A) and (B). We also know that acceleration is not zero, which eliminates choice (C). This leaves choice (D) as the only possible remaining answer. Choice (D) correctly shows a constant negative acceleration, because the slope of the first graph is negative, and the y-value of the second graph is also negative.

Use this space for summary and/or additional notes:

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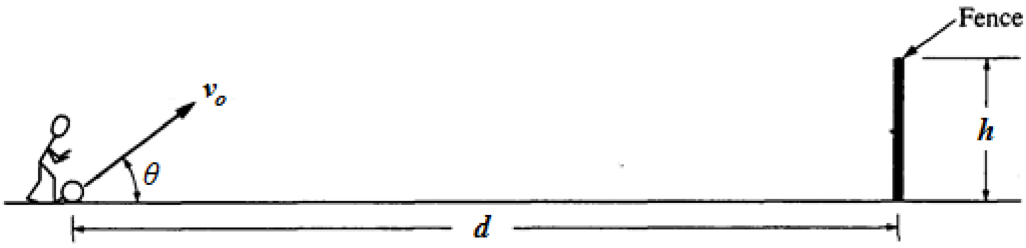
Q: A ball of mass *m*, initially at rest, is kicked directly toward a fence from a point that is a distance *d* away, as shown above. The velocity of the ball as it leaves the kicker's foot is v_o at an angle of θ above the horizontal. The ball just clears the top of the fence, which has a height of *h*. The ball hits nothing while in flight and air resistance is negligible.



a. Determine the time, *t*, that it takes for the ball to reach the plane of the fence, in terms of v_o, θ, *d*, and appropriate physical constants.

The horizontal component of the velocity is $v_h = v_o \cos\theta = \frac{d}{t}$.

Solving this expression for t gives $t = \frac{d}{v_o \cos\theta}$.

b. What is the vertical velocity of the ball when it passes over the top of the fence?

The initial vertical component of the velocity is $v_{v,o} = v_o \sin\theta$. *The equation for velocity is* $v_v = v_{o,v} + at$. *Substituting* $a = -g$, *and* $t = \frac{d}{v_o \cos\theta}$ *into this expression gives the answer of:*

$$v_v = v_o \sin\theta - \frac{gd}{v_o \cos\theta}$$

Use this space for summary and/or additional notes:

Homework Problems

Horizontal (level) projectile problems:

1. **(M)** A diver running $1.6\,\frac{m}{s}$ dives out horizontally from the edge of a vertical cliff and reaches the water below 3.0 s later.
   a. **(M)** How high was the cliff?

      Answer: 45 m

   b. **(M)** How far from the base did the diver hit the water?

      Answer: 4.8 m

2. **(S)** A ball is thrown horizontally from the roof of a building 56 m tall and lands 45 m from the base. What was the ball's initial speed?

   Answer: $13.4\,\frac{m}{s}$

Use this space for summary and/or additional notes:

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3. **(M – honors & AP®; A – CP1)** A tiger leaps horizontally from a rock with height h at a speed of v_o. What is the distance, d, from the base of the rock where the tiger lands?

   (*If you are not sure how to do this problem, do #4 below and use the steps to guide your algebra.*)

   Answer: $d = v_o\sqrt{\frac{2h}{g}}$

4. **(S – honors & AP®; M – CP1)** A tiger leaps horizontally from a 7.5 m high rock with a speed of $4.5\,\frac{m}{s}$. How far from the base of the rock will he land?

   (<u>You must start with the equations in your Physics Reference Tables *and show all of the steps of GUESS.*</u> *You may only use the answer to question #3 above as a starting point if you have already solved that problem.*)

   Answer: 5.5 m

5. **(M)** The pilot of an airplane traveling $45\,\frac{m}{s}$ wants to drop supplies to flood victims isolated on a patch of land 160 m below. The supplies should be dropped when the plane is how far from the island?

   Answer: 255 m

Use this space for summary and/or additional notes:

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Problems involving projectiles launched at an angle:

6. **(M – honors & AP®; A – CP1)** A ball is shot out of a slingshot with a velocity of $10.0\,\frac{\text{m}}{\text{s}}$ at an angle of 40.0° above the horizontal. How far away does it land?

   Answer: 9.85 m

7. **(S – honors & AP®; A – CP1)** The 12 Pounder Napoleon Model 1857 was the primary cannon used during the American Civil War. If the cannon had a muzzle velocity of $439\,\frac{\text{m}}{\text{s}}$ and was fired at a 5.00° angle, what was the effective range of the cannon (the distance it could fire)? (Neglect air resistance.)

   Answer: 3 347 m (Note that this is more than 2 miles!)

Use this space for summary and/or additional notes:

honors & AP®

8. **(M – AP®; S – honors; A – CP1)** A physics teacher is designing a ballistics event for a science competition. The ceiling is 3.00 m high, and the maximum velocity of the projectile will be $20.0\,\frac{m}{s}$.

   a. What is the maximum that the vertical component of the projectile's initial velocity could have?

      Answer: $7.75\,\frac{m}{s}$

   b. At what angle should the projectile be launched in order to achieve this maximum height?

      Answer: 22.8°

   c. What is the maximum horizontal distance that the projectile could travel?

      Answer: 28.6 m

Use this space for summary and/or additional notes:

AP®

Angular Motion, Speed and Velocity

Unit: Kinematics (Motion) in Multiple Dimensions

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3

Mastery Objective(s): (Students will be able to…)

- Solve problems that involve angular position and velocity.

Success Criteria:

- Correct quantities are chosen in each dimension (*r, ω* & *ϑ*).
- Positive direction is chosen for each dimension and vector quantities in each dimension have the appropriate sign (+ or −).
- Time (scalar) is correct, positive, and the same in both dimensions.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Correctly identify quantities with respect to type of quantity and direction in word problems.
- Assign variables correctly in word problems.

Tier 2 Vocabulary: rotation, angular

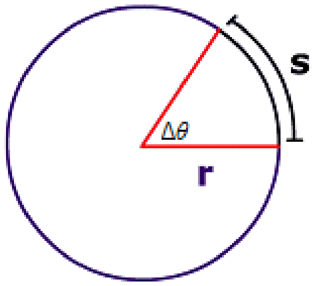
Labs, Activities & Demonstrations:

- Swing an object on a string.

Notes:

If an object is rotating (traveling in a circle), then its position at any given time can be described using polar coördinates by its distance from the center of the circle (*r*) and its angle (θ) relative to some reference angle (which we will call $\theta = 0$).

arc length (*s*): the length of an arc; the distance traveled around part of a circle.



$$s = r\Delta\theta$$

Use this space for summary and/or additional notes:

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angular velocity (ω): the rotational velocity of an object as it travels around a circle, *i.e.,* its change in angle per unit of time. (For purposes of comparison, the definition of angular velocity is presented along with its linear counterpart.)

$$\vec{\boldsymbol{v}} = \frac{\vec{\boldsymbol{d}}}{t} = \frac{\Delta\vec{\boldsymbol{x}}}{\Delta t} = \frac{\vec{\boldsymbol{x}} - \vec{\boldsymbol{x}}_o}{t} \qquad \vec{\boldsymbol{\omega}} = \frac{\Delta\vec{\boldsymbol{\theta}}}{\Delta t} = \frac{\vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\theta}}_o}{t}$$

linear **angular**

In general, physicists use Greek letters for angular variables. The variable for angular velocity is the lower case Greek letter omega (ω). Be careful to distinguish in your writing between the Greek letter "ω" and the Roman letter "w".

tangential velocity: the linear velocity of a point on a rigid, rotating body. The term tangential velocity is used because the instantaneous direction of the velocity is tangential to the direction of rotation.

To find the tangential velocity of a point on a rotating (rigid) body, the point travels an arc length of s in time t. If angle θ is in radians, then $s = r\Delta\theta$. This means:

$$\vec{\boldsymbol{v}}_{T,\,ave.} = \frac{\Delta\vec{\boldsymbol{s}}}{\Delta t} = \frac{r\Delta\vec{\boldsymbol{\theta}}}{\Delta t} = r\vec{\boldsymbol{\omega}}_{ave.} \quad \text{and therefore} \quad \vec{\mathbf{v}}_T = r\vec{\boldsymbol{\omega}}$$

Sample Problems:

Q: What is the angular velocity ($\frac{\text{rad}}{\text{s}}$) in of a car engine that is spinning at 2400 rpm?

A: 2400 rpm means 2400 revolutions per minute.

$$\left(\frac{2400\,\cancel{\text{rev}}}{1\,\cancel{\text{min}}}\right)\left(\frac{1\,\cancel{\text{min}}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\,\cancel{\text{rev}}}\right) = \frac{4800\pi}{60} = 80\pi\,\frac{\text{rad}}{\text{s}} = 251\,\frac{\text{rad}}{\text{s}}$$

Use this space for summary and/or additional notes:

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Q: Find the total distance traveled in 10 s by a penny sitting on a spinning disc with a radius of 0.25 m that is rotating at a rate of 1 revolution per 2 s.

A: We are looking for the distance around the circle, which is the arc length. (This means we need to work in radians.)

We know that:

$$\vec{s} = r\Delta\vec{\theta}$$

and we know:

$$\Delta\vec{\theta} = \vec{\omega}t$$

Substituting the second equation into the first gives:

$$\vec{s} = r\Delta\vec{\theta} = r\vec{\omega}t$$

We need to convert $\vec{\omega}$ to $\frac{\text{rad}}{\text{s}}$:

1 revolution per 2 s means $\vec{\omega} = \left(\frac{1\,\cancel{\text{rev}}}{2\,\text{s}}\right)\left(\frac{2\pi\,\text{rad}}{1\,\cancel{\text{rev}}}\right) = \frac{2\pi}{2} = \pi\,\frac{\text{rad}}{\text{s}}$

Now we can substitute and solve:

$$\vec{s} = r\vec{\omega}t = (0.25)(\pi)(10) = 2.5\pi = (2.5)(3.14) = 7.85\,\text{m}$$

Extension

Just as jerk is the rate of change of linear acceleration, angular jerk is the rate of change of angular acceleration. $\vec{\zeta} = \frac{\Delta\vec{\alpha}}{\Delta t}$. (ζ is the Greek letter "zeta". Many college professors cannot draw it correctly and just call it "squiggle".) Angular jerk has not been seen on AP® Physics exams.

Use this space for summary and/or additional notes:

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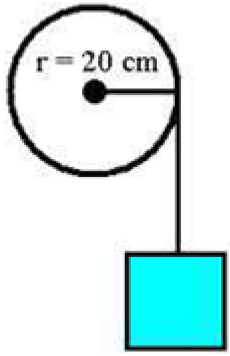
Homework Problems

1. **(M – AP®; A – honors & CP1)** Through what angle must the wheel shown at the right turn in order to unwind 40 cm of string?



Answer: 2 rad

2. **(M – AP®; A – honors & CP1)** Find the average angular velocity of a softball pitcher's arm (in $\frac{\text{rad}}{\text{s}}$) if, in throwing the ball, her arm rotates one-third of a revolution in 0.1 s.

Answer: $20.9\ \frac{\text{rad}}{\text{s}}$

3. **(M – AP®; A – honors & CP1)** A golfer swings a nine iron (radius = 1.1 m for the combination of the club and his arms) with an average angular velocity of $5.0\ \frac{\text{rad}}{\text{s}}$. Find the tangential velocity of the club head.

Answer: $5.5\ \frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

AP®

Angular Acceleration

Unit: Kinematics (Motion) in Multiple Dimensions

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3

Mastery Objective(s): (Students will be able to…)

- Solve problems that involve angular acceleration.

Success Criteria:

- Correct quantities are chosen in each dimension (r, ω, ω_o, α and θ).
- Positive direction is chosen for each dimension and vector quantities in each dimension have the appropriate sign (+ or −).
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Language Objectives:

- Correctly identify quantities with respect to type of quantity and direction in word problems.
- Assign variables correctly in word problems.

Tier 2 Vocabulary: rotation, angular

Labs, Activities & Demonstrations:

- Swing an object on a string and then change its angular velocity.

Use this space for summary and/or additional notes:

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Notes:

If a rotating object starts rotating faster or slower, this means its rotational velocity is changing.

angular acceleration (α): the change in angular velocity with respect to time. (Again, the definition is presented with the linear equation for comparison.)

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_o}{t} \qquad \vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t} = \frac{\vec{\omega} - \vec{\omega}_o}{t}$$

linear **angular**

As before, be careful to distinguish between the lower case Greek letter "α" and the lower case Roman letter "a".

As with linear acceleration, if the object has angular velocity and then accelerates, the position equation looks like this:

$$\vec{x} - \vec{x}_o = \vec{d} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2 \qquad \vec{\theta} - \vec{\theta}_o = \Delta\vec{\theta} = \vec{\omega}_o t + \frac{1}{2}\vec{\alpha}t^2$$

linear **angular**

tangential acceleration: the linear acceleration of a point on a rigid, rotating body. The term tangential acceleration is used because the instantaneous direction of the acceleration is tangential to the direction of rotation.

The tangential acceleration of a point on a rigid, rotating body is:

$$\vec{a}_T = r\vec{\alpha}$$

Use this space for summary and/or additional notes:

AP®

Sample Problem:

Q: A bicyclist is riding at an initial (linear) velocity of $7.5\,\frac{\text{m}}{\text{s}}$, and accelerates to a velocity of $10.0\,\frac{\text{m}}{\text{s}}$ over a duration of 5.0 s. If the wheels on the bicycle have a radius of 0.343 m, what is the angular acceleration of the bicycle wheels?

A: First we need to find the initial and final angular velocities of the bike wheel. We can do this from the tangential velocity, which equals the velocity of the bicycle.

$$\vec{v}_{o,T} = r\vec{\omega}_o$$
$$\frac{\vec{v}_{o,T}}{r} = \vec{\omega}_o$$
$$\frac{7.5}{0.343} = \vec{\omega}_o = 21.87\,\tfrac{\text{rad}}{\text{s}}$$

$$\vec{v}_T = r\vec{\omega}$$
$$\frac{\vec{v}_T}{r} = \vec{\omega}$$
$$\frac{10.0}{0.343} = \vec{\omega} = 29.15\,\tfrac{\text{rad}}{\text{s}}$$

Then we can use the equation:

$$\vec{\omega} - \vec{\omega}_o = \vec{\alpha}t$$
$$\frac{\vec{\omega} - \vec{\omega}_o}{t} = \vec{\alpha}$$
$$\frac{29.15 - 21.87}{5.0} = \vec{\alpha} = 1.46\,\tfrac{\text{rad}}{\text{s}^2}$$

An alternative method is to solve the equation by finding the linear acceleration first:

$$\vec{v} - \vec{v}_o = \vec{a}t$$
$$\frac{\vec{v} - \vec{v}_o}{t} = \vec{a}$$
$$\frac{10.0 - 7.5}{5} = \vec{a} = 0.5\,\tfrac{\text{m}}{\text{s}^2}$$

Then we can use the relationship between tangential acceleration and angular acceleration:

$$\vec{a}_T = r\vec{\alpha}$$
$$\frac{\vec{a}_T}{r} = \vec{\alpha}$$
$$\frac{0.5}{0.343} = \vec{\alpha} = 1.46\,\tfrac{\text{rad}}{\text{s}^2}$$

Use this space for summary and/or additional notes:

AP®

Homework Problems

1. **(M – AP®; A – honors & CP1)** A turntable rotating with an angular velocity of ω_o is shut off. It slows down at a constant rate and coasts to a stop in time t. What is its angular acceleration, α?
(*If you are not sure how to do this problem, do #2 below and use the steps to guide your algebra.*)

Answer: $\alpha = \dfrac{-\omega_o}{t}$

2. **(S – AP®; A – honors & CP1)** A turntable rotating at 33⅓ RPM is shut off. It slows down at a constant rate and coasts to a stop in 26 s. What is its angular acceleration?
(*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #1 above as a starting point if you have already solved that problem.*)

Answer: $-0.135\ \frac{\text{rad}}{\text{s}^2}$

Use this space for summary and/or additional notes:

Centripetal Motion

Unit: Kinematics (Motion) in Multiple Dimensions

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3

Mastery Objective(s): (Students will be able to…)

- Calculate the tangential and angular velocity and acceleration of an object moving in a circle.

Success Criteria:

- Correct quantities are chosen in each dimension (r, ω, ω_o, α, a and/or ϑ).
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why an object moving in a circle must be accelerating toward the center.
- Correctly identify quantities with respect to type of quantity and direction in word problems.
- Assign variables correctly in word problems.

Tier 2 Vocabulary: centripetal, centrifugal

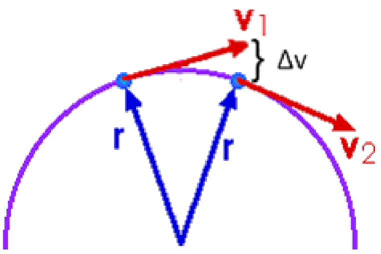
Labs, Activities & Demonstrations:

- Have students swing an object and let it go at the right time to try to hit something. (Be sure to observe the trajectory.)
- Swing a bucket of water in a circle.

Notes:

If an object is moving at a constant speed around a circle, its speed is constant, its direction keeps changing as it goes around. Because *velocity* is a vector (speed and direction), this means its velocity is constantly changing. (To be precise, the magnitude is staying the same, but the direction is changing.)

Because a change in velocity over time is acceleration, this means the object is constantly accelerating. This continuous change in velocity is toward the center of the circle, which means *there is continuous acceleration toward the center of the circle.*



Use this space for summary and/or additional notes:

centripetal acceleration (a_c): the constant acceleration of an object toward the center of rotation that keeps it rotating around the center at a fixed distance.

The equation* for centripetal acceleration (a_c) is:

$$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

(The derivation of this equation requires calculus, so it will not be presented here.)

Sample Problem:

Q: A weight is swung from the end of a string that is 0.65 m long at a rate of rotation of 10 revolutions in 6.5 s. What is the centripetal acceleration of the weight? How many "g's" is that? (*I.e.,* how many times the acceleration due to gravity is the centripetal acceleration?)

A: There are two ways to solve this problem.

Without using angular velocity:

In each revolution, the object travels a distance of $2\pi r$:

$$s_{rev} = 2\pi r = (2)(3.14)(0.65) = 4.08\,\text{m}$$

The total distance for 10 revolutions is therefore: $s = (4.08)(10) = 40.8\,\text{m}$

The velocity is the distance divided by the time: $v = \frac{d}{t} = \frac{40.8}{6.5} = 6.28\,\frac{\text{m}}{\text{s}}$

Finally, $a_c = \frac{v^2}{r} = \frac{(6.28)^2}{0.65} = 60.7\,\frac{\text{m}}{\text{s}^2}$

This is $\frac{60.7}{10} = 6.07$ times the acceleration due to gravity.

AP®

Using angular velocity:

The angular velocity is:

$$\left(\frac{10\,\cancel{\text{rev}}}{6.5\,\text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\,\cancel{\text{rev}}}\right) = \frac{20\pi}{6.5} = 9.67\,\frac{\text{rad}}{\text{s}}$$

The centripetal acceleration is therefore:

$$a_c = r\omega^2$$

$$a_c = (0.65)(9.67)^2 = (0.65)(93.44) = 60.7\,\frac{\text{m}}{\text{s}^2}$$

This is $\frac{60.7}{10} = 6.07$ times the acceleration due to gravity.

* Centripetal motion relates to angular motion (which is studied in AP® Physics but not in the CP1 or honors courses). Equations or portions of equations with angular velocity (ω) and angular acceleration (α) apply only to the AP® course.

Use this space for summary and/or additional notes:

Homework Problem

1. One of the demonstrations we saw in class was swinging a bucket of water in a vertical circle without spilling any of the water.

   a. **(M)** Explain why the water stayed in the bucket.

   b. **(M)** If the combined length of your arm and the bucket is 0.90 m, what is the minimum tangential velocity that the bucket must have in order to not spill any water?

   Answer: $3.0\,\frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

AP®

Solving Linear & Rotational Motion Problems

Unit: Kinematics (Motion) in Multiple Dimensions

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 3.A.1.3

Mastery Objective(s): (Students will be able to…)

- Solve problems involving any combination of linear and/or angular motion.

Success Criteria:

- Correct quantities are identified and correct variables are chosen for each dimension.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Correctly identify quantities with respect to type of quantity and direction in word problems.
- Assign variables correctly in word problems.

Tier 2 Vocabulary: N/A

Notes:

The following is a summary of the variables used for motion problems. Note the correspondence between the linear and angular quantities.

Linear			Angular		
Var.	**Unit**	**Description**	**Var.**	**Unit**	**Description**
x	m	position	θ	rad (—)	angle; angular position
$\vec{d}, \Delta x$	m	displacement	$\Delta\theta$	rad (—)	angular displacement
$\vec{v}$	$\frac{\text{m}}{\text{s}}$	velocity	$\vec{\omega}$	$\frac{\text{rad}}{\text{s}} \left(\frac{1}{\text{s}}\right)$	angular velocity
$\vec{a}$	$\frac{\text{m}}{\text{s}^2}$	acceleration	$\vec{\alpha}$	$\frac{\text{rad}}{\text{s}^2} \left(\frac{1}{\text{s}^2}\right)$	angular acceleration
t	s	time	t	s	time

Notice that each of the linear variables has an angular counterpart.

Note also that "radian" is a dimensionless quantity. A radian is a ratio that describes an angle as the ratio of the arc length to the radius of the circle. This ratio is dimensionless (has no unit), because the units cancel—an angle is the same regardless of the distance units used.

Of course, the same would be true if we measured angles in degrees (or gradians* or anything else), but using radians makes many of the calculations particularly convenient.

* A gradian is $\frac{9}{10}$ of a degree, which means a right angle measures 100 gradians. It is sometimes called a "metric degree" because it was introduced as part of the metric system in France in the 1790s.

Use this space for summary and/or additional notes:

AP®

We have learned the following equations for solving motion problems. Again, note the correspondence between the linear and angular equations.

Linear Equation	**Angular Equation**	**Relationship**	**Comments**
$\vec{\boldsymbol{d}} = \boldsymbol{\Delta}\vec{\boldsymbol{x}} = \vec{\boldsymbol{x}} - \vec{\boldsymbol{x}}_o$	$\boldsymbol{\Delta}\vec{\boldsymbol{\theta}} = \vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\theta}}_o$	$s = r\Delta\theta$	Definition of displacement.
$\vec{\boldsymbol{v}}_{ave.} = \frac{\vec{\boldsymbol{d}}}{t} = \frac{\Delta\vec{\boldsymbol{x}}}{t} = \frac{\vec{\boldsymbol{v}}_o + \vec{\boldsymbol{v}}}{2}$	$\vec{\boldsymbol{\omega}}_{ave.} = \frac{\Delta\vec{\boldsymbol{\theta}}}{t} = \frac{\vec{\boldsymbol{\omega}}_o + \vec{\boldsymbol{\omega}}}{2}$	$v_T = r\omega$	Definition of *average* velocity. Note that you can't use $\vec{\boldsymbol{v}}_{ave.}$ or $\vec{\boldsymbol{\omega}}_{ave.}$ if there is acceleration.
$\vec{\boldsymbol{a}} = \frac{\Delta\vec{\boldsymbol{v}}}{t} = \frac{\vec{\boldsymbol{v}} - \vec{\boldsymbol{v}}_o}{t}$	$\vec{\boldsymbol{\alpha}} = \frac{\Delta\vec{\boldsymbol{\omega}}}{t} = \frac{\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\omega}}_o}{t}$	$a_T = r\alpha$	Definition of acceleration.
$\vec{\boldsymbol{x}} - \vec{\boldsymbol{x}}_o = \vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}_o t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$	$\vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\theta}}_o = \Delta\vec{\boldsymbol{\theta}} = \vec{\boldsymbol{\omega}}_o t + \frac{1}{2}\vec{\boldsymbol{\alpha}}t^2$		Position/ displacement formula.
$\vec{\boldsymbol{v}}^2 - \vec{\boldsymbol{v}}_o^2 = 2\vec{\boldsymbol{a}}\vec{\boldsymbol{d}}$ $\vec{\boldsymbol{v}}^2 - \vec{\boldsymbol{v}}_o^2 = 2\vec{\boldsymbol{a}}(\Delta\vec{\boldsymbol{x}})$	$\vec{\boldsymbol{\omega}}^2 - \vec{\boldsymbol{\omega}}_o^2 = 2\vec{\boldsymbol{\alpha}}\Delta\vec{\boldsymbol{\theta}}$		Relates velocities, acceleration and distance. Useful if time is not known.
$a_c = \frac{v^2}{r}$	$a_c = r\omega^2$		Centripetal acceleration (toward the center of a circle.)

Note that vector quantities can be positive or negative, depending on direction.

Note that $\vec{\boldsymbol{r}}$, $\vec{\boldsymbol{\omega}}$ and $\vec{\boldsymbol{\alpha}}$ are vector quantities. However, the equations that relate linear and angular motion and the centripetal acceleration equations apply to magnitudes only, because of the differences in coordinate systems and changing frames of reference.

Note that the relationships $s = r\Delta\theta$, $v_T = r\omega$, and $a_T = r\alpha$ are not listed on the AP® Physics exam sheet, so ***you need to memorize these***!

Use this space for summary and/or additional notes:

• *AP®*

Selecting the Right Equation

(This is the same as the list from page 191, with the addition of angular velocity.)

When you are faced with a problem, choose an equation based on the following criteria:

- The equation *must* contain the variable you are looking for.
- All other quantities in the equation must be either given in the problem or assumed from the description of the problem.

Linear

- If an object starts at rest (not moving), then $\vec{v}_o = 0$.
- If an object comes to a stop, then $\vec{v} = 0$.
- If an object is moving at a constant velocity, then $\vec{v} = \text{constant} = \vec{v}_{ave.}$ and $\vec{a} = 0$.
- If an object is in free fall, then $\vec{a} = \vec{g} \approx 10\,\frac{\text{m}}{\text{s}^2}$.

Angular

- If an object's rotation starts from rest (not rotating), then $\vec{\omega}_o = 0$.
- If an object stops rotating, then $\vec{\omega} = 0$.
- If an object is rotating at a constant rate (angular velocity), then $\vec{\omega} = \text{constant} = \vec{\omega}_{ave.}$ and $\vec{\alpha} = 0$.

This means you can choose the appropriate equation by making a list of what you are looking for and what you know. The equation in which you know everything except what you are looking for is the one to use.

Use this space for summary and/or additional notes:

Introduction: Forces in One Dimension

Unit: Forces in One Dimension

Topics covered in this chapter:

In this chapter you will learn about different kinds of forces and how they relate.

- *Newton's Laws* and *Types of Forces* describe basic scientific principles of how objects affect each other.
- *Gravitational Fields* introduces the concept of a force field and how gravity is an example of one.
- *Free-Body Diagrams* describes a way of drawing a picture that represents forces acting on an object.
- *Tension, Friction* and *Drag* describe situations in which a force is created by the action of another force.

One of the first challenges will be working with variables that have subscripts. Each type of force uses the variable ***F***. Subscripts will be used to keep track of the different kinds of forces. This chapter also makes extensive use of vectors.

Another challenge in this chapter will be to "chain" equations together to solve problems. This involves finding the equation that has the quantity you need, and then using a second equation to find the quantity that you are missing from the first equation.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-1. Analyze data to support the claim that Newton's second law of motion is a mathematical model describing change in motion (the acceleration) of objects when acted on by a net force.

Use this space for summary and/or additional notes:

HS-PS2-3. Apply scientific principles of motion and momentum to design, evaluate, and refine a device that minimizes the force on a macroscopic object during a collision.

HS-PS2-10(MA). Use free-body force diagrams, algebraic expressions, and Newton's laws of motion to predict changes to velocity and acceleration for an object moving in one dimension in various situations.

AP®

AP® Physics 1 Learning Objectives:

1.C.1.1: The student is able to design an experiment for collecting data to determine the relationship between the net force exerted on an object its inertial mass and its acceleration. **[SP 4.2]**

1.C.3.1: The student is able to design a plan for collecting data to measure gravitational mass and to measure inertial mass and to distinguish between the two experiments. **[SP 4.2]**

2.B.1.1: The student is able to apply to calculate the gravitational force on an object with mass *m* in a gravitational field of strength $\vec{g}$ in the context of the effects of a net force on objects and systems. **[SP 2.2, 7.2]**

3.A.2.1: The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **[SP 1.1]**

3.A.3.1: The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. **[SP 6.4, 7.2]**

3.A.3.2: The student is able to challenge a claim that an object can exert a force on itself. **[SP 6.1]**

3.A.3.3: The student is able to describe a force as an interaction between two objects and identify both objects for any force. **[SP 1.4]**

3.A.4.1: The student is able to construct explanations of physical situations involving the interaction of bodies using Newton's third law and the representation of action-reaction pairs of forces. **[SP 1.4, 6.2]**

3.A.4.2: The student is able to use Newton's third law to make claims and predictions about the action-reaction pairs of forces when two objects interact. **[SP 6.4, 7.2]**

3.A.4.3: The student is able to analyze situations involving interactions among several objects by using free-body diagrams that include the application of Newton's third law to identify forces. **[SP 1.4]**

3.B.1.1: The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations with acceleration in one dimension. **[SP 6.4, 7.2]**

Use this space for summary and/or additional notes:

AP®

3.B.1.2: The student is able to design a plan to collect and analyze data for motion (static, constant, or accelerating) from force measurements and carry out an analysis to determine the relationship between the net force and the vector sum of the individual forces. **[SP 4.2, 5.1]**

3.B.1.3: The student is able to re-express a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. **[SP 1.5, 2.2]**

3.B.2.1: The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. **[SP 1.1, 1.4, 2.2]**

3.C.4.1: The student is able to make claims about various contact forces between objects based on the microscopic cause of those forces. **[SP 6.1]**

3.C.4.2: The student is able to explain contact forces (tension, friction, normal, buoyant, spring) as arising from interatomic electric forces and that they therefore have certain directions. **[SP 6.2]**

3.G.1.1: The student is able to articulate situations when the gravitational force is the dominant force and when the electromagnetic, weak, and strong forces can be ignored. **[SP 7.1]**

4.A.1.1: The student is able to use representations of the center of mass of an isolated two-object system to analyze the motion of the system qualitatively and semi-quantitatively. **[SP 1.2, 1.4, 2.3, 6.4]**

4.A.2.1: The student is able to make predictions about the motion of a system based on the fact that acceleration is equal to the change in velocity per unit time, and velocity is equal to the change in position per unit time. **[SP 6.4]**

4.A.2.2: The student is able to evaluate using given data whether all the forces on a system or whether all the parts of a system have been identified. **[SP 5.3]**

4.A.2.3: The student is able to create mathematical models and analyze graphical relationships for acceleration, velocity, and position of the center of mass of a system and use them to calculate properties of the motion of the center of mass of a system. **[SP 1.4, 2.2]**

4.A.3.1: The student is able to apply Newton's second law to systems to calculate the change in the center-of-mass velocity when an external force is exerted on the system. **[SP 2.2]**

4.A.3.2: The student is able to use visual or mathematical representations of the forces between objects in a system to predict whether or not there will be a change in the center-of-mass velocity of that system. **[SP 1.4]**

Use this space for summary and/or additional notes:

Topics from this chapter assessed on the SAT Physics Subject Test:

Dynamics, such as force, Newton's laws, statics, and friction.

1. What are Forces?
2. Types of Forces
3. Newton's Laws
4. Problem Solving With Newton's Laws
5. Pulleys

Skills learned & applied in this chapter:

- Solving chains of equations.
- Working with material-specific constants (coëfficients of friction) from a table.
- Solving systems of equations (pulley problems).

Use this space for summary and/or additional notes:

Newton's Laws of Motion

Unit: Forces in One Dimension

MA Curriculum Frameworks (2016): HS-PS2-10(MA), HS-PS2-1

AP® Physics 1 Learning Objectives: 3.A.3.1, 3.A.3.2, 3.A.3.3, 3.A.4.1, 3.A.4.2, 3.4.C.1, 3.4.C.2

Mastery Objective(s): (Students will be able to…)

- Define and give examples of Newton's laws of motion.

Success Criteria:

- Examples illustrate the selected law appropriately.

Language Objectives:

- Explain each of Newton's laws in plain English and give illustrative examples.

Tier 2 Vocabulary: at rest, opposite, action, reaction, inert

Labs, Activities & Demonstrations:

- Mass with string above & below
- Tablecloth with dishes (or equivalent)
- "Levitating" globe.
- Fan cart
- Fire extinguisher & skateboard
- Forces on two masses hanging (via pulleys) from the same rope

Notes:

force: a push or pull on an object.

In the MKS system, force is measured in newtons, named after Sir Isaac Newton:

$$1\,\text{N} \equiv 1\,\tfrac{\text{kg}\cdot\text{m}}{\text{s}^2} \approx 3.6\,\text{oz}$$
$$4.45\,\text{N} \approx 1\,\text{lb.}$$

net force: the amount of force that remains in effect after the effects of opposing forces cancel.

Mathematically, the net force is the result of combining (adding) all of the forces on an object. (Remember that in one dimension, we use positive and negative numbers to indicate direction, which means forces in opposite directions need to have opposite signs.)

$$\vec{F}_{net} = \sum \vec{F}$$

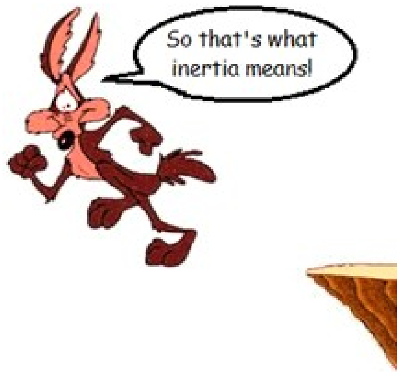
(The mathematical symbol Σ means "sum", which means "There are probably several of the thing after the Σ sign. Add them all up.")

Use this space for summary and/or additional notes:

Newton's First Law: (the law of inertia) Everything keeps doing what it was doing unless a net force acts to change it. "An object at rest remains at rest, unless acted upon by a net force. An object in motion remains in motion at a constant velocity, unless acted upon by a net force."

No net force ↔ no change in motion (no acceleration).

If an object's motion is not changing, then there must be no net force on it, which means any forces on it must cancel.



For example, a brick sitting on the floor will stay at rest on the floor forever unless an outside force moves it. Wile E. Coyote, on the other hand, remains in motion…

Inertia (resistance to change) is a property of mass. Everything with mass has inertia. The more mass an object has, the more inertia it has.

Newton's Second Law: Forces cause a change in velocity (acceleration). "A net force, $\vec{\boldsymbol{F}}$, acting on an object causes the object to accelerate in the direction of the net force."

Net force ↔ change in motion (acceleration).

If there is a net force on an object, the object's motion must change (accelerate), and if an object's motion has changed, there must have been a net force on it.

In equation form:

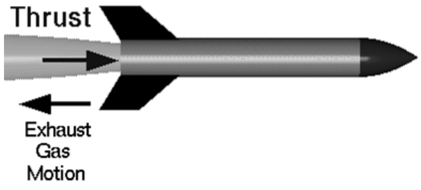
$$\vec{\boldsymbol{a}} = \frac{\vec{\boldsymbol{F}}_{\text{net}}}{m} = \frac{\sum \vec{\boldsymbol{F}}}{m} \quad \text{or} \quad \sum \vec{\boldsymbol{F}} = \vec{\boldsymbol{F}}_{net} = m\vec{\boldsymbol{a}}$$

This equation represents one of the most important relationships in physics.

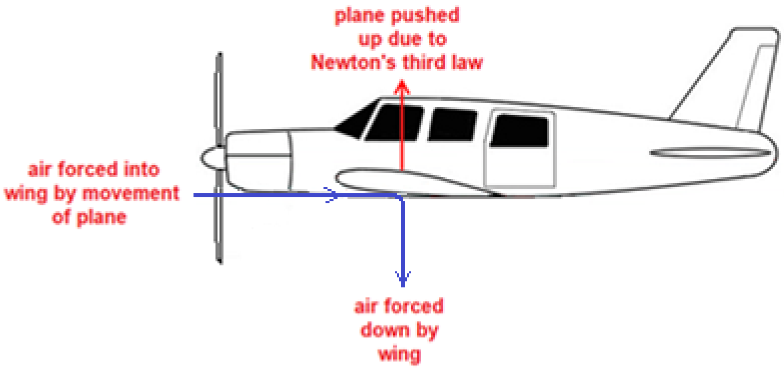
Use this space for summary and/or additional notes:

Newton's Third Law: Every force produces an equal and opposite reaction force of the same type. The first object exerts a force on the second, which causes the second object to exert the same force back on the first. "For every action, there is an equal and opposite reaction."

- When you pull on a rope (the action), the rope also pulls on you (the reaction).
- Burning fuel in a rocket causes exhaust gases to escape from the back of the rocket. The force from the gases exiting the rocket (the action) causes an equal and opposite thrust force to the rocket (the reaction), which propels the rocket forward.



- The wings of an airplane are angled (the "angle of attack"), which deflects air downwards as the plane moves forward. The wing pushing the air down (the action) causes the air to push the wing (and therefore the plane) up (the reaction). This reaction force is called "lift".



- If you punch a hole in a wall and break your hand, your hand applied a force to the wall (the action). This caused a force from the wall, which broke your hand (the reaction). This may seem obvious, although you will find that someone who has just broken his hand by punching a wall is unlikely to be receptive to a physics lesson!

Use this space for summary and/or additional notes:

Systems

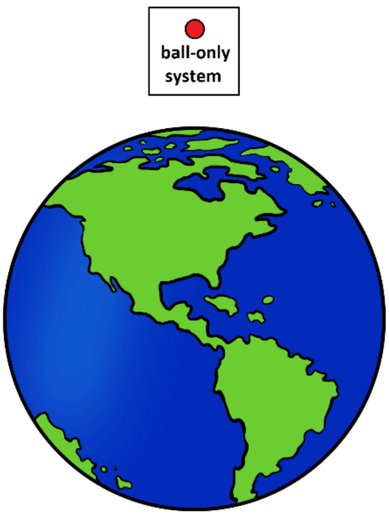
<u>system</u>: a specific object or set of objects considered together as a way to understand, model or predict the behaviors of those objects.

<u>surroundings</u>: the objects that are not part of the system.

The system may be considered as a group or unit. According to Newton's Second Law, ***a net force on an object in a system caused by an object outside of the system will cause the entire system to accelerate as if the system were a single object.***

According to Newton's Third Law, forces between objects that are both in the same system may affect each other, but their effects cancel with respect to the system as a whole. This means that ***forces within a system do not affect the motion of the system.***

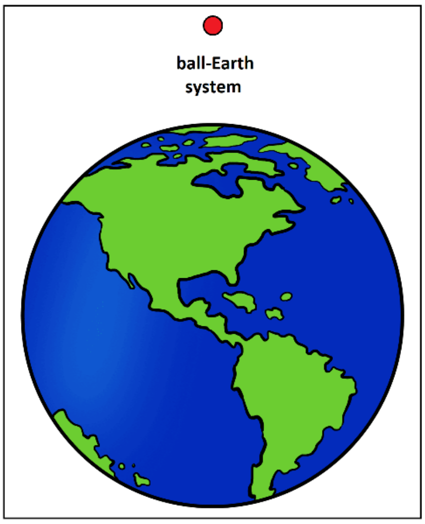
For example, gravity is the force of attraction between two objects because of their mass. If a student drops a ball off the roof of the school, the Earth attracts the ball and the ball attracts the Earth. (Because the Earth has a lot more mass than the ball, the ball moves much farther toward the Earth than the Earth moves toward the ball.)



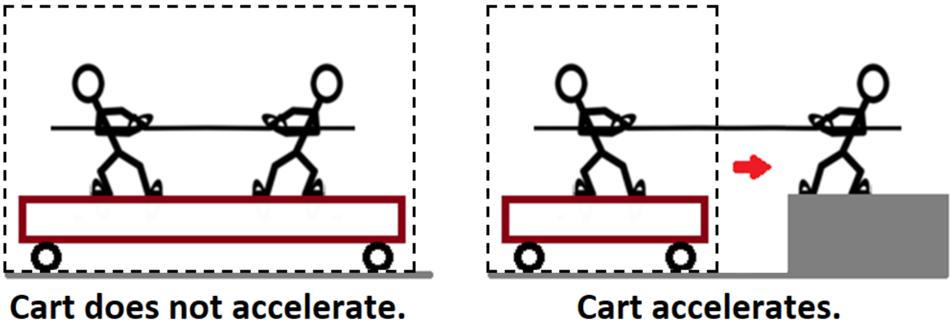
- **Ball-Only System**: If the system under consideration is only the ball, then the gravity field of the Earth exerts a net force on the ball, causing the ball to move.

Use this space for summary and/or additional notes:

- **Ball-Earth System**: If the system is the ball and the Earth, the force exerted by the Earth on the ball is equal to the force exerted by the ball on the Earth. Because the forces are equal in strength but in opposite directions ("equal and opposite"), their effects cancel, which means there is no net force on the system. (Yes, there are forces within the system, but that's not the same thing.) This is why, for example, if all 7.5 billion people on the Earth jumped at once, an observer on the moon would not be able to detect the Earth moving.



A demonstration of this concept is to have two students standing on a cart (a platform with wheels), playing "tug of war" with a rope. In the student-rope-student-cart system, the forces of the students pulling on the rope are all within the system. There is no net force (from outside of the system) on the cart, which means the cart does not move. However, if one student moves off the cart (outside of the system), then the student outside of the system can exert an external net force on the student-cart system, which causes the system (the student and cart) to accelerate.



One of the important implications of this concept is that ***an object cannot apply a net force to itself***. This means that "pulling yourself up by your bootstraps" is impossible according to the laws of physics.

Later, in the section on potential energy on page 392, we will see that potential energy is a property of systems, and that a single isolated object cannot have potential energy.

Use this space for summary and/or additional notes:

Types of Forces

Unit: Forces in One Dimension

MA Curriculum Frameworks (2016): HS-PS2-10(MA)

AP® Physics 1 Learning Objectives: 3.A.2.1, 3.A.4.1, 3.C.4.1, 3.C.4.2, 4.A.2.2

Mastery Objective(s): (Students will be able to…)

- Identify the forces acting on an object.

Success Criteria:

- Students correctly identify all forces, including contact forces such as friction, tension and the normal force.

Language Objectives:

- Identify and describe the forces acting on an object.

Tier 2 Vocabulary: force, tension, normal

Labs, Activities & Demonstrations:

- Tie a rope to a chair or stool and pull it.

Notes:

force: ($\vec{F}$, vector quantity) a push or pull on an object.

reaction force: a force that is created in reaction to the action of another force, as described by Newton's Third Law. Examples include friction and the normal force. Tension is both an applied force and a reaction force.

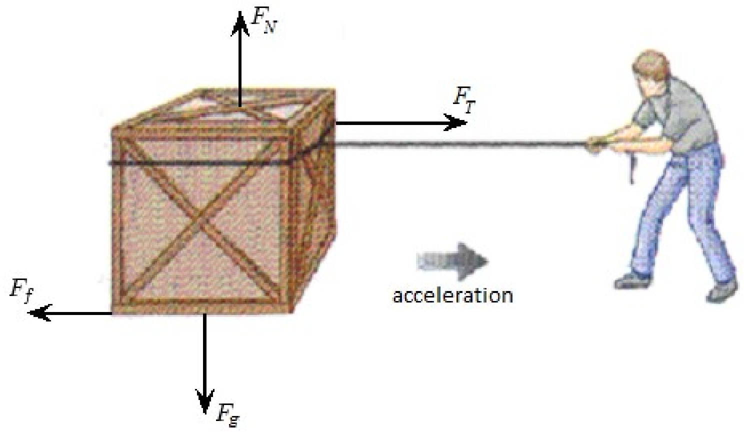
opposing force: a force in the opposite direction of another force, which reduces the effect of the original force. Examples include friction, the normal force, and the spring force (the force exerted by a spring).

contact force: a force that is caused directly by the action of another force, and exists *only* while the objects are in contact and the other force is in effect. Contact forces are generally reaction forces and also opposing forces. Examples include friction and the normal force.

net force: the amount of force that remains on an object after the effects of all opposing forces cancel.

Use this space for summary and/or additional notes:

An object can have several forces acting on it at once:



On the box in the above diagram, the forces are gravity ($\vec{F}_g$), the normal force ($\vec{F}_N$), the tension in the rope ($\vec{F}_T$), and friction ($\vec{F}_f$). Notice that in this problem, the arrow for tension is longer than the arrow for friction, because the force of tension is stronger than the force of friction.

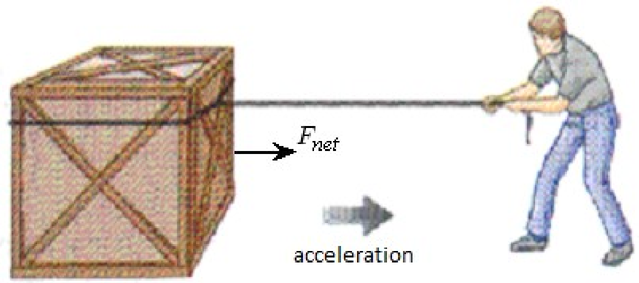
<u>net force</u>: the amount of force that remains on an object after the effects of all opposing forces cancel.

You can think of forces as the participants in a tug-of-war:

The net force is the amount of force that is not canceled by the other forces. It determines which direction the object will move, and with how much force.

Use this space for summary and/or additional notes:

In the situation with the box above (after canceling out gravity and the normal force, and subtracting friction from the tension) the net force would be:



Because there is a ***net force*** to the right, the box will ***accelerate*** to the right as a result of the force.

Forces are what cause acceleration. If a net force acts on an object, the object will speed up, slow down or change direction. Remember that *if the object's velocity is not changing, there is no net force, which means all of the forces on the object must cancel.*

Physics problems are sometimes classified in the following categories:

statics: situations in which there is no net force on an object. (*I.e.,* the object is not accelerating.)

dynamics: situations in which there is a net force on an object. (*I.e.,* the object is accelerating.)

In the MKS system, the unit of force is the newton (N). One newton is defined as the amount of force that it would take to cause a 1 kg object to accelerate at a rate of $1\,\frac{\text{m}}{\text{s}^2}$.

$$1\,\text{N} \equiv 1\,\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

Because the acceleration due to gravity on Earth is approximately $10\,\frac{\text{m}}{\text{s}^2}$, $\vec{F} = m\vec{a}$ becomes $\vec{F}_g = m\vec{g}$, which indicates that a 1 kg mass on Earth has a weight of approximately 10 N.

In more familiar units, one newton is approximately 3.6 ounces, which happens to be the weight of an average-sized apple. One pound is approximately 4.5 N.

Use this space for summary and/or additional notes:

Types of Forces

Weight ($\vec{F}_g$)

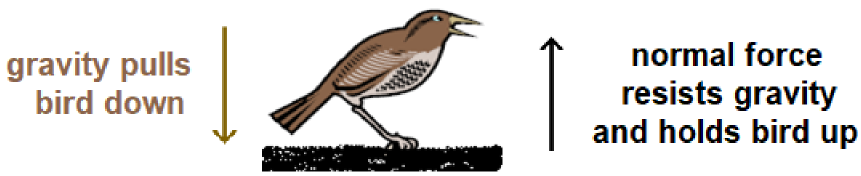
Weight ($\vec{F}_g, \vec{w}, m\vec{g}$) is what we call the action of the gravitational force. It is the downward force on an object that has mass, caused by the gravitational attraction between the object and another massive object, such as the Earth. The direction (assuming Earth) is always toward the center of the Earth.

In physics, we represent weight as the vector $\vec{F}_g$. Note that from Newton's second law, $\vec{F}_g = m\vec{g}$, which means on Earth, $\vec{F}_g = m(10)$. The unit for $\vec{g}$ is $\frac{\text{N}}{\text{kg}}$.

Normal Force ($\vec{F}_N$)

The normal force ($\vec{F}_N, \vec{N}$) is a force exerted by a surface (such as the ground or a wall) that resists a force exerted on that surface. The normal force is always perpendicular to the surface. (This use of the word "normal" comes from mathematics, and means "perpendicular".) The normal force is both a *contact* force and a *reaction* force.

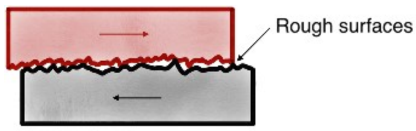
For example, if you push on a wall with a force of 10 N and the wall doesn't move, that means the force you apply causes the wall to apply a normal force of 10 N pushing back. The normal force is created by your pushing force, and it continues for as long as you continue pushing.



Friction ($\vec{F}_f$)

Friction ($\vec{F}_f, \vec{f}$) is a force that opposes sliding (or attempted sliding) of one surface along another. Friction is both a *contact* force and a *reaction* force. Friction is always parallel to the interface between the two surfaces.

Friction is caused by the roughness of the materials in contact, deformations in the materials, and/or molecular attraction between materials. Frictional forces are parallel to the plane of contact between two surfaces, and opposite to the direction of motion or applied force.



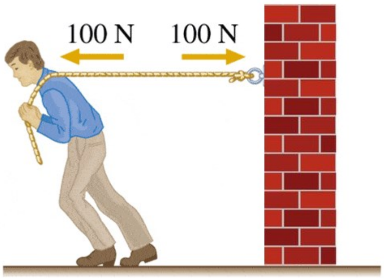
Friction is discussed in more detail in the Friction section, starting on page 297.

Use this space for summary and/or additional notes:

Tension ($\vec{F}_T$)

Tension ($\vec{F}_T$, $\vec{T}$) is the pulling force on a rope, string, chain, cable, *etc.* Tension is its own reaction force; ***tension always applies in both directions at once***. The direction of any tension force is along the rope, chain, *etc.*

For example, in the following picture the person pulls on the rope with a force of 100 N. The rope transmits the force to the wall, which causes a reaction force (also tension) of 100 N in the opposite direction. The reaction force pulls on the person.



This means that there is a force of 100 N exerted on the wall by the person (to the left), and a force of 100 N exerted on the person by the wall (to the right). The two forces cancel, which means there is no net force, and the total tension in the rope adds up to zero.

Thrust ($\vec{F}_t$)

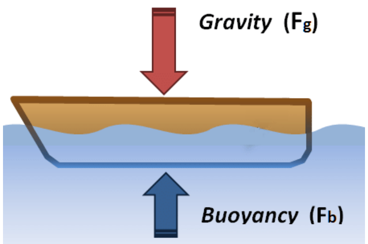
Thrust is any kind pushing force, which can be anything from a person pushing on a cart to the engine of an airplane pushing the plane forward. The direction is the direction of the push.

Spring Force ($\vec{F}_s$)

The spring force is an elastic force exerted by a spring, elastic (rubber band), *etc.* The spring force is a *reaction* force and a *restorative* force; if you pull or push a spring away from its equilibrium (rest) position, it will exert a force that attempts to return itself to that position. The direction is toward the equilibrium point.

Buoyancy ($\vec{F}_b$)

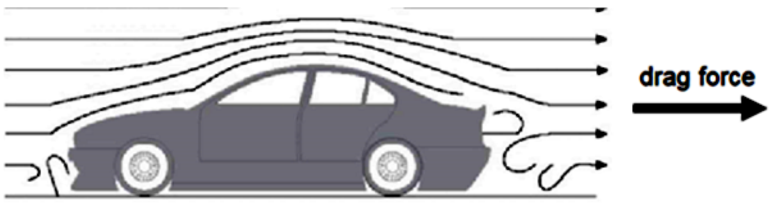
Buoyancy, or the buoyant force, is an upward force exerted by a fluid. The buoyant force causes (or attempts to cause) objects to float. The buoyant force is caused when an object displaces a fluid (pushes it out of the way). This causes the fluid level to rise. Gravity pulls down on the fluid, and the weight of the fluid causes a lifting force on the object. The direction of the buoyant force is always opposite to gravity. Buoyancy is discussed in detail in Physics 2.



Use this space for summary and/or additional notes:

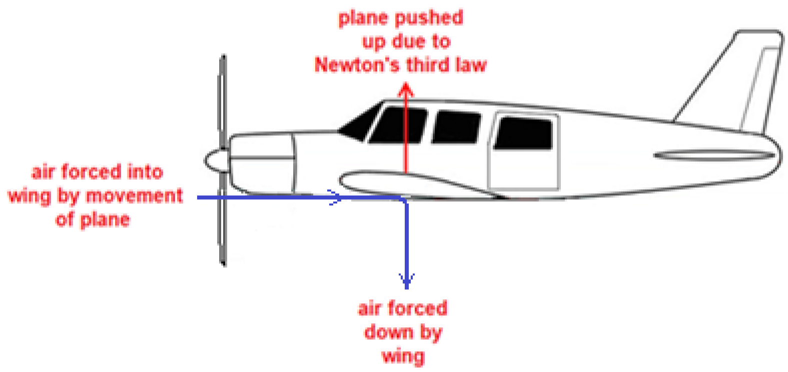
Drag $(\vec{F}_D)$

Drag is the opposing force from the particles of a fluid (liquid or gas) as an object moves through it. Drag is similar to friction; it is a contact force and a reaction force because it is caused by the relative motion of the object through the fluid, and it opposes the motion of the object. The direction is therefore opposite to the direction of motion of the object relative to the fluid. An object at rest does not push through any particles and therefore does not create drag. The drag force is described in more detail in the Drag section starting on page 306.



Lift $(\vec{F}_L)$

Lift is a reaction force caused by an object moving through a fluid at an angle. The object pushes the fluid downward, which causes a reaction force pushing the object upward. The term is most commonly used to describe the upward force on an airplane wing.



Electrostatic Force $(\vec{F}_e)$

The electrostatic force is a force of attraction or repulsion between objects that have an electrical charge. Like charges repel and opposite charges attract. The electrostatic force is described in more detail in the *Introduction*: Electrostatics unit starting on page 481.

Magnetic Force $(\vec{F}_B)$

The magnetic force is a force of attraction or repulsion between objects that have the property of magnetism. Magnetism is caused by the "spin" property of electrons. Like magnetic poles repel and opposite magnetic poles attract. Magnetism is described in more detail in the *Introduction: Magnetism* & Electromagnetism unit starting on page 549.

Use this space for summary and/or additional notes:

Summary of Common Forces

Force	Symbol	Definition	Direction
weight (gravitational force)	$\vec{F}_g$	pull by the Earth (or some other very large object) on an object with mass	toward the ground (or center of mass of the large object)
tension	$\vec{F}_T$	pull by a rope/string/cable	along the string/rope/cable
normal (perpendicular)	$\vec{F}_N$	contact/reaction force by a surface on an object	perpendicular to and away from surface
friction	$\vec{F}_f$	contact/reaction force that opposes sliding between surfaces	parallel to surface; opposite to direction of motion or applied force
thrust	$\vec{F}_t$	push that accelerates objects such as rockets, planes & cars	in the direction of the push
spring	$\vec{F}_s$	push or pull reaction force exerted by a spring	opposite to the displacement from equilibrium
buoyancy	$\vec{F}_b$	upward reaction force by a fluid on partially/completely submerged objects	opposite to gravity
drag (air/water resistance)	$\vec{F}_D$	reaction force caused by the molecules of a gas or liquid as an object moves through it	opposite to direction of motion
lift	$\vec{F}_L$	upward reaction force by a fluid (liquid or gas) on an object (such as an airplane wing) moving through it very fast at an angle	opposite to gravity
electrostatic force	$\vec{F}_e$	attractive or repulsive force between objects with electric charge	like charges repel; opposite charges attract
magnetic force	$\vec{F}_B$	attractive or repulsive force between objects with magnetism	like magnetic poles repel; opposite poles attract

CP1 & honors (not AP®)

Extension

The rate of change of force with respect to time is called "yank": $\vec{Y} = \frac{\Delta \vec{F}}{\Delta t}$. Just as $\vec{F} = m\vec{a}$, yank is the product of mass times jerk: $\vec{Y} = m\vec{j}$.

Use this space for summary and/or additional notes:

Gravitational Force

Unit: Forces in One Dimension

MA Curriculum Frameworks (2016): HS-PS2-10(MA)

AP® Physics 1 Learning Objectives: 2.B.1.1

Mastery Objective(s): (Students will be able to…)

- Explain gravity as a force field that acts on objects with mass.

Success Criteria:

- Explanation accounts for all terms in the field equation $\vec{F}_g = m\vec{g}$.

Language Objectives:

- Explain the concept of a force field that acts on objects with a certain property.

Tier 2 Vocabulary: gravity, force field

Labs, Activities & Demonstrations:

- Miscellaneous falling objects

Notes:

weight: the gravitational force acting on an object.

The gravitational force is an attractive force between objects that have mass. (This is caused by the action of a theoretical sub-atomic particle called a graviton mediating an interaction among Higgs bosons.) The amount of gravitational force between any two objects with mass can be calculated using the equation:

$$F_g = \frac{Gm_1m_2}{r^2}$$

where:

$F_g =$ gravitational force (N)

$G =$ universal gravitational constant $= 6.67\times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

$m_1 =$ mass of object #1 (kg)

$m_2 =$ mass of object #2 (kg)

$r =$ distance between the objects (m)

Use this space for summary and/or additional notes:

In this equation, suppose object #1 is the Earth and object #2 is some other object that has mass. This means m_1 is the mass of the Earth, m_2 is the mass of the object in question, and *r* is the distance from the center of the Earth[*] to the surface of the Earth, which means *r* is the radius of the Earth. The gravitation equation is therefore:

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{Gm_{Earth}m_{object}}{r_{Earth}^2}$$

Gravitational Field

On the surface of the Earth, we can model the gravitational force as a force field.

<u>force field</u>: a region in which a force acts upon objects or that have some particular characteristic or property.

The strength of this force field is based on the gravitational constant *G*, the mass of the Earth and the radius of the Earth. Because those values are all constant in any small region (within a few miles) on the surface of the Earth, we can combine them into a single constant, *g*:

$$g = \frac{Gm_{Earth}}{r_{Earth}^2} \quad \text{which means} \quad F_g = \frac{Gm_{Earth}m_{object}}{r_{Earth}^2} = gm_{object}$$

We can rewrite this equation, replacing m_{object} with just *m*. Also, because force is a vector and the force of gravity on an object is toward the other object (in this case, toward the center of mass of the Earth), we can write the equation in the following format:

$$\vec{\boldsymbol{F}}_g = m\vec{\boldsymbol{g}}$$

At different points on the surface of the Earth, the value of $\vec{\boldsymbol{g}}$ varies from approximately $9.76\,\frac{\text{N}}{\text{kg}}$ to $9.83\,\frac{\text{N}}{\text{kg}}$. In this course, unless otherwise noted, ***we will use the approximation that*** $\vec{\boldsymbol{g}} = 10\,\frac{\text{N}}{\text{kg}}$ ***.***

Don't worry about the equation for gravitation at this point—that concept and equation will be discussed further in the section on *Universal Gravitation,* starting on page 377. The equation $\vec{\boldsymbol{F}}_g = m\vec{\boldsymbol{g}}$ will be sufficient for the gravitational force in this unit.

[*] This should be the ***center of mass*** of the Earth. For the purposes of this section, we will assume that the Earth's center of mass is in its physical center.

Use this space for summary and/or additional notes:

Other types of force fields include electric fields, in which an electric force acts on all objects that have electric charge, and magnetic fields, in which a magnetic force acts on all objects that have magnetic susceptibility (the property that causes them to be attracted to or repelled by a magnet).

Units for Force Fields

The equation for the force due to any force field is that the force equals the quantity that the field acts on times the strength of the field:

$$\underbrace{\vec{F}_g}_{\text{force}} = \underbrace{m}_{\text{quantity that the field acts on}} \underbrace{\vec{g}}_{\text{strength of field}}$$

Because force is measured in newtons, the unit for a force field must therefore be newtons divided by the unit for the quantity that the force acts on. This means that the unit for $\vec{g}$ must be $\frac{\text{N}}{\text{kg}}$. Note that $1\frac{\text{N}}{\text{kg}} \equiv 1\frac{\text{m}}{\text{s}^2}$, *i.e.,* the unit $\frac{\text{N}}{\text{kg}}$ is mathematically equivalent to the unit $\frac{\text{m}}{\text{s}^2}$. Thus, a gravitational field of $10\frac{\text{N}}{\text{kg}}$ produces an acceleration of $10\frac{\text{m}}{\text{s}^2}$.

In physics, we use $\vec{g}$ to represent ***both*** the strength of the gravitational force near the surface of the Earth (in $\frac{\text{N}}{\text{kg}}$) ***and*** the acceleration due to gravity near the surface of the Earth (in $\frac{\text{m}}{\text{s}^2}$). Therefore, what $\vec{g}$ actually means *and the units used for it* depend on context!

Sample Problem:

Q: What is the weight of (*i.e.,* the force of gravity acting on) a 7 kg block?

A: weight = $\vec{F}_g = m\vec{g} = (7)(10) = 70\,\text{N}$

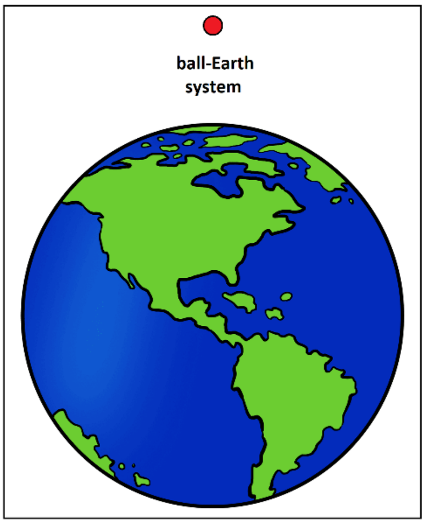
Use this space for summary and/or additional notes:

Force Fields and Systems

For the purposes of this course, we usually think of a force field as external to a system, which means the field can be considered to act on the system as a whole, as well as every component of the system that the field acts upon. (In the case of the gravitational field, that means every component of the system that has mass.)

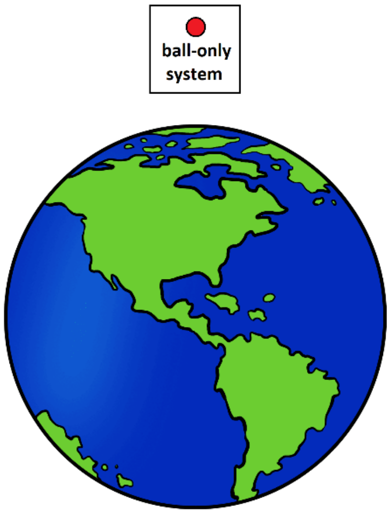
When we define a system of objects in order to make a situation or problem easier to understand (see *Systems* on page 254), the system can either include or exclude the Earth. This means that we would only use the force field definition for a single object or for a system that does not include the Earth.

If the system includes the Earth, we need to consider the gravitational force to be a force between two objects, one of which is the Earth.



two objects (one of which is Earth)

$$F_g = \frac{Gm_{Earth}m_{object}}{r^2_{Earth}} = mg$$



gravitational field

$$F_g = mg$$

Note that the gravitational force is the same no matter which way we calculate it. This is important—the strength of the gravitational force cannot depend on how we choose to look at it!

Use this space for summary and/or additional notes:

g-Forces

Acceleration is often described in terms of "g-force". "g-force" represents the acceleration to which an object (usually a person) is subjected as a fraction/multiple of Earth's gravity. A force of 1 g is equivalent to the $10\,\frac{\text{N}}{\text{kg}} \equiv 10\,\frac{\text{m}}{\text{s}^2}$ that would apply to an object in free fall near the surface of the Earth.

Vertical Frame of Reference Accelerations*

g-force	Description
−5	limit of sustained human tolerance
−2	severe blood congestion; throbbing headache; reddening of vision (redout)
−1	congestion of blood in head
0	free fall; orbit (apparent weightlessness)
+⅙	surface of the moon (not accelerating)
+⅓	surface of Mars (not accelerating)
+1	surface of the Earth (not accelerating)
+4.5	roller coaster maximum at bottom of first dip
+3.4–4.8	partial loss of vision (grayout)
+3.9−5.5	complete loss of vision (blackout)
+4.5−6.3	loss of consciousness for most people

Horizontal Frame of Reference Accelerations*

g-force	Description
0	at rest or moving at constant velocity
0.4	maximum acceleration of typical American car
0.8	maximum acceleration in a high-performance sports car
2	"Extreme Launch" roller coaster at start
3	space shuttle, maximum at takeoff
8	limit of sustained human tolerance
60	chest acceleration limit during car crash at 30 mph with airbag
3400	impact acceleration limit of "black box" flight data recorder

* Data are from the Physics Hypertextbook, https://physics.info

Use this space for summary and/or additional notes:

Free-Body Diagrams

Unit: Forces in One Dimension

MA Curriculum Frameworks (2016): HS-PS2-10(MA)

AP® Physics 1 Learning Objectives: 3.A.2.1, 3.A.3.1, 3.A.4.3, 3.B.1.1, 3.B.1.2, 3.B.2.1, 4.A.2.2, 4.A.3.2

Mastery Objective(s): (Students will be able to…)

- Draw a free-body diagram that represents all of the forces on an object and their directions.

Success Criteria:

- Each force starts from the dot representing the object.
- Each force is represented as a separate arrow pointing in the direction that the force acts.

Language Objectives:

- Explain how a dot with arrows can be used to represent an object with forces.

Tier 2 Vocabulary: force, free, body

Labs, Activities & Demonstrations:

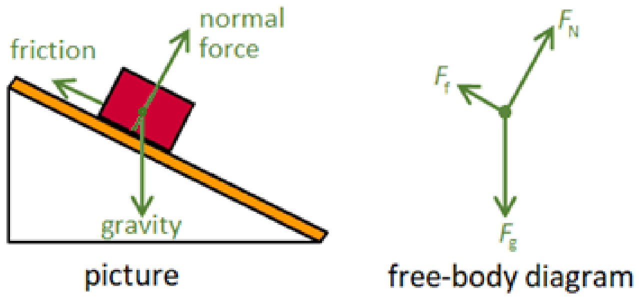
- Human free-body diagram activity.

Notes:

<u>free-body diagram</u> (force diagram): a diagram representing all of the forces acting on an object.

In a free-body diagram, we represent the object as a dot, and each force as an arrow. The direction of the arrow represents the direction of the force, and the relative lengths of the arrows represent the relative magnitudes of the forces.

Consider the following situation:

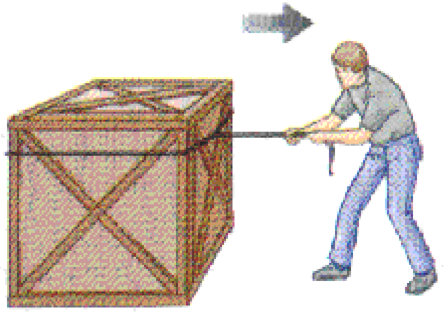


In the picture, a block is sitting on a ramp. The forces on the block are gravity (straight down), the normal force (perpendicular to and away from the ramp), and friction (parallel to the ramp).

In the free-body diagram, the block is represented by a dot. The forces, represented by arrows, are gravity (F_g), the normal force (F_N), and friction (F_f).

Use this space for summary and/or additional notes:

Now consider the following situation of a box that *accelerates* to the right as it is pulled across the floor by a rope:

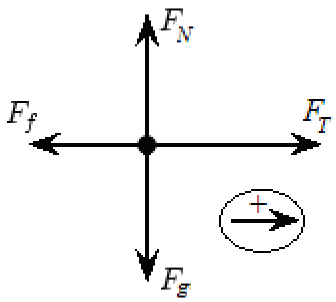


From the picture and description, we can assume that:

- The box has weight, which means gravity is pulling down on it.
- The floor is holding up the box.
- The rope is pulling on the box.
- Friction between the box and the floor is resisting the force from the rope.
- Because the box is accelerating to the right, the force applied by the rope must be stronger than the force from friction.

In the free-body diagram for the accelerating box, we again represent the object (the box) as a dot, and the forces (vectors) as arrows. Because there is a net force, we should also include a legend that shows which direction is positive.

The forces are:



- $\vec{F}_g$ = the force of gravity pulling down on the box
- $\vec{F}_N$ = the normal force (the floor holding the box up)
- $\vec{F}_T$ = the force of tension from the rope. (This might also be designated $\vec{F}_a$ because it is the force *applied* to the object.)
- $\vec{F}_f$ = the force of friction resisting the motion of the box.

Notice that the arrows for the normal force and gravity are equal in length, because in this problem, these two forces are equal in magnitude.

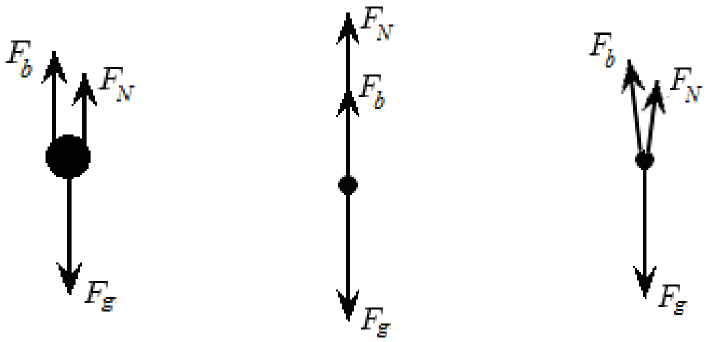
Notice that the arrow for friction is shorter than the arrow for tension, because in this problem the tension is stronger than the force of friction. The difference between the lengths of these two vectors would be the net force, which is what causes the box to accelerate to the right.

In general, if the object is moving, it is easiest to choose the positive direction to be the direction of motion. In our free-body diagram, the legend in the bottom right corner of the diagram shows an arrow with a "+" sign, meaning that we have chosen to make the positive direction to the right.

Use this space for summary and/or additional notes:

If you have multiple forces in the same direction, each force vector must originate from the point that represents the object, and must be as close as is practical to the *exact* direction of the force.

For example, consider a rock sitting at the bottom of a pond. The rock has three forces on it: the buoyant force ($\vec{F}_b$) and the normal force ($\vec{F}_N$), both acting upwards, and gravity ($\vec{F}_g$) acting downwards.



correct incorrect incorrect

The first representation is correct because all forces originate from the dot that represents the object, the directions represent the exact directions of the forces, and the length of each is proportional to its strength.

The second representation is incorrect because it is unclear whether $\vec{F}_N$ starts from the object (the dot), or from the tip of the $\vec{F}_b$ arrow.

The third representation is incorrect because it implies that $\vec{F}_b$ and $\vec{F}_N$ each have a slight horizontal component, which is not true.

Because there is no net force (the rock is just sitting on the bottom of the pond), the forces must all cancel. This means that the lengths of the arrows for $\vec{F}_b$ and $\vec{F}_N$ need to add up to the length of the arrow for $\vec{F}_g$.

Use this space for summary and/or additional notes:

Steps for Drawing Free-Body Diagrams

In general, the following are the steps for drawing most free-body diagrams.

1. Is gravity involved? (In most physics problems that take place on Earth near the planet's surface, the answer is yes.)
    - Represent gravity as $\vec{F}_g$ pointing straight down.

2. Is something holding the object up?
    - If it is a flat surface, it is the normal force ($\vec{F}_N$), perpendicular to the surface.
    - If it is a rope, chain, *etc.,* it is the force of tension ($\vec{F}_T$) acting along the rope, chain, *etc.*

3. Is there a force pulling or pushing on the object?
    - If the pulling force involves a rope, chain, *etc.*, the force is tension ($\vec{F}_T$) and the direction is along the rope, chain, *etc*.
    - A pushing force is called thrust ($\vec{F}_t$).
    - Only include forces that are acting currently. (Do not include forces that acted in the past but are no longer present.)

4. Is there friction?
    - If there are two surfaces in contact, there is almost always friction ($\vec{F}_f$), unless the problem specifically states that the surfaces are frictionless. (In physics problems, ice is almost always assumed to be frictionless.)
    - At low velocities, air resistance is very small and can usually be ignored unless the problem explicitly states otherwise.
    - Usually, all sources of friction are shown as one combined force. *E.g.,* if there is sliding friction along the ground and also air resistance, the $\vec{F}_f$ vector includes both.

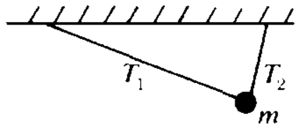
5. Do we need to choose positive & negative directions?
    - If the problem requires calculations involving opposing forces, you need to indicate which direction is positive. If the problem does not require calculations or if there is no net force, you do not need to do so.

Use this space for summary and/or additional notes:

AP®

What AP® Free-Body Diagram Problems Look Like

AP® force problems almost always involve free-body diagrams of a stationary object with multiple forces on it. Here are a couple of examples:



Q: A ball of mass m is suspended from two strings of unequal length as shown above. The magnitudes of the tensions T_1 and T_2 in the strings must satisfy which of the following relations?

(A) $T_1 = T_2$ (B) $T_1 > T_2$ (C) $T_1 < T_2$ (D) $T_1 + T_2 = mg$

A: Remember that forces are vectors, which have direction as well as magnitude. This means that T_1 and T_2 must each have a vertical and horizontal component. The ball is not moving, which means there is no acceleration and therefore $F_{net} = 0$. For F_{net} to be zero, the components of all forces must cancel overall, *and separately in every dimension*. This means, the vertical components of T_1 and T_2 must add up to mg, and the horizontal components of T_1 and T_2 must cancel (add up to zero). Therefore, answer choice (D) $T_1 + T_2 = mg$ is correct.

Use this space for summary and/or additional notes:

Homework Problems

For each picture, draw a free-body diagram that shows all of the forces acting upon the object (represented by the underlined word) in the picture.

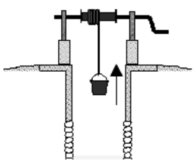
1. **(M)** A bird sits motionless on a perch.

2. **(M)** A hockey player glides at ***constant velocity*** across the ice. (*Ignore friction.*)

3. **(M)** A baseball player slides into a base.

4. **(M)** A chandelier hangs from the ceiling, suspended by a chain.

5. **(M)** A bucket of water is raised out of a well at ***constant velocity***.



Use this space for summary and/or additional notes:

6. **(M)** A <u>skydiver</u> has just jumped out of an airplane and begins ***accelerating*** toward the ground.

7. **(M)** A <u>skydiver</u> falls through the air at ***terminal velocity***. (*Terminal velocity* means the velocity has stopped changing and is constant.)

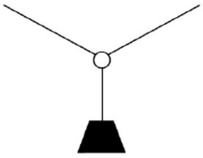
8. **(M)** A <u>hurdler</u> is moving horizontally as she clears a hurdle. (*Ignore air resistance.*)

9. **(M)** An <u>airplane</u> moves through the air in ***level flight*** at ***constant velocity***.

10. **(M)** A <u>sled</u> is pulled through the snow at ***constant velocity***. (*Note that the rope is at an angle.*)

Use this space for summary and/or additional notes:

11. **(M)** A stationary metal ring is held by three ropes, one of which has a mass hanging from it. (*Draw the free-body diagram for the metal ring.*)



12. **(M)** A child swings on a swing.
(*Ignore all sources of friction, including air resistance.*)

13. **(M)** A squirrel sits motionless on a sloped roof.

14. **(M)** A skier moves down a slope at ***constant velocity***.

15. **(M)** A skier ***accelerates*** down a slope.

Use this space for summary and/or additional notes:

Newton’s Second Law

Unit: Forces in One Dimension

MA Curriculum Frameworks (2016): HS-PS2-1, HS-PS2-10(MA)

AP® Physics 1 Learning Objectives: 1.C.1.1, 2.B.1.1, 3.A.2.1, 3.B.1.1, 3.B.1.3, 4.A.2.1, 4.A.3.1, 4.A.3.2

Mastery Objective(s): (Students will be able to…)

- Solve problems relating to Newton’s second law $(\vec{F}_{net} = m\vec{a})$.
- Solve problems that combine kinematics (motion) and forces.

Success Criteria:

- Free-body diagram is correct.
- Vector quantities position, velocity, and acceleration are correct, including sign (direction).
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Identify the quantities in a word problem and assign the correct variables to them.
- Select equations that relate the quantities given in the problem.

Tier 2 Vocabulary: force, free, body, displacement, acceleration

Labs, Activities & Demonstrations:

- Handstands in an elevator.

Notes:

Newton’s Second Law: Forces cause acceleration (a change in velocity). “A net force, $\vec{F}$, acting on an object causes the object to accelerate in the direction of the net force.”

If there is a net force, the object accelerates (its velocity changes). If there is no net force, the object’s velocity remains the same.

If an object accelerates (its velocity changes), there was a net force on it. If an object’s velocity remains the same, there was no net force on it.

Remember that forces are vectors. “No net force” can either mean that there are no forces at all, or it can mean that there are equal forces in opposite directions and their effects cancel.

static equilibrium: when all of the forces on an object cancel each other’s effects (resulting in a net force of zero) and the object remains stationary.

dynamic equilibrium: when all of the forces on an object cancel each other’s effects (resulting in a net force of zero) and the object remains in motion with constant velocity.

Use this space for summary and/or additional notes:

In equation form:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{\sum \vec{F}}{m} \quad \text{or} \quad \vec{F}_{\text{net}} = \sum \vec{F} = m\vec{a}$$

The first form is preferred for teaching purposes, because acceleration is what results from a force applied to a mass. (*I.e.,* force and mass are the manipulated variables and acceleration is the responding variable. Forces cause acceleration, not the other way around.) However, the equation is more commonly written in the second form, which makes the typesetting and the algebra easier.

Sample Problems

Most of the physics problems involving forces require the application of Newton's Second Law, $\vec{F}_{\text{net}} = \sum \vec{F} = m\vec{a}$.

Q: A net force of 50 N in the positive direction is applied to a cart that has a mass of 35 kg. How fast does the cart accelerate?

A: Applying Newton's Second Law:

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{50}{35} = 1.43 \tfrac{\text{m}}{\text{s}^2}$$

Q: Two children are fighting over a toy.

Jamie pulls to the left with a force of 40 N, and Edward pulls to the right with a force of 60 N. If the toy has a mass of 0.6 kg, what is the resulting acceleration of the toy?

A: The free-body diagram looks like this:

(We chose the positive direction to the right because it makes more intuitive sense for the positive direction to be the direction that the toy will move.)

$$\sum \vec{F} = m\vec{a}$$

$$-40 + 60 = (0.6)\vec{a}$$

$$\vec{a} = \frac{+20}{0.6} = +33.3 \tfrac{\text{m}}{\text{s}^2} \text{ (to the right)}$$

Use this space for summary and/or additional notes:

Q: A person applies a net force of 100. N to cart full of books that has a mass of 75 kg. If the cart starts from rest, how far will the cart have moved by the time it gets to a speed of $4.0\,\frac{\text{m}}{\text{s}}$?

A: Using the GUESS system, we can see that only two of the quantities are known (initial velocity and final velocity). However, we can find acceleration from $\vec{F}_{net} = m\vec{a}$, at which point we have the quantities that we need to solve the motion problem. This means we need to add a second GUESS chart for Newton's second law. Because $\vec{a}$ appears in both equations, we connect it in the two charts.

Motion Equations

var.	dir.	value
$\vec{d}$	→	$\vec{d}$
$\vec{v}_o$	N/A	0
$\vec{v}$	→	$+4\,\frac{\text{m}}{\text{s}}$
$\vec{a}$	→	$\vec{a}$
t	–	–

$$\frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$$
$$\vec{v} - \vec{v}_o = \vec{a}t$$
$$\vec{d} = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2$$
$$\boxed{\vec{v}^2 - \vec{v}_o^{\,2} = 2\vec{a}\vec{d}}$$

Newton's Second Law

var.	dir.	value
$\vec{F}_{net}$	→	$\vec{F}_{net}$
m	N/A	5 kg
$\vec{a}$	→	$\vec{a}$

$$\vec{F}_{net} = m\vec{a}$$

Our strategy is therefore:

1. Find acceleration from $\vec{F}_{net} = m\vec{a}$:

$$\vec{F}_{net} = m\vec{a}$$
$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{100}{75} = 1.\overline{3}\,\frac{\text{m}}{\text{s}^2}$$

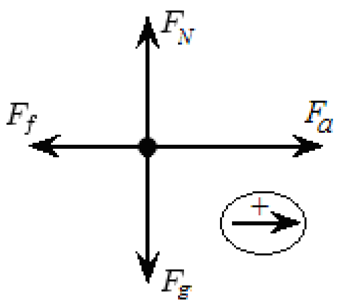
2. Now that we have $\vec{a}$ we can use the last motion equation to solve the problem:

$$\vec{v}^2 - \vec{v}_o^{\,2} = 2\vec{a}\vec{d}$$
$$\frac{\vec{v}^2 - \vec{v}_o^{\,2}}{2a} = \vec{d}$$
$$\vec{d} = \frac{4^2 - 0^2}{(2)(1.\overline{3})} = \frac{16}{2.\overline{6}} = 6\,\text{m}$$

Use this space for summary and/or additional notes:

Q: A 5.0 kg block is resting on a horizontal, flat surface. How much force is needed to overcome a force of 2.0 N of friction and accelerate the block from rest to a velocity of $6.0\,\frac{\text{m}}{\text{s}}$ over a 1.5-second interval?

A: This is a combination of a Newton's second law problem, and a motion problem. There are multiple forces in the problem, so we should draw a free-body diagram so we can visualize what's going on.



We are trying to find the applied force, $\vec{F}_a$.

Again using the GUESS system, we now have three connected equations. Our strategy is to start with the equation that contains the quantity we need ($\vec{F}_a$). Each time we need a quantity that we don't have, we tack on an additional GUESS chart that enables us to calculate that quantity.

List of Forces

var.	dir.	value
$\vec{F}_{net}$	→	$\vec{F}_{net}$
$\vec{F}_a$	→	$\vec{F}_a$
$\vec{F}_f$	←	−2 N

$$\vec{F}_{net} = \sum \vec{F} = \vec{F}_a + \vec{F}_f$$

Newton's Second Law

var.	dir.	value
$\vec{F}_{net}$	→	$\vec{F}_{net}$
m	N/A	5 kg
$\vec{a}$	→	$\vec{a}$

$$\vec{F}_{net} = m\vec{a}$$

Motion Equations

var.	dir.	value
$\vec{d}$	–	–
$\vec{v}_o$	N/A	0
$\vec{v}$	→	$+6\,\frac{\text{m}}{\text{s}}$
$\vec{a}$	→	$\vec{a}$
t	N/A	1.5 s

$$\frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$$

$$\boxed{\vec{v} - \vec{v}_o = \vec{a}t}$$

$$\vec{d} = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2$$

$$\vec{v}^2 - \vec{v}_o^{\,2} = 2\vec{a}\vec{d}$$

Use this space for summary and/or additional notes:

Based on our GUESS charts, our strategy is therefore:

1. Use motion equations to find acceleration:

$$\vec{\boldsymbol{v}} - \vec{\boldsymbol{v}}_o = \vec{\boldsymbol{a}}t$$
$$\frac{\vec{\boldsymbol{v}} - \vec{\boldsymbol{v}}_o}{t} = \vec{\boldsymbol{a}}$$
$$\frac{6-0}{1.5} = \vec{\boldsymbol{a}} = 4\,\tfrac{\text{m}}{\text{s}^2}$$

2. Use $\vec{\boldsymbol{F}}_{net} = m\vec{\boldsymbol{a}}$ to find $\vec{\boldsymbol{F}}_{net}$:

$$\vec{\boldsymbol{F}}_{net} = m\vec{\boldsymbol{a}} = (5)(4) = 20\,\text{N}$$

3. Use $\vec{\boldsymbol{F}}_{net} = \sum\vec{\boldsymbol{F}}$ to find $\vec{\boldsymbol{F}}_a$. We need to remember that $\vec{\boldsymbol{F}}_f$ is negative because it is in the negative direction.

$$\vec{\boldsymbol{F}}_{net} = \sum\vec{\boldsymbol{F}} = \vec{\boldsymbol{F}}_a + \vec{\boldsymbol{F}}_f$$
$$20 = \vec{\boldsymbol{F}}_a + (-2)$$
$$\vec{\boldsymbol{F}}_a = 22\,\text{N}$$

Use this space for summary and/or additional notes:

Homework Problems

1. **(S)** Two horizontal forces, 225 N and 165 N are exerted on a canoe. If these forces are both applied eastward, what is the net force on the canoe?

2. **(S)** Two horizontal forces are exerted on a canoe, 225 N westward and 165 N eastward. What is the net force on the canoe?

3. **(M)** Three confused sled dogs are trying to pull a sled across the snow in Alaska. Alutia pulls to the east with a force of 135 N. Seward pulls to the east with a force of 143 N. Kodiak pulls to the west with a force of 153 N.

   a. **(M)** What is the net force on the sled?

   Answer: 125 N east

   b. **(M)** If the sled has a mass of 150. kg and the driver has a mass of 100. kg, what is the acceleration of the sled? (Assume there is no friction between the runners of the sled and the snow.)

   Answer: $0.500\,\frac{m}{s^2}$

Use this space for summary and/or additional notes:

4. **(S)** When a net force of 10. N acts on a hockey puck, the puck accelerates at a rate of $50.\,\frac{m}{s^2}$. Determine the mass of the puck.

Answer: 0.20 kg

5. **(S)** A 15 N net force is applied for 6.0 s to a 12 kg box initially at rest. What is the speed of the box at the end of the 6.0 s interval?

Answer: $7.5\,\frac{m}{s}$

6. **(S)** A cart with a mass of 0.60 kg is propelled by a fan. The cart starts from rest, and travels 1.2 m in 4.0 s. What is the net force applied by the fan?

Answer: 0.09 N

7. **(M)** A child with a mass of 44 kg stands on a scale that reads in newtons.
   a. **(M)** What is the child's weight?

   b. **(M)** The child now places one foot on each of two scales side-by-side. If the child distributes equal amounts of weight between the two scales, what is the reading on each scale?

Use this space for summary and/or additional notes:

8. **(S)** A 70.0 kg astronaut pushes on a spacecraft with a force $\vec{F}$ in space. The spacecraft has a total mass of 1.0×10^4 kg. The push causes the astronaut to accelerate to the right with an acceleration of $0.36\,\frac{m}{s^2}$. Determine the magnitude of the acceleration of the spacecraft.

   Answer: $0.0025\,\frac{m}{s^2}$

honors & AP®

9. **(M – honors & AP®; A – CP1)** How much force will it take to accelerate a student with mass *m*, wearing special frictionless roller skates, across the ground from rest to velocity *v* in time *t*?

   (*If you are not sure how to do this problem, do #10 below and use the steps to guide your algebra.*)

   Answer: $F=\frac{mv}{t}$

10. **(S – honors & AP®; M – CP1)** How much force will it take to accelerate a 60 kg student, wearing special frictionless roller skates, across the ground from rest to $16\,\frac{m}{s}$ in 4 s?

    (<u>You must start with the equations in your Physics Reference Tables *and show all of the steps of GUESS.*</u> *You may only use the answer to question #9 above as a starting point if you have already solved that problem.*)

    Answer: 240 N

Use this space for summary and/or additional notes:

11. **(M)** How much force would it take to accelerate a 60 kg student upwards at $2\,\frac{\text{m}}{\text{s}^2}$?

    Hint: you need to account for gravity. Draw a free-body diagram.

    Answer: 720 N

12. **(S)** An air conditioner weighs 400 N on Earth. How much would the air conditioner weigh on the planet Mercury, where the value of $\vec{g}$ is only $3.6\,\frac{\text{N}}{\text{kg}}$?

    Answer: 144 N

13. **(M – honors & AP®; S – CP1)** A person pushes a 500 kg crate with a force of 1200 N and the crate accelerates at $0.5\,\frac{\text{m}}{\text{s}^2}$. What is the force of friction acting on the crate?
    Hint: draw the free-body diagram.

    Answer: 950N

Use this space for summary and/or additional notes:

Tension

Unit: Forces in One Dimension

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 1.C.1.1, 2.B.1.1, 3.A.2.1, 3.B.1.1, 3.B.1.2, 3.B.1.3, 3.B.2.1, 4.A.2.3, 4.A.3.1, 4.A.3.2

Mastery Objective(s): (Students will be able to…)

- Set up and solve problems involving pulleys and ropes under tension.

Success Criteria:

- Expressions involving tension and acceleration are correct including the sign (direction).
- Equations for all parts of the system are combined correctly algebraically.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how the sign of all of the forces in a pulley system relate to the direction that the system will move.

Tier 2 Vocabulary: pulley, tension

Labs, Activities & Demonstrations:

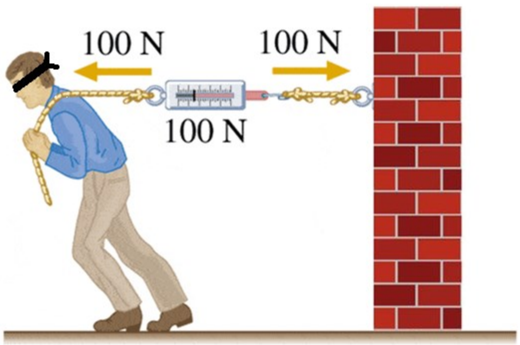
- Atwood machine

Notes:

<u>tension</u> ($\vec{F}_T$, $\vec{T}$): the pulling force on a rope, string, chain, cable, *etc.*

Tension is its own reaction force; tension always travels through the rope in both directions at once, and unless there are additional forces between one end of the rope and the other, the tension at every point along the rope is the same. The direction of tension is always along the rope.

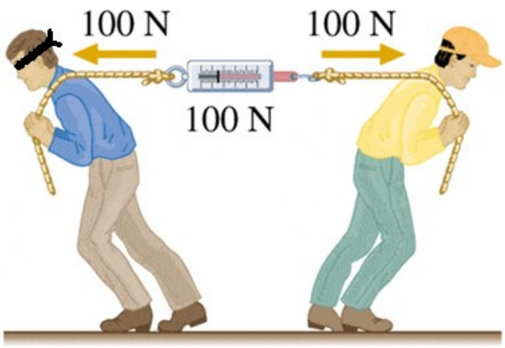
For example, in the following picture a blindfolded person pulls on a rope with a force of 100 N. The rope transmits the force to the scale, which transmits the force to the other rope and then to the wall. This causes a reaction force (also tension) of 100 N in the opposite direction.



The scale attached to the rope measures 100 N, because that is the amount of force (tension) that is stretching the spring in the scale.

Use this space for summary and/or additional notes:

If we replace the brick wall with a person who is pulling with a force of 100 N, the blindfolded person has no idea whether the 100 N of resistance is coming from a brick wall or another person. Thus, the forces acting on the blindfolded person (and the scale) are the same.

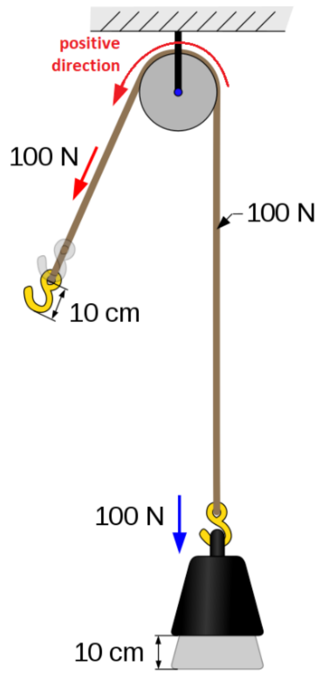


Of course, the scale doesn't "know" either, so it still reads 100 N.

Pulleys

pulley: a wheel used to change the direction of tension on a rope

The tension remains the same in all parts of the rope.



In the example at the right (with one pulley), it takes 100 N of force to lift a 100 N weight. The pulley changes the direction of the force, but the amount of force does not change. If the rope is pulled 10 cm, the weight is lifted by the same 10 cm.

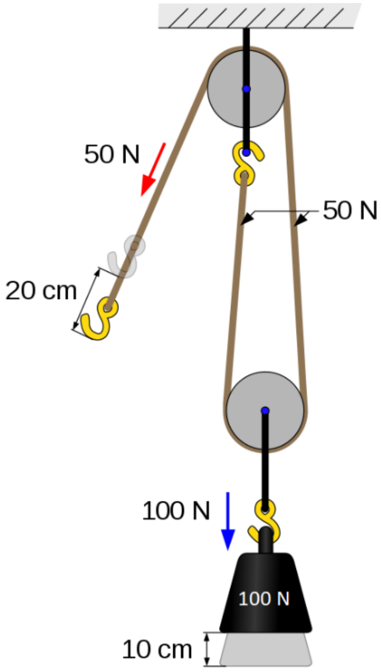
Up to this point, we have chosen a single direction (left/right or up/down) to be the positive direction. With pulleys, we usually define the positive and negative directions to follow the rope. In this example, we would most likely choose the positive direction to be the direction that the rope is pulled. Instead of saying that positive is upward for the weight and downward for the hook, we would usually say that the positive direction is counter-clockwise (↺), because that is the direction that the pulley will turn.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Mechanical Advantage

If we place a second pulley just above the weight that we want to lift, two things will happen when we pull on the rope:



1. As we pull on the rope, there is less rope between the two pulleys. This means the lower pulley will move upward.

2. Both of the sections of rope that go around the lower pulley will be lifting the 100 N weight. This means each side will hold half of the weight (50 N). Therefore, the tension in every part of the rope is 50 N, which means it takes half as much force to lift the weight.

3. The length of rope that is pulled is divided between the two sections that go around the lower pulley. This means that pulling the rope 20 cm will raise the weight half as much (10 cm).

Notice that when the force is cut in half, the length of rope is doubled. The double pulley is effectively trading force for distance. Later, in the *Introduction: Energy, Work* & Power unit starting on page 384, we will see that force times distance is work (change in energy). This means using half as much force but pulling the rope twice as much distance takes the same amount of energy to lift the weight.

As you would expect, as we add more pulleys, the force needed is reduced and the distance increases. This reduction in force is called mechanical advantage.

<u>mechanical advantage</u>: the ratio of the force applied by a machine divided by the force needed to operate it.

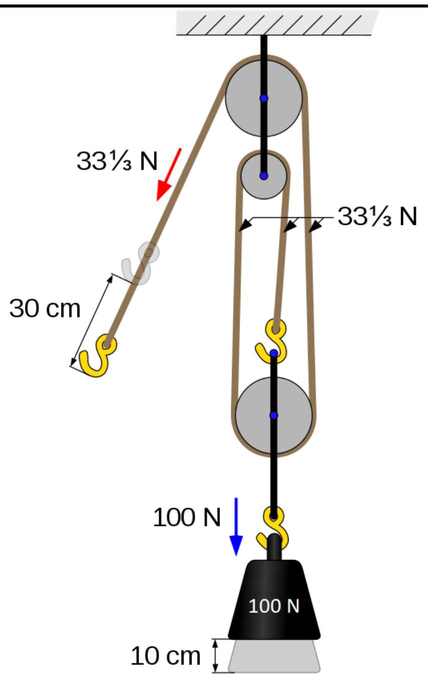
The mechanical advantage is equal to the number of ropes supporting the hanging weight, and therefore also equal to the number of pulleys.

The mechanical advantage of the above system is 2.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

If we add a third pulley, we can see that there are now three sections of the rope that are lifting the 100 N weight. This means that each section is holding up ⅓ of the weight. This means that the tension in the rope is ⅓ of 100 N, or 33⅓ N, but we now need to pull three times as much rope to lift the weight the same distance.



A two-pulley system has a mechanical advantage of 2, because it applies twice as much force to the weight as you need to apply to the rope. Similarly, a 3-pulley system has a mechanical advantage of 3, and so on.

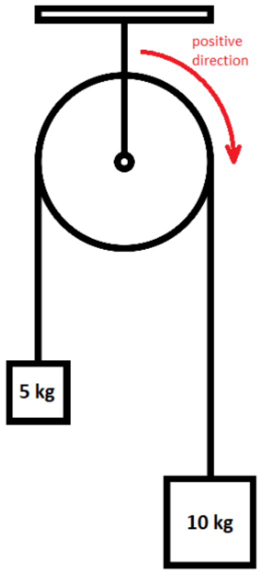
The mechanical advantage of any pulley system equals the number of ropes participating in the lifting.

<u>block and tackle</u>: a system of two or more pulleys (which therefore have a mechanical advantage of 2 or more) used for lifting heavy loads.

Use this space for summary and/or additional notes:

Atwood's Machine

Atwood's machine is named for the English mathematician George Atwood. The machine is a device with a single pulley in which one weight, which is pulled down by gravity, is used to lift a second weight. Atwood invented the machine in 1784 to verify Isaac Newton's equations of motion with constant acceleration.



To illustrate how Atwood's experiment works, consider the system to the right. To simplify the problem, we will assume that the pulley has negligible mass and operates with no friction. Let us choose the positive direction as the direction that turns the pulley clockwise (↻). We could have chosen either direction to be positive, but it makes intuitive sense to choose the direction that the system will move when we release the weights.

The force on the mass on the right is its weight, which is $m\vec{g} = (10)(+10) = 100\,\text{N}$. (We use a positive value for $\vec{g}$ because gravity is attempting to pull this weight in the positive direction.)

The force on the mass on the left is $m\vec{g} = (5)(-10) = -50\,\text{N}$. (We use a negative value for $\vec{g}$ because gravity is attempting to pull this weight in the positive direction.)

The net force on the system is therefore $\vec{F}_{net} = \sum \vec{F} = 100 + (-50) = 50\,\text{N}$.

The masses are connected by a rope, which means both masses will accelerate together. The total mass is 15 kg.

Newton's Second Law says:

$$\begin{aligned}\vec{F}_{net} &= \sum \vec{F} = m\vec{a}\\ +50 &= 15\vec{a}\\ \vec{a} &= \frac{50}{15} = +3.\bar{3}\,\tfrac{\text{m}}{\text{s}^2}\end{aligned}$$

I.e., the system will accelerate at $3.3\,\frac{\text{m}}{\text{s}^2}$ in the positive direction (clockwise).

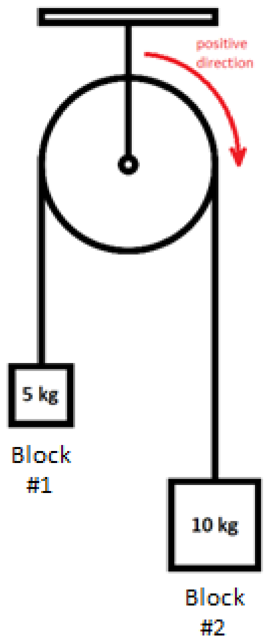
Atwood performed experiments with different masses and observed behavior that was consistent with both Newton's second law, and with Newton's equations of motion.

Notice that the solution to finding acceleration in a problem involving Atwood's machine is to simply find the net force, add up the total mass, and use $\vec{F}_{net} = m\vec{a}$.

Use this space for summary and/or additional notes:

An important feature of Newton's second law is that it can be applied to an <u>entire system</u>, or to <u>any component</u> of the system.

For the Atwood's machine pictured, we found that:



Entire system:

$$\vec{F}_{net} = m\vec{a}$$
$$+50\,\text{N} = (5\,\text{kg} + 10\,\text{kg})(3.\overline{3}\,\tfrac{\text{m}}{\text{s}^2})$$

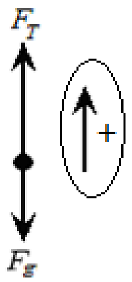
We can apply Newton's second law to each block separately:

$$\vec{F}_{1,net} = m_1\vec{a}$$
$$\vec{F}_{2,net} = m_2\vec{a}$$

Because the blocks are connected via the same rope, the acceleration is the same for both blocks.

This means that we can apply Newton's second law to either of the blocks to determine the tension in the rope:

Block #1:

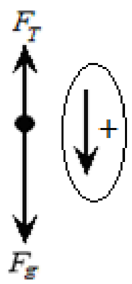


$$\vec{F}_{1,net} = (\vec{F}_T - \vec{F}_{g,1}) \qquad \vec{F}_{1,net} = m_1\vec{a}$$
$$(\vec{F}_T - m_1\vec{g}) = m_1\vec{a}$$
$$[\vec{F}_T - (5)(10)] = (5)(3.\overline{3})$$
$$\vec{F}_T - (-50) = 16.\overline{6}$$
$$\vec{F}_T = 66.\overline{6}\,\text{N}$$

Block #2: (same calculation; yields same result)

We can do the same calculation for Block #2, with the same result for $\vec{F}_T$.

(Remember that we chose the positive direction to be the direction that the system moves. This means the positive direction is up for block #1, but down for block #2.)



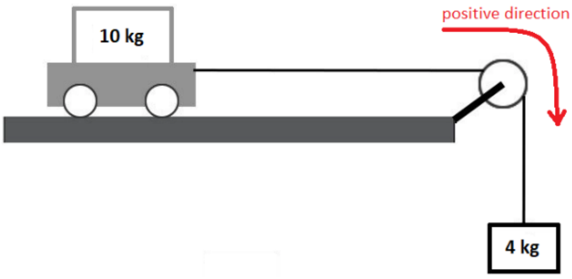
$$\vec{F}_{2,net} = (\vec{F}_{g,2} - \vec{F}_T) \qquad \vec{F}_{2,net} = m_2\vec{a}$$
$$(m_2\vec{g} - \vec{F}_T) = m_2\vec{a}$$
$$[(10)(10) - \vec{F}_T] = (10)(3.\overline{3})$$
$$100 - \vec{F}_T = 33.\overline{3}$$
$$\vec{F}_T = 66.\overline{6}\,\text{N}$$

Q.E.D.

Use this space for summary and/or additional notes:

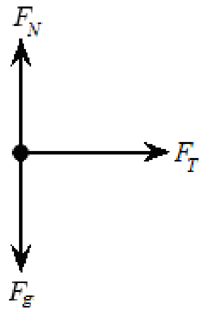
A variation of Atwood's machine is to have one of the masses on a horizontal table (possibly on a cart to reduce friction). This means that the net force is only the action of gravity on the hanging mass.

Consider the problem illustrated to the right. To simplify the problem, we will assume that the pulley has negligible mass, and that both the pulley and the cart are frictionless. The 10 kg for the mass on the left includes the mass of the cart.



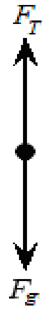
The forces on the two masses are:

FBD for the cart:



$$\vec{F}_{1,net} = \vec{F}_T$$

FBD for the hanging mass:



$$\vec{F}_{2,net} = \vec{F}_g - \vec{F}_T$$

Gravity and the normal force cancel for the cart. The tensions cancel because they are equal (it's the same rope) and are in opposite directions. This means that the only uncancelled force is the force of gravity on the 4 kg mass. This uncancelled force is the net force, which is $\vec{F}_{net} = mg = (4)(10) = 40\,\text{N}$.

The total mass is 10 + 4 = 14 kg.

Now that we have the net force and the total mass, we can find the acceleration using Newton's Second Law:

$$\vec{F}_{net} = m\vec{a}$$
$$40 = 14\vec{a}$$
$$\vec{a} = \frac{40}{14} = 2.86\,\tfrac{\text{m}}{\text{s}^2}$$

Use this space for summary and/or additional notes:

If we also need to find the tension, we can apply Newton's second law to the cart:

$$F_{net,cart} = F_T$$
$$m_{cart}a = F_T$$
$$(10)(2.86) = 28.6\,\text{N}$$

Again, we can get the same result by applying Newton's second law to the hanging mass:

$$F_{net,hang} = F_g - F_T$$
$$m_{hang}a = F_g - F_T$$
$$(4)(2.86) = (4)(10) - F_T$$
$$11.4 = 40 - F_T$$
$$F_T = 40 - 11.4 = 28.6\,\text{N}$$

Alternative Approach

In most physics textbooks, the solution to Atwood's machine problems is presented as a system of equations. The strategy is:

- Draw a free-body diagram for each block.
- Apply Newton's 2nd Law to each block separately, giving $F_{net} = m_1 a$ for block 1 and $F_{net} = F_g - F_T = m_2 a$, which becomes $F_{net} = m_2 g - F_T = m_2 a$ for block 2.
- Set the two F_{net} equations equal to each other, eliminate one of F_T or *a*, and solve for the other.

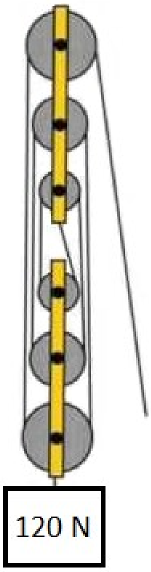
This is really just a different presentation of the same approach, but most students find it less intuitive.

Use this space for summary and/or additional notes:

Homework Problems

CP1 & honors (not AP®)

1. **(M)** For the pulley system shown at the right:



   a. **(M)** What is the mechanical advantage of the system?

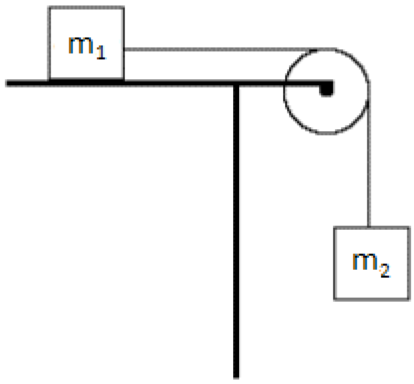
   b. **(M)** How much force will it take to lift the hanging mass?

   c. **(M)** If the 120 N weight is to be lifted 0.5 m, how far will the rope need to be pulled?

Use this space for summary and/or additional notes:

honors & AP®

2. **(M – AP® & honors; A – CP1)** A block with a mass of m_1 sitting on a frictionless horizontal table is connected to a hanging block of mass m_2 by a string that passes over a pulley, as shown in the figure below.



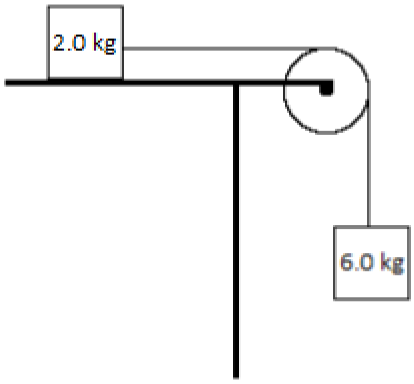
Assuming that friction, the mass of the string, and the mass of the pulley are negligible, derive expressions for the rate at which the blocks accelerate and the tension in the rope.

(*If you are not sure how to do this problem, do #3 below and use the steps to guide your algebra.*)

Answer: $a = \dfrac{m_2 g}{m_1 + m_2}$; $F_T = \dfrac{m_1 m_2 g}{m_1 + m_2}$

Use this space for summary and/or additional notes:

3. **(S – AP® & honors; M – CP1)** A block with a mass of 2.0 kg sitting on a frictionless horizontal table is connected to a hanging block of mass 6.0 kg by a string that passes over a pulley, as shown in the figure below.



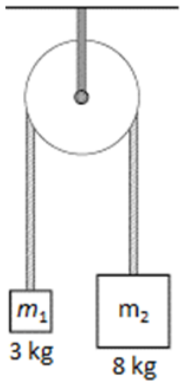
Assuming that friction, the mass of the string, and the mass of the pulley are negligible, at what rate do the blocks accelerate? What is the tension in the rope?

(You must start with the equations in your Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #2 above as a starting point if you have already solved that problem.*)

Answer: $a = 7.5\,\frac{\text{m}}{\text{s}^2}$; $F_T = 15\,\text{N}$

4. **(M)** Two masses, m_1 = 3 kg and m_2 = 8 kg, are connected by an ideal (massless) rope over an ideal pulley (massless and frictionless).

What is the acceleration of the system? What is the tension in the rope?

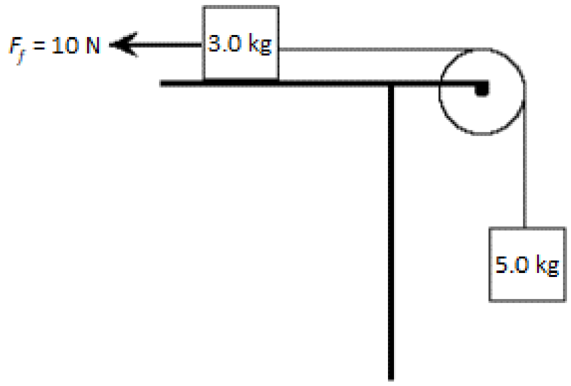


Answer: $a = 4.5\,\frac{\text{m}}{\text{s}^2}$; $F_T = 43.6\,\text{N}$

Use this space for summary and/or additional notes:

honors & AP®

5. **(S)** A block with a mass of 3.0 kg sitting on a frictionless horizontal table is connected to a hanging block of mass 5.0 kg by a string that passes over a pulley, as shown in the figure below. The force of friction between the upper block and the table is 10 N.



At what rate do the blocks accelerate? What is the tension in the rope?

Answer: $a = 5\,\frac{\text{m}}{\text{s}^2}$; $F_T = 25\,\text{N}$

Use this space for summary and/or additional notes:

Friction

Unit: Forces in One Dimension

MA Curriculum Frameworks (2016): HS-PS2-1, HS-PS2-10(MA)

AP® Physics 1 Learning Objectives: 3.A.2.1, 3.A.3.1, 3.A.3.3, 3.A.4.1, 3.A.4.2, 3.B.1.3, 3.B.2.1, 3.4.C.1, 3.4.C.2, 4.A.3.2

Mastery Objective(s): (Students will be able to…)

- Calculate the frictional force on an object.
- Calculate the net force in problems that involve friction.

Success Criteria:

- Free-body diagram is correct.
- Frictional force is correctly identified as static or kinetic and correct coëfficient of friction is chosen.
- Vector quantities (force & acceleration) are correct, including sign (direction).
- Algebra is correct and correct units are included.

Language Objectives:

- Explain how to identify the type of friction (static or kinetic) and how to choose the correct coëfficient of friction.

Tier 2 Vocabulary: friction, static, kinetic, force

Labs, Activities & Demonstrations:

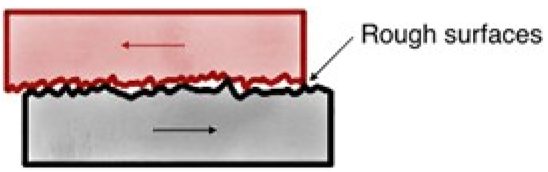
- Drag a heavy object attached to a spring scale.
- Friction board (independent of surface area of contact).

Notes:

Most people understand the concept of friction. If you say, "The wheel is too hard to turn because there's too much friction," people will know what you mean.

<u>friction</u>: a contact force that resists sliding of surfaces against each other.

Friction is caused by the roughness of the materials in contact, deformations of the materials, and/or molecular attraction between the materials.



If you slide (or try to slide) either or both of the objects in the direction of the arrows, the applied force would need to be enough to occasionally lift the upper object so that the rough parts of the surfaces have enough room to pass.

Frictional forces are parallel to the plane of contact between two surfaces, and opposite to the direction of motion or applied force.

Use this space for summary and/or additional notes:

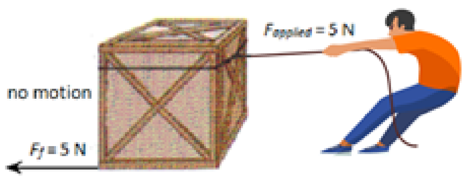
There are two types of friction:

<u>static friction</u>: friction between surfaces that *<u>are not</u>* moving relative to each other. Static friction resists the surfaces' ability to *<u>start</u>* sliding against each other.

<u>kinetic friction</u>: friction between surfaces that *<u>are</u>* moving relative to each other. Kinetic friction resists the surfaces' ability to *<u>keep</u>* sliding against each other.

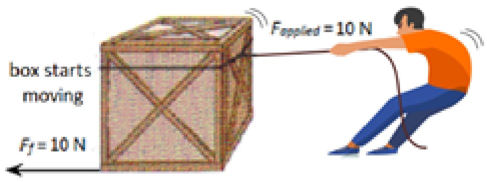
Consider the situations below. Suppose that it takes 10 N of force to overcome static friction and get the box moving. Suppose that once the box is moving, it takes 9 N of force to keep it moving.

Static Friction



When the person applies 5 N of force, it creates 5 N of friction, which is less than the maximum amount of static friction. The forces cancel, so there is ***no net force*** and the box ***remains at rest***.

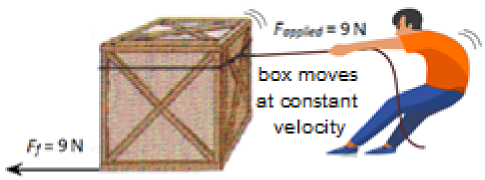
$$\vec{F}_{net} = 0 \rightarrow \vec{a} = 0$$



When the person applies 10 N of force, it creates 10 N of friction. That is the ***maximum amount of static friction***, *i.e.,* exactly the amount of force that it takes to get the box moving. The friction immediately changes to kinetic friction (which is less than static friction). There is now a ***net force***, so the box ***accelerates***.

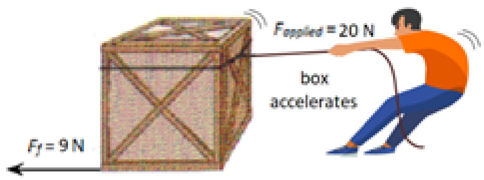
Kinetic Friction

Once the box is moving, the *kinetic friction remains constant regardless of the force applied*. Notice that the amount of kinetic friction (9 N) is less than the maximum amount of static friction (10 N). This is almost always the case; it takes more force to start an object moving than to keep it moving.



When the person applies exactly 9 N of force, there is ***no net force*** and the box ***moves at a constant velocity***.

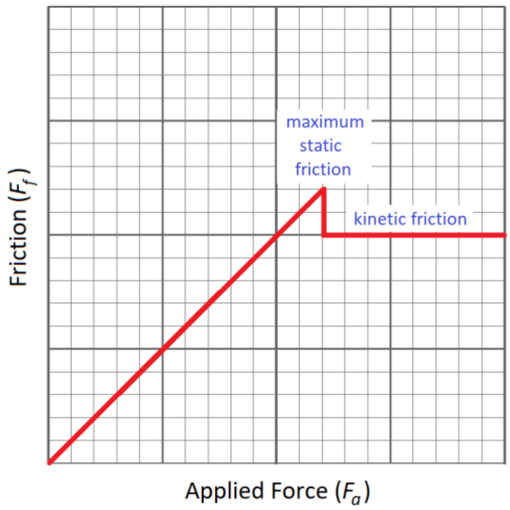
$$\vec{F}_{net} = 0 \rightarrow \vec{a} = 0$$



If the person applies more than 9 N of force, there is a ***net force*** and the box ***accelerates***.

Use this space for summary and/or additional notes:

For the above situations, a graph of the applied force *vs.* friction would look approximately like this:



While the object is not moving, the force of friction is always equal to the applied force. (*I.e.,* the first part of the graph has a slope of 1.) As soon as the applied force is enough to start the object moving, the friction changes to kinetic friction. Once the object is moving, additional applied force does not increase the amount of friction. (Instead, the additional force causes acceleration.)

The factors that affect friction are:

- Roughness and other qualities of the surfaces that affect how difficult it is to slide them against each other. This is described by a number called the coëfficient of friction (μ).
- The amount of force that is pressing the surfaces together. This is, of course, the normal force $(\vec{F}_N)$.

<u>coëfficient of friction</u> (μ): a material-specific constant that is the ratio of friction to the normal force.

$$\mu = \frac{\vec{F}_f}{\vec{F}_N}$$

The coëfficient of friction is a dimensionless number, which means that it has no units. (This is because μ is a ratio of two forces, which means the units cancel.)

The coëfficient of friction takes into account the surface characteristics of the objects in contact.

Use this space for summary and/or additional notes:

Because static friction and kinetic friction are different situations, their coëfficients of friction are different.

coëfficient of static friction (μ_s): the coëfficient of friction between two surfaces when the surfaces are *not moving* relative to each other.

coëfficient of kinetic friction (μ_k): the coëfficient of friction between two surfaces when the surfaces are *sliding* against each other.

The force of friction on an object is given by rearranging the equation for the coëfficient of friction:

$F_f \le \mu_s F_N$ for an object that is stationary, and

$F_f = \mu_k F_N$ for an object that is moving

Where F_f is the magnitude of the force of friction, μ_s and μ_k are the coëfficients of static and kinetic friction, respectively, and F_N is the magnitude of the normal force.

Note that the force of static friction is an inequality. As described above, when an object is at rest the force that resists sliding is, of course, equal to the force applied.

Friction as a Vector Quantity

Like other forces, the force of friction is, of course, actually a vector. Its direction is:

- parallel to the interface between the two surfaces and opposite to the direction of motion (kinetic friction)
- opposite to the component of the applied force that is parallel to the interface between the surfaces (static friction)

Whether the force of friction is represented by a positive or negative number depends on the above and on which direction you have chosen to be positive. As always, whenever multiple forces are involved it is helpful to draw a free-body diagram.

Use this space for summary and/or additional notes:

Solving Simple Friction Problems

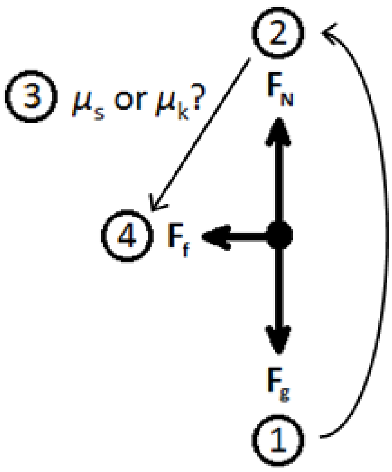
Because friction is a contact force, all friction problems involve friction in addition to some other (usually externally applied) force.

To calculate the force from friction, you need to:

1. Calculate the force of gravity. On Earth, $F_g = mg = m(10)$

2. Calculate the normal force. If the object is resting on a horizontal surface (which is usually the case), the normal force is usually equal in magnitude to the force of gravity. This means that for an object sliding across a horizontal surface:

$$F_N = F_g$$



3. Figure out whether the friction is static (there is an applied force, but the object is not moving), or kinetic (the object is moving). Look up the appropriate coëfficient of friction (μ_s for static friction, or for kinetic friction).

4. Calculate the force of friction from the equation:

$$F_f \leq \mu_s F_N \quad \text{or} \quad F_f = \mu_k F_N$$

Make the force of friction positive or negative, as appropriate. (This will depend on which direction you have chosen to be positive; refer to the free-body diagram.)

5. If the problem is asking for net force, remember to go back and calculate it now that you have calculated the force of friction.

If friction is the only uncancelled force, and it is causing the object to slow down and eventually stop, then:

$$F_{net} = F_f$$

However, if there is an applied force and friction is opposing it, then the net force would be:

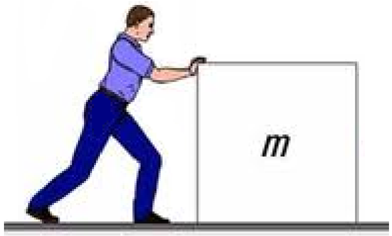
$$F_{net} = \sum F = F_{applied} + F_f$$

(Note, however, that in the above situation, $F_{applied}$ and F_f are in opposite directions, so they need to have opposite signs. In most cases, this will make F_f negative.)

Use this space for summary and/or additional notes:

Sample Problem:

Q: A person pushes a box at a constant velocity across a floor:



The box has a mass of 40 kg, and the coefficient of kinetic friction between the box and the floor is 0.35. What is the magnitude of the force that the person exerts on the box?

A: The box is moving at a *constant velocity*, which means there is *no acceleration*, and therefore *no net force* on the box. This means the force exerted by the person is exactly equal to the force of friction.

The force of friction between the box and the floor is given by the equation:

$$F_f = \mu_k F_N$$

The normal force is equal in magnitude to the weight of the box (F_g), which is given by the equation:

$$F_N = F_g = mg = (40)(10) = 400\,\text{N}$$

Therefore, the force of friction is:

$$F_f = \mu_k F_N$$
$$F_f = (0.35)(400) = 140\,\text{N}$$

Use this space for summary and/or additional notes:

Homework Problems

For these problems, you will need to look up coëfficients of friction in *Table E. Approximate Coëfficients of Friction* on page 684 of your Physics Reference Tables).

1. **(M)** A student wants to slide a steel 15 kg mass across a steel table.

   a. **(M)** How much force must the student apply in order to start the box moving?

   Answer: 111 N

   b. **(M)** Once the mass is moving, how much force must the student apply to keep it moving at a constant velocity?

   Answer: 85.5 N

2. **(S)** A wooden desk has a mass of 74 kg.

   a. **(S)** How much force must be applied to the desk to start it moving across a wooden floor?

   Answer: 310.8 N

   b. **(S)** Once the desk is in motion, how much force must be used to keep it moving at a constant velocity?

   Answer: 222 N

Use this space for summary and/or additional notes:

3. A large sport utility vehicle has a mass of 1850 kg and is traveling at $15\,\frac{m}{s}$ (a little over 30 MPH). The driver slams on the brakes, causing the vehicle to skid.

   a. **(M)** How far would the SUV travel before it stops on dry asphalt? (*Hint: this is a combination of a motion problem and a Newton's Second Law problem with friction.*)

   Answer: 16.8 m

   b. **(S)** How far would the SUV travel if it were skidding to a stop on ice? (*This is the same problem as part (a), but with a different coëfficient of friction.*)

   Answer: 75 m

Use this space for summary and/or additional notes:

honors & AP®

4. **(M – AP® & honors; A – CP1)** A curling stone with a mass of *m* slides a distance *d* across a sheet of ice in time *t* before it stops because of friction. What is the coëfficient of kinetic friction between the ice and the stone?

   (*If you are not sure how to do this problem, do #5 below and use the steps to guide your algebra.*)

   Answer: $\mu_k = \dfrac{2d}{gt^2}$

5. **(S – AP® & honors; M – CP1)** A curling stone with a mass of 18 kg slides 38 m across a sheet of ice in 8.0 s before it stops because of friction. What is the coëfficient of kinetic friction between the ice and the stone?

   (*<u>You must start with the equations in your Physics Reference Tables and show all of the steps of GUESS.</u> You may only use the answer to question #4 above as a starting point if you have already solved that problem.*)

   Answer: 0.12

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Drag

Unit: Forces in One Dimension

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Calculate the drag force on an object.

Success Criteria:

- Correct drag coëfficient is chosen.
- Variables are correctly identified and substituted correctly into the equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why aerodynamic drag depends on each of the variables in the equation.

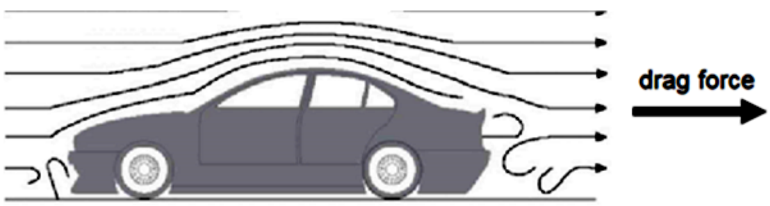
Tier 2 Vocabulary: drag

Labs, Activities & Demonstrations:

- Crumpled piece of paper or tissue *vs.* golf ball (drag force doesn't depend on mass).
- Projectiles with same mass but different shapes.

Notes:

Drag is the force exerted by particles of a fluid* resisting the motion of an object relative to a fluid. The drag force is essentially friction between the object and particles of the fluid.



Most of the problems that involve drag fall into three categories:

1. The drag force is small enough that we ignore it.
2. The drag force is equal to some other force that we can measure or calculate.
3. The question asks only for a qualitative comparison of forces with and without drag.

* A fluid is any substance whose particles can separate easily, allowing it to flow (does not have a definite shape) and allowing objects to pass through it. Fluids can be liquids or gases.

Use this space for summary and/or additional notes:

Calculating drag is complicated, because the effects of drag change dramatically at different flow rates.

The drag force can be estimated in simple situations, given the velocity, shape, and cross-sectional area of the object and the density of the fluid it is moving through.

For these situations, the drag force is given by the following equation:

$$\vec{\boldsymbol{F}}_D = -\frac{1}{2}\rho\vec{\boldsymbol{v}}^2 C_D A$$

where:

$\vec{\boldsymbol{F}}_D$ = drag force

ρ = density of the fluid that the object is moving through

$\vec{\boldsymbol{v}}$ = velocity of the object (relative to the fluid)

C_D = drag coëfficient of the object (based on its shape)

A = cross-sectional area of the object in the direction of motion

This equation can be applied when:

- the object has a blunt form factor
- the object's velocity relative to the properties of the fluid causes turbulence in the object's wake
- the fluid is in laminar (not turbulent) flow before it interacts with the object
- the fluid has a relatively low viscosity*

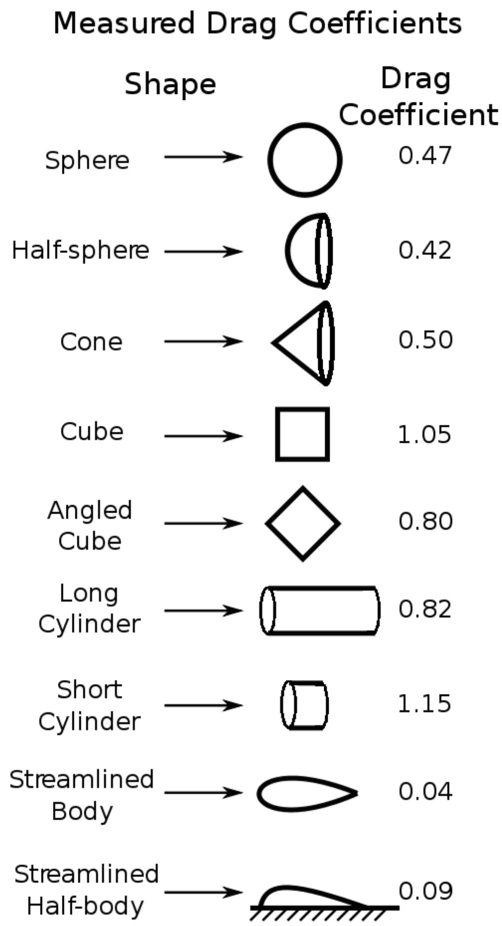
However, fluid flow is a lot more complicated than the above equation would suggest, and there are few situations in which the above equation gives a good result.

* Viscosity is a measure of how "gooey" a fluid is, meaning how much it resists flow and hinders the motion of objects through itself. Water has a low viscosity; honey and ketchup are more viscous.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

The drag coëfficient, C_D, is a dimensionless number (meaning that it has no units). The drag coëfficient encompasses all of the types of friction associated with drag, including form drag and skin drag. It serves the same purpose in drag problems that the coëfficient of friction (μ) serves in problems involving friction between solid surfaces.



Approximate drag coëfficients for simple shapes are given in the table to the right, assuming that the fluid is moving (relative to the object) in the direction of the arrow.

The reason that raindrops have their characteristic shape ("streamlined body") is because the drag force changes their shape until they have the shape with the least amount of drag.

The reason that many cars have rooves that slope downward from the front of the car to the back is to reduce the drag force.

Drag coefficients of some vehicles and other objects:

Vehicle	C_D
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram truck	0.43
Hummer H2	0.64

Object	C_D
skydiver (vertical)	0.70
skydiver (horizontal)	1.0
parachute	1.75
bicycle & rider	0.90

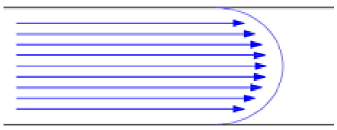
Use this space for summary and/or additional notes:

To highlight some of the problems with the drag equation presented here, it is necessary to explain more about fluid flow.

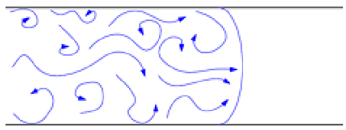
Fluid flow is often characterized by a dimensionless number (*i.e.,* one that has no units because all of the units cancel) called the Reynolds number.

Reynolds number (*Re*): the ratio of inertial forces (remember that inertia = resistance to movement) to the viscous forces in a fluid.

There are two basic types of fluid flow:



laminar flow



turbulent flow

laminar flow: occurs when the velocity of the fluid (or the object moving through it) is relatively low, and the particles of fluid generally move in a straight line in an organized fashion. Generally, flow is laminar if $Re < 2300$.

Turbulent flow: occurs when the velocity of the fluid (or the object moving through it) is high, and the particles move in a more jumbled, random manner. In general, turbulent flow causes higher drag forces. Generally, flow is turbulent if $Re > 2900$.

The type of flow affects the drag coëfficient, C_D :

- In laminar flow, the drag coëfficient is roughly proportional to $\frac{1}{Re}$. Because velocity is a component of the Reynolds number, this means the drag coëfficient is roughly proportional to $\frac{1}{v}$.
- In turbulent flow, the drag coëfficient depends greatly on the characteristics of the system. In many systems, for turbulent flow the drag coefficient is proportional to $\frac{1}{Re^7}$.

Note also that the viscosity of a fluid drops steeply with temperature, which means temperature also affects the Reynolds number and therefore the drag coëfficient.

This is all to say that a reasonable quantitative treatment of fluid flow and drag is beyond the scope of this course.

Use this space for summary and/or additional notes:

honors & AP®

Introduction: Forces in Multiple Dimensions

Unit: Forces in Multiple Dimensions

Topics covered in this chapter:

In this chapter you will learn about different kinds of forces and how they relate.

- *Force Applied at an Angle*, *Ramp Problems*, and *Pulleys & Tension* describe some common situations involving forces and how to calculate the forces involved.
- *Centripetal Force* describes the forces experienced by an object moving in a circle.
- *Center of Mass, Rotational Inertia,* and *Torque* describe the relationship between forces and rotation.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-1. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.

HS-PS2-10(MA). Use free-body force diagrams, algebraic expressions, and Newton's laws of motion to predict changes to velocity and acceleration for an object moving in one dimension in various situations.

AP®

AP® Physics 1 Learning Objectives:

3.A.2.1: The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. **[SP 1.1]**

3.A.3.1: The student is able to analyze a scenario and make claims (develop arguments, justify assertions) about the forces exerted on an object by other objects for different types of forces or components of forces. **[SP 6.4, 7.2]**

3.A.3.2: The student is able to challenge a claim that an object can exert a force on itself. **[SP 6.1]**

3.A.3.3: The student is able to describe a force as an interaction between two objects and identify both objects for any force. **[SP 1.4]**

Use this space for summary and/or additional notes:

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3.A.4.3: The student is able to analyze situations involving interactions among several objects by using free-body diagrams that include the application of Newton's third law to identify forces. **[SP 1.4]**

3.B.1.1: The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton's second law in a variety of physical situations with acceleration in one dimension. **[SP 6.4, 7.2]**

3.B.1.2: The student is able to design a plan to collect and analyze data for motion (static, constant, or accelerating) from force measurements and carry out an analysis to determine the relationship between the net force and the vector sum of the individual forces. **[SP 4.2, 5.1]**

3.B.1.3: The student is able to re-express a free-body diagram representation into a mathematical representation and solve the mathematical representation for the acceleration of the object. **[SP 1.5, 2.2]**

3.B.2.1: The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. **[SP 1.1, 1.4, 2.2]**

3.C.4.1: The student is able to make claims about various contact forces between objects based on the microscopic cause of those forces. **[SP 6.1]**

3.C.4.2: The student is able to explain contact forces (tension, friction, normal, buoyant, spring) as arising from interatomic electric forces and that they therefore have certain directions. **[SP 6.2]**

4.A.1.1: The student is able to use representations of the center of mass of an isolated two-object system to analyze the motion of the system qualitatively and semi-quantitatively. **[SP 1.2, 1.4, 2.3, 6.4]**

4.A.2.1: The student is able to make predictions about the motion of a system based on the fact that acceleration is equal to the change in velocity per unit time, and velocity is equal to the change in position per unit time. **[SP 6.4]**

4.A.2.2: The student is able to evaluate using given data whether all the forces on a system or whether all the parts of a system have been identified. **[SP 5.3]**

4.A.1.1: The student is able to use representations of the center of mass of an isolated two-object system to analyze the motion of the system qualitatively and semi-quantitatively. **[SP 1.2, 1.4, 2.3, 6.4]**

4.A.2.1: The student is able to make predictions about the motion of a system based on the fact that acceleration is equal to the change in velocity per unit time, and velocity is equal to the change in position per unit time. **[SP 6.4]**

4.A.2.2: The student is able to evaluate using given data whether all the forces on a system or whether all the parts of a system have been identified. **[SP 5.3]**

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Topics from this chapter assessed on the SAT Physics Subject Test:

Dynamics, such as force, Newton's laws, statics, and friction.

1. Inclined Planes

Skills learned & applied in this chapter:

- Solving chains of equations.
- Using geometry and trigonometry to combine forces (vectors).
- Using trigonometry to split forces (vectors) into components.

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Force Applied at an Angle

Unit: Forces in Multiple Dimensions

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 3.A.2.1, 3.B.1.1, 3.B.1.2, 3.B.1.3

Mastery Objective(s): (Students will be able to…)

- Calculate forces applied at different angles, using trigonometry.

Success Criteria:

- Forces are split or combined correctly using the Pythagorean Theorem and trigonometry.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the concept of a component of a force.
- Explain why it is incorrect to just add together the vertical and horizontal components of a force.

Tier 2 Vocabulary: force

Labs, Activities & Demonstrations:

- Mass hanging from one or two scales. Change angle and observe changes in force.
- Fan cart with fan at an angle.
- For rope attached to heavy object, pull *vs.* anchor rope at both ends & push middle.

Notes:

An important property of vectors is that a vector has no effect on a second vector that is perpendicular to it. As we saw with projectiles, this means that the velocity of an object in the horizontal direction has no effect on the velocity of the same object in the vertical direction. This allowed us to solve for the horizontal and vertical velocities as separate problems.

The same is true for forces. If forces are perpendicular to each other, they act independently, and the two can be separated into separate, independent mathematical problems:

In the x-direction: $\vec{F}_{net,x} = m\vec{a}_x$

In the y-direction: $\vec{F}_{net,y} = m\vec{a}_y$

Note that the above is for linear situations. Two-dimensional rotational problems require calculus, and are therefore outside the scope of this course.

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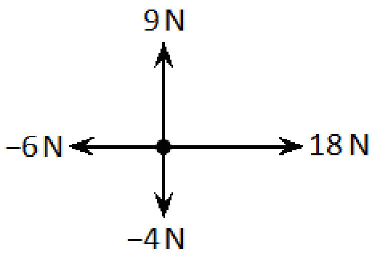
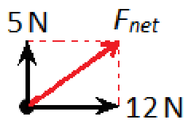
For example, if we have the following forces acting on an object:



The net horizontal force (F_x) would be 18 N + (−6 N) = +12 N, and the net vertical force (F_y) would be 9 N + (−4 N) = +5 N. The total net force would be the resultant of the net horizontal and net vertical forces:



Using the Pythagorean Theorem:

$$a^2 + b^2 = c^2 \qquad 169 = F_{net}^2$$

$$5^2 + 12^2 = F_{net}^2 \qquad \sqrt{169} = F_{net} = 13\,\text{N}$$

We can get the angle from trigonometry:

$$\tan\theta = \frac{opposite}{adjacent} = \frac{5}{12} = 0.417$$

$$\theta = \tan^{-1}(\tan\theta) = \tan^{-1}(0.417) = 22.6°$$

(Of course, because you have just figured out the length of the hypotenuse, you could get the same answer by using $\sin^{-1}$ or $\cos^{-1}$.)

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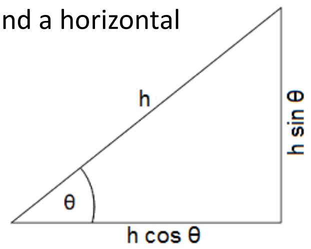
If we have one or more forces that is neither vertical nor horizontal, we can use trigonometry to split the force into a vertical component and a horizontal component.

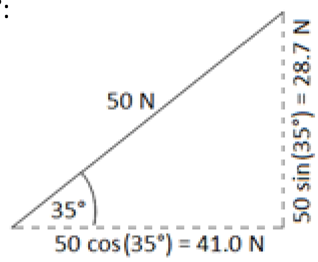
Recall the following relationships from trigonometry:



Suppose we have a force of 50 N at a direction of 35° above the horizontal. In the above diagram, this would mean that *h* = 50 N and *θ* = 35°:

The horizontal force is $\vec{F}_x = h\cos(\theta) = 50\cos(35°) = 41.0\,\text{N}$

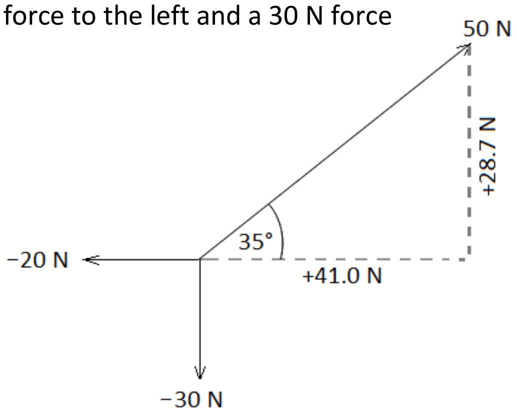
The vertical force is $\vec{F}_y = h\sin(\theta) = 50\sin(35°) = 28.7\,\text{N}$



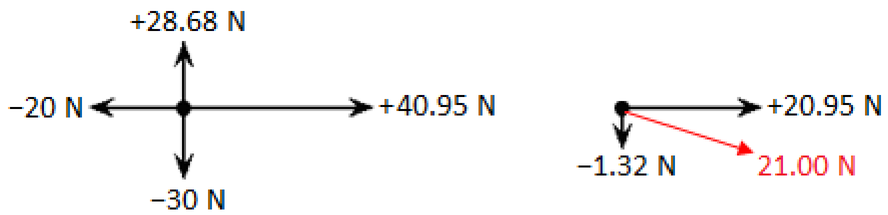
Now, suppose that same object was subjected to the same 50 N force at an angle of 35° above the horizontal, but also a 20 N force to the left and a 30 N force downward.

The net horizontal force would therefore be 41 + (−20) = 21 N to the right.

The net vertical force would therefore be 28.7 + (−30) = −1.3 N upwards (which equals 1.3 N downwards).



Once you have calculated the net vertical and horizontal forces, you can resolve them into a single net force, as in the previous example. (Because the vertical component of the net force is so small, an extra digit is necessary in order to see the difference between the total net force and its horizontal component.)



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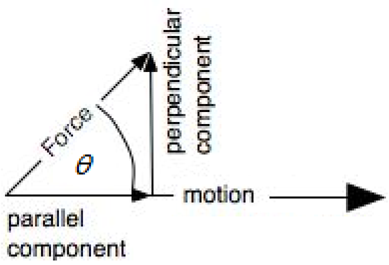
In some physics problems, a force is applied at an angle but the object can move in only one direction. A common problem is a force applied at an angle to an object resting on a flat surface, which causes the object to move horizontally:



In this situation, only the horizontal (parallel) component of the applied force $(F_{||})$ actually causes the object to move. If magnitude of the total force is *F*, then the horizontal component of the force is given by:

$$F_x = F_{||} = F\cos\theta$$

If the object accelerates horizontally, that means *__only__* the horizontal component is causing the acceleration, which means the net force must be $F_{||} = F\cos\theta$ and we can ignore the vertical component.

For example, suppose the worker in the diagram at the right pushes on the hand truck with a force of 200 N at an angle of 60°.

The force in the direction of motion (horizontally) would be:

$$\begin{aligned} F_{||} &= F\cos\theta = 200\cos(60^\circ) \\ &= (200)(0.5) = 100\,\text{N} \end{aligned}$$

In other words, if the worker applies 200 N of force at an angle of 60°, the resulting *__horizontal__* force will be 100 N.

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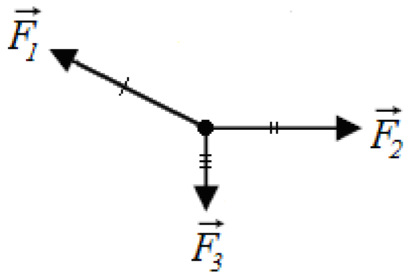
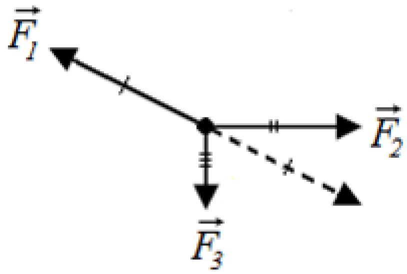
Static Problems Involving Forces at an Angle

Many problems involving forces at an angle are based on an object with no net force (either a stationary object or an object moving at constant velocity) that has three or more forces acting at different angles. In the following diagram, the forces are $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$.



$\vec{F}_1$ needs to cancel the resultant of $\vec{F}_2$ and $\vec{F}_3$:



Of course, $\vec{F}_2$ will also cancel the resultant of $\vec{F}_1$ and $\vec{F}_3$, and $\vec{F}_3$ will also cancel the resultant of $\vec{F}_1$ and $\vec{F}_2$.

Strategy

1. Resolve all known forces into their horizontal and vertical components.
2. Add the horizontal and vertical components separately.
3. Use the Pythagorean Theorem to find the magnitude of forces that are neither horizontal nor vertical.
4. Because you know the vertical and horizontal components of the resultant force, use arcsine ($\sin^{-1}$), arccosine ($\cos^{-1}$) or arctangent ($\tan^{-1}$) to find the angle.

Use this space for summary and/or additional notes:

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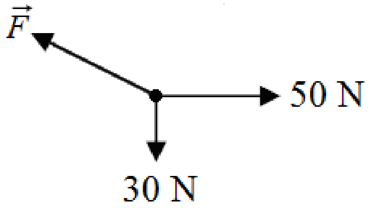
Sample Problems:

Q: A stationary object has three forces acting on it, as shown in the diagram below (which is not to scale):



What are the magnitude and direction of $\vec{F}$?

A: $\vec{F}$ is equal and opposite to the resultant of the other two vectors. The magnitude of the resultant is:

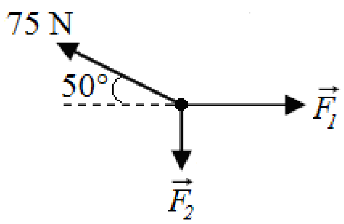
$$\|\vec{F}'\| = \sqrt{30^2 + 50^2} = \sqrt{3400} = 58.3\,\text{N}$$

The direction is:

$$\tan\theta = \frac{30}{50} = 0.6$$

$$\theta = \tan^{-1}(\tan\theta) = \tan^{-1}(0.6) = 31.0^\circ \text{ up from the left (horizontal)}$$

Q: A stationary object has three forces acting on it, as shown in the diagram below (which is not to scale):



What are the magnitudes of $\vec{F}_1$ and $\vec{F}_2$?

A: $\vec{F}_1$ and $\vec{F}_2$ are equal and opposite to the vertical and horizontal components of the 75 N force, which we can find using trigonometry:

$$\|\vec{F}_1\| = \text{horizontal} = 75\cos(50^\circ) = (75)(0.643) = 48.2\,\text{N}$$

$$\|\vec{F}_2\| = \text{vertical} = 75\sin(50^\circ) = (75)(0.766) = 57.5\,\text{N}$$

Use this space for summary and/or additional notes:

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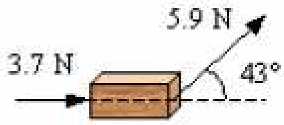
Homework Problems

1. **(M – honors; A – CP1)** An object has three forces acting on it, a 15 N force pushing to the right, a 10. N force pushing to the right, and a 20. N force pushing to the left.

   a. **(M – honors; A – CP1)** Draw a free-body diagram for the object showing each of the forces that acts on the object (including a legend showing which direction is positive).

   b. **(M – honors; A – CP1)** Calculate the magnitude of the net force on the object.

2. **(M – honors; A – CP1)** A force of 3.7 N horizontally and a force of 5.9 N at an angle of 43° act on a 4.5-kg block that is resting on a frictionless surface, as shown in the following diagram:



What is the magnitude of the horizontal acceleration of the block?

Answer: $1.8\,\frac{\text{m}}{\text{s}^2}$

Use this space for summary and/or additional notes:

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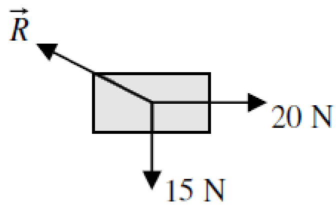
3. **(S – honors; A – CP1)** A stationary block has three forces acting on it: a 20. N force to the right, a 15 N force downwards, and a third force, $\vec{R}$ of unknown magnitude and direction, as shown in the diagram to the right:



   a. **(S – honors; A – CP1)** What are the horizontal and vertical components of $\vec{R}$?

   b. **(S – honors; A – CP1)** What is the magnitude of $\vec{R}$?

   Answer: 25 N

   c. **(S – honors; A – CP1)** What is the direction (angle up from the horizontal) of $\vec{R}$?

   Answer: 36.9°

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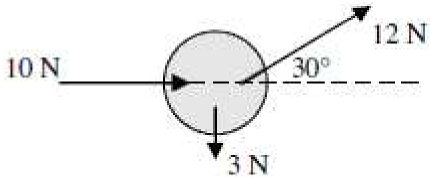
4. **(S – honors; A – CP1)** Three forces act on an object. One force is 10. N to the right, one force is 3.0N downwards, and one force is 12 N at an angle of 30.° above the horizontal, as shown in the diagram below.



   a. **(S – honors; A – CP1)** What are the net vertical and horizontal forces on the object?

   Answer: positive directions are up and to the right.
   vertical: +3.0 N; horizontal: +20.4 N

   b. **(S – honors; A – CP1)** What is the net force (magnitude and direction) on the object?

   Answer: 20.6 N at an angle of +8.4°

Use this space for summary and/or additional notes:

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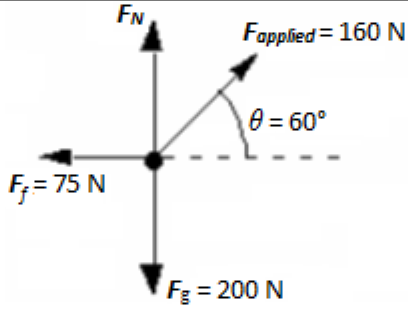
5. **(M – AP®; S – honors; A – CP1)** An applied force of 160 N ($\vec{F}_{applied}$) pulls at an angle of 60° (θ) on a crate that is sitting on a rough surface. The weight of the crate ($\vec{F}_g$) is 200 N. The force of friction on the crate ($\vec{F}_f$) is 75 N. These forces are shown in the diagram to the right.



Using the variables but not the quantities from the diagram, derive an expression for the magnitude of the normal force ($\vec{F}_N$) on the crate, in terms of the given quantities $\vec{F}_{applied}$, $\vec{F}_g$, $\vec{F}_f$, θ, and natural constants (such as $\vec{g}$).

(*If you are not sure how to do this problem, do #6 below and use the steps to guide your algebra.*)

Answer: $\vec{F}_N = \vec{F}_g - \vec{F}_{applied} \sin\theta$

Use this space for summary and/or additional notes:

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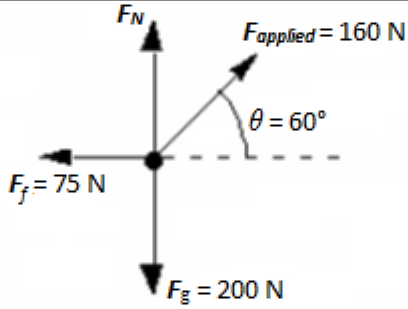
6. **(M – honors; A – CP1)** An applied force of 160 N ($\vec{F}_{applied}$) pulls at an angle of 60° (θ) on a crate that is sitting on a rough surface. The weight of the crate ($\vec{F}_g$) is 200 N. The force of friction on the crate ($\vec{F}_f$) is 75 N. These forces are shown in the diagram to the right.



(*<u>You must start with the equations in your</u>* <u>Physics Reference Tables</u> *<u>and show all of the steps of GUESS.</u> You may only use the answer to question #5 above as a starting point if you have already solved that problem.*)

a. What is the magnitude of the normal force ($\vec{F}_N$) on the crate?

Answer: 61 N

b. What is the acceleration of the crate?

Answer: $0.25\,\frac{\text{m}}{\text{s}^2}$

Use this space for summary and/or additional notes:

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Ramp Problems

Unit: Forces in Multiple Dimensions

MA Curriculum Frameworks (2016): N/A

AP Physics 1 Learning Objectives: 1.C.1.1, 2.B.1.1, 3.A.2.1, 3.B.1.1, 3.B.1.2, 3.B.1.3, 3.B.2.1, 4.A.2.3, 4.A.3.1, 4.A.3.2

Mastery Objective(s): (Students will be able to…)

- Calculate forces on an object on a ramp.

Success Criteria:

- Forces are split or combined correctly using the Pythagorean Theorem and trigonometry.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how the forces on an object on a ramp depend on the angle of inclination of the ramp.

Tier 2 Vocabulary: force, ramp, inclined, normal

Labs, Activities & Demonstrations:

- Objects sliding down a ramp at different angles.
- Set up ramp with cart & pulley and measure forces at different angles.

Notes:

The direction of the normal force does not always directly oppose gravity. For example, if a block is resting on a (frictionless) ramp, the weight of the block is $\vec{F}_g$, in the direction of gravity. However, the normal force is perpendicular to the ramp, not to gravity.

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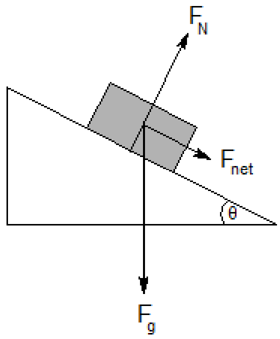
If we were to add the vectors representing the two forces, we would see that the resultant—the net force—acts down the ramp:



Intuitively, we know that if the ramp is horizontal $(\theta = 0)$, the net force is zero and $\vec{F}_N = \vec{F}_g$, because they are equal and opposite.

We also know intuitively that if the ramp is vertical $(\theta = 90°)$, the net force is $\vec{F}_g$ and $\vec{F}_N = 0$.

If the angle is between 0 and 90°, the net force must be between 0 and $\vec{F}_g$, and the proportion must be related to the angle (trigonometry!). Note that $\sin(0°) = 0$ and $\sin(90°) = 1$. Intuitively, it makes sense that the steeper the angle, the greater the net force, and therefore multiplying $\vec{F}_g$ by the sine of the angle should give the net force down the ramp for any angle between 0 and 90°.

Similarly, If the angle is between 0 and 90°, the normal force must be between $\vec{F}_g$ (at 0) and 0 (at 90°). Again, the proportion must be related to the angle (trigonometry!). Note that $\cos(0°) = 1$ and $\cos(90°) = 0$. Intuitively, it makes sense that the shallower the angle, the greater the normal force, and therefore multiplying $\vec{F}_g$ by the cosine of the angle should give the normal force for any angle 0 and 90°.

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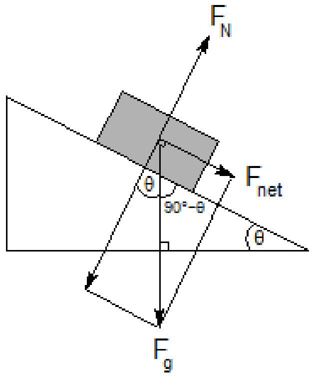
Let's look at a geometric explanation:

From geometry, we can determine that the angle of the ramp, θ, is the same as the angle between gravity and the normal force.

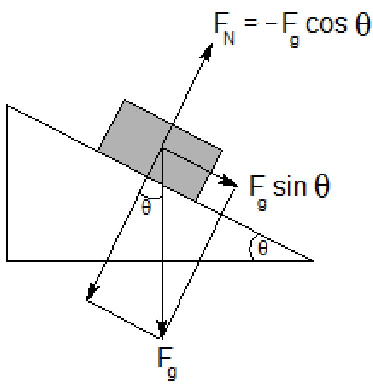


From trigonometry, we can calculate that the component of gravity parallel to the ramp (which equals the net force down the ramp) is the side *opposite* angle θ. This means:

$$F_{net} = F_g \sin\theta$$

The component of gravity perpendicular to the ramp is $F_g \cos\theta$, which means the normal force is:

$$F_N = -F_g \cos\theta$$



(The negative sign is because the normal force is in the opposite direction from $F_g \cos\theta$.)

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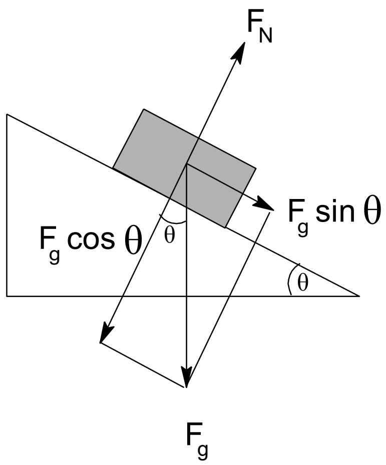
Sample Problem:

Q: A block with a mass of 2.5 kg sits on a frictionless ramp with an angle of inclination of 35°. How fast does the block accelerate down the ramp?

A: The weight of the block is $F_g = ma = (2.5)(10) = 25\,\text{N}$, directed straight down. However, the net force must be in the same direction as the acceleration. Therefore, the net force is the component of the force of gravity in the direction that the block can move (down the ramp), which is $F_g \sin\theta$:



$$F_{net} = F_g \sin\theta = 25 \sin 35^\circ = (25)(0.574) = 14.3\,\text{N}$$

Now that we know the net force (in the direction of acceleration), we can apply Newton's Second Law:

$$F_{net} = ma$$
$$14.3 = 2.5\,a$$
$$a = 5.7\,\tfrac{\text{m}}{\text{s}^2}$$

Use this space for summary and/or additional notes:

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Homework Problem

1. **(M – honors & AP®; A – CP1)** A 10. kg block sits on a frictionless ramp with an angle of inclination of 30°. What is the rate of acceleration of the block?

   Answer: $5.0\,\frac{\text{m}}{\text{s}^2}$

2. **(S – AP®; A – honors & CP1)** A skier is skiing down a slope at a constant and fairly slow velocity (meaning that air resistance is negligible). What is the angle of inclination of the slope?

   Hints:

   - *You will need to look up the coëfficient of kinetic friction for a* <u>*waxed ski on snow*</u> in *Table E. Approximate Coëfficients of Friction* on page 684 *of your Physics Reference Tables.*
   - *You do not need to know the mass of the skier because it drops out of the equation.*
   - *If the velocity is constant, that means there is no net force, which means the force down the slope (ramp) is equal to the opposing force (friction).*

   Answer: 2.9°

Use this space for summary and/or additional notes:

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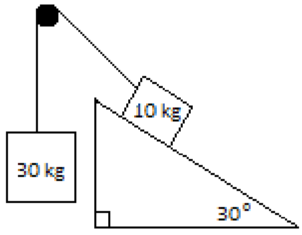
Homework Problems: Ramps & Pulleys

These problems combine ramp problems with pulley problems, which were covered in the section on Tension, starting on page 285.

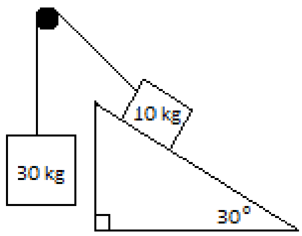
1. **(M – honors & AP®; A – CP1)** A mass of 30. kg is suspended from a massless rope on one side of a massless, frictionless pulley. A mass of 10. kg is connected to the rope on the other side of the pulley and is sitting on a ramp with an angle of inclination of 30°. The system is shown in the diagram to the right.



   a. Assuming the ramp is frictionless, determine the acceleration of the system.

   Answer: $a = 6.25\,\frac{\text{m}}{\text{s}^2}$

   b. **(M – honors & AP®; A – CP1)** Assuming instead that the ramp has a coëfficient of kinetic friction of $\mu_k = 0.3$, determine the acceleration of the system once the blocks start to move.



   Answer: $a = 5.60\,\frac{\text{m}}{\text{s}^2}$

Use this space for summary and/or additional notes:

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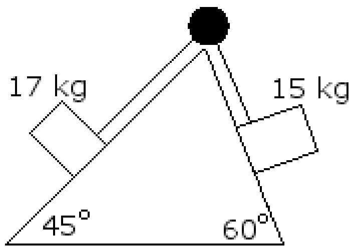
2. **(S – honors & AP®; A – CP1)** Two boxes with masses 17 kg and 15 kg are connected by a light string that passes over a frictionless pulley of negligible mass as shown in the figure below. The surfaces of the planes are frictionless.



a. **(S – honors & AP®; A – CP1)** When the blocks are released, which direction will the blocks move?

b. **(S – honors & AP®; A – CP1)** Determine the acceleration of the system.

Answer: $0.303\,\frac{m}{s^2}$

Use this space for summary and/or additional notes:

Introduction: Rotational Statics & Dynamics

Unit: Rotational Statics & Dynamics

Topics covered in this chapter:

Use this space for summary and/or additional notes:

In this chapter you will learn about different kinds of forces and how they relate.

- *Forces Applied at an Angle*, *Ramp Problems*, and *Pulleys & Tension* describe some common situations involving forces and how to calculate the forces involved.
- *Centripetal Force* describes the forces experienced by an object moving in a circle.
- *Center of Mass, Rotational Inertia,* and *Torque* describe the relationship between forces and rotation.
- *Solving Linear & Rotational Force/Torque Problems* discusses situations where torque is converted to linear motion and *vice versa*.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-1. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.

HS-PS2-10(MA). Use free-body force diagrams, algebraic expressions, and Newton's laws of motion to predict changes to velocity and acceleration for an object moving in one dimension in various situations.

AP®

AP® Physics 1 Learning Objectives:

3.F.1.1: The student is able to use representations of the relationship between force and torque. [**SP 1.4**]

3.F.1.2: The student is able to compare the torques on an object caused by various forces. [**SP 1.4**]

3.F.1.3: The student is able to estimate the torque on an object caused by various forces in comparison to other situations. [**SP 2.3**]

3.F.1.4: The student is able to design an experiment and analyze data testing a question about torques in a balanced rigid system. [**SP 4.1, 4.2, 5.1**]

3.F.1.5: The student is able to calculate torques on a two-dimensional system in static equilibrium, by examining a representation or model (such as a diagram or physical construction). [**SP 1.4, 2.2**]

3.F.2.1: The student is able to make predictions about the change in the angular velocity about an axis for an object when forces exerted on the object cause a torque about that axis. [**SP 6.4**]

3.F.2.2: The student is able to plan data collection and analysis strategies designed to test the relationship between a torque exerted on an object and the change in angular velocity of that object about an axis. [**SP 4.1, 4.2, 5.1**]

Use this space for summary and/or additional notes:

AP®

4.A.1.1: The student is able to use representations of the center of mass of an isolated two-object system to analyze the motion of the system qualitatively and semi-quantitatively. [**SP 1.2, 1.4, 2.3, 6.4**]

4.D.1.1: The student is able to describe a representation and use it to analyze a situation in which several forces exerted on a rotating system of rigidly connected objects change the angular velocity ~~and angular momentum~~ of the system. [**SP 1.2, 1.4**]

4.D.1.2: The student is able to plan data collection strategies designed to establish that torque, angular velocity, angular acceleration, ~~and angular momentum~~ can be predicted accurately when the variables are treated as being clockwise or counterclockwise with respect to a well-defined axis of rotation, and refine the research question based on the examination of data. [**SP 3.2, 4.1, 4.2, 5.1, 5.3**]

4.D.2.1: The student is able to compare the torques on an object caused by various forces. [**SP 2.2**]

Topics from this chapter assessed on the SAT Physics Subject Test:

Dynamics, such as force, Newton's laws, statics, and friction.

1. Balanced Torques

Skills learned & applied in this chapter:

- Solving chains of equations.
- Using geometry and trigonometry to combine forces (vectors).

Use this space for summary and/or additional notes:

Centripetal Force

Unit: Rotational Statics & Dynamics

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 4.D.1.1

Mastery Objective(s): (Students will be able to…)

- Explain qualitatively the forces involved in circular motion.
- Describe the path of an object when it is released from circular motion.
- Calculate the velocity and centripetal force of an object that is in uniform circular motion.

Success Criteria:

- Explanations account for constant change in direction.
- Variables are correctly identified and substituted correctly into the equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why centripetal force is always toward the center of the circle.

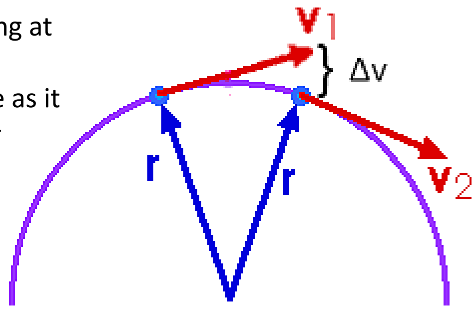
Tier 2 Vocabulary: centripetal, centrifugal

Labs, Activities & Demonstrations:

- Swing a bucket of water in a circle.
- Golf ball loop-the-loop.
- Spin a weight on a string and have the weight pull up on a mass or spring scale.

Notes:

As we saw previously, when an object is moving at a constant speed around a circle, its direction keeps changing toward the center of the circle as it goes around, which means *there is continuous acceleration toward the center of the circle.*



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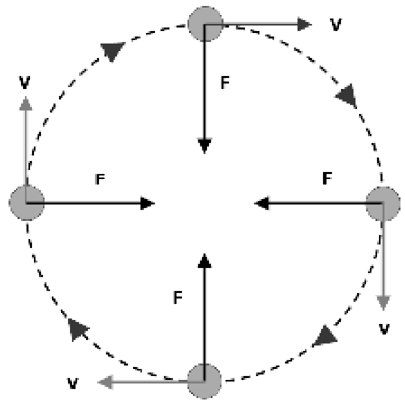

Because acceleration is caused by a net force (Newton's second law of motion), if there is continuous acceleration toward the center of the circle, then there must be a continuous force toward the center of the circle.

This force is called "centripetal force".

<u>centripetal force</u>: the inward force that keeps an object moving in a circle. If the centripetal force were removed, the object would fly away from the circle in a straight line that starts from a point tangent to the circle.

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Recall that the equation* for centripetal acceleration (a_c) is:

$$a_c = \frac{v^2}{r} = r\omega^2$$

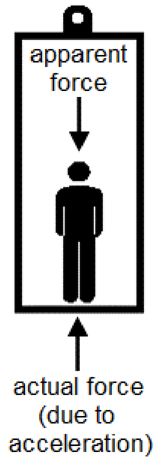
Given that $F = ma$, the equation for centripetal force is therefore:

$$F_c = ma_c = \frac{mv^2}{r} = mr\omega^2$$

If you are in the reference frame of the object that is moving in a circle, you are being accelerated toward the center of the circle. You feel a force that appears to be pushing or pulling you away from the center of the circle. This is called "centrifugal force".

<u>centrifugal force</u>: the outward force felt by an object that is moving in a circle.

Centrifugal force is called a "fictitious force" because it does not exist in an inertial reference frame. However, centrifugal force does exist in a rotating reference frame; it is the inertia of objects resisting acceleration as they are continuously pulled toward the center of a circle by centripetal acceleration.

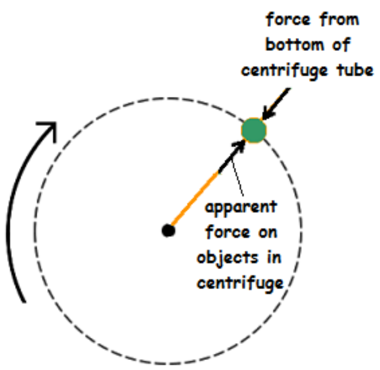


This is the same as the feeling of increased weight that you feel when you are in an elevator and it starts to move upwards (which is also a moving reference frame). An increase in the normal force from the floor because of the upward acceleration of the elevator feels the same as an increase in the downward force of gravity.

* Recall that centripetal motion and centripetal force relates to angular/rotational motion and forces (which are studied in AP® Physics but not in the CP1 or honors courses). Equations or portions of equations with angular velocity (ω) and angular acceleration (α) apply only to the AP® course.

Use this space for summary and/or additional notes:

Similarly, a sample being spun in a centrifuge is subjected to the force *from the bottom of the centrifuge tube* as the tube is accelerated toward the center. The faster the rotation, the stronger the force. An increase in the normal force from the bottom of the centrifuge tube would feel like a downward force in the reference frame of the centrifuge tube.



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Sample Problems:

Q: A 300 kg roller coaster car reaches the bottom of a hill traveling at a speed of $20\,\frac{\text{m}}{\text{s}}$. If the track curves upwards with a radius of 50 m, what is the total force exerted by the track on the car?

A: The total force on the car is the normal force needed to resist the force of gravity on the car (equal to the weight of the car) plus the centripetal force exerted on the car as it moves in a circular path.

$$F_g = mg = (300)(10) = 3\,000\text{ N}$$
$$F_c = \frac{mv^2}{r} = \frac{(300)(20)^2}{50} = 2\,400\text{ N}$$
$$F_N = F_g + F_c = 3\,000 + 2\,400 = 5\,400\text{ N}$$

Q: A 20 g ball attached to a 60 cm long string is swung in a horizontal circle 80 times per minute. Neglecting gravity, what is the tension in the string?

A: Converting to MKS units, the mass of the ball is 0.02 kg and the string is 0.6 m long.

We can solve this two ways: we can convert revolutions either to meters by multiplying by $2\pi r$, or to radians by multiplying by 2π:

$$\omega = \frac{80\,\cancel{\text{revolutions}}}{1\,\cancel{\text{min}}} \times \frac{(2\pi)(0.60\text{ m})}{1\,\cancel{\text{revolution}}} \times \frac{1\,\cancel{\text{min}}}{60\text{ s}} = \frac{96\pi\text{ m}}{60\text{ s}} = 5.03\,\frac{\text{m}}{\text{s}}$$
$$F_T = F_c = \frac{mv^2}{r} = \frac{(0.02)(5.03)^2}{0.6} = 0.842\text{ N}$$

AP®

$$\omega = \frac{80\,\cancel{\text{revolutions}}}{1\,\cancel{\text{min}}} \times \frac{2\pi\text{ rad}}{1\,\cancel{\text{revolution}}} \times \frac{1\,\cancel{\text{min}}}{60\text{ s}} = \frac{160\pi\text{ rad}}{60\text{ s}} = 8.38\,\frac{\text{rad}}{\text{s}}$$
$$F_T = F_c = mr\omega^2$$
$$F_T = F_c = (0.02)(0.6)(8.38)^2 = 0.842\text{ N}$$

Use this space for summary and/or additional notes:

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Homework Problems

1. **(M – AP®; A – honors & CP1)** Find the force needed to keep a 0.5 kg ball traveling in a 0.70 m radius circle with an angular velocity of 15 revolutions every 10 s.

   Answer: 31.1 N

2. **(M – honors & AP®; A – CP1)** Find the force of friction needed to keep a 3 000 kg car traveling with a speed of $22\,\frac{m}{s}$ around a level highway exit ramp curve that has a radius of 100 m.

   Answer: 14 520 N

3. **(S – honors & AP®; A – CP1)** A passenger on an amusement park ride is cresting a hill in the ride at $15\,\frac{m}{s}$. If the top of the hill has a radius of 30 m, what force will a 50 kg passenger feel from the seat? What fraction of the passenger's weight is this?

   Answer: 125 N; $\frac{1}{4}$

Use this space for summary and/or additional notes:

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4. **(M – honors & AP®; A – CP1)** A roller coaster has a vertical loop with a 40 m radius. What speed at the top of the loop will make a 60 kg rider feel "weightless?"

   Answer: $20\,\frac{m}{s}$

5. **(S – AP®; A – honors & CP1)** A ride called "The Rotor" at Six Flags is a cylinder that spins at 56 RPM, which is enough to "stick" people to the walls. What force would a 90 kg rider feel from the walls of the ride, if the ride has a diameter of 6 m?

   Answer: 9 285 N

Use this space for summary and/or additional notes:

Center of Mass

Unit: Rotational Statics & Dynamics

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 4.A.1.1

Mastery Objective(s): (Students will be able to…)

- Find the center of mass of an object.

Success Criteria:

- Object balances at its center of mass.

Language Objectives:

- Explain why an object balances at its center of mass.

Tier 2 Vocabulary: center

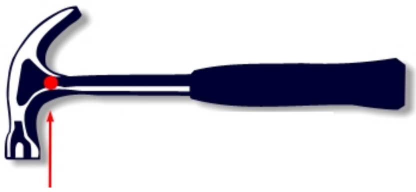
Labs, Activities & Demonstrations:

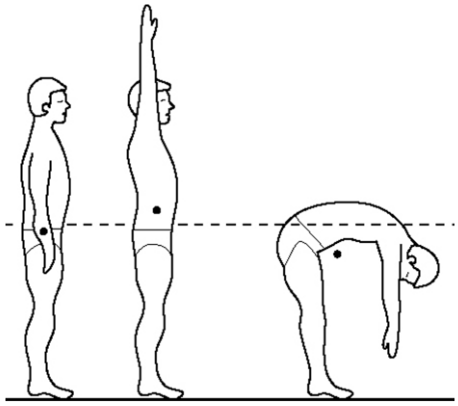
- Spin an object (*e.g.,* a hammer or drill team rifle) with its center of mass marked.

Notes:

center of mass: the point where all of an object's mass could be placed without changing the overall forces on the object or its rotational inertia.

Objects have nonzero volumes. For any object, some of the mass of the object will always be closer to the center of rotation, and some of the mass will always be farther away. In most of the problems that you will see in this course, we can simplify the problem by pretending that all of the mass of the object is at a single point.





Use this space for summary and/or additional notes:

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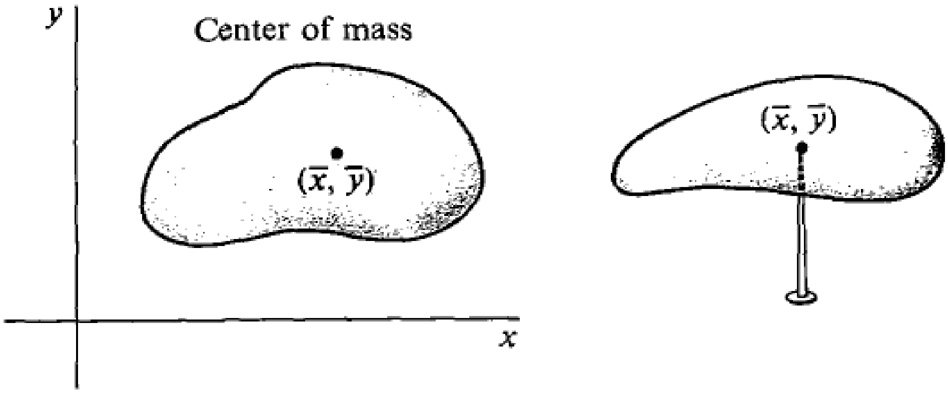
You can find the location of the center of mass of an object from the following formula:

$$r_{cm} = \frac{\sum_i m_i r_i}{\sum_i m_i}$$

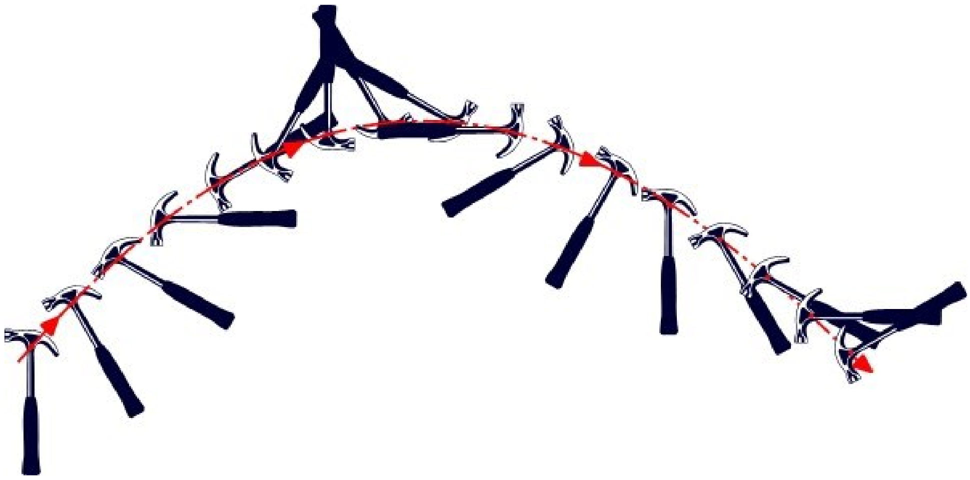
In this equation, the symbol Σ means "summation." When this symbol appears in a math equation, calculate the equation to the right of the symbol for each set of values, then add them up.

In this case, for each object (designated by a subscript), first multiply *mr* for that object, and then add up each of these products to get the numerator. Add up the masses to get the denominator. Then divide.

Because an object at rest remains at rest, this means that an object's center of mass is also the point at which the object will balance on a sharp point. (Actually, because gravity is involved, the object balances because the torques around the center of mass cancel. We will discuss that in detail later.)



Finally, note that an object that is rotating freely in space will always rotate about its center of mass:



Use this space for summary and/or additional notes:

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Sample Problem:

Q: Two people sit at the ends of a massless 3.5 m long seesaw. One person has a mass of 59 kg, and the other has a mass of 71 kg. Where is their center of mass?

A: (Yes, there's no such thing as a massless seesaw. This is an idealization to make the problem easy to solve.)

In order to make this problem simple, let us place the 59-kg person at a distance of zero.

$$r_{cm} = \frac{\sum_i m_i r_i}{\sum_i m_i}$$

$$r_{cm} = \frac{(59)(0) + (71)(3.5)}{(59 + 71)}$$

$$r_{cm} = \frac{248.5}{130} = 1.91\,\text{m}$$

Their center of mass is 1.91 m away from the 59-kg person.

Use this space for summary and/or additional notes:

Rotational Inertia

Unit: Rotational Statics & Dynamics

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A, but needed for torque and angular momentum

Mastery Objective(s): (Students will be able to…)

- Calculate the moment of (rotational) inertia of a system that includes one or more masses at different radiï from the center of rotation.

Success Criteria:

- Correct formula for moment of inertia of each basic shape is correctly selected.
- Variables are correctly identified and substituted correctly into the equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how an object's moment of inertia affects its rotation.

Tier 2 Vocabulary: moment

Labs, Activities & Demonstrations:

- Try to stop a bicycle wheel with different amounts of mass attached to it.

Notes:

inertia: the tendency for an object to continue to do what it is doing (remain at rest or remain in motion).

rotational inertia (or angular inertia): the tendency for a rotating object to continue rotating.

moment of inertia (I): a quantitative measure of the rotational inertia of an object. Moment of inertia is measured in units of kg·m^2.

Inertia in linear systems is a fairly easy concept to understand. The more mass an object has, the more it tends to remain at rest or in motion, and the more force is required to change its motion. *I.e.,* in a linear system, inertia depends only on mass.

Use this space for summary and/or additional notes:

Rotational Inertia

Rotational inertia is somewhat more complicated than the inertia in a non-rotating system. Suppose we have a mass that is being rotated at the end of a string. (Let's imagine that we're doing this in space, so we can neglect the effects of gravity.) The mass's inertia keeps it moving around in a circle at the same speed. If you suddenly shorten the string, the mass continues moving at the *same speed through the air*, but because the radius is shorter, the mass makes more revolutions around the circle in a given amount of time.

In other words, the object has the same linear speed (*not* the same *velocity* because its *direction* is constantly changing), but its angular velocity (degrees per second) has increased.

This must mean that an object's moment of inertia (its tendency to continue moving at a constant angular velocity) must depend on its distance from the center of rotation as well as its mass.

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The formula for moment of inertia is:

$$I = \sum_i m_i r_i^2$$

I.e., for each object or component (designated by a subscript), first multiply mr^2 for the object and then add up the rotational inertias for each of the objects to get the total.

For a point mass (a simplification that assumes that the entire mass exists at a single point):

$$I = mr^2$$

This means the rotational inertia of the point-mass is the same as the rotational inertia of the object.

Use this space for summary and/or additional notes:

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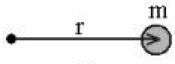
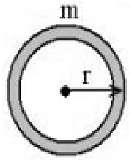
Calculating the moment of inertia for an arbitrary shape requires calculus. However, for solid, regular objects with well-defined shapes, their moments of inertia can be reduced to simple formulas:

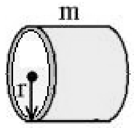
Point Mass at a Distance:
$I = mr^2$



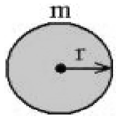
Hollow Sphere:
$I = \frac{2}{3}mr^2$



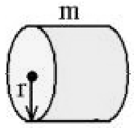
Hollow Cylinder:
$I = mr^2$



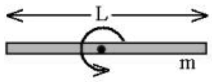
Solid Sphere:
$I = \frac{2}{5}mr^2$



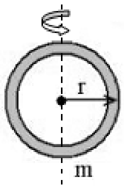
Solid Cylinder:
$I = \frac{1}{2}mr^2$



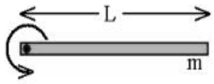
Rod about the Middle:
$I = \frac{1}{12}mL^2$



Hoop about Diameter:
$I = \frac{1}{2}mr^2$



Rod about the End:
$I = \frac{1}{3}mL^2$



In the above table, note that a rod can have a cross-section of any shape; for example a door hanging from its hinges is considered a rod rotated about the end for the purpose of determining its moment of inertia.

Use this space for summary and/or additional notes:

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Sample Problem:

Q: A solid brass cylinder has a density of $8\,500\,\frac{\text{kg}}{\text{m}^3}$, a radius of 0.10 m and a height of 0.20 m and is rotated about its center. What is its moment of inertia?

A: In order to find the mass of the cylinder, we need to use the volume and the density.

$$V = \pi r^2 h = (3.14)(0.1)^2(0.2)$$
$$V = 0.00628\,\text{m}^3$$

$$\rho = \frac{m}{V}$$
$$8\,500 = \frac{m}{0.006\,28}$$
$$m = 53.4\,\text{kg}$$

Now that we have its mass, we can find the moment of inertia of the cylinder:

$$I = \frac{1}{2} m r^2$$
$$I = \frac{1}{2}(53.4)(0.1)^2 = 0.534\,\text{kg}\cdot\text{m}^2$$

Use this space for summary and/or additional notes:

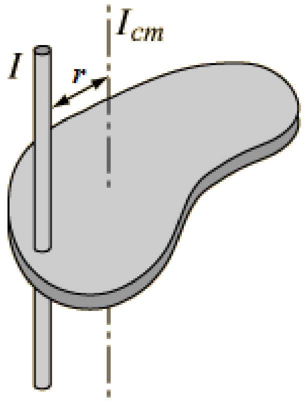
honors only (not AP®)

Parallel Axis Theorem

The moment of inertia of any object about an axis through its center of mass is always the minimum moment of inertia for any axis in that direction in space.

The moment of inertia about any axis that is parallel to the axis through the center of mass is given by:

$$I_{\text{parallel axis}} = I_{\text{cm}} + mr^2$$



Note that the formula for the moment of inertia of a point mass at a distance *r* from the center of rotation comes from the parallel axis theorem. The radius of the point mass itself is zero, which means:

$$I_{cm} = 0$$

$$I = I_{cm} + mr^2$$
$$I = 0 + mr^2$$

The parallel axis theorem is used in AP® Physics C: Mechanics, but is outside the scope of this course.

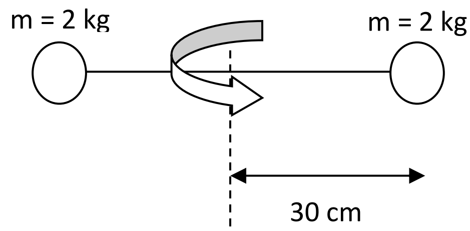
Use this space for summary and/or additional notes:

honors & AP®

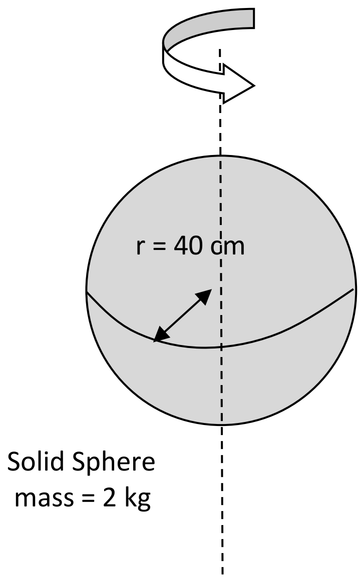
Homework Problems

Find the moment of inertia of each of the following objects. (Note that you will need to convert distances to meters.)

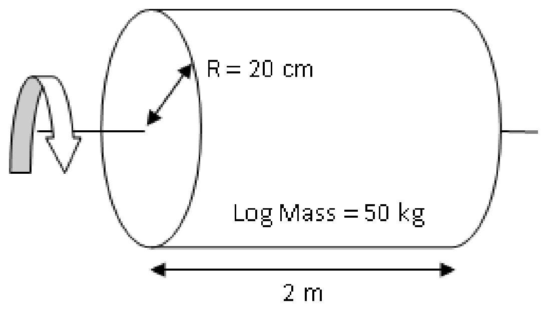
1. **(M – honors & AP®; A – CP1)** Answer: $0.36\,\text{kg}\cdot\text{m}^2$



2. **(M – honors & AP®; A – CP1)** Answer: $0.128\,\text{kg}\cdot\text{m}^2$



3. **(M – honors & AP®; A – CP1)** Answer: $1\,\text{kg}\cdot\text{m}^2$



Use this space for summary and/or additional notes:

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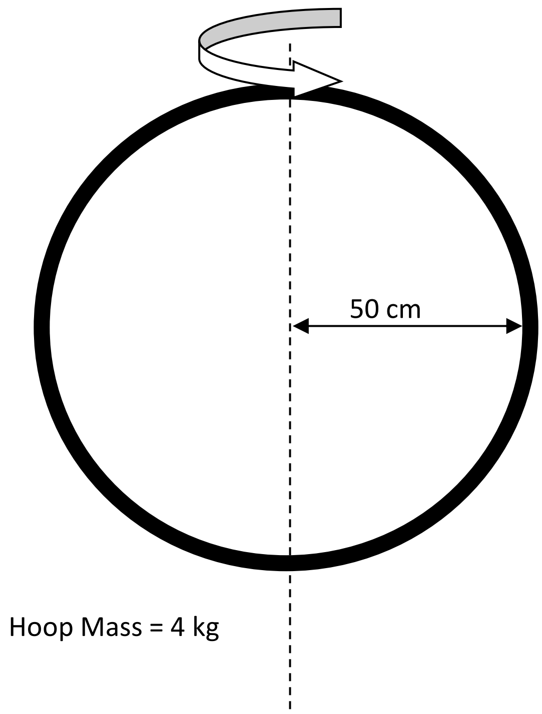
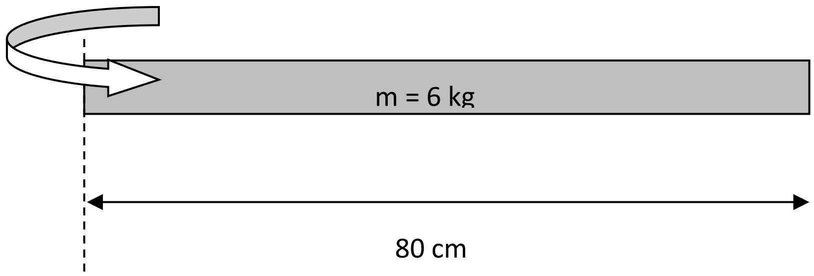
4. **(M – honors & AP®; A – CP1)** Answer: $0.5\,\text{kg}\cdot\text{m}^2$



5. **(M – honors & AP®; A – CP1)** Answer: $1.28\,\text{kg}\cdot\text{m}^2$



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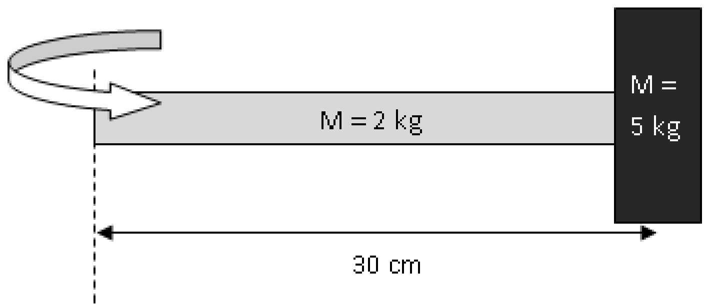
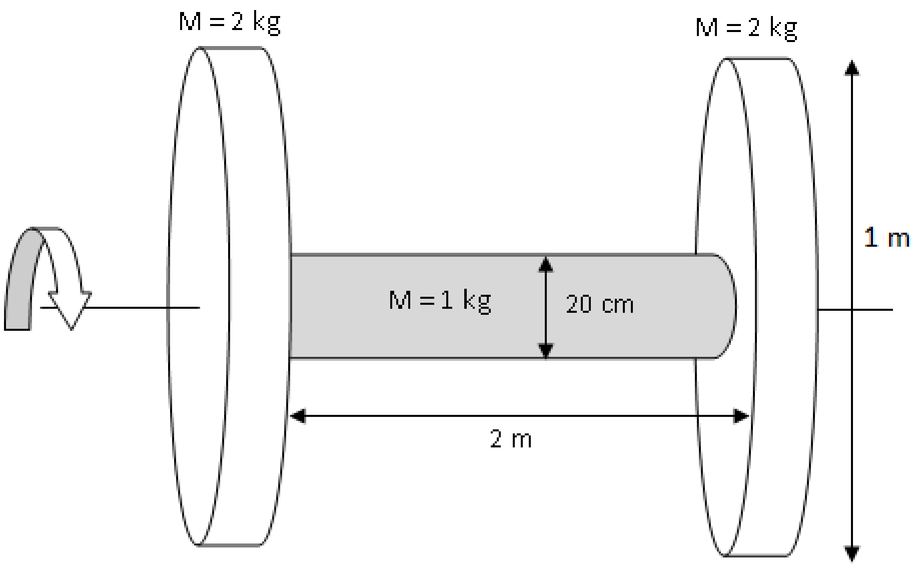
Find the moment of inertia of each of the following compound objects. (Be careful to note when the diagram gives a diameter instead of a radius.)

6. **(M – honors & AP®; A – CP1)** Sledge hammer: Answer: $0.51\,\text{kg}\cdot\text{m}^2$



7. **(M – honors & AP®; A – CP1)** Wheels and axle: Answer: $0.505\,\text{kg}\cdot\text{m}^2$



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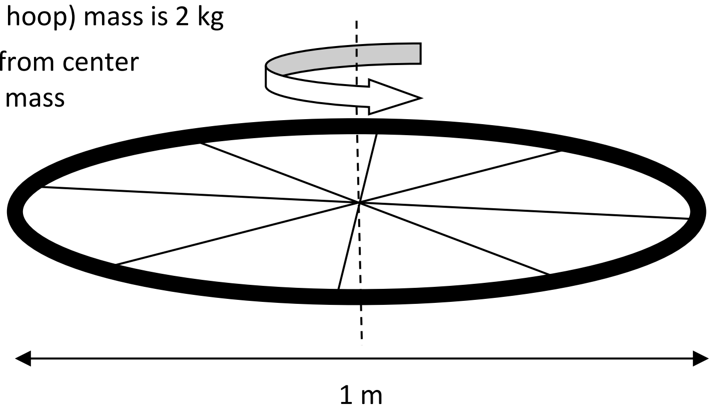
8. **(S – honors & AP®; A – CP1)** Wheel: Answer: $0.8\overline{3}\,\text{kg}\cdot\text{m}^2$

- Rim (outside hoop) mass is 2 kg
- Each spoke (from center to rim) has a mass of 0.5 kg



Use this space for summary and/or additional notes:

Torque

Unit: Rotational Statics & Dynamics

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 3.F.1.1, 3.F.1.2, 3.F.1.3, 3.F.1.4, 3.F.1.5, 3.F.2.2, 4.D.1.2

Mastery Objective(s): (Students will be able to…)

- Calculate the torque on an object.
- Calculate the location of the fulcrum of a system using balanced torques.
- Calculate the amount and distance from the fulcrum of the mass needed to balance a system.

Success Criteria:

- Variables are correctly identified and substituted correctly into equations.
- Equations for torques on different masses are combined correctly algebraically.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why a longer lever arm is more effective.

Tier 2 Vocabulary: balance, torque

Labs, Activities & Demonstrations:

- Balance an object on two fingers and slide both toward the center.
- Clever wine bottle stand.

Notes:

torque ($\vec{\boldsymbol{\tau}}$): a vector quantity that measures the effectiveness of a force in causing rotation. Take care to distinguish the Greek letter "τ" from the Roman letter "t". Torque is measured in units of newton-meters:

$$1\,\mathrm{N\cdot m} = 1\,\tfrac{\mathrm{kg\cdot m^2}}{\mathrm{s^2}}$$

Note that work and energy (which we will study later) are also measured in newton-meters. However, work and energy are different quantities from torque, and are not interchangeable. (Among other differences, work and energy are scalar quantities, and torque is a vector quantity.)

axis of rotation: the point around which an object rotates.

fulcrum: the point around which a lever pivots. Also called the pivot.

lever arm: the distance from the axis of rotation that a force is applied, causing a torque.

Use this space for summary and/or additional notes:

Just as force is the quantity that causes linear acceleration, torque is the quantity that causes a change in the speed of rotation (rotational acceleration).

Because inertia is a property of mass, Newton's second law is the relationship between force and inertia. Newton's second law in rotational systems looks similar to Newton's second law in linear systems:

linear	rotational
$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$	$\vec{\alpha} = \frac{\Sigma \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$
$\vec{F}_{net} = m\vec{a}$	$\vec{\tau}_{net} = I\vec{\alpha}$ *

As you should remember, a net force of zero, that means all forces cancel in all directions and there is no acceleration. If there is no acceleration $(\vec{\mathbf{a}} = 0)$, the velocity remains constant (which may or may not equal zero).

Similarly, if the net torque is zero, then the torques cancel in all directions and there is no angular acceleration. If there is no angular acceleration $(\vec{\boldsymbol{\alpha}} = 0)$, then the angular velocity remains constant (which may or may not equal zero).

rotational equilibrium: when all of the torques on an object cancel each other's effects (resulting in a net force of zero) and the object either does not rotate or rotates with a constant angular velocity.

Torque is also the cross product of distance from the center of rotation ("lever arm") × force:

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} \qquad \text{which gives:} \qquad \|\vec{\boldsymbol{\tau}}\| = \tau = rF\sin\theta = rF_{\perp}$$

where θ is the angle between the lever arm and the applied force.

We use the variable *r* for the lever arm (which is a distance) because torque causes rotation, and *r* is the distance from the center of the circle (radius) at which the force is applied.

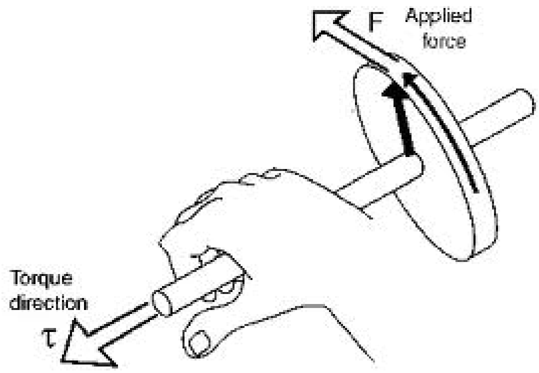
$F\sin\theta$ is sometimes written as $F_{\perp}$ (the component of the force that is perpendicular to the radius) and sometimes $F_{||}$ (the component of the force that is parallel to the direction of motion). These notes will use $F_{\perp}$, because in many cases the force is applied to a lever, and the component of the force that causes the torque is perpendicular to the lever itself, so it is easy to think of it as "the amount of force that is perpendicular to the lever". This gives the equation:

$$\tau = rF_{\perp}$$

* In this equation, $\vec{\alpha}$ is angular acceleration, which is studied in AP® Physics 1, but is beyond the scope of the CP1 and honors physics course. Qualitatively, angular acceleration is a change in how fast something is rotating.

Use this space for summary and/or additional notes:

Of course, because torque is the cross product of two vectors, it is a vector whose direction is perpendicular to both the lever arm and the force.



This is an application of the "right hand rule." If your fingers of your right hand curl from the first vector ($\vec{r}$) to the second ($\vec{F}$), then your thumb points in the direction of the resultant vector ($\vec{\tau}$). Note that the direction of the torque vector is parallel to the axis of rotation.

Note, however, that you can't "feel" torque; you can only "feel" force. Most people think of the "direction" of a torque as the direction of the rotation that the torque would produce (clockwise or counterclockwise). In fact, the College Board usually uses this convention.

Mathematically, the direction of the torque vector is needed only to give torques a positive or negative sign, so torques in the same direction add and torques in opposite directions subtract. In practice, most people find it easier to define the positive direction for rotation (clockwise ↻ or counterclockwise ↺) and use those for positive or negative torques in the problem, regardless of the direction of the torque vector.

Sample Problem:

Q: If a perpendicular force of 20 N is applied to a wrench with a 25 cm handle, what is the torque applied to the bolt?

A: $\tau = r F_{\perp}$

$\tau = (0.25\,\text{m})(20\,\text{N})$

$\tau = 4\,\text{N}\cdot\text{m}$

Use this space for summary and/or additional notes:

Seesaw Problems

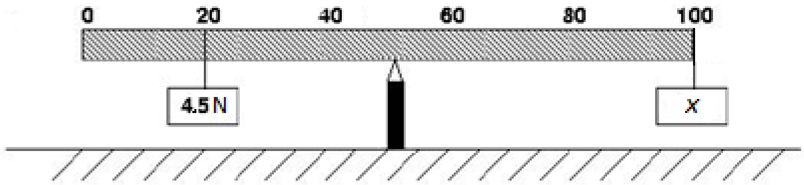
A seesaw problem is one in which objects on opposite sides of a lever (such as a seesaw) balance one another.

To solve seesaw problems, if the seesaw is not moving, then the torques must balance and the net torque must be zero.

The total torque on each side is the sum of the separate torques caused by the separate masses. Each of these masses can be considered as a point mass (infinitely small object) placed at the object's *center of mass*.

Sample Problems:

Q: A 100 cm meter stick is balanced at its center (the 50-cm mark) with two objects hanging from it, as shown below:



One of the objects weighs 4.5 N, and is hung from the 20-cm mark (30 cm = 0.3 m from the fulcrum). A second object is hung at the opposite end (50 cm = 0.5 m from the fulcrum). What is the weight of the second object?

A: In order for the ruler to balance, the torque on the left side (which is trying to rotate the ruler counter-clockwise) must be equal to the torque on the right side (which is trying to rotate the ruler clockwise). The torques from the two halves of the ruler are the same (because the ruler is balanced in the middle), so this means the torques applied by the objects also must be equal.

The torque applied by the object on the left is:

$$\tau = rF = (0.30)(4.5) = 1.35\,\mathrm{N\cdot m}$$

The torque applied by the object on the right must also be 1.35 N·m, so we can calculate the force:

$$\tau = rF$$
$$1.35 = 0.50F$$
$$F = \frac{1.35}{0.50} = 2.7\,\mathrm{N}$$

Use this space for summary and/or additional notes:

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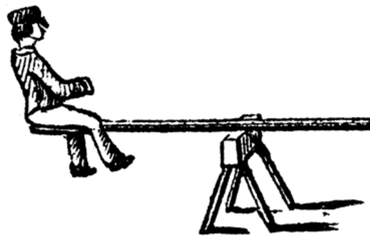
Q: In the following diagram, the mass of the person on the left is 90. kg and the mass of the person on the right is 50. kg. The board is 6.0 m long and has a mass of 20. kg.



Where should the board be positioned in order to balance the seesaw?

A: When the seesaw is balanced, the torques on the left have to equal the torques on the right.

This problem is more challenging because the board has mass and is not balanced at its center. This means the two sides of the board apply different (unequal) torques, so we have to take into account the torque applied by each fraction of the board as well as the torque by each person.

Let's say that the person on the left is sitting a distance of *x* meters from the fulcrum. The board is 6 m long, which means the person on the right must be $(6 - x)$ meters from the fulcrum.

Our strategy is:

1. Calculate and add up the counter-clockwise (CCW = ↺) torques. These are the torques that would turn the seesaw in a counter-clockwise direction, which are on the left side. They are caused by the force of gravity acting on the person, at distance *x*, and the left side of the board, centered at distance $\frac{x}{2}$.
2. Calculate and add up the clockwise (CW = ↻) torques. These are caused by the force of gravity acting on the person, at distance $(6 - x)$, and the left side of the board, centered at distance $\frac{6-x}{2}$.
3. Set the two torques equal to each other and solve for *x*.

Use this space for summary and/or additional notes:

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Left Side (CCW = ↺)

Person

The person has a mass of 90 kg and is sitting at a distance *x* from the fulcrum:

$$\tau_{LP} = rF$$

$$\tau_{LP} = x(mg) = x(90\,\text{kg})(10\,\tfrac{\text{m}}{\text{s}^2})$$

$$\tau_{LP} = 900x$$

Board

The center of mass of the left part of the board is at a distance of $\frac{x}{2}$.

The weight (F_g) of the board to the left of the fulcrum is $\left(\frac{x}{6}\right)(20)(10)$

$$\tau_{LB} = rF$$

$$\tau_{LB} = r(mg) = \left(\frac{x}{2}\right)\left(\frac{x}{6.0}\right)(20)(10)$$

$$\tau_{LB} = 16.\overline{6}\,x^2$$

Total

$$\tau_{ccw} = \tau_{LB} + \tau_{LP}$$

$$\tau_{ccw} = 16.\overline{6}x^2 + 900x$$

Right Side (CW = ↻)

Person

The person on the right has a mass of 50 kg and is sitting at a distance of 6 − x from the fulcrum:

$$\tau_{RP} = rF$$

$$\tau_{RP} = r(mg) = (6-x)(50\,\text{kg})(10\,\tfrac{\text{m}}{\text{s}^2})$$

$$\tau_{RP} = 500\,(6-x)$$

$$\tau_{RP} = 3000 - 500x$$

Board

The center of mass of the right part of the board is at a distance of $\frac{6-x}{2}$.

The weight (F_g) of the board to the right of the fulcrum is $\left(\frac{6-x}{6}\right)(20)(10)$

$$\tau_{RB} = rF$$

$$\tau_{RB} = r(mg) = \left(\frac{6-x}{2}\right)\left(\frac{6-x}{6}\right)(20)(10)$$

$$\tau_{RB} = 16.\overline{6}\,(36 - 12x + x^2)$$

$$\tau_{RB} = 600 - 200x + 16.\overline{6}x^2$$

Total

$$\tau_{cw} = \tau_{RB} + \tau_{RP}$$

$$\tau_{cw} = 16.\overline{6}x^2 - 200x + 600 + 3000 - 500x$$

$$\tau_{cw} = 16.\overline{6}x^2 - 700x + 3600$$

Because the seesaw is not rotating, the net torque must be zero. So we need to define the positive and negative directions. A common convention is to define counter-clockwise as the positive direction. (Most math classes already do this—a positive angle means counter-clockwise starting from zero at the *x*-axis.)

This gives:

$$\tau_{ccw} = 16.\overline{6}x^2 + 900x \qquad \tau_{cw} = -(16.\overline{6}x^2 - 700x + 3600) = -16.\overline{6}x^2 + 700x - 3600$$

Use this space for summary and/or additional notes:

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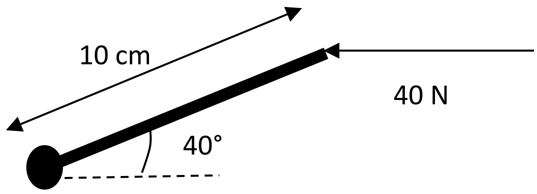
Because the seesaw is not rotating we set $\tau_{CCW} + \tau_{CW} = 0$ and solve:

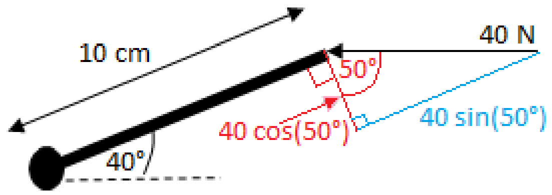
$$0 = \tau_{net} = \sum \tau = \tau_{ccw} + \tau_{cw} = \cancel{16.\bar{6}x^2} + 900x - \cancel{16.\bar{6}x^2} + 700x - 3600$$
$$0 = 900x + 700x - 3600$$
$$0 = 1600x - 3600$$
$$1600x = 3600$$
$$x = \frac{3600}{1600} = 2.25\,\text{m}$$

The board should therefore be placed with the fulcrum 2.25 m away from the person on the left.

Q: Calculate the torque on the following 10 cm lever. The lever is angled 40° up from horizontal, and a force of 40 N force is applied parallel to the ground.



A: This is an exercise in geometry. We need the component of the 40 N force that is perpendicular to the lever $(F_{\perp})$. To find this, we draw a right triangle in which the hypotenuse is the applied force. (R*emember that the hypotenuse is the longest side, and the total force must be greater than or equal to any of its components.*)



Now, we simply use our calculators (or trigonometry tables):

$$40\cos(50°) = (40)(0.643) = 25.7\,\text{N}$$

Extension

Just as yank is the rate of change of force with respect to time, the rate of change of torque with respect to time is called rotatum: $\vec{\boldsymbol{P}} = \frac{\Delta \vec{\boldsymbol{\tau}}}{\Delta t} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{Y}}$. Rotatum is also sometimes called the "moment of a yank," because it is the rotational analogue to yank.

Use this space for summary and/or additional notes:

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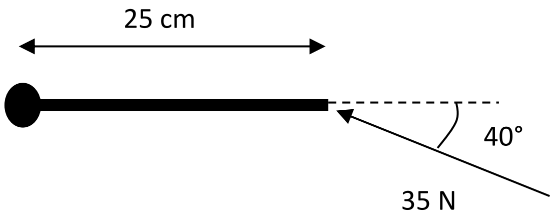
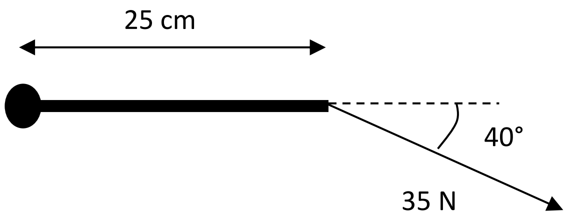
Homework Problems

For each of the following diagrams, find the torque about the axis indicated by the black dot. Assume that the lever itself has negligible mass.

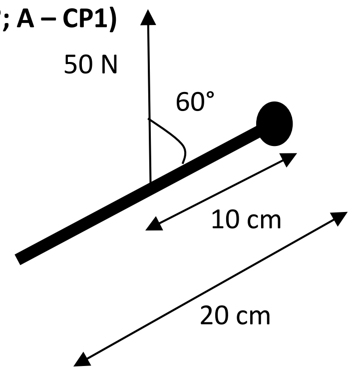
1. **(M – honors & AP®; A – CP1)** Answer: 5.62 N·m CCW (↺)



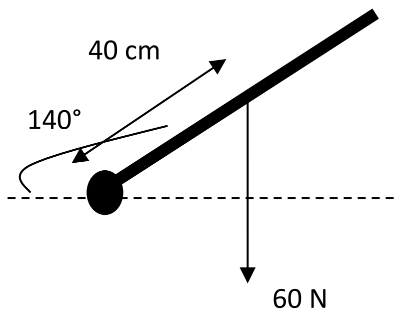
2. **(M – honors & AP®; A – CP1)** Answer: 5.62 N·m CW (↻)



3. **(M – honors & AP®; A – CP1)** Answer: 4.33 N·m CW (↻)

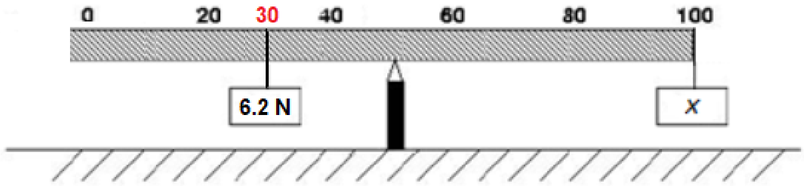


4. **(S – honors & AP®; A – CP1)** Answer: 18.4 N·m CW (↻)



Use this space for summary and/or additional notes:

5. **(M)** In the following diagram, a meter stick is balanced in the center (at the 50 cm mark). A 6.2 N weight is hung from the meter stick at the 30 cm mark. How much weight must be hung at the 100 cm mark in order to balance the meter stick?



Hints:

- *The meter stick has the same amount of mass on both sides of the fulcrum. This means it applies the same amount of torque in both directions and you don't need to include it in your calculations.*
- *The 30 cm mark is 20 cm = 0.2 m from the fulcrum; the 100 cm mark is 50 cm = 0.5 m from the fulcrum.*

Answer: 0.25 kg

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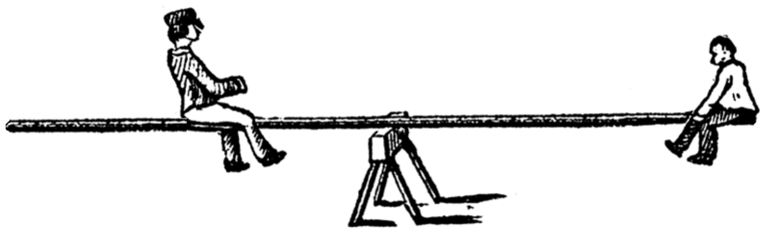
6. **(M – AP®; S – honors; A – CP1)** The seesaw shown in the following diagram balances when no one is sitting on it. The child on the right has a mass of 35 kg and is sitting 2.0 m from the fulcrum. If the adult on the left has a mass of 85 kg, how far should the adult sit from the fulcrum in order for the seesaw to be balanced?



Answer: 0.82 m

Use this space for summary and/or additional notes:

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Solving Linear & Rotational Force/Torque Problems

Unit: Rotational Statics & Dynamics

MA Curriculum Frameworks (2016): HS-PS2-1, HS-PS2-10(MA)

AP® Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3, 3.F.1.1, 3.F.1.2, 3.F.1.3, 3.F.1.4, 3.F.1.5, 3.F.2.1, 3.F.2.2, 4.A.1.1, 4.D.1.1, 4.D.1.2

Mastery Objective(s): (Students will be able to…)

- Set up and solve problems involving combinations of linear and rotational dynamics.

Success Criteria:

- Variables are correctly identified and substituted correctly into equations.
- Equations are combined correctly algebraically.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Identify which parts of a problem are linear and which parts are rotational.

Tier 2 Vocabulary: force, rotation, balance, torque

Notes:

Newton's second law—that forces produce acceleration—applies in both linear and rotational contexts. In fact, you can think of the equations as exactly the same, except that one set uses Cartesian coördinates, and the other uses polar or spherical coördinates.

You can substitute rotational variables for linear variables in all of Newton's equations (motion and forces), and the equations are still valid.

Use this space for summary and/or additional notes:

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The following is a summary of the variables used for dynamics problems:

Linear			Angular		
Var.	**Unit**	**Description**	**Var.**	**Unit**	**Description**
$\vec{\boldsymbol{x}}$	m	position	$\vec{\boldsymbol{\theta}}$	— (rad)	angle; angular position
$\vec{\boldsymbol{d}}, \Delta\vec{\boldsymbol{x}}$	m	displacement	$\Delta\vec{\boldsymbol{\theta}}$	— (rad)	angular displacement
$\vec{\boldsymbol{v}}$	$\frac{\text{m}}{\text{s}}$	velocity	$\vec{\boldsymbol{\omega}}$	$\frac{1}{\text{s}}\left(\frac{\text{rad}}{\text{s}}\right)$	angular velocity
$\vec{\boldsymbol{a}}$	$\frac{\text{m}}{\text{s}^2}$	acceleration	$\vec{\boldsymbol{\alpha}}$	$\frac{1}{s^2}\left(\frac{\text{rad}}{\text{s}^2}\right)$	angular acceleration
t	s	time	t	s	time
m	kg	mass	I	$\text{kg}\cdot\text{m}^2$	moment of inertia
$\vec{\boldsymbol{F}}$	N	force	$\vec{\boldsymbol{\tau}}$	N·m	torque

Notice that each of the linear variables has an angular counterpart.

Keep in mind that "radian" is not a unit. A radian is a ratio that describes an angle as the ratio of the arc length to the radius. This ratio is dimensionless (has no unit), because the units cancel. This means that an angle described in radians has no unit, and therefore never needs to be converted from one unit to another. However, we often write "rad" after an angle measured in radians to remind ourselves that the quantity describes an angle.

We have learned the following equations for solving motion problems:

Linear Equation	**Angular Equation**	**Relation**	**Comments**
$\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$	$\vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}}$	$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} = rF_\perp$	Quantity that produces acceleration
$\vec{\boldsymbol{F}}_c = m\vec{\boldsymbol{a}}_c = \dfrac{m\vec{\boldsymbol{v}}^2}{r}$	$\vec{\boldsymbol{F}}_c = m\vec{\boldsymbol{a}}_c = mr\vec{\boldsymbol{\omega}}^2$		Centripetal force (which causes centripetal acceleration)

Note that vector quantities (shown in bold) can be positive or negative, depending on direction.

Use this space for summary and/or additional notes:

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Problems Involving Linear and Rotational Dynamics

The main points of the linear Dynamics (Forces) & Gravitation chapter were:

a. A net force produces acceleration. $\vec{F}_{net} = m\vec{a}$

b. If there is no acceleration, then there is no net force, which means all forces must cancel in all directions. No acceleration may mean a static situation (nothing is moving) or constant velocity.

c. Forces are vectors. Perpendicular vectors do not affect each other, which means perpendicular forces do not affect each other.

The analogous points hold true for torques:

1. A net torque produces angular acceleration. $\vec{\boldsymbol{\tau}}_{net} = I\vec{\boldsymbol{\alpha}}$

2. If there is no angular acceleration, then there is no net torque, which means all torques must cancel. No angular acceleration may mean a static situation (nothing is rotating) or it may mean that there is rotation with constant angular velocity.

3. Torques are vectors. Perpendicular torques do not affect each other.

4. Torques and linear forces act independently.

Use this space for summary and/or additional notes:

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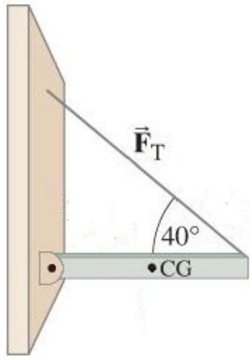
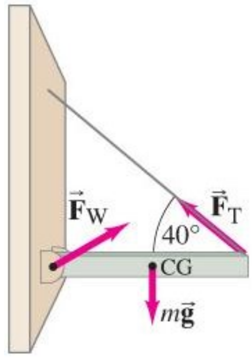
One of the most common types of problem involves a stationary object that has both linear forces and torques, both of which are in balance.

In the diagrams at the right, a beam with a center of gravity (center of mass) in the middle (labeled "CG") is attached to a wall with a hinge. The end of the beam is held up with a rope at an angle of 40° above the horizontal.



The rope applies a torque to the beam at the end at an angle of rotation with a radius equal to the length of the beam. Gravity applies a force straight down on the beam.



1. Because the beam is not rotating, we know that $\vec{\tau}_{net}$ must be zero, which means the wall must apply a torque that counteracts the torque applied by the rope. (Note that the axis of rotation for the torque from the wall is the opposite end of the beam.)

2. Because the beam is not moving (translationally), we know that $\vec{F}_{net}$ must be zero in both the vertical and horizontal directions. This means that the wall must apply a force $\vec{F}_W$ to balance the vertical and horizontal components of $\vec{F}_T$ and $m\vec{g}$. Therefore, the vertical component of $\vec{F}_W$ plus the vertical component of $\vec{F}_T$ must add up to $m\vec{g}$, and the horizontal components of $\vec{F}_T$ and $\vec{F}_W$ must cancel.

AP questions often combine pulleys with torque. (See the section on Tension starting on page 285.) These questions usually require you to combine the following concepts/equations:

1. A torque is the action of a force acting perpendicular to the radius at some distance from the axis of rotation: $\tau = rF_{\perp}$

2. Net torque produces angular acceleration according to the formula: $\tau_{net} = I\alpha$

3. The relationships between tangential and angular velocity and acceleration are: $v_T = r\omega$ and $a_T = r\alpha$ ← ***Memorize these!***

AP free-response problems are always scaffolded, meaning that each part leads to the next.

Use this space for summary and/or additional notes:

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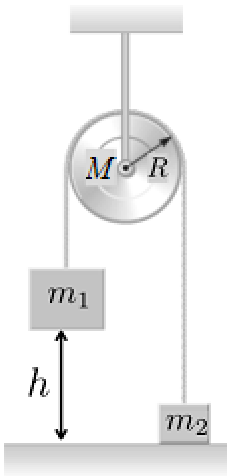
Sample AP-Style Problem



Q: Two masses, $m_1 = 23.0\,\text{kg}$ and $m_2 = 14.0\,\text{kg}$ are suspended by a rope that goes over a pulley that has a radius of R = 0.350 m and a mass of M = 40 kg, as shown in the diagram to the right. (You may assume that the pulley is a solid cylinder.) Initially, mass m_2 is on the ground, and mass m_1 is suspended at a height of h = 0.5 m above the ground.

a. What is the net torque on the pulley?

CCW: the torque is caused by mass m_1 at a distance of R, which is given by:

$$\tau_1 = m_1 gR = (23.0)(10)(0.350) = 80.5\,\text{N}\cdot\text{m}$$

(Note that we are using positive numbers for counter-clockwise torques and negative numbers for clockwise torques.)

CW: the torque is caused by mass m_2 at a distance of R, so:

$$\tau_2 = m_2 gR = -(14.0)(10)(0.350) = -49.0\,\text{N}\cdot\text{m}$$

Net: The net torque is just the sum of all of the torques:

$$\tau_{net} = 80.5 + (-49.0) = +31.5\,\text{N}\cdot\text{m}\ (\text{CCW})$$

b. What is the angular acceleration of the pulley?

Now that we know the net torque, we can use the equation $\tau_{\text{net}} = I\alpha$ to calculate α (but we have to calculate I first).

$$I = \tfrac{1}{2}MR^2 = (\tfrac{1}{2})(40)(0.35)^2 = 2.45\,\text{N}\cdot\text{m}^2$$

$$\tau_{\text{net}} = I\alpha$$
$$31.5 = 2.45\alpha$$
$$\alpha = 12.9\,\tfrac{\text{rad}}{\text{s}^2}$$

c. What is the linear acceleration of the blocks?

The linear acceleration of the blocks is the same as the acceleration of the rope, which is the same as the tangential acceleration of the pulley:

$$a_T = r\alpha = (0.35)(12.9) = 4.5\,\tfrac{\text{m}}{\text{s}^2}$$

d. How much time does it take for mass m_1 to hit the floor?

We never truly get away from kinematics problems!

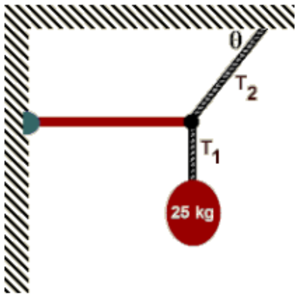
$$d = v_o t + \tfrac{1}{2}at^2 \qquad t^2 = 0.222$$
$$0.5 = (\tfrac{1}{2})(4.5)t^2 \qquad t = \sqrt{0.222} = 0.47\,\text{s}$$

Use this space for summary and/or additional notes:

AP®

Homework Problems

1. **(M – AP®; A – honors & CP1)** A 25 kg bag is suspended from the end of a uniform 100 N beam of length *L*, which is attached to the wall by an ideal (freely-swinging, frictionless) hinge, as shown in the figure to the right. The angle of rope hanging from the ceiling is $\theta = 30°$.



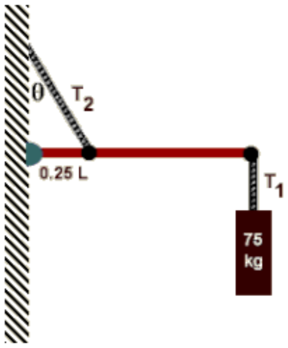
What is the tension, T_2, in the rope that hangs from the ceiling?

Answer: 600 N

Use this space for summary and/or additional notes:

AP®

2. **(M – AP®; A – honors & CP1)** A 75 kg block is suspended from the end of a uniform 100 N beam of length *L*, which is attached to the wall by an ideal hinge. A support rope is attached ¼ of the way to the end of the beam at an angle from the wall of θ = 30°.



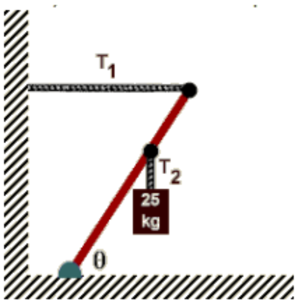
What is the tension in the support rope (T_2)?

Answer: 3 695 N

Use this space for summary and/or additional notes:

AP®

3. **(M – AP®; A – honors & CP1)** A 25 kg box is suspended $^{2}/_{3}$ of the way up a uniform 100 N beam of length L, which is attached to the floor by an ideal hinge, as shown in the picture to the right. The angle of the beam above the horizontal is $\theta = 37°$.



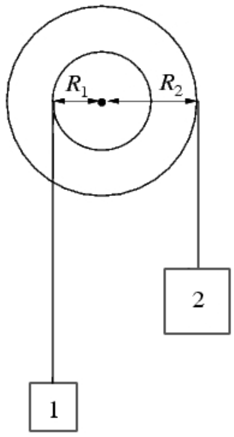
What is the tension, T_1, in the horizontal support rope?

Answer: 288 N

Use this space for summary and/or additional notes:

AP®

4. **(M – AP®; A – honors & CP1)** Two blocks are suspended from a double pulley as shown in the picture to the right. Block #1 has a mass of 2 kg and is attached to a pulley with radius $R_1 = 0.25$ m. Block #2 has a mass of 3.5 kg and is attached to a pulley with radius $R_2 = 0.40$ m. The pulley has a moment of inertia of 1.5 kg·m^2.



When the weights are released and are allowed to fall,

a. **(M – AP®; A – honors & CP1)** What will be the net torque on the system?

Answer: 9 N·m CW (↻)

b. **(M – AP®; A – honors & CP1)** What will be the angular acceleration of the pulley?

Answer: $6\ \frac{\text{rad}}{\text{s}^2}$

c. **(M – AP®; A – honors & CP1)** What will be the linear accelerations of blocks #1 and #2?

Answer: block #1: $1.5\ \frac{\text{m}}{\text{s}^2}$; block #2: $2.4\ \frac{\text{m}}{\text{s}^2}$

Use this space for summary and/or additional notes:

Introduction: Gravitation

Unit: Gravitation

Topics covered in this chapter:

In this chapter you will learn about different kinds of forces and how they relate.

- *Early Theories of the Universe* describes the geocentric (Earth-centered) model of the universe, and the theories of Ptolemy and Copernicus.
- *Kepler's Laws of Planetary Motion* describes the motion of planets and other celestial bodies and the time period that it takes for planets to revolve around stars throughout the universe.
- *Universal Gravitation* describes how to calculate the force of mutual gravitational attraction between massive objects such as planets and stars.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-4: Use mathematical representations of Newton's Law of Gravitation ~~and Coulomb's Law~~ to describe and predict the gravitational ~~and electrostatic~~ forces between objects.

AP®

AP® Physics 1 Learning Objectives:

2.B.1.1: The student is able to apply to calculate the gravitational force on an object with mass m in a gravitational field of strength g in the context of the effects of a net force on objects and systems. [**SP 2.2**, **7.2**]

2.B.2.1: The student is able to apply to calculate the gravitational field due to an object with mass M, where the field is a vector directed toward the center of the object of mass M. [**SP 2.2**]

2.B.2.2: The student is able to approximate a numerical value of the gravitational field (g) near the surface of an object from its radius and mass relative to those of the Earth or other reference objects. [**SP 2.2**]

3.C.1.1: The student is able to use Newton's law of gravitation to calculate the gravitational force the two objects exert on each other and use that force in contexts other than orbital motion. [**SP 2.2**]

Use this space for summary and/or additional notes:

AP®

3.C.1.2: The student is able to use Newton's law of gravitation to calculate the gravitational force between two objects and use that force in contexts involving orbital motion [**SP 2.2**]

3.C.2.2: The student is able to connect the concepts of gravitational force and electric force to compare similarities and differences between the forces. [**SP 7.2**]

3.G.1.1: The student is able to articulate situations when the gravitational force is the dominant force and when the electromagnetic, weak, and strong forces can be ignored. [**SP 7.1**]

Topics from this chapter assessed on the SAT Physics Subject Test:

Gravity, such as the law of gravitation, orbits, and Kepler's laws.

1. Kepler's Laws
2. Weightlessness
3. Newton's Law of Universal Gravitation

Skills learned & applied in this chapter:

- Estimating the effect of changing one variable on other variables in the same equation.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Early Theories of the Universe

Unit: Gravitation

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Describe early models of the planets and stars, including Copernicus's heliocentric model

Success Criteria:

- Description accounts for observations of the time.

Language Objectives:

- Explain the primary differences between the geocentric (Earth-in-the-center) and heliocentric (sun-in-the-center) model.

Tier 2 Vocabulary: sphere, cycle, revolve

Notes:

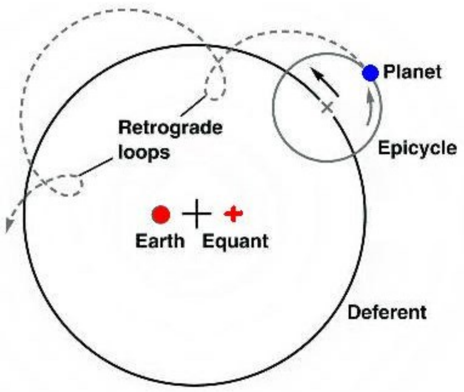
Early Observations

Prior to the renaissance in Europe, most people believed that the Earth was the center of the universe. Early astronomers observed objects moving across the night sky. Objects that were brighter and changed their position significantly from one night to the next were called planets. Objects that were dimmer and more constant (changed their position more gradually from one night to the next) were called stars.

Retrograde Motion and Epicycles

Early astronomers observed that planets sometimes moved "backwards" as they moved across the sky.

<u>retrograde</u>: apparent "backwards" motion of a planet as it appears to move across the sky.



The ancient astronomer Claudius Ptolemy theorized that this retrograde motion was caused by the planets moving in small circles, called *epicycles*, as they moved in their large circular path around the Earth, called the *deferent*.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Heliocentric Theory

In 1532, Polish mathematician and astronomer Nicolaus Copernicus published a new heliocentric theory of the universe that placed the sun at the center and designated the Earth as one of the planets that revolve around the sun.

heliocentric theory: the theory that the sun (not the Earth) is the center of the universe.

The assumptions of Copernicus's theory were:

1. There is no one center of all the celestial circles or *spheres*.*
2. The center of the Earth is the center towards which heavy objects move†, and the center of the lunar sphere (the moon's orbit). However, the center of the Earth is not the center of the universe.
3. All the spheres surround the sun as if it were in the middle of them, and therefore the center of the universe is near the sun.
4. The spheres containing the stars are much farther from the sun than the sphere in which the Earth moves. This far-away sphere that contained the stars was called the *firmament*.
5. The firmament does not move. The stars appear to move because the Earth is rotating.
6. The sun appears to move because of a combination of the Earth rotating and revolving around the sun. This means the Earth is just a planet, and nothing special (as far as the universe is concerned).
7. The apparent motion of the planets (both direct and retrograde) is explained by the Earth's motion.

* At the time, it was thought that planets and stars were somehow attached to the surface of a hollow sphere, and that they moved along that sphere.

† Remember that Copernicus published this theory more than 150 years before Isaac Newton published his theory of gravity.

Use this space for summary and/or additional notes:

Kepler's Laws of Planetary Motion

Unit: Gravitation

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Set up and solve problems involving Kepler's Laws.

Success Criteria:

- All variables are identified and substituted correctly.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how the speed that a planet is moving changes as it revolves around the sun.

Tier 2 Vocabulary: focus

Notes:

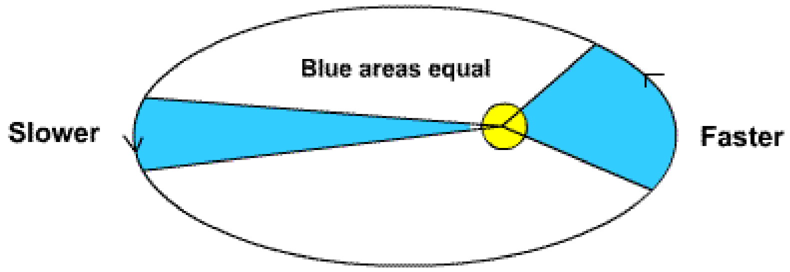
The German mathematician and astronomer Johannes Kepler lived about 100 years after Copernicus. Kepler derived three laws and equations that govern planetary motion, which were published in three volumes between 1617 and 1621.

Kepler's First Law

The orbit of a planet is an ellipse, with the sun at one focus.

Kepler's Second Law

A line that joins a planet with the sun will sweep out equal areas in equal amounts of time.



I.e., the planet moves faster as it moves closer to the sun and slows down as it gets farther away. If the planet takes exactly 30 days to sweep out one of the blue areas above, then it will take exactly 30 days to sweep out the other blue area, and any other such area in its orbit.

While we now know that the planet's change in speed is caused by the force of gravity, Kepler's Laws were published fifty years before Isaac Newton published his theory of gravity.

Use this space for summary and/or additional notes:

Kepler's Third Law

If T is the period of time that a planet takes to revolve around a sun and $r_{ave.}$ is the average radius of the planet from the sun (the length of the semi-major axis of its elliptical orbit) then:

$$\frac{T^2}{r_{ave.}^3} = \text{constant} \text{ for every planet in that solar system}$$

We now know that, $\frac{T^2}{r_{ave.}^3} = \frac{4\pi^2}{GM}$, where G is the universal gravitational constant and M is the mass of the star in question, which means this ratio is different for every planetary system. For our solar system, the value of $\frac{T^2}{r_{ave.}^3}$ is approximately $9.5 \times 10^{-27}\,\frac{\text{s}^2}{\text{m}^3}$ or $3 \times 10^{-34}\,\frac{\text{years}^2}{\text{m}^3}$.

Kepler's third law allows us to estimate the mass of a planet in some distant solar system, based on the mass of its sun and the time it takes for the planet to make one revolution.

Use this space for summary and/or additional notes:

Universal Gravitation

Unit: Gravitation

MA Curriculum Frameworks (2016): HS-PS2-4

AP® Physics 1 Learning Objectives: 2.B.2.1, 2.B.2.2, 3.A.3.3, 3.C.1.1, 3.C.1.2, 3.G.1.1

Mastery Objective(s): (Students will be able to…)

- Set up and solve problems involving Newton's Law of Universal Gravitation.
- Assess the effect on the force of gravity of changing one of the parameters in Newton's Law of Universal Gravitation.

Success Criteria:

- All variables are identified and substituted correctly.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how changing each of the parameters in Newton's Law of Universal Gravitation affects the result.

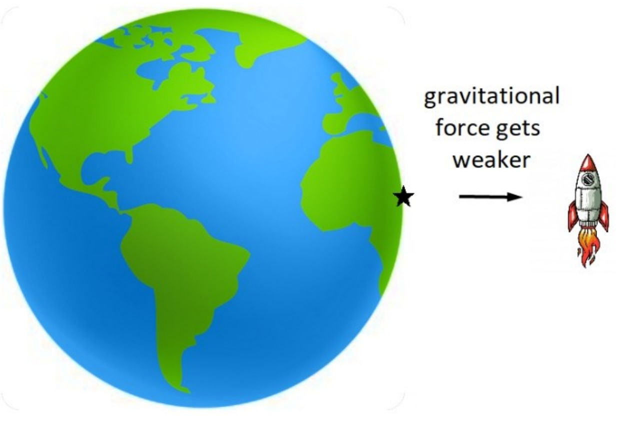
Tier 2 Vocabulary: gravity

Notes:

Gravity is a force of attraction between two objects because of their mass. The cause of this attraction is not currently known, though the most popular theory is that it is a force mediated by an elementary particle called a graviton.

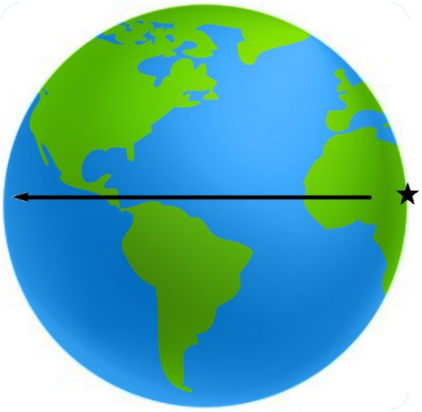
An object with more mass causes a stronger gravitational force, which means "the more mass you have, the more attractive you are."

However, the force gets weaker as the object gets farther away.



Use this space for summary and/or additional notes:

If we are on the Earth, our distance from the part of the Earth that we are standing on is zero, but our distance from the opposite side of the Earth would be the diameter of the Earth, which is about 8 000 miles.



This means that we need to measure distance from the *center of mass of the Earth*, which is approximately the center of the Earth. If we are on the surface of the Earth, this distance would be the radius of the Earth, which is 6.37×10^{6} m (a little less than 4 000 miles).

If we represent the gravitational pull of the Earth as a fraction, it would be proportional to:

$$\frac{\text{mass of Earth}}{\text{distance from center of Earth}}$$

If we write this as an equation, using the mathematical symbol $\propto$, which "is proportional to", it would look like:

$$F_g \propto \frac{m}{r}$$

However, all objects with mass have gravity. If we have two objects, such as the Earth and the sun, they pull on each other. This means that the total gravitational pull between the Earth and the sun would be:

$$\left(\frac{\text{mass of sun}}{\text{distance from the sun}}\right)\cdot\left(\frac{\text{mass of Earth}}{\text{distance from the Earth}}\right)$$

If we call the sun #1 and the Earth #2, this would give us:

$$F_g \propto \frac{m_1}{r_1}\cdot\frac{m_2}{r_2}$$

However, because the distance from the sun to the Earth is the same as the distance from the Earth to the sun, $r_1 = r_2 = r$, which means:

$$F_g \propto \frac{m_1\, m_2}{r_1\, r_2} = \frac{m_1\, m_2}{r\cdot r} = \frac{m_1\, m_2}{r^2}$$

Use this space for summary and/or additional notes:

However, we want an equation, not a proportion.

If we multiply the right side of the equation, using the masses in kilograms and the distance in meters, we would get a much larger number than the actual force (in newtons). This means we have to include the conversion factor, which is called the "universal gravitational constant". This constant turns out to be $6.67\times10^{-11}\,\frac{\text{N·m}^2}{\text{kg}^2}$.

(The units are because they cancel the m^2 and kg^2 from the formula and give newtons, which is the desired unit.) The symbol used for this constant is *G*. Thus our formula becomes:

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11})m_1m_2}{r^2}$$

This relationship is the universal gravitation equation, which we saw in the section on the *Gravitational Force*, starting on page 263. Sir Isaac Newton first published this equation in *Philosophiæ Naturalis Principia Mathematica* in 1687.

Discovery of Neptune

In the 1820s, irregularities were discovered in the orbit of Uranus. In 1845, the French mathematician and astronomer Urbain Le Verrier theorized that the gravitational force from another undiscovered planet must be causing Uranus' unusual behavior. Based on calculations using Kepler's and Newton's laws, Le Verrier predicted the existence and location of this new planet and sent his calculations to astronomer Johann Galle at the Berlin Observatory. Based on Le Verrier's work, Galle found the new planet on the night that he received Le Verrier's letter—September 23–24, 1846—within one hour of starting to look, and within 1° of its predicted position. Le Verrier's feat—predicting the existence and location of Neptune using only mathematics, was one of the most remarkable scientific achievements of the 19th century and a dramatic validation of celestial mechanics.

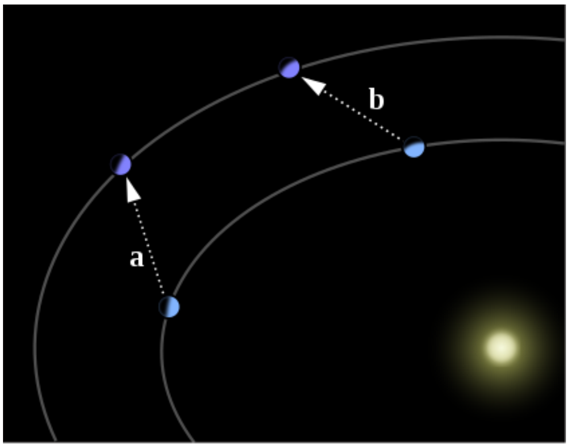


Diagram by R.J. Hall. Used with permission.

This diagram shows the orbits of Uranus (inner arc) and Neptune (outer arc). The planets are both orbiting from the top right to the bottom left.

At position *b*, the gravitational force from Neptune pulls Uranus ahead of its predicted location. At position *a*, the gravitational force pulls back on Uranus, leaving it behind its predicted location.

Use this space for summary and/or additional notes:

Relationship between *G* and *g*

As we saw in the section on the *Gravitational Force, t*he strength of the gravitational field anyplace in the universe can be calculated from the universal gravitation equation.

If m_1 is the mass of the planet (moon, star, *etc.*) that we happen to be standing on and m_2 is the object that is being attracted by it, we can divide the universal gravitation equation by m_2, which gives us:

$$\frac{F_g}{m_2} = \frac{Gm_1 \cancel{m_2}}{r^2 \cancel{m_2}} = \frac{Gm_1}{r^2}$$

as we saw previously.

Therefore, $g = \frac{Gm_1}{r^2}$ where m_1 is the mass of the planet in question and r is its radius.

If we wanted to calculate the value of g on Earth, m_1 would be the mass of the Earth $(5.97\times10^{24}\text{ kg})$ and r would be the radius of the Earth $(6.38\times10^{6}\text{ m})$. Substituting these numbers into the equation gives:

$$g = \frac{Gm_1}{r^2} = \frac{(6.67\times10^{-11})(5.97\times10^{24})}{(6.38\times10^{6})^2} = 9.81\,\tfrac{\text{N}}{\text{kg}}\ ^{*}$$

* In most places in this book, we round g to $10\,\tfrac{\text{N}}{\text{kg}}$ to simplify the math. However, if we are using 3 significant figures for the terms in this equation, we can express g to 3 significant figures as well. However, note that the value of g varies because the distance to the center (of mass) of the Earth varies. The Earth is a heterogeneous mixture, not a single solid object; the inertia of the particles as the Earth spins causes its equator to bulge ("equatorial bulge"), which takes mass from the poles ("polar flattening"). For example, the value of g in Boston, Massachusetts is approximately $9.80\,\tfrac{\text{N}}{\text{kg}}$.

Use this space for summary and/or additional notes:

Sample Problems:

Q: Find the force of gravitational attraction between the Earth and a person with a mass of 75 kg. The mass of the Earth is 5.97×10^{24} kg, and its radius is 6.37×10^{6} m.

A: $$F_g = \frac{Gm_1m_2}{r^2}$$
$$F_g = \frac{(6.67\times10^{-11})(5.97\times10^{24})(75)}{(6.38\times10^{6})^2}$$
$$F_g = 736\,\text{N}$$

This is the same number that we would get using $F_g = mg$, with $g = 9.81\,\frac{\text{N}}{\text{kg}}$.

If we use the approximation $g = 10\,\frac{\text{N}}{\text{kg}}$ (which is about 2 % higher), we get $F_g = 750\,\text{N}$.

Q: Find the acceleration due to gravity on the moon.

A: $$g_{moon} = \frac{Gm_{moon}}{r^2_{moon}}$$
$$g_{moon} = \frac{(6.67\times10^{-11})(7.35\times10^{22})}{(1.74\times10^{6})^2} = 1.62\,\frac{\text{N}}{\text{kg}} \equiv 1.62\,\frac{\text{m}}{\text{s}^2}$$

Q: If the distance between an object and the center of mass of a planet is tripled, what happens to the force of gravity between the planet and the object?

A: There are two ways to solve this problem.

Starting with $F_g = \frac{Gm_1m_2}{r^2}$, if we replace r with $3r$, we would get:

$$F'_g = \frac{Gm_1m_2}{(3r)^2} = \frac{Gm_1m_2}{9r^2} = \frac{1}{9}\cdot\frac{Gm_1m_2}{r^2}$$

A useful shortcut for these kinds of problems is to set them up as "before and after" problems, using the number 1 for every quantity on the "before" side, and replacing the ones that change with their new values on the "after" side. This shortcut is often called "the rule of 1s":

Before	**After**
$F_g = \frac{1\cdot1\cdot1}{1^2} = 1$	$F'_g = \frac{1\cdot1\cdot1}{3^2} = \frac{1}{9}$

Thus F'_g is $\frac{1}{9}$ of the original F_g.

Use this space for summary and/or additional notes:

Homework Problems

You will need to use data from *Table T. Planetary Data* and *Table U. Sun & Moon Data* on page 691 of your Physics Reference Tables.

1. **(M)** Find the force of gravity between the earth and the sun.

   Answer: 3.52×10^{22} N

2. **(M)** Find the acceleration due to gravity (the value of *g*) on the planet Mars.

   Answer: $3.70\,\frac{\text{m}}{\text{s}^2}$

3. **(S)** A mystery planet in another part of the galaxy has an acceleration due to gravity of $5.0\,\frac{\text{m}}{\text{s}^2}$. If the radius of this planet is 2.0×10^6 m, what is its mass?

   Answer: 3.0×10^{23} kg

Use this space for summary and/or additional notes:

4. A person has a mass of 80. kg.

   a. **(S)** What is the weight of this person on the surface of the Earth?

      You may use $\vec{F}_g = m\vec{g}$ *for this problem, but use* $\vec{g} = 9.81\,\frac{\text{N}}{\text{kg}}$ *instead of* $\vec{g} = 10\,\frac{\text{N}}{\text{kg}}$ *so you will get the same answer as you would get with the universal gravitation equation.)*

      Answer: 785 N

   b. **(M – honors & AP®; S – CP1)** What is the weight of the same person when orbiting the Earth at a height of 4.0×10^{6} m above its surface?

      (*Hint: Remember that Earth's gravity is calculated from the center of mass of the Earth. Therefore, the "radius" in this problem is the distance from the center of the Earth to the spaceship, which includes both the radius of the Earth and the distance from the Earth's surface to the spaceship. It may be helpful to draw a sketch.*)

      Answer: 296N

Use this space for summary and/or additional notes:

Introduction: Energy, Work & Power

Unit: Energy, Work & Power

Topics covered in this chapter:

This chapter deals with the ability of a moving object (or potential for an object to move) to affect other objects.

- *Energy* describes different types of energy, particularly potential and kinetic energy.
- *Work* describes changes in energy through the application of a force over a distance.
- *Conservation of Energy* explains and gives examples of the principle that "energy cannot be created or destroyed, only changed in form".
- *Rotational Work* and *Rotational Kinetic Energy* describe how these principles apply in rotating systems.
- *Escape Velocity* describes the application of the conservation of energy energy to calculate the velocity need to launch an object into orbit.
- *Power* describes the rate at which energy is applied

New challenges in this chapter involve keeping track of the same quantity applied to the same object, but at different times.

Use this space for summary and/or additional notes:

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS3-1. Use algebraic expressions and the principle of energy conservation to calculate the change in energy of one component of a system when the change in energy of the other component(s) of the system, as well as the total energy of the system including any energy entering or leaving the system, is known. Identify any transformations from one form of energy to another, including thermal, kinetic, gravitational, magnetic, or electrical energy, in the system.

HS-PS3-2. Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

HS-PS3-3. Design, build, and refine a device that works within given constraints to convert one form of energy into another form of energy.

AP®

AP® Physics 1 Learning Objectives:

3.E.1.1: The student is able to make predictions about the changes in kinetic energy of an object based on considerations of the direction of the net force on the object as the object moves. **[SP 6.4, 7.2]**

3.E.1.2: The student is able to use net force and velocity vectors to determine qualitatively whether kinetic energy of an object would increase, decrease, or remain unchanged. **[SP 1.4]**

3.E.1.3: The student is able to use force and velocity vectors to determine qualitatively or quantitatively the net force exerted on an object and qualitatively whether kinetic energy of that object would increase, decrease, or remain unchanged. **[SP 1.4, 2.2]**

3.E.1.4: The student is able to apply mathematical routines to determine the change in kinetic energy of an object given the forces on the object and the displacement of the object. **[SP 2.2]**

4.C.1.1: The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy. **[SP 1.4, 2.1, 2.2]**

4.C.1.2: The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system. **[SP 6.4]**

4.C.2.1: The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass. **[SP 6.4]**

Use this space for summary and/or additional notes:

AP®

4.C.2.2: The student is able to apply the concepts of Conservation of Energy and the Work-Energy theorem to determine qualitatively and/or quantitatively that work done on a two-object system in linear motion will change the kinetic energy of the center of mass of the system, the potential energy of the systems, and/or the internal energy of the system. **[SP 1.4, 2.2, 7.2]**

4.D.1.1: The student is able to describe a representation and use it to analyze a situation in which several forces exerted on a rotating system of rigidly connected objects change the angular velocity and angular momentum of the system. **[SP 1.2, 1.4]**

4.D.1.2: The student is able to plan data collection strategies designed to establish that torque, angular velocity, angular acceleration, ~~and angular momentum~~ can be predicted accurately when the variables are treated as being clockwise or counterclockwise with respect to a well-defined axis of rotation, and refine the research question based on the examination of data. **[SP 3.2, 4.1, 4.2, 5.1, 5.3]**

5.A.2.1: The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, ~~and linear momentum~~ to those situations. **[SP 6.4, 7.2]**

5.B.1.1: The student is able to set up a representation or model showing that a single object can only have kinetic energy and use information about that object to calculate its kinetic energy. **[SP 1.4, 2.2]**

5.B.1.2: The student is able to translate between a representation of a single object, which can only have kinetic energy, and a system that includes the object, which may have both kinetic and potential energies. **[SP 1.5]**

5.B.2.1: The student is able to calculate the expected behavior of a system using the object model (*i.e.,* by ignoring changes in internal structure) to analyze a situation. Then, when the model fails, the student can justify the use of conservation of energy principles to calculate the change in internal energy due to changes in internal structure because the object is actually a system. **[SP 1.4, 2.1]**

5.B.3.1: The student is able to describe and make qualitative and/or quantitative predictions about everyday examples of systems with internal potential energy. **[SP 2.2, 6.4, 7.2]**

5.B.3.2: The student is able to make quantitative calculations of the internal potential energy of a system from a description or diagram of that system. **[SP 1.4, 2.2]**

5.B.3.3: The student is able to apply mathematical reasoning to create a description of the internal potential energy of a system from a description or diagram of the objects and interactions in that system. **[SP 1.4, 2.2]**

5.B.4.1: The student is able to describe and make predictions about the internal energy of systems. **[SP 6.4, 7.2]**

Use this space for summary and/or additional notes:

AP®

5.B.4.2: The student is able to calculate changes in kinetic energy and potential energy of a system, using information from representations of that system. **[SP 1.4, 2.1, 2.2]**

5.B.5.1: The student is able to design an experiment and analyze data to examine how a force exerted on an object or system does work on the object or system as it moves through a distance. **[SP 4.2, 5.1]**

5.B.5.2: The student is able to design an experiment and analyze graphical data in which interpretations of the area under a force-distance curve are needed to determine the work done on or by the object or system. **[SP 4.2, 5.1]**

5.B.5.3: The student is able to predict and calculate from graphical data the energy transfer to or work done on an object or system from information about a force exerted on the object or system through a distance. **[SP 1.4, 2.2, 6.4]**

5.B.5.4: The student is able to make claims about the interaction between a system and its environment in which the environment exerts a force on the system, thus doing work on the system and changing the energy of the system (kinetic energy plus potential energy). **[SP 6.4, 7.2]**

5.B.5.5: The student is able to predict and calculate the energy transfer to (*i.e.,* the work done on) an object or system from information about a force exerted on the object or system through a distance. **[SP 2.2, 6.4]**

5.D.1.1: The student is able to make qualitative predictions about natural phenomena based on ~~conservation of linear momentum and~~ restoration of kinetic energy in elastic collisions. **[SP 6.4, 7.2]**

5.D.3.1: The student is able to predict the velocity of the center of mass of a system when there is no interaction outside of the system but there is an interaction within the system (*i.e.,* the student simply recognizes that interactions within a system do not affect the center of mass motion of the system and is able to determine that there is no external force). **[SP 6.4]**

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Energy**, such as potential and kinetic energy, work, power, and conservation laws.

1. Work
2. Energy
3. Forms of Energy
4. Power
5. Center of Mass

Skills learned & applied in this chapter:

- Conservation laws (before/after problems).

Use this space for summary and/or additional notes:

Energy

Unit: Energy, Work & Power

MA Curriculum Frameworks (2016): HS-PS3-1

AP® Physics 1 Learning Objectives: 3.E.1.1, 3.E.1.2, 3.E.1.3, 3.E.1.4, 4.C.1.1, 4.C.1.2, 4.C.2.2, 5.A.2.1, 5.B.1.1, 5.B.1.2, 5.B.2.1, 5.B.3.1, 5.B.3.2, 5.B.3.3, 5.B.4.1, 5.B.5.4, 5.B.5.5, 5.D.3.1

Mastery Objective(s): (Students will be able to…)

- Calculate the gravitational potential energy of an object.
- Calculate the kinetic energy of an object.

Success Criteria:

- Correct equation(s) are chosen for the situation.
- Variables are correctly identified and substituted correctly into equation(s).
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain when & why an object has potential energy.
- Explain when & why an object has kinetic energy.

Tier 2 Vocabulary: work, energy

Labs, Activities & Demonstrations:

- "Happy" and "sad" balls.
- Popper.

Notes:

energy: the ability to cause macroscopic objects or microscopic particles to increase their velocity; or their ability to increase their velocity due to the effects of a force field.

If we apply mechanical energy to a physical object, the object will either move faster (think of pushing a cart), heat up, or have the ability to suddenly move when we let go of it (think of stretching a rubber band).

Energy is a scalar quantity, meaning that it does not have a direction. Energy can be transferred from one object (or collection of objects) to another.

Energy is a "conserved" quantity in physics, which means it cannot be created or destroyed, only changed in form.*

Energy is measured in joules (J):

$$1\,\text{J} \equiv 1\,\text{N}\cdot\text{m} \equiv 1\,\tfrac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

* More properly, the combination of mass and energy is conserved. Einstein's equation states the equivalence between mass and energy: $E = mc^2$.

Use this space for summary and/or additional notes:

Kinetic Energy

Because energy is a conserved quantity, if energy is used to cause a macroscopic object to increase its velocity, that energy is then contained within the moving object. We call this energy "kinetic energy", and the amount of kinetic energy that an object has is related to its mass and velocity. An object has translational kinetic energy (the kinetic energy of an object or system that is moving in the *xy* plane or *xyz* space) if its center of mass is moving. Translational kinetic energy is given by the equation:*

$$K = \frac{1}{2}mv^2$$

Note that a single object can have kinetic energy. An entire system can also have kinetic energy if the center of mass of the system is moving (has nonzero mass and nonzero velocity).

The above equation is for translational kinetic energy only. Kinetic energy also exists in rotating systems; an object can have rotational kinetic energy whether or not its center of mass is moving. *Rotational Kinetic Energy* will de discussed in a later topic, starting on page 417.

Potential Energy

Potential energy is "stored" energy due to an object's position, properties, and/or forces acting on the object. Potential energy is energy that is available to be turned into some other form, such as kinetic energy, internal (thermal) energy, *etc.*

Potential Energy from Force Fields

Potential energy can be caused by the action of a force field. (Recall that a force field is a region in which an object experiences a force because of some property of that object.) Some fields that can cause an object to have potential energy include:

- gravitational field (or "gravity field"): a force field in which an object experiences a force because of (and proportional to) its mass. (See page 263 for more information.)
- electric field: a force field in which an object experiences a force because of (and proportional to) its electric charge.

* In these notes, K without a subscript is assumed to be translational kinetic energy. In problems involving both translational and rotational kinetic energy, translational kinetic energy will be denoted as K_t and rotational kinetic energy as K_r.

Use this space for summary and/or additional notes:

Potential Energy from Forces that are not Fields

Potential energy can also come from non-field-related sources. Some examples include:

- gravitational force: two objects that exert gravitational attraction on each other are separated. When the objects are released and allowed to come together, the potential energy due to the separation becomes kinetic energy of one or more of the objects.
- springs: an object that is attached to a spring that has been stretched or compressed has potential energy. When the spring is released and allowed to move, the potential energy stored in the spring becomes the kinetic energy of the object.
- chemical potential: when chemicals react spontaneously, energy is released, usually in the form of heat.
- electric potential: the energy that causes electrons to move through an object that has electrical resistance.

As you have probably noticed, gravitational forces are listed above, both under Fields and Not Fields. That is because gravitational potential energy can be viewed as an interaction between an object and the Earth's gravitational field or an interaction between two objects with mass. Either of these ways of looking at the interaction is correct and yields the same results.

Gravitational Potential Energy (GPE) for Objects Close to the Earth

We can think of gravitational potential energy (GPE) as the action of the gravitational force in a way that can increase an object's kinetic energy. Because acceleration due to gravity on Earth is approximately $\vec{\boldsymbol{g}} = 10\,\frac{\text{m}}{\text{s}^2}$ near the surface of the Earth, this means that the acceleration caused by the gravitational force is:

$$\vec{\boldsymbol{F}}_g = m\vec{\boldsymbol{a}} = m\vec{\boldsymbol{g}}$$

Because kinetic energy is $K = \frac{1}{2}mv^2$ and $v^2 - v_o^2 = 2ad$, if an object starts from rest $(v_o = 0)$, and is only accelerated by the gravitational force, then:

$$v^2 = 2ad \qquad \text{and} \qquad \Delta K = \tfrac{1}{2}m(2ad) = mad$$

Remember that $\vec{\boldsymbol{a}} = \vec{\boldsymbol{g}}$. If the distance is vertical, we usually call it height, and we use the variable *h* instead of *d*, which means $\Delta K = mgh$. Therefore, the GPE (U_g) is the amount of kinetic energy that could be added by the object falling from a height:

$$U_g = mgh$$

Use this space for summary and/or additional notes:

Deriving Potential Energy from the Gravitational Force as a Force Field

A discussed earlier, we think of the gravitational force as the force caused by a gravitational force field—a region (near a massive object like the Earth) in which a gravitational force acts on all objects that have mass (*m*). If the strength of the gravitational force is $\vec{g}$ (approximately $10\,\frac{\text{N}}{\text{kg}}$ near the surface of the Earth), then the force is $\vec{F}_g = m\vec{g}$. Using the reasoning above, this gives the same equation, $U_g = mgh$.

GPE between Objects that are Far Apart Compared with Their Size

Considering the gravitational force as a force field of constant strength does not work when the objects are very far apart. In that case, we need to consider GPE as the result of a gravitational force between two objects.

As we saw in the section on *Universal Gravitation*, starting on page 377, when two objects with mass (m_1 and m_2) objects are separated, the gravitational force between them is given by:

$$F_g = \frac{Gm_1m_2}{r^2}$$

If *r* is the distance between the objects' centers of mass, then we can apply the same reasoning as above, but using *r* instead of *h* as the distance between the objects. Thus, $U_g = mgh$ becomes $U_g = mgr$, which means:

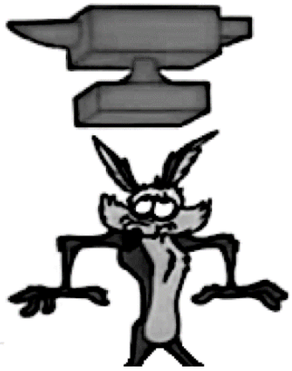
$$U_g = F_g h = \frac{Gm_1m_2}{r^2}\cdot r = \frac{Gm_1m_2}{r}$$

Use this space for summary and/or additional notes:

Systems and Potential Energy

Recall that a _system_ is a collection of objects for the purpose of describing the interaction of objects within *vs.* outside of that collection. The surroundings is all of the objects outside of the system ("everything else"). (Systems are explained in more detail on page 254.)

Potential energy is an energy relationship between two objects within a system. A single, isolated object cannot have potential energy.



For example, in the coyote-anvil system pictured to the right, both Wile E. Coyote and the anvil have negligible potential energy. (There is a tiny amount of gravitational attraction between them—assuming the anvil has a mass of 200 kg and the coyote has a mass of 20 kg, the gravitational attraction between them would be 3×10^{-7} N.) However, the Earth can attract the entire coyote-anvil system toward itself.

In the coyote-anvil-Earth system, the anvil and the coyote each have GPE with respect to the Earth. As the coyote and anvil both fall toward the Earth, that GPE changes to kinetic energy for both objects, causing both the coyote and the anvil to fall faster and faster…

Remember that potential energy requires:

- Two objects that exert some sort of attractive or repulsive force on each other. (In the case of GPE, this is the gravitational force, which is attractive.)
- A distance between the two objects over which at least one of the objects can move. (In the case of gravitational potential energy, this is the height above the ground.)

This means that a single, isolated object cannot have potential energy.

Mechanical Energy

Mechanical energy is gravitational potential energy (GPE) plus kinetic energy. Because GPE and kinetic energy are easily interconverted, it is convenient to have a term that represents the combination of the two. There is no single variable for mechanical energy; in this text, we will sometimes use the abbreviation *TME*:

$$TME = U_g + K$$

Use this space for summary and/or additional notes:

Internal (Thermal) Energy

Kinetic energy is both a macroscopic property of a large object (*i.e.,* something that is at least large enough to see), and a microscopic property of the individual particles (atoms or molecules) that make up an object. Internal (thermal) energy is the aggregate microscopic energy that an object (often an enclosed sample of a gas) has due to the combined kinetic energies of its individual particles. (Heat is thermal energy added to or removed from a system.)

As we will see when we study thermal physics, temperature is the average of the microscopic kinetic energies of the individual particles that an object is made of. Kinetic energy can be converted to internal energy if the kinetic energy of a macroscopic object is turned into the individual kinetic energies of the particles of that object and/or some other object. Processes that can convert kinetic energy to internal energy include friction and collisions.

Chemical Potential Energy

In chemistry, chemical potential energy comes from the forces between particles (atoms or molecules), largely the electromagnetic forces attracting the atoms in a chemical bond. The energy absorbed or given off in a chemical reaction is the difference between the energies contained in the molecules before *vs.* after the reaction. If energy is given off by a reaction, it is absorbed by the particles, increasing their kinetic energy, which means the temperature increases. If energy is absorbed by a reaction, that energy must come from the kinetic energy of the particles, which means the temperature decreases.

Electric Potential

Electric potential is the energy that causes electrically charged particles to move through an electric circuit. The energy for this ultimately comes from some other source, such as chemical potential energy (*i.e.,* a battery), mechanical energy (*i.e.,* a generator), *etc.*

Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** Calculate the kinetic energy of a car with a mass of 1200 kg moving at a velocity of $15\,\frac{\text{m}}{\text{s}}$.

   Answer: 135 000 J

2. **(M)** Calculate the gravitational potential energy of a person with a mass of 60. kg at the top of a 10. m flight of stairs.

   Answer: 6 000 J

3. **(M)** Calculate the gravitational potential energy between the Earth and the moon. (You will need to use information from *Table T. Planetary Data* and *Table U. Sun & Moon Data* on page 691 of your *Appendix: Physics* Reference Tables.)

   Answer: 7.62×10^{28} J

Use this space for summary and/or additional notes:

Work

Unit: Energy, Work & Power

MA Curriculum Frameworks (2016): HS-PS3-1

MA Curriculum Frameworks (2006): 2.3

Mastery Objective(s): (Students will be able to…)

- Calculate the work done when a force displaces an object .

Success Criteria:

- Variables are correctly identified and substituted correctly into equation(s).
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why a longer lever arm is more effective.

Tier 2 Vocabulary: work, energy

Notes:

In this course, there are two ways that we will study of transferring energy into or out of a system:

work (W): mechanical *energy transferred* into or out of a system *by a net force acting over a distance*.

heat (Q): thermal energy transferred into or out of a system. Heat is covered in more detail in the *Introduction: Thermal Physics* (Heat) unit that starts on page 613, and in much more detail in Physics 2.

If you lift a heavy object off the ground, you are giving the object gravitational potential energy (in the object-Earth system). The Earth's gravitational field can now cause the object to fall, turning the potential energy into kinetic energy. Therefore, we would say that you are doing work against the force of gravity.

Work is the amount of energy that was added to the object ($W = \Delta E$)*. (In this case, because the work was turned into *potential* energy, we would say that $W = \Delta U$.)

* Many texts start with work as the application of force over a distance, and then discuss energy. Those texts then derive the work-energy theorem, which states that the two quantities are equivalent. In these notes, we instead started with energy, and then defined work as the change in energy. This presentation makes the concept of work more intuitive, especially when studying other energy-related topics such as thermodynamics.

Use this space for summary and/or additional notes:

Mathematically, work is also the effect of a force applied over a distance (the dot product of the force vector and the displacement vector). Therefore:

$$\Delta E = W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}}$$

Recall that the dot product is one of three ways of multiplying vectors. The dot product is a scalar quantity (a number, including its units, but without a direction), and is equal to the product of the magnitudes of the force and distance, and the cosine of the angle between them. This means:

$$W = Fd\cos\theta = F_{\parallel}d$$

Where F is the magnitude of the force vector $\vec{\boldsymbol{F}}$, d is the magnitude of the displacement vector $\vec{\boldsymbol{d}}$, and θ is the angle between the two vectors. Sometimes $F\cos\theta$ is written as $F_{\parallel}$, which means "the component of the force that is parallel to the direction of motion."

Note that when the force and the displacement are in the same direction, the angle $\theta = 0°$ which means $\cos\theta = \cos(0°) = 1$. In this case, $F_{||} = F\cos\theta = (F)(1) = F$ and the equation reduces to $W = Fd$.

Work is measured in newton-meters (N·m) or joules (J).

$$1\,\text{N}\cdot\text{m} \equiv 1\,\text{J} \equiv 1\,\tfrac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

Positive *vs.* Negative Work

Recall that in physics, we use positive and negative numbers to indicate direction. So far, we have used positive and negative numbers for one-dimensional vector quantities (*e.g.,* velocity, acceleration, force) to indicate the direction of the vector. We can also use positive and negative numbers to indicate the direction for energy (and other scalar quantities), to indicate whether the energy is being transferred into or out of a system.

- If the *energy* of an object or system *increases* because of work (energy is transferred ***into*** the object or system), then the *work* is *positive* with respect to that object or system.
- If the *energy* of an object or system *decreases* because of work (energy is transferred ***out of*** the object or system), then the *work* is *negative* with respect to that object or system.

However, we often discuss work using the prepositions ***on*** (into) and ***by*** (out of).

- If energy is transferred ***into*** an object or system, then we can say that work was done ***on*** (into) the object or system, or that work was done ***by*** (out of) the surroundings.
- If energy was transferred ***out of*** an object or system, we can say that work was done ***by*** (out of) the object, or we can say that work was done ***on*** (into) the surroundings.

Use this space for summary and/or additional notes:

Example:

A truck pushes a 1000 kg car up a 50 m hill. The car gained $U_g = mgh = (1000)(10)(50) = 500\,000\text{ J}$ of potential energy. We could say that:

- 500 000 J of work was done ***on*** the car (by the truck).
- 500 000 J of work was done ***by*** the truck (on the car).
- −500 000 J of work was done ***on*** the truck (by the car).

A simple way to tell if a force does positive or negative work on an object is to use the equation $W = \vec{F} \bullet \vec{d}$. If the force and the displacement are in the ***same*** direction, then the work done by the force is ***positive***. If the force and displacement are in ***opposite*** directions, then the work done by the force is ***negative***.

Example:

Suppose a force of 750 N is used to push a cart against 250 N of friction for a distance of 20 m. The work done ***by*** the force is $W = F_{||}d = (750)(20) = 15\,000\text{ J}$. The work done ***by*** friction is $W = F_{||}d = (-250)(20) = -5\,000\text{ J}$ (negative because friction is in the negative direction). The total (net) work done on the cart is $15\,000 + (-5\,000) = 10\,000\text{ J}$.

We could also figure out the net work done on the cart directly by using the net force: $W_{net} = F_{net,||}d = (750 - 250)(20) = (500)(20) = 10\,000\text{ J}$

Notes:

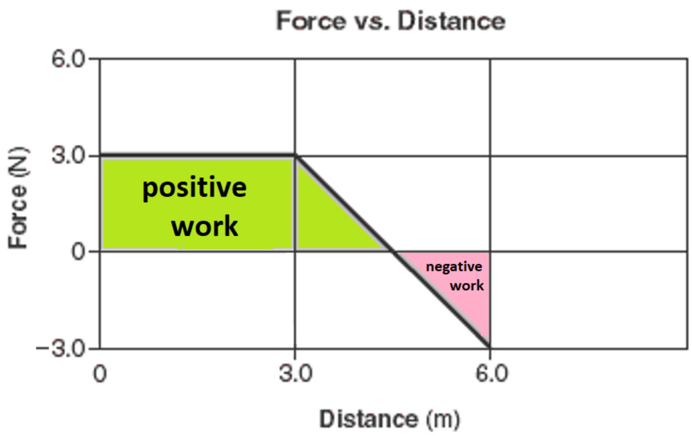
- If the displacement is zero, no work is done by the force. *E.g.,* if you hold a heavy box without moving it, you are exerting a force (counteracting the force of gravity) but you are not doing work.
- If the net force is zero, no work is done by the displacement (change in location) of the object. *E.g.,* if a cart is sliding across a frictionless air track at a constant velocity, the net force on the cart is zero, which means no work is being done.
- If the displacement is perpendicular to the direction of the applied force (θ = 90°, which means cos θ = 0), no work is done by the force. *E.g.,* you can slide a very heavy object along a roller conveyor, because the force of gravity is acting vertically and the object's displacement is horizontal, which means gravity and the normal force cancel, and you therefore do not have to do any work against gravity.

Use this space for summary and/or additional notes:

Force *vs.* Distance Graphs

Recall that on a graph, the area "under the graph" (between the graph and the *x*-axis)[*] represents what you get when you multiply the quantities on the *x* and *y*-axes by each other.

Because $W = F_{||}d$, if we plot force *vs.* distance, the area "under the graph" is therefore the work:



In the above example, $(3\,\text{N})(3\,\text{m}) = 9\,\text{N}\cdot\text{m} = 9\,\text{J}$ of work was done on the object in the interval from 0–3 s, 2.25 J of work was done on the object in the interval from 3–4.5 s, and −2.25 J of work was done on the object in the interval from 4.5–6 s. (Note that the work from 4.5–6 s is negative, because the force was applied in the negative direction during that interval.) The total work is therefore $9 + 2.25 + (-2.25) = +9\,\text{J}$.

[*] In most physics and calculus textbooks, the term "area under the graph" is used. This term ***always*** means the area ***between the graph and the x-axis***.

Use this space for summary and/or additional notes:

Sample Problems:

Q: How much work does it take to lift a 60. kg box 1.5 m off the ground at a constant velocity over a period of 3.0 s?

A: The box is being lifted, which means the work is done against the force of gravity.

$$W = F_{||} \cdot d = F_g d$$
$$W = F_g d = [mg]d = [(60)(10)](1.5) = [600](1.5) = 900\text{ J}$$

Note that the amount of time it took to lift the box has nothing to do with the amount of work done.

It may be tempting to try to use the time to calculate velocity and acceleration in order to calculate the force. However, because the box is lifted at a constant velocity, the only force needed to lift the box is enough to overcome the weight of the box (F_g).

In general, if work is done to move an object vertically, the work is done against gravity, and you need to use $a = g = 10\,\frac{\text{m}}{\text{s}^2}$ for the acceleration when you calculate $F = ma$.

Similarly, if work is done to move an object horizontally, the work is *not* against gravity and either you need to know the force applied or you need to find it from the acceleration of the object using $F = ma$.

Q: In the picture to the right, the adult is pulling on the handle of the wagon with a force of 150. N at an angle of 60.0°.

If the adult pulls the wagon a distance of 500. m, how much work does he do?

A: $W = F_{||} d$

$$W = [F\cos\theta]d = [(150.)\cos 60.0°](500.) = [(150.)(0.500)](500.) = 37500\text{ J}$$

Use this space for summary and/or additional notes:

Homework Problems

1. **(S)** How much work is done against gravity by a weightlifter lifting a 30. kg barbell 1.5 m upwards at a constant speed?

   Answer: 450 J

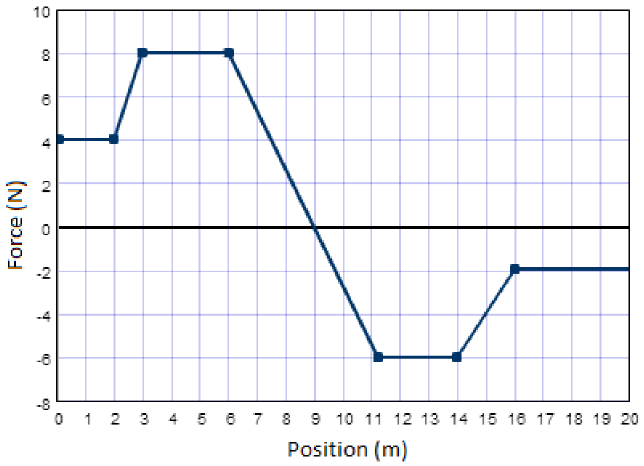
2. **(M)** A 3000. kg car is moving across level ground at $5.0\,\frac{\text{m}}{\text{s}}$ when it begins an acceleration that ends with the car moving at $15.0\,\frac{\text{m}}{\text{s}}$. Is work done in this situation? How do you know?

3. **(S)** A 60. kg man climbs a 3.0 m tall flight of stairs. How much work was done by the man against the force of gravity?

   Answer: 1 800 J

Use this space for summary and/or additional notes:

4. **(M)** The following graph shows the force on a 2.0 kg object *vs.* its position on a level surface.



The object has a velocity of $+4.0\,\frac{\text{m}}{\text{s}}$ at time $t = 0$.

a. **(M)** What is the kinetic energy of the object at time $t = 0$?

Answer: 16 J

b. **(M)** How much work was done on the object by the force during the interval from 0–2 m? What are the kinetic energy and velocity of the object at position $x = 2$ m?

Answer: $W = 8$ J; $K = 24$ J; $\vec{\boldsymbol{v}} = +4.9\,\frac{\text{m}}{\text{s}}$

c. **(M)** How much work was done on the object by the force during the interval from 0–9 m? What are the kinetic energy and velocity of the object at position $x = 9$ m?

Answer: $W = 50$ J; $K = 66$ J; $\vec{\boldsymbol{v}} = +8.1\,\frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

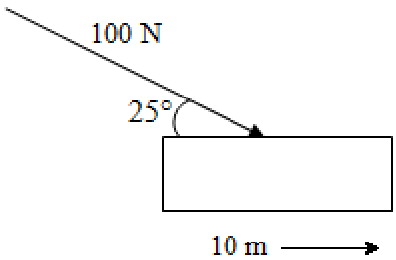
d. **(M)** How much work was done on the block during the interval from 9–20 m? What are the kinetic energy and velocity of the block at position x = 20 m?

Answer: W = −40 J; K = 26 J; $\vec{v} = +5.1\,\frac{\text{m}}{\text{s}}$

5. **(M)** A dog pulls a sled using a 500. N force across a 10. m wide street. The force of friction on the 90. kg sled is 200. N. How much work is done by the dog? How much work is done by friction? How much work is done on the sled? How much work is done by gravity?

honors & AP®

6. **(M – honors & AP®; A – CP1)** Find the work done when a 100. N force at an angle of 25° pushes a cart 10. m to the right, as shown in the diagram to the right.



Answer: 906 N·m

Use this space for summary and/or additional notes:

Conservation of Energy

Unit: Energy, Work & Power

MA Curriculum Frameworks (2016): HS-PS3-1

AP® Physics 1 Learning Objectives: 3.E.1.1, 3.E.1.2, 3.E.1.3, 3.E.1.4, 4.C.1.1, 4.C.1.2, 4.C.2.2, 5.A.2.1, 5.B.1.1, 5.B.1.2, 5.B.2.1, 5.B.3.1, 5.B.3.2, 5.B.3.3, 5.B.4.1, 5.B.5.4, 5.B.5.5, 5.D.3.1

Mastery Objective(s): (Students will be able to…)

- Solve problems that involve the conversion of energy from one form to another.

Success Criteria:

- Correct equations are chosen for the situation.
- Variables are correctly identified and substituted correctly into equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Describe the type(s) of energy that an object has in different situations.

Tier 2 Vocabulary: work, energy, potential

Labs, Activities & Demonstrations:

- Golf ball loop-the-loop.
- Marble raceways.
- Bowling ball pendulum.

Notes:

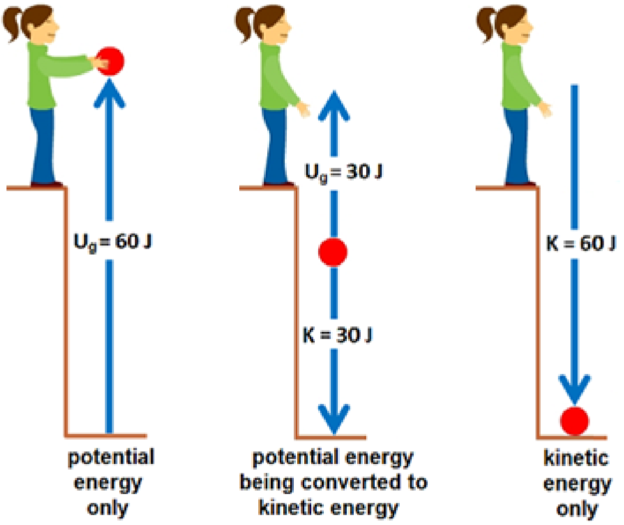
In a *closed system* (meaning a system in which there is no exchange of matter or energy between the system and the surroundings), the total energy is constant. Energy can be converted from one form to another. When this happens, the increase in any one form of energy is the result of a corresponding decrease in another form of energy.

In a system that has potential energy and kinetic energy, the total mechanical energy is given by:

$$TME = U + K$$

Use this space for summary and/or additional notes:

In the following diagram, suppose that a student drops a ball with a mass of 2 kg from a height of 3 m.



Before the student lets go of the ball, it has 60 J of potential energy. As the ball falls to the ground, potential energy is gradually converted to kinetic energy. The potential energy continuously decreases and the kinetic energy continuously increases, but the total energy is always 60 J. After the ball hits the ground, 60 J of work was done by gravity, and the 60 J of kinetic energy is converted to other forms. For example, if the ball bounces back up, some of the kinetic energy is converted back to potential energy. If the ball does not reach its original height, that means the rest of the energy was converted into other forms, such as thermal energy (the temperatures of the ball and the ground increase infinitesimally), sound, *etc.*

Use this space for summary and/or additional notes:

Work-Energy Theorem

We have already seen that work is the action of a force applied over a distance. A broader and more useful definition is that work is the change in the energy of an object or system. If we think of a system as having imaginary boundaries, then work is the flow of energy across those boundaries, either into or out of the system.

For a system that has only mechanical energy, work changes the amount of potential and/or kinetic energy in the system.

$$W = \Delta K + \Delta U$$

As mentioned earlier, although work is a scalar quantity, we ***use generally use a positive number for work coming into the system*** ("work is done on the system"), and ***a negative number for work going out of the system*** ("work is done by the system on the surroundings").

The units for work are sometimes shown as newton-meters (N·m). Because work is equivalent to energy, the units for work and energy—newton-meters and joules—are equivalent.

$$1\,\text{J} \equiv 1\,\text{N}\cdot\text{m} \equiv 1\,\tfrac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

Work-energy theorem problems will give you information related to the gravitational potential and/or kinetic energy of an object (such as its mass and a change in velocity) and ask you how much work was done.

A simple rule of thumb (meaning that it's helpful, though not always strictly true) is:

- Potential energy is energy in the *future* (energy that is available for use).
- Kinetic energy is energy in the *present* (the energy of an object that is currently in motion).
- Work is the result of energy in the *past* (energy that has already been added to or taken from an object).

Use this space for summary and/or additional notes:

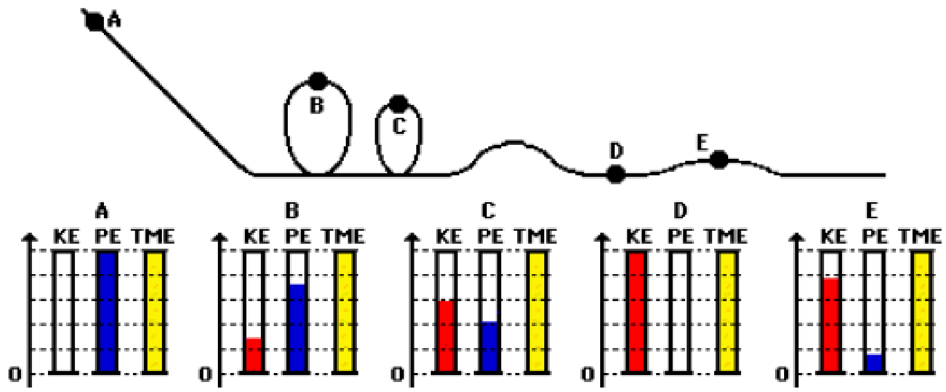
Conservation of Energy

In physics, if a quantity is "conserved", that means when some change happens to a system, there is the same amount of that quantity after the change as there was before.

Energy Bar Charts

A useful way to represent conservation of energy is through bar graphs that represent kinetic energy (*K* or "KE"), gravitational potential energy (U_g or "PE"), and total mechanical energy (TME). (We use the term "chart" rather than "graph" because the scale is usually arbitrary and the chart is not meant to be used quantitatively.)

The following is an energy bar chart for a roller coaster, starting from point A and traveling through points B, C, D, and E.



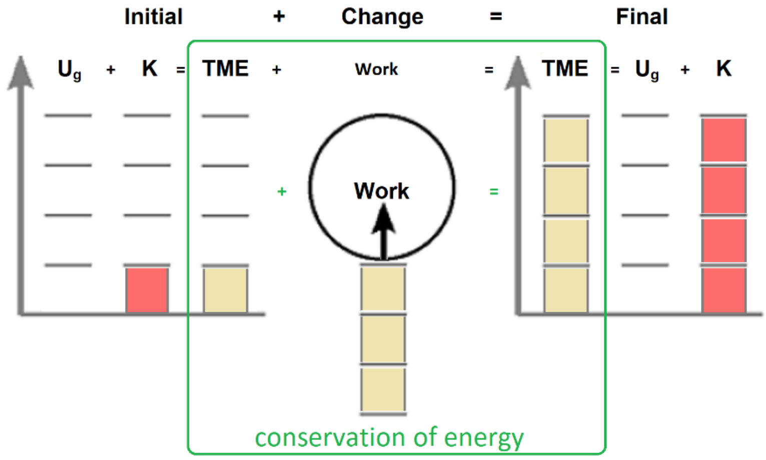
Notice, in this example, that:

1. The total mechanical energy always remains the same. (This the case in conservation of energy problems if there is no work added to or removed from the system.)
2. KE is zero at point **A** because the roller coaster is not moving. All of the energy is PE, so PE = TME.
3. PE is zero at point **D** because the roller coaster is at its lowest point. All of the energy is KE, so KE = TME.
4. At all points (including points **A** and **D**), KE + PE = TME

It can be helpful to sketch energy bar charts representing the different points in complicated conservation of energy problems. If energy is being added to or removed from the system, add an Energy Flow diagram to show energy that is being added to or removed from the system.

Use this space for summary and/or additional notes:

If we draw initial ("before") and final ("after") energy bar charts, and we represent the system in the center as a circle with work available to go in or out ("change"), the diagram would look like the following:



Charts like this are called "LOL charts" or "LOL diagrams," because the axes on the left and right side resemble the letter "L", and the circle for the system resembles the letter "O".

Solving Conservation of Energy Problems

Conservation of energy problems involve recognizing that energy is changing from one form to another. Once you have figured out what is being converted, calculate the amount energy that is converted, and use the equation for the new form to calculate the desired quantity.

In mechanics, conservation of energy problems usually involve work, gravitational potential energy, and kinetic energy:

$W = F \bullet d = Fd\cos\theta$ ($= Fd$ if force & displacement are in the same direction)

$U_g = mgh$

$K = \frac{1}{2}mv^2$

The general form of work-energy problems is:

Total Energy Before + *Work* = *Total Energy After*

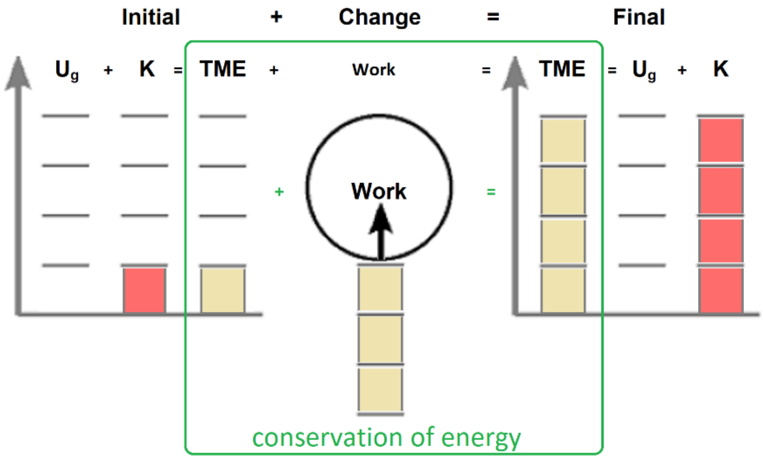
The strategy is to identify the types of energy before and after the change and write the above equation. Then replace each type of energy with its formula and solve.

Use this space for summary and/or additional notes:

Sample Problems:

Q: An 875 kg car accelerates from $22\,\frac{m}{s}$ to $44\,\frac{m}{s}$ on level ground.

a. Draw an LOL chart representing the initial and final energies and the flow of energy into or out of the system.



Notice that:

- *The height doesn't change, which means the gravitational potential energy is zero, both before and after, and the only type of energy the car has in this problem is kinetic.*
- *The car is moving, both before and after, so it has kinetic energy. The car is moving faster at the end, so it has more kinetic energy at the end than at the beginning, and therefore more total mechanical energy at the end than at the beginning.*
- *Because the total energy at the end was more than at the beginning, work must have gone into the system.*

b. What were the initial and final kinetic energies of the car? How much work did the engine do to accelerate it?

$$\textbf{Before} + \textbf{Work} = \textbf{After}$$
$$TME_{before} + W = TME_{after}$$
$$K_i + W = K_f$$
$$\tfrac{1}{2}mv_i^2 + W = \tfrac{1}{2}mv_f^2$$
$$\tfrac{1}{2}(875)(22)^2 + W = \tfrac{1}{2}(875)(44)^2$$
$$211750 + W = 847000$$
$$W = 847000 - 211750 = 635250\text{ J}$$

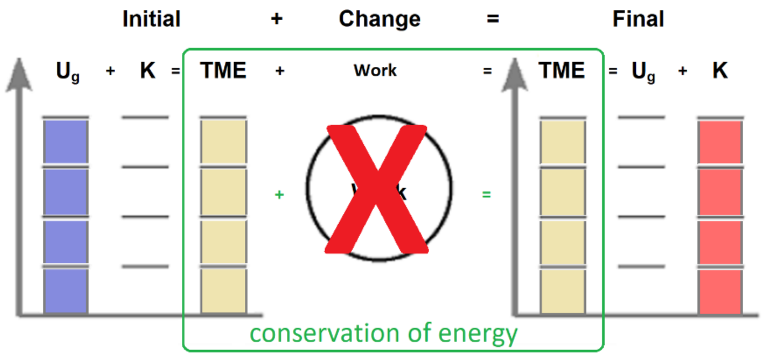
Answers: K_i = 211 750 J; K_f = 847 000 J; W = 635 250 J

Use this space for summary and/or additional notes:

Q: An 80 kg physics student falls off the roof of a 15 m high school building. How much kinetic energy does he have when he hits the ground? What is his final velocity?

A: There are two approaches to answer this question.

1. Recognize that the student's potential energy at the top of the building is entirely converted to kinetic energy when he hits the ground.



Notice that:

- *No work is done on the student.** *Total mechanical energy therefore is the same at the beginning and end.*
- *Initially, the student has only gravitational potential energy. At the end, the student has no potential energy and all of his energy has been converted to kinetic.*

$$\textbf{Before} + \textbf{Work} = \textbf{After}$$

$$TME_{before} + W = TME_{after}$$

$$U_i + K_i + W = U_f + K_f$$

$$mgh_i + 0 + 0 = 0 + \tfrac{1}{2}mv_f^2$$

$$(80)(10)(15) = \tfrac{1}{2}(80)v_f^2$$

$$12\,000 = 40v_f^2 \qquad \frac{12\,000}{40} = 300 = v_f^2 \qquad v_f = \sqrt{300} = 17.3\,\tfrac{\text{m}}{\text{s}}$$

Answers: $K_f = 12\,000$ J; $v_f = 17.3\,\frac{\text{m}}{\text{s}}$

* Actually, we have two options. If we consider the Earth-student system, no outside energy is added or removed, which means there is no work, and gravitational potential energy is converted to kinetic energy. If we consider the student-only system, then there is no potential energy, and gravity does work on the student to increase their kinetic energy: $W = \vec{F}_g \bullet \vec{d} = mgh$. The two situations are equivalent and give the same answer.

Use this space for summary and/or additional notes:

2. Use motion equations to find the student's velocity when he hits the ground, based on the height of the building and acceleration due to gravity. Then use the formula $K=\frac{1}{2}mv^2$.

$$d=\frac{1}{2}at^2$$
$$15=\frac{1}{2}(10)t^2$$
$$t^2=3$$
$$t=\sqrt{3}=1.732$$
$$v=at$$
$$v=(10)(1.732)=17.32\,\tfrac{\text{m}}{\text{s}}$$

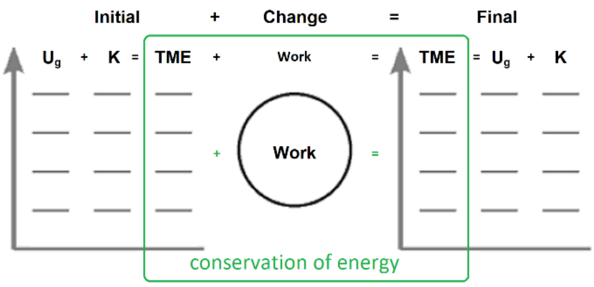
$$K=\frac{1}{2}mv^2$$
$$K=\frac{1}{2}(80)(17.32)^2$$
$$K=12000\text{ J}$$

Answers: K_f = 12 000 J; v_f = 17.3 $\tfrac{\text{m}}{\text{s}}$ as before.

Use this space for summary and/or additional notes:

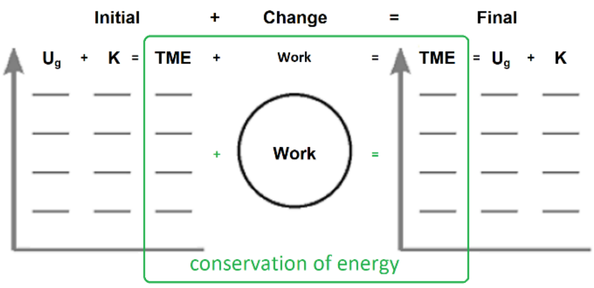
Homework Problems

1. **(S)** A 70. kg pole vaulter converts the kinetic energy of running at ground level into the potential energy needed to clear the crossbar at a height of 4.0 m above the ground. What is the minimum velocity that the pole vaulter must have when taking off from the ground in order to clear the bar?



Answer: $8.9\,\frac{\text{m}}{\text{s}}$

2. **(S)** A 10.0 kg lemur swings on a vine from a point which is 40.0 m above the jungle floor to a point which is 15.0 m above the floor. If the lemur was moving at $2.0\,\frac{\text{m}}{\text{s}}$ initially, what will be its velocity at the 15.0 m point?

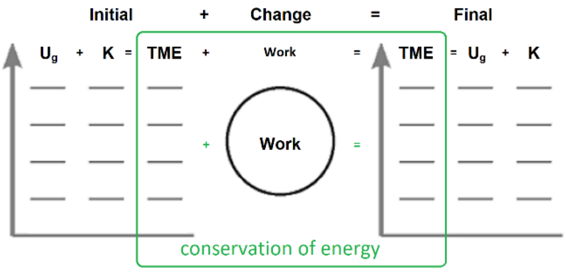


Answer: $22.4\,\frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

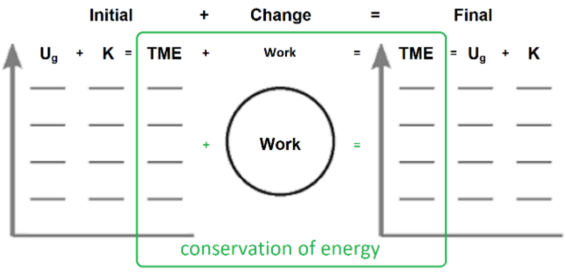
3. **(M)** A 7.25 kg bowling ball hanging from a chain is held against a student's chin, which is 1.45 m above the floor. The student releases the bowling ball, which swings across the room and back, stopping just before it hits the student.

   a. **(M)** What is the maximum velocity of the bowling ball?



   Answer: 5.39 $\frac{m}{s}$

   b. **(M)** What is the velocity of the bowling ball when it is 0.25 m below the person's chin (*i.e.,* 1.2 m above the floor)?



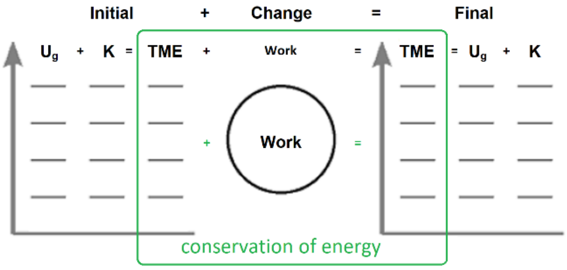
   Answer: 2.24 $\frac{m}{s}$

Use this space for summary and/or additional notes:

honors & AP®

4. **(M – honors & AP®; A – CP1)** A roller coaster car with mass *m* is launched, from ground level with a velocity of v_o. Neglecting friction, how fast will it be moving when it reaches the top of a loop, which a distance of *h* above the ground?

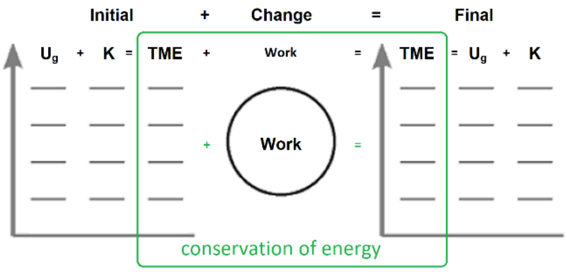
   (*If you are not sure how to do this problem, do #5 below and use the steps to guide your algebra.*)



   Answer: $\sqrt{v_o^2 - 2gh}$

5. **(S – honors & AP®; M – CP1)** A 500. kg roller coaster car is launched, from ground level, at $20.\frac{\text{m}}{\text{s}}$. Neglecting friction, how fast will it be moving when it reaches the top of a loop, which is 15 m above the ground?

   (*<u>You must start with the equations in your</u>* <u>Physics Reference Tables</u> *<u>and show all of the steps of GUESS.</u> You may only use the answer to question #3 above as a starting point if you have already solved that problem.*)



   Answer: $10\frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

6. **(S)** The engine of a 0.200 kg model rocket provides a constant thrust of 10. N for 1.0 s.

   a. **(S)** What is the net force that the engine applies to the rocket?
   (*Hint: draw a free-body diagram.*)

   Answer: 8.0 N

   b. **(S)** What is the velocity of the rocket when the engine shuts off? What is its height at that time?
   (*Hint: Use* $F_{net} = ma$ *to find the acceleration. Then use motion equations to find the velocity and height.*)

   Answer: $v = 40.\,\frac{\text{m}}{\text{s}}$; h = 20. m

   c. **(S)** What is the final height of the rocket?
   (*Hint: calculate the kinetic energy of the rocket when the engine shuts off. This will become additional potential energy when the rocket reaches its highest point. Add this to the work from part b above to get the total energy at the end, which is all potential. Finally, use* $U_g = mgh$ *to find the height.*)

   Answer: 100 m

   d. **(S)** How much work did the engine do on the rocket?

   Answer: 200 N·m

Use this space for summary and/or additional notes:

AP®

Rotational Work

Unit: Energy, Work & Power

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 3.F.2.1, 3.F.2.2

Mastery Objective(s): (Students will be able to…)

- Solve problems that involve work on a rotating object.

Success Criteria:

- Correct equations are chosen for the situation.
- Variables are correctly identified and substituted correctly into equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Describe how an object can have both rotational and translational work.

Tier 2 Vocabulary: work, energy, translational

Notes:

Just as work is done when a force causes an object to translate (move in a straight line), work is also done when a torque causes an object to rotate.

As with other equations for rotational motion, the rotational equation for work looks just like the linear (translational) equation, with each variable from the linear equation replaced by its analogue from the rotational equation.

In the equation for work, force is replaced by torque, and (translational) distance is replaced by rotational distance (angle):

$W = F_{\parallel} d$	$W = \tau \Delta\theta$
translational	**rotational**

Use this space for summary and/or additional notes:

AP®

Sample Problem

Q: How much work is done on a bolt when it is turned 30° by applying a perpendicular force of 100 N to the end of a 36 cm long wrench?

A: The equation for work is:

$$W = \tau \Delta\theta$$

The torque is:

$$\tau = rF_{\perp}$$
$$\tau = (0.36)(100) = 36\,\text{N}\cdot\text{m}$$

The angle, in radians, is:

$$\theta = 30° \times \frac{2\pi\,\text{rad}}{360°} = \frac{\pi}{6}\,\text{rad}$$

The work done on the bolt is therefore:

$$W = \tau \Delta\theta$$
$$W = (36)\left(\frac{\pi}{6}\right)$$
$$W = 6\pi = (6)(3.14) = 18.8\,\text{N}\cdot\text{m}$$

Note that torque and work are different, unrelated quantities that both happen to have the same unit (N·m). However, *torque and work are not interchangeable*! Notice that 36 N·m of *torque* produced 18.8 N·m of *work* because of the angle through which the torque was applied. If the angle had been different, the amount of work would have been different.

This is an example of why you cannot rely exclusively on dimensional analysis to set up and solve problems!

Use this space for summary and/or additional notes:

AP®

Rotational Kinetic Energy

Unit: Energy, Work & Power

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 3.F.2.1, 3.F.2.2

Mastery Objective(s): (Students will be able to…)

- Solve problems that involve kinetic energy of a rotating object.

Success Criteria:

- Correct equations for *both* translational *and* rotational kinetic energy are used in the problem.
- Variables are correctly identified and substituted correctly into the appropriate equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Describe how an object can have both rotational and translational kinetic energy.
- Explain the relationship between rotational and translational kinetic energy for a rolling object.

Tier 2 Vocabulary: energy, translational, rotational

Labs, Activities & Demonstrations:

- Calculate the exact landing spot of golf ball rolling down a ramp.

Notes:

Just as an object that is moving in a straight line has kinetic energy, a rotating object also has kinetic energy.

The angular velocity (rate of rotation) and the translational velocity are related, because distance that the object must travel (the arclength) is the object's circumference $(s = 2\pi r)$, and the object must make one complete revolution $(\Delta\theta = 2\pi \text{ radians})$ in order to travel this distance. This means that for a rolling object:

$$\Delta\theta = 2\pi r$$

Just as energy can be converted from one form to another and transferred from one object to another, rotational kinetic energy can be converted into any other form of energy, including translational kinetic energy.

Use this space for summary and/or additional notes:

AP®

This is the principle behind log rolling. The two contestants get the log rolling quite fast. When one contestant fails to keep up with the log, some of the log's rotational kinetic energy is converted to that contestant's translational kinetic energy, which catapults them into the water:

In a rotating system, the formula for kinetic energy looks similar to the equation for kinetic energy in linear systems, with mass (translational inertia) replaced by moment of inertia (rotational inertia), and linear (translational) velocity replaced by angular velocity:

$K_t = \tfrac{1}{2}mv^2$	$K_r = \tfrac{1}{2}I\omega^2$
translational	**rotational**

In the rotational equation, *I* is the object's moment of inertia (see Rotational Inertia starting on page 344), and ω is the object's angular velocity.

Note: these problems make use of three relationships that you need to *memorize*:

$$s = r\Delta\theta \qquad v_t = r\omega \qquad a_t = r\alpha$$

Use this space for summary and/or additional notes:

AP®

Sample Problem:

Q: What is the rotational kinetic energy of a tenpin bowling ball that has a mass of 7.25 kg and a radius of 10.9 cm as it rolls down a bowling lane at $8.0\,\frac{m}{s}$?

A: The equation for rotational kinetic energy is:

$$K_r = \frac{1}{2} I\omega^2$$

We can find the angular velocity from the translational velocity:

$$v = r\omega$$
$$8.0 = (0.109)\omega$$
$$\omega = \frac{8.0}{0.109} = 73.3\,\tfrac{\text{rad}}{\text{s}}$$

The bowling ball is a solid sphere. The moment of inertia of a solid sphere is:

$$I = \tfrac{2}{5} mr^2$$
$$I = \left(\tfrac{2}{5}\right)(7.25)(0.109)^2$$
$$I = 0.0345\,\text{kg}\cdot\text{m}^2$$

To find the rotational kinetic energy, we plug these numbers into the equation:

$$K_r = \tfrac{1}{2} I\omega^2$$
$$K_r = (\tfrac{1}{2})(0.0345)(73.3)^2$$
$$K_r = 185.6\text{ J}$$

Use this space for summary and/or additional notes:

AP®

Total Kinetic Energy

If an object (such as a ball) is rolling, then it is rotating and also moving (translationally). Its total kinetic energy must therefore be the sum of its translational kinetic energy and its rotational kinetic energy:

$$K_{total} = K_t + K_r$$
$$K_{total} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

Sample problem:

Q: A standard Type 2 (medium) tennis ball is hollow and has a mass of 58 g and a diameter of 6.75 cm. If the tennis ball rolls 5.0 m across a floor in 1.25 s, how much total energy does the ball have?

A: The translational velocity of the tennis ball is:

$$v = \frac{d}{t} = \frac{5.0}{1.25} = 4.0\,\tfrac{\text{m}}{\text{s}}$$

The translational kinetic energy of the ball is therefore:

$$K_t = \tfrac{1}{2}mv^2 = (\tfrac{1}{2})(0.058)(4)^2 = 0.464\text{ J}$$

The angular velocity of the tennis ball can be calculated from:

$$v = r\omega$$
$$4 = (0.03375)\,\omega$$
$$\omega = \frac{4}{0.03375} = 118.5\,\tfrac{\text{rad}}{\text{s}}$$

The moment of inertia of a hollow sphere is:

$$I = \tfrac{2}{3}mr^2 = \left(\frac{2}{3}\right)(0.058)(0.03375)^2 = 4.40\times 10^{-5}\text{ kg}\cdot\text{m}^2$$

The rotational kinetic energy is therefore:

$$K_r = \tfrac{1}{2}I\omega^2 = (\tfrac{1}{2})(4.40\times 10^{-5})(118.5)^2 = 0.309\text{ J}$$

Finally, the total kinetic energy is the sum of the translational and rotational kinetic energies:

$$K = K_t + K_r = 0.464 + 0.309 = 0.773\text{ J}$$

Use this space for summary and/or additional notes:

AP®

Homework Problems

1. **(M – AP®; A – honors & CP1)** A solid ball with a mass of 100 g and a radius of 2.54 cm rolls with a rotational velocity of $1.0\,\frac{\text{rad}}{\text{s}}$.

   a. **(M – AP®; A – honors & CP1)** What is its rotational kinetic energy?

   Answer: 1.29×10^{-5} J

   b. **(M – AP®; A – honors & CP1)** What is its translational kinetic energy?

   Answer: 3.23×10^{-5} J

   c. **(M – AP®; A – honors & CP1)** What is its total kinetic energy?

   Answer: 4.52×10^{-5} J

Use this space for summary and/or additional notes:

AP®

2. **(M – AP®; A – honors & CP1)** How much work is needed to stop a 25 cm diameter solid cylindrical flywheel rotating at 3 600 RPM? The flywheel has a mass of 2 000 kg.

   (*Hint: Note that the problem gives the diameter, not the radius, and that the diameter is in centimeters, not meters.*)

   Answer: $1.11\times10^{6}\ \text{N}\cdot\text{m}$

3. **(M – AP®; A – honors & CP1)** An object is initially at rest. When $250\,\text{N}\cdot\text{m}$ of work is done on the object, it rotates through 20 revolutions in 4.0 s. What is its moment of inertia?

   Answer: $5.066\,\text{kg}\cdot\text{m}^2$

Use this space for summary and/or additional notes:

AP®

4. **(M – AP®; A – honors & CP1)** How much work is required to slow a 20 cm diameter solid ball that has a mass of 2.0 kg from $5.0\,\frac{\text{m}}{\text{s}}$ to $1.0\,\frac{\text{m}}{\text{s}}$?

   (*Hint: Again, note that the problem gives the diameter, not the radius, and that the diameter is in centimeters, not meters.*)

   Answer: 33.6 J

5. **(M – AP®; A – honors & CP1)** A flat disc that has a mass of 1.5 kg and a diameter of 10 cm rolls down a 1 m long incline with an angle of 15°. What is its linear speed at the bottom?

   (*Hint: Again, note that the problem gives the diameter, not the radius, and that the diameter is in centimeters, not meters.*)

   Answer: $1.86\,\frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Escape Velocity

Unit: Energy, Work & Power

MA Curriculum Frameworks (2016): HS-PS2-4

AP® Physics 1 Learning Objectives: 2.B.2.1, 2.B.2.2, 3.A.3.3, 3.C.1.1, 3.C.1.2, 3.G.1.1

Mastery Objective(s): (Students will be able to…)

- Calculate the velocity that a rocket or spaceship needs in order to escape the pull of gravity of a planet.

Success Criteria:

- Variables are correctly identified and substituted correctly into the appropriate equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why we can't simply use $\vec{g}=10\,\frac{\text{m}}{\text{s}^2}$ to calculate escape velocity.

Tier 2 Vocabulary: escape

Notes:

If you want to send a rocket or space ship to explore the rest of the solar system or beyond, the rocket needs enough kinetic energy to escape from the force of Earth's gravity.

To explain the calculation, we measure height from Earth's surface and use $\vec{g}=10\,\frac{\text{m}}{\text{s}^2}$ for the strength of the gravitational field. However, when we calculate the escape velocity of a rocket, the rocket has to go from the surface of the Earth to a point where $\vec{g}$ is small enough to be negligible.

We can still use the conservation of energy, but we need to calculate the potential energy that the rocket has based on its distance from the center of the Earth instead of the surface of the Earth. (When the distance from the Earth is great enough, the gravitational potential energy becomes zero, and the rocket has escaped.) Therefore, the spaceship needs to turn kinetic energy into this much potential energy.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

To solve this, we need to turn to Newton's Law of Universal Gravitation. Recall from Universal Gravitation starting on page 377 that:

$$F_g = \frac{Gm_1m_2}{r^2}$$

The potential energy equals the work that gravity could theoretically do on the rocket, based on the force of gravity and the distance to the center of the Earth:

$$W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}} = F_g h = \left(\frac{Gm_1m_2}{r^2}\right)h$$

Because h is the distance to the <u>center</u> of the Earth, $h = r$ and we can cancel, giving the equation:

$$U_g = \frac{Gm_1m_2}{r}$$

Now, we can use the law of conservation of energy. The kinetic energy that the rocket needs to have at launch needs equals the potential energy that the rocket has due to gravity. Using m_1 for the mass of the Earth and m_2 for the mass of the spaceship:

$$\textbf{Before} = \textbf{After}$$
$$TME_i = TME_f$$
$$K_i = U_f$$
$$\tfrac{1}{2}\cancel{m_2}v_e^2 = \frac{Gm_1\cancel{m_2}}{r}$$
$$v_e^2 = \frac{2Gm_E}{r}$$
$$v_e = \sqrt{\frac{2Gm_E}{r}}$$

Therefore, at the surface of the Earth, where $m_E = 5.97\times10^{24}$ kg and $r = 6.37\times10^{6}$ m, this gives $v_e = 1.12\times10^{4}\ \frac{\text{m}}{\text{s}} = 11200\ \frac{\text{m}}{\text{s}}$. (If you're curious, this equals just over 25 000 miles per hour.)

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Sample Problem:

Q: When Apollo 11 went to the moon, the space ship needed to achieve the Earth's escape velocity of $11200\,\frac{\text{m}}{\text{s}}$ to escape Earth's gravity. What velocity did the spaceship need to achieve in order to escape the moon's gravity and return to Earth? (*I.e.,* what is the escape velocity on the surface of the moon?)

A:
$$v_e = \sqrt{\frac{2Gm_{moon}}{d_{moon}}}$$
$$v_e = \sqrt{\frac{(2)(6.67\times10^{-11})(7.35\times10^{22})}{1.74\times10^{6}}}$$
$$v_e = 2370\,\tfrac{\text{m}}{\text{s}}$$

Use this space for summary and/or additional notes:

Power

Unit: Energy, Work & Power

MA Curriculum Frameworks (2016): N/A

AP Physics 1 Learning Objectives: N/A, but power calculations and problems have appeared on the AP® exam.

Mastery Objective(s): (Students will be able to…)

- Calculate power as a rate of energy consumption.

Success Criteria:

- Variables are correctly identified and substituted correctly into the appropriate equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the difference between total energy and power.

Tier 2 Vocabulary: power

Notes:

power: a measure of the rate at which energy is applied or work is done. Power is calculated by dividing work (or energy) by time.

$$P = \frac{W}{t} = \frac{\Delta K + \Delta U}{t}$$

Power is a scalar quantity and is measured in Watts (W).

$$1\,\text{W} = 1\,\tfrac{\text{J}}{\text{s}} = 1\,\tfrac{\text{N}\cdot\text{m}}{\text{s}} = 1\,\tfrac{\text{kg}\cdot\text{m}^2}{\text{s}^3}$$

Note that utility companies measure energy in kilowatt-hours. This is because $P = \frac{W}{t}$, which means energy = $W = Pt$.

Because 1 kW = 1000 W and 1 h = 3600 s, this means
1 kWh = (1000 W)(3600 s) = 3 600 000 J

Because $W = Fd$, this means $P = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv$

Use this space for summary and/or additional notes:

AP®

Power in Rotational Systems

In a rotational system, the formula for power looks similar to the equation for power in linear systems, with force replaced by torque and linear velocity replaced by angular velocity:

$P = Fv$	$P = \tau\omega$
linear	**rotational**

Solving Power Problems

Many power problems require you to calculate the amount of work done or the change in energy, which you should recall is:

$W = F_{\parallel} d$	if the force is caused by linear displacement
$\Delta K_t = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2$ * $= \frac{1}{2}m(v^2 - v_o^2)$	if the change in energy was caused by a change in velocity
$\Delta U_g = mgh - mgh_o = mg\Delta h$	if the change in energy was caused by a change in height

Solving Rotational Power Problems

AP®

Power is also applicable to rotating systems:

$W = \tau\Delta\theta$	if the work is produced by a torque
$\Delta K_r = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_o^2 = \frac{1}{2}I(\omega^2 - \omega_o^2)$	if the change in energy was caused by a change in angular velocity

Once you have the work or energy, you can plug it in for either W, ΔK or ΔU, use the appropriate parts of the formula:

$$P = \frac{W}{t} = \frac{\Delta K + \Delta U}{t} = Fv = \tau\omega$$

and solve for the missing variable.

* K_t is translational kinetic energy. This is the only form of kinetic energy used in CP1 and honors physics. The subscript *t* is used here to distinguish translational kinetic energy from rotational kinetic energy (K_r), because both are used in AP® Physics.

Use this space for summary and/or additional notes:

Sample Problems:

Q: What is the power output of an engine that pulls with a force of 500. N over a distance of 100. m in 25 s?

A: $W = Fd = (500)(100) = 50\,000 \text{ J}$

$$P = \frac{W}{t} = \frac{50\,000}{25} = 2000 \text{ W}$$

Q: A 60. W incandescent light bulb is powered by a generator that is powered by a falling 1.0 kg mass on a rope. Assuming the generator is 100 % efficient (*i.e.,* no energy is lost when the generator converts its motion to electricity), how far must the mass fall in order to power the bulb at full brightness for 1.0 minute?

A:
$$P = \frac{\Delta U_g}{t} = \frac{mg\,\Delta h}{t}$$
$$60 = \frac{(1)(10)\,\Delta h}{60}$$
$$3600 = 10\,\Delta h$$
$$\Delta h = \frac{3600}{10} = 360 \text{ m}$$

Note that 360 m is approximately the height of the Empire State Building. This is why changing from incandescent light bulbs to more efficient compact fluorescent or LED bulbs can make a significant difference in energy consumption!

Use this space for summary and/or additional notes:

Homework Problems

1. **(S)** A small snowmobile has a 9 000 W (12 hp) engine. It takes a force of 300. N to move a sled load of wood along a pond. How much time will it take to tow the wood across the pond if the distance is measured to be 850 m?

   Answer: 28.3 s

2. **(M)** A winch, which is rated at 720 W, is used to pull an all-terrain vehicle (ATV) out of a mud bog for a distance of 2.3 m. If the average force applied by the winch is 1 500 N, how long will the job take?

   Answer: 4.8 s

3. **(S)** What is your power output if you have a mass of 65 kg and you climb a 5.2 m vertical ladder in 10.4 s?

   Answer: 325 W

4. **(M)** Jack and Jill went up the hill. (The hill was 23m high.) Jack was carrying a 21 kg pail of water.

   a. **(M)** Jack has a mass of 75 kg and he carried the pail up the hill in 45 s. How much power did he apply?

      Answer: 490.7 W

   b. **(M)** Jill has a mass of 55 kg, and she carried the pail up the hill in 35 s. How much power did she apply?

      Answer: 499.4 W

Use this space for summary and/or additional notes:

honors & AP®

5. **(M – honors & AP®; A – CP1)** The maximum power output of a particular crane is P. What is the fastest time, t, in which this crane could lift a crate with mass m to a height h?

   (*If you are not sure how to do this problem, do #6 below and use the steps to guide your algebra.*)

   Answer: $t = \dfrac{mgh}{P}$

6. **(S – honors & AP®; M – CP1)** The maximum power output of a particular crane is 12 kW. What is the fastest time in which this crane could lift a 3 500 kg crate to a height of 6.0 m?

   (*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #5 above as a starting point if you have already solved that problem.*)

   Hint: Remember to convert kilowatts to watts.

   Answer: 17.5 s

AP®

7. **(M – AP®; A – honors & CP1)** A 30 cm diameter solid cylindrical flywheel with a mass of 2 500 kg was accelerated from rest to an angular velocity of 1 800 RPM in 60 s.

   a. How much work was done on the flywheel?

      Answer: $5.0\times10^{5}\ \mathrm{N\cdot m}$

   b. How much power was exerted?

      Answer: $8.3\times10^{3}\ \mathrm{W}$

Use this space for summary and/or additional notes:

Introduction: Momentum

Unit: Momentum

Topics covered in this chapter:

This chapter deals with the ability of a moving object (or potential for an object to move) to affect other objects.

- *Linear Momentum* describes a way to represent the movement of an object and what happens when objects collide, and the equations that relate to it.
- *Impulse* describes changes in momentum.
- *Conservation of Linear Momentum* explains and gives examples of the law that total momentum before a collision must equal total momentum after a collision.
- *Angular Momentum* describes momentum and conservation of momentum in rotating systems, and the transfer between linear and angular momentum.

New challenges in this chapter involve keeping track of the same quantity applied to the same object, but at different times.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-2. Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.

HS-PS2-3. Apply scientific principles of motion and momentum to design, evaluate, and refine a device that minimizes the force on a macroscopic object during a collision.

AP®

AP® Physics 1 Learning Objectives:

3.D.1.1: The student is able to justify the selection of data needed to determine the relationship between the direction of the force acting on an object and the change in momentum caused by that force. **[SP 4.1]**

3.D.2.1: The student is able to justify the selection of routines for the calculation of the relationships between changes in momentum of an object, average force, impulse, and time of interaction. **[SP 2.1]**

Use this space for summary and/or additional notes:

AP®

3.D.2.2: The student is able to predict the change in momentum of an object from the average force exerted on the object and the interval of time during which the force is exerted. **[SP 6.4]**

3.D.2.3: The student is able to analyze data to characterize the change in momentum of an object from the average force exerted on the object and the interval of time during which the force is exerted. **[SP 5.1]**

3.D.2.4: The student is able to design a plan for collecting data to investigate the relationship between changes in momentum and the average force exerted on an object over time. **[SP 4.2]**

3.F.3.1: The student is able to predict the behavior of rotational collision situations by the same processes that are used to analyze linear collision situations using an analogy between impulse and change of linear momentum and angular impulse and change of angular momentum. **[SP 6.4, 7.2]**

3.F.3.2: In an unfamiliar context or using representations beyond equations, the student is able to justify the selection of a mathematical routine to solve for the change in angular momentum of an object caused by torques exerted on the object. **[SP 2.1]**

3.F.3.3: The student is able to plan data collection and analysis strategies designed to test the relationship between torques exerted on an object and the change in angular momentum of that object. **[SP 4.1, 4.2, 5.1, 5.3]**

4.B.1.1: The student is able to calculate the change in linear momentum of a two-object system with constant mass in linear motion from a representation of the system (data, graphs, *etc.*). **[SP 1.4, 2.2]**

4.B.1.2: The student is able to analyze data to find the change in linear momentum for a constant-mass system using the product of the mass and the change in velocity of the center of mass. **[SP 5.1]**

4.B.2.1: The student is able to apply mathematical routines to calculate the change in momentum of a system by analyzing the average force exerted over a certain time on the system. **[SP 2.2]**

4.B.2.2: The student is able to perform analysis on data presented as a force-time graph and predict the change in momentum of a system. **[SP 5.1]**

4.D.1.1: The student is able to describe a representation and use it to analyze a situation in which several forces exerted on a rotating system of rigidly connected objects change the angular velocity and angular momentum of the system. **[SP 1.2, 1.4]**

4.D.1.2: The student is able to plan data collection strategies designed to establish that torque, angular velocity, angular acceleration, and angular momentum can be predicted accurately when the variables are treated as being clockwise or counterclockwise with respect to a well-defined axis of rotation, and refine the research question based on the examination of data. **[SP 3.2, 4.1, 4.2, 5.1, 5.3]**

Use this space for summary and/or additional notes:

AP®

4.D.2.1: The student is able to describe a model of a rotational system and use that model to analyze a situation in which angular momentum changes due to interaction with other objects or systems. **[SP 1.2, 1.4]**

4.D.2.2: The student is able to plan a data collection and analysis strategy to determine the change in angular momentum of a system and relate it to interactions with other objects and systems. **[SP 4.2]**

4.D.3.1: The student is able to use appropriate mathematical routines to calculate values for initial or final angular momentum, or change in angular momentum of a system, or average torque or time during which the torque is exerted in analyzing a situation involving torque and angular momentum. **[SP 2.2]**

4.D.3.2: The student is able to plan a data collection strategy designed to test the relationship between the change in angular momentum of a system and the product of the average torque applied to the system and the time interval during which the torque is exerted. **[SP 4.1, 4.2]**

5.A.2.1: The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. **[SP 6.4, 7.2]**

5.D.1.1: The student is able to make qualitative predictions about natural phenomena based on conservation of linear momentum and restoration of kinetic energy in elastic collisions. **[SP 6.4, 7.2]**

5.D.1.2: The student is able to apply the principles of conservation of momentum and restoration of kinetic energy to reconcile a situation that appears to be isolated and elastic, but in which data indicate that linear momentum and kinetic energy are not the same after the interaction, by refining a scientific question to identify interactions that have not been considered. Students will be expected to solve qualitatively and/or quantitatively for one-dimensional situations and only qualitatively in two-dimensional situations. **[SP 2.2, 3.2, 5.1, 5.3]**

5.D.1.3: The student is able to apply mathematical routines appropriately to problems involving elastic collisions in one dimension and justify the selection of those mathematical routines based on conservation of momentum and restoration of kinetic energy. **[SP 2.1, 2.2]**

5.D.1.4: The student is able to design an experimental test of an application of the principle of the conservation of linear momentum, predict an outcome of the experiment using the principle, analyze data generated by that experiment whose uncertainties are expressed numerically, and evaluate the match between the prediction and the outcome. **[SP 4.2, 5.1, 5.3, 6.4]**

5.D.1.5: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values. **[SP 2.1, 2.2]**

Use this space for summary and/or additional notes:

AP®

5.D.2.1: The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic. **[SP 6.4, 7.2]**

5.D.2.2: The student is able to plan data collection strategies to test the law of conservation of momentum in a two-object collision that is elastic or inelastic and analyze the resulting data graphically. **[SP 4.1, 4.2, 5.1]**

5.D.2.3: The student is able to apply the conservation of linear momentum to a closed system of objects involved in an inelastic collision to predict the change in kinetic energy. **[SP 6.4, 7.2]**

5.D.2.4: The student is able to analyze data that verify conservation of momentum in collisions with and without an external friction force. **[SP 4.1, 4.2, 4.4, 5.1, 5.3]**

5.D.2.5: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum as the appropriate solution method for an inelastic collision, recognize that there is a common final velocity for the colliding objects in the totally inelastic case, solve for missing variables, and calculate their values. **[SP 2.1, 2.2]**

5.E.1.1: The student is able to make qualitative predictions about the angular momentum of a system for a situation in which there is no net external torque. **[SP 6.4, 7.2]**

5.E.1.2: The student is able to make calculations of quantities related to the angular momentum of a system when the net external torque on the system is zero. **[SP 2.1, 2.2]**

5.E.2.1: The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses. **[SP 2.2]**

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Momentum**, including impulse, and conservation laws.

1. What is Linear Momentum?
2. Impulse
3. Conservation of Momentum
4. Collisions
5. Center of Mass

Skills learned & applied in this chapter:

- Working with more than one instance of the same quantity in a problem.
- Conservation laws (before/after problems).

Use this space for summary and/or additional notes:

Linear Momentum

Unit: Momentum

MA Curriculum Frameworks (2016): HS-PS2-2

AP® Physics 1 Learning Objectives: 4.B.1.1, 4.B.1.2, 5.A.2.1, 5.D.1.1, 5.D.1.2, 5.D.2.1, 5.D.2.3, 5.D.3.1

Mastery Objective(s): (Students will be able to…)

- Calculate the momentum of an object.
- Solve problems involving collisions in which momentum is conserved.

Success Criteria:

- Masses and velocities are correctly identified as before and after the collision.
- Variables are correctly identified and substituted correctly into the correct part of the equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the difference between momentum and kinetic energy.

Tier 2 Vocabulary: momentum

Labs, Activities & Demonstrations:

- Collisions on air track.
- Newton's Cradle.
- Ballistic pendulum.

Notes:

In the 17th century, the German mathematician Gottfried Leibnitz recognized the fact that in some cases, the mass and velocity of objects before and after a collision were related by kinetic energy ($\frac{1}{2}mv^2$, which he called the "quantity of motion"); in other cases, however, the "quantity of motion" was not preserved but another quantity (mv, which he called the "motive force") was the same before and after. Debate about whether "quantity of motion" or "motive force" was the correct quantity to use for these types of problems continued through the 17th and 18th centuries.

We now realize that both quantities are relevant. "Quantity of motion" is what we now call kinetic energy, and "motive force" is what we now call momentum. The two quantities are different but related.

Use this space for summary and/or additional notes:

<u>momentum</u> ($\vec{p}$): the amount of force that a moving object could transfer in a given amount of time in a collision. (Formerly called "motive force".)

Momentum is a vector quantity given by the formula:

$$\vec{p} = m\vec{v}$$

and is measured in units of $N \cdot s$, or $\frac{kg \cdot m}{s}$.

Because momentum is a vector, its sign (positive or negative) indicates its direction. An object's momentum is in the same direction as (and therefore has the same sign as) its velocity.

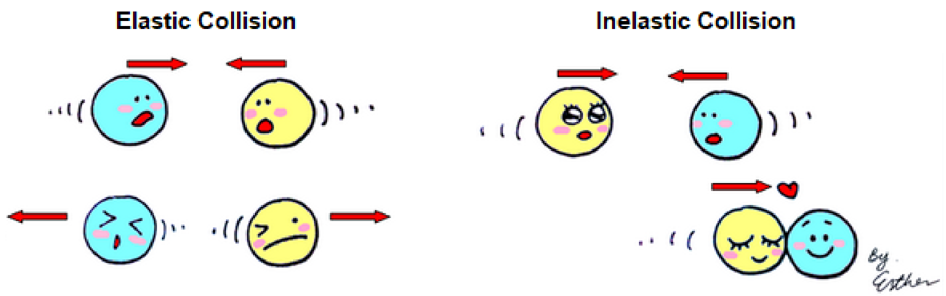
An object at rest has a momentum of zero because $\vec{v} = 0$.

<u>collision</u>: when two or more objects come together, at least one of which is moving, and make contact with each other. Momentum is always transferred in a collision.

<u>elastic collision</u>: when two or more objects collide and then separate.

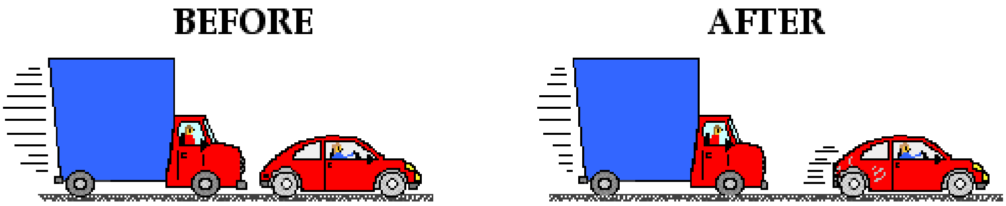
<u>perfectly elastic collision</u>: when two or more objects collide and separate, and none of the kinetic energy of any of the objects is "lost" (turned into any other form of energy).

<u>inelastic collision</u>: when two or more objects collide and remain together. (Or when one object separates into two or more objects.)



Use this space for summary and/or additional notes:

Momentum is the quantity that is transferred between objects in a collision.



Before the above collision, the truck was moving, so it had momentum; the car was not moving, so it did not have any momentum. After the collision, some of the truck's momentum was transferred to the car. After the collision, both vehicles were moving, which means both vehicles had momentum.

Of course, *total energy* is also conserved in a collision. However, the form of energy can change. Before the above collision, all of the energy in the system was the initial kinetic energy of the truck. Afterwards, some of the energy is the final kinetic energy of the truck, some of the energy is the kinetic energy of the car, but some of the energy is converted to heat, sound, *etc.* during the collision.

inertia: an object's ability to resist the action of a force.

Recall that a net force causes acceleration, which means the inertia of an object is its ability to resist a change in velocity. This means that in linear (translational) systems, inertia is simply mass. In rotating systems, inertia is the moment of inertia, which depends on the mass and the distance from the center of rotation. (See Rotational Inertia on page 344.)

Inertia and momentum are related, but are not the same thing; an object has inertia even at rest, when its momentum is zero. An object's momentum changes if either its mass or its velocity changes, but an the inertia of an object can change only if either its mass changes or its distance from the center of rotation changes.

Use this space for summary and/or additional notes:

Momentum and Kinetic Energy

We have the following equations, both of which relate mass and velocity:

momentum: $\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}}$

kinetic energy: $K = \frac{1}{2}mv^2$

We can combine these equations to eliminate *v*, giving the equation:

$$K = \frac{p^2}{2m}$$

The relationship between momentum and kinetic energy explains why the velocities of objects after a collision are determined by the collision.

Because kinetic energy and momentum must *both* be conserved in an elastic collision, the two final velocities are actually determined by the masses and the initial velocities. The masses and initial velocities are determined before the collision. The only variables are the two velocities after the collision. This means there are two equations (conservation of momentum and conservation of kinetic energy) and two unknowns (and $\vec{\boldsymbol{v}}_{2,f}$).

For a perfectly elastic collision, conservation of momentum states:

$$m_1\vec{\boldsymbol{v}}_{1,i} + m_2\vec{\boldsymbol{v}}_{2,i} = m_1\vec{\boldsymbol{v}}_{1,f} + m_2\vec{\boldsymbol{v}}_{2,f}$$

and conservation of kinetic energy states:

$$\tfrac{1}{2}m_1v_{1,i}^2 + \tfrac{1}{2}m_2v_{2,i}^2 = \tfrac{1}{2}m_1v_{1,f}^2 + \tfrac{1}{2}m_2v_{2,f}^2$$

If we use these two equations to solve for $\vec{\boldsymbol{v}}_{1,f}$ and $\vec{\boldsymbol{v}}_{2,f}$ in terms of the other variables, the result is the following:

$$\vec{\boldsymbol{v}}_{1,f} = \frac{\vec{\boldsymbol{v}}_{1,i}(m_1 - m_2) + 2m_2\vec{\boldsymbol{v}}_{2,i}}{m_1 + m_2}$$

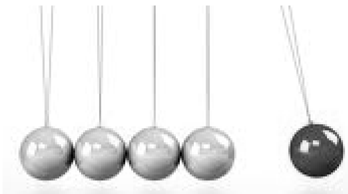
$$\vec{\boldsymbol{v}}_{2,f} = \frac{\vec{\boldsymbol{v}}_{2,i}(m_2 - m_1) + 2m_1\vec{\boldsymbol{v}}_{1,i}}{m_1 + m_2}$$

For an inelastic collision, there is no solution that satisfies both the conservation of momentum and the conservation of kinetic energy; the total kinetic energy after the collision is always less than the total kinetic energy before. This matches what we observe, which is that momentum is conserved, but some of the kinetic energy is converted to heat during the collision.

Use this space for summary and/or additional notes:

Newton's Cradle

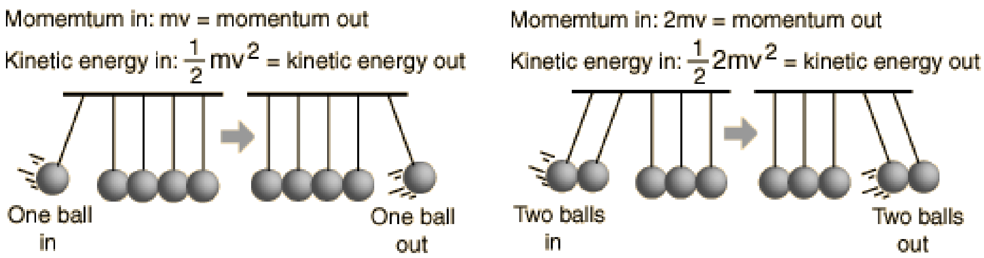
Newton's Cradle is the name given to a set of identical balls that are able to swing suspended from wires, as shown at the right.



When one ball is swung and allowed to collide with the rest of the balls, the momentum transfers through the balls and one ball is knocked out from the opposite end. When two balls are swung, two balls are knocked out from the opposite end, and so on.

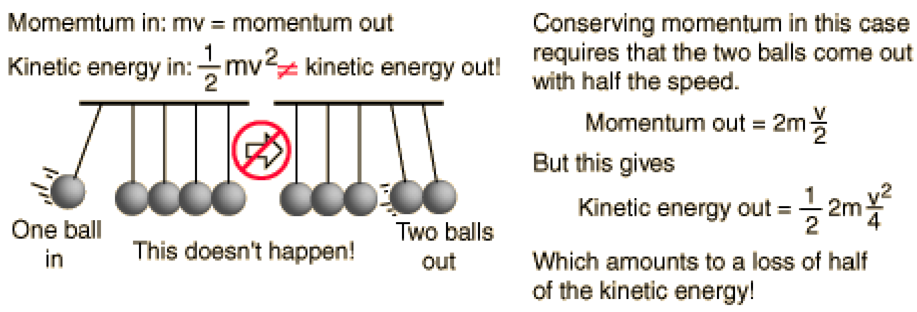
This apparatus demonstrates the relationship between the conservation of momentum and conservation of kinetic energy. When the balls collide, the collision is mostly elastic collision, meaning that all of the momentum and most of the kinetic energy are conserved.

Before the collision, the moving ball(s) have momentum (mv) and kinetic energy ($\frac{1}{2}mv^2$). There are no external forces, which means *momentum* must be conserved. The collision is mostly elastic, which means *kinetic energy* is mostly also conserved. The only way for the same momentum and kinetic energy to be present after the collision is for the same number of balls to swing away from the opposite end with the same velocity.



Use this space for summary and/or additional notes:

If only momentum had to be conserved, it would be possible to pull back one ball but for two balls to come out the other side at ½ of the original velocity. However, this can't actually happen.



Note also that if there were no friction, the balls would continue to swing forever. However, because of friction (between the balls and air molecules, within the strings as they stretch, *etc.*) and conversion of some of the kinetic energy to other forms (such as heat), the balls in a real Newton's Cradle will, of course, slow down and eventually stop.

Use this space for summary and/or additional notes:

Impulse

Unit: Momentum

MA Curriculum Frameworks (2016): HS-PS2-2

AP® Physics 1 Learning Objectives: 3.D.1.1, 3.D.2.1, 3.D.2.2, 3.D.2.3, 3.D.2.4, 4.B.1.1, 4.B.1.2, 4.B.2.1, 4.B.2.2, 5.A.2.1, 5.D.1.1, 5.D.1.2, 5.D.2.1, 5.D.2.3, 5.D.3.1

Mastery Objective(s): (Students will be able to…)

- Calculate the change in momentum of (impulse applied to) an object.
- Calculate impulse as a force applied over a period of time.
- Calculate impulse as the area under a force-time graph.

Success Criteria:

- Masses and velocities are correctly identified as before and after the collision.
- Variables are correctly identified and substituted correctly into the correct part of the equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the similarities and differences between impulse and work.

Tier 2 Vocabulary: momentum, impulse

Notes:

impulse ($\vec{\boldsymbol{J}}$): the effect of a force applied over a period of time; the accumulation of momentum.

Mathematically, impulse is a change in momentum, and is also equal to force times time:

$$\Delta\vec{\boldsymbol{p}} = \vec{\boldsymbol{J}} = \vec{\boldsymbol{F}}t \quad \text{and} \quad \vec{\boldsymbol{F}} = \frac{\vec{\boldsymbol{J}}}{t} = \frac{\Delta\vec{\boldsymbol{p}}}{t} = \frac{d\vec{\boldsymbol{p}}}{dt}$$

Where $\vec{\boldsymbol{F}}$ is the force vector and, t is time.

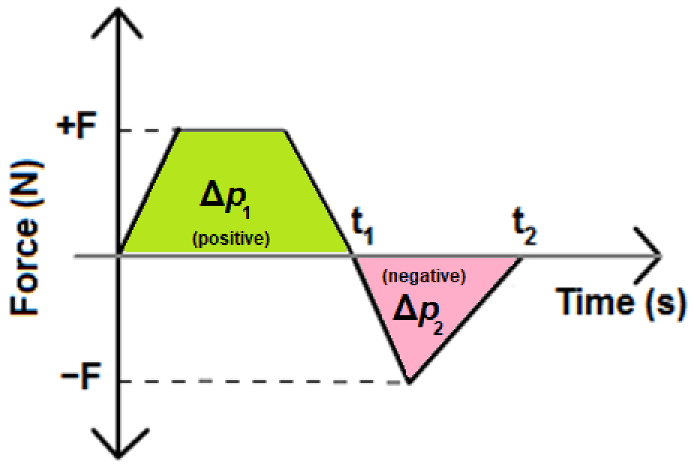
Impulse is measured in newton-seconds (N·s), just like momentum.

Impulse is analogous to work:

- Work is a change in energy;
  impulse is a change in momentum.
- Work is the accumulation of force over a distance $(W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}})$;
  Impulse is the accumulation of force over a time $(\vec{\boldsymbol{J}} = \vec{\boldsymbol{F}}t)$

Use this space for summary and/or additional notes:

Just as work is the area of a graph of force *vs.* distance, impulse is the area under a graph of force *vs.* time:



In the above graph, the impulse from time zero to t_1 would be Δp_1. The impulse from t_1 to t_2 would be Δp_2, and the total impulse would be $\Delta p_1 + \Delta p_2$ (keeping in mind that Δp_2 is negative).

Sample Problem:

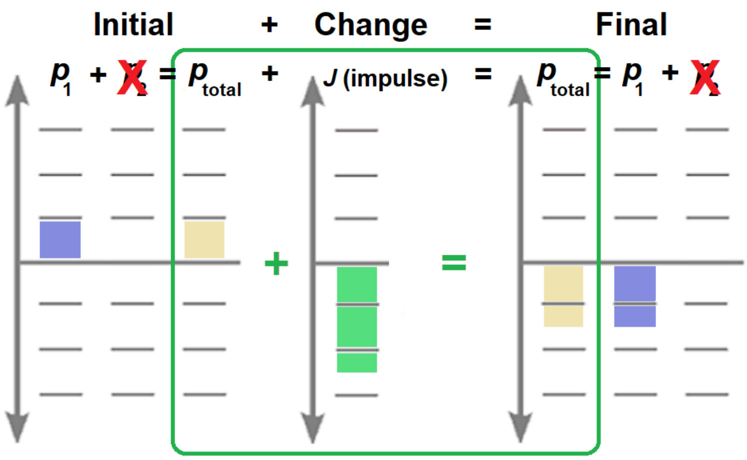
Q: A baseball has a mass of 0.145 kg and is pitched with a velocity of $38\,\frac{m}{s}$ toward home plate. After the ball is hit, its velocity is $52\,\frac{m}{s}$ in the opposite direction, toward the center field fence. If the impact between the ball and bat takes place over an interval of 3.0 ms (0.0030 s), find the impulse given to the ball by the bat, and the force applied to the ball by the bat.

A: The ball starts out moving toward home plate. The bat applies an impulse in the ***opposite*** direction. As with any vector quantity, opposite directions means we will have opposite signs. If we choose the initial direction of the ball (toward home plate) as the positive direction, then the initial velocity is $+38\,\frac{m}{s}$, and the final velocity is $-52\,\frac{m}{s}$. Because mass is scalar and always positive, this means the initial momentum is positive and the final momentum is negative.

Furthermore, because the final velocity is about 1½ times as much as the initial velocity (in the opposite direction) and the mass doesn't change, this means the impulse needs to be enough to negate the ball's initial momentum plus enough in addition to give the ball about 1½ times as much momentum in the opposite direction.

Use this space for summary and/or additional notes:

Just like the energy bar charts that we used for conservation of energy problems, we can create a momentum bar chart. Note that because momentum is a vector that can be positive or negative, our momentum bar chart needs to be able to accommodate positive and negative values.



Now, solving the problem mathematically:

$$p_i + J = p_f$$
$$mv_i + J_{bat} = mv_f$$
$$(0.145)(38) + J = (0.145)(-52)$$
$$5.51 + J = -7.54$$
$$J = -13.05\,\text{N}\cdot\text{s}$$

$$J = Ft$$
$$-13.05 = F(0.003)$$
$$F = \frac{-13.05}{0.003} = -4350\,\text{N}$$

The negative values for force and impulse mean that they are directed toward center field, which makes sense.

Answers: $\vec{J} = 13.05\,\text{N}\cdot\text{s}$ toward center field; $\vec{F} = 4\,350\,\text{N}$ toward center field

Use this space for summary and/or additional notes:

Homework Problems

1. **(S)** A teacher who was standing still in a corridor was run into by a student rushing to class. The teacher has a mass of 100. kg and the collision lasted for 0.020 s. After the collision, the teacher's velocity was $0.67\,\frac{\text{m}}{\text{s}}$. What were the impulse and force applied to the teacher?

   Answer: impulse: $67\,\text{N}\cdot\text{s}$; force: $3\,350\,\text{N}$

2. **(M)** An 800 kg car travelling at $10\,\frac{\text{m}}{\text{s}}$ comes to a stop in 0.50 s in an accident.

   a. **(M)** What was the impulse applied to the car?

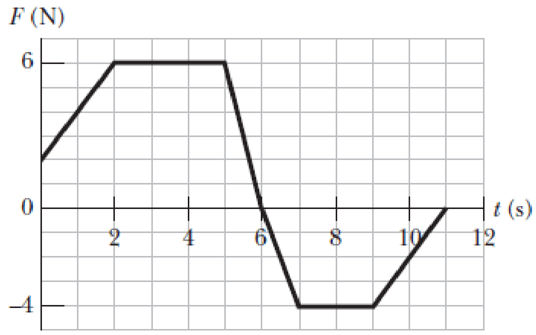
      Answer: $-8\,000\,\text{N}\cdot\text{s}$

   b. **(M)** What was the average net force on the car as it came to a stop?

      Answer: $-16\,000\,\text{N}$

Use this space for summary and/or additional notes:

3. Force is applied to a 2.0 kg block on a frictionless surface, as shown on the graph below.



At time t = 0, the block has a velocity of $+3.0\,\frac{\text{m}}{\text{s}}$.

a. **(M)** What is the momentum of the block at time t = 0?

Answer: $6\,\text{N}\cdot\text{s}$

b. **(S)** What is the impulse applied to the block during the interval from 0–2 s? What are the momentum and velocity of the block at time t = 2 s?

Answer: $\vec{J} = +8.0\,\text{N}\cdot\text{s}$; $\vec{p} = +14.0\,\text{N}\cdot\text{s}$; $\vec{v} = +7.0\,\frac{\text{m}}{\text{s}}$

c. **(M)** What is the impulse applied to the block during the interval from
0–6 s? What are the momentum and velocity of the block at time t = 6 s?

Answer: $\vec{J} = +29.0\,\text{N}\cdot\text{s}$; $\vec{p} = +35.0\,\text{N}\cdot\text{s}$; $\vec{v} = +17.5\,\frac{\text{m}}{\text{s}}$

d. **(S)** What is the impulse applied to the block during the interval from 6–11 s? What are the momentum and velocity of the block at time t = 11 s?

Answer: $\vec{J} = -14.0\,\text{N}\cdot\text{s}$; $\vec{p} = +21\,\text{N}\cdot\text{s}$; $\vec{v} = +10.5\,\frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

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4. **(S – honors & AP®; A – CP1)** A ball with mass *m* was dropped from a height h_o above the ground. It rebounded to a height of *h*. The contact between the ball and the ground lasted for time *t*.
[*Hint: this problem combines impulse and conservation of energy.*]

(*If you are not sure how to do this problem, do #5 below and use the steps to guide your algebra.*)

a. **(S – honors & AP®; A – CP1)** What was the impulse applied to the ball?

Answer: $\vec{\boldsymbol{J}} = \Delta\vec{\boldsymbol{p}} = m\sqrt{2g}\left(\sqrt{\vec{\boldsymbol{h}}_o} - \sqrt{\vec{\boldsymbol{h}}}\right)$

b. **(S – honors & AP®; A – CP1)** What was the average net force on the ball?

Answer: $F = \dfrac{m\sqrt{2g}\left(\sqrt{\vec{\boldsymbol{h}}_o} - \sqrt{\vec{\boldsymbol{h}}}\right)}{t}$

5. **(S)** A 0.80 kg ball was dropped from a height of 2.0 m above the ground. It rebounded to a height of 1.6 m. The contact between the ball and the ground lasted for 0.045 s.
[*Hint: this problem combines impulse and conservation of energy.*]

(*<u>You must start with the equations in your</u>* <u>Physics Reference Tables</u> *<u>and show all of the steps of GUESS.</u> You may only use the answer to question #4 above as a starting point if you have already solved that problem.*)

a. **(S)** What was the impulse applied to the ball?

Answer: $9.59\,\text{N}\cdot\text{s}$

b. **(S)** What was the average net force on the ball?

Answer: $213\,\text{N}$ upwards

Use this space for summary and/or additional notes:

Conservation of Linear Momentum

Unit: Momentum

MA Curriculum Frameworks (2016): HS-PS2-2

AP® Physics 1 Learning Objectives: 5.A.2.1, 5.D.1.1, 5.D.1.3, 5.D.1.4, 5.D.1.5, 5.D.2.1, 5.D.2.2, 5.D.2.3, 5.D.2.4, 5.D.2.5, 5.D.3.1

Mastery Objective(s): (Students will be able to…)

- Solve problems involving collisions in which momentum is conserved, with or without an external impulse.

Success Criteria:

- Masses and velocities are correctly identified for each object, both before and after the collision.
- Variables are correctly identified and substituted correctly into the correct part of the equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what happens before, during, and after a collision from the point of view of one of the objects participating in the collision.

Tier 2 Vocabulary: momentum, collision

Labs, Activities & Demonstrations:

- Collisions on air track.
- "Happy" and "sad" balls knocking over a board.
- Students riding momentum cart.

Notes:

<u>collision</u>: when two or more objects come together and hit each other.

<u>elastic collision</u>: a collision in which the objects bounce off each other (remain separate) after they collide, without any loss of kinetic energy.

<u>inelastic collision</u>: a collision in which the objects remain together after colliding. In an inelastic collision, total energy is still conserved, but some of the energy is changed into other forms, so the amount of kinetic energy is different before *vs.* after the collision.

Any macroscopic collision (meaning a collision between objects that are larger than individual atoms or molecules) will convert some of the kinetic energy into internal energy and other forms of energy. This means that no large-scale impacts are ever perfectly elastic.

Use this space for summary and/or additional notes:

Conservation of Momentum

Recall that in physics, if a quantity is "conserved", that means when some change happens to a system, there is the same amount of that quantity after the change as there was before.

In a closed system in which objects are free to move before and after a collision, momentum is ***conserved***. This means that unless there is an outside force, *the combined momentum of all of the objects after they collide is equal to the combined momentum of all of the objects before the collision*.

Solving Conservation of Momentum Problems

In plain English, the conservation of momentum law means that the total momentum before a collision, plus any momentum that we add (positive or negative impulse), must add up to the total momentum after.

In equation form, the conservation of momentum looks like this:

$$\textbf{Before} + \textbf{Impulse} = \textbf{After}$$
$$\sum \vec{\boldsymbol{p}}_i + \vec{\boldsymbol{J}} = \sum \vec{\boldsymbol{p}}_f$$
$$\sum \vec{\boldsymbol{p}}_i + \Delta\vec{\boldsymbol{p}} = \sum \vec{\boldsymbol{p}}_f$$
$$\sum m\vec{\boldsymbol{v}}_i + \Delta\vec{\boldsymbol{p}} = \sum m\vec{\boldsymbol{v}}_f$$

The symbol $\sum$ is the Greek capital letter "sigma". In mathematics, the symbol $\sum$ means "summation". $\sum \vec{\boldsymbol{p}}$ means the sum of the momentums. The subscript "*i*" means initial (before the collision), and the subscript "*f*" means final (after the collision). In plain English, $\sum \vec{\boldsymbol{p}}$ means find each individual value of $\vec{\boldsymbol{p}}$ (positive or negative, depending on the direction) and then add them all up to find the total.

In the last step, we replaced each $\vec{\boldsymbol{p}}$ with $m\vec{\boldsymbol{v}}$, because we are usually given the masses and velocities in collision problems.

(Note that most momentum problems do not mention the word "momentum." The problems usually give information about masses and velocities before and after some sort of collision, and it is up to you to realize that any problem involving collisions is almost always a conservation of momentum problem.)

The problems that we will see in this course involve two objects. These objects will either bounce off each other and remain separate (elastic collision), or they will either start out or end up together (inelastic collision).

Use this space for summary and/or additional notes:

Elastic Collisions

An elastic collision occurs when two or more object come together in a collision and then separate. There are the same number of separate objects before and after the collision.

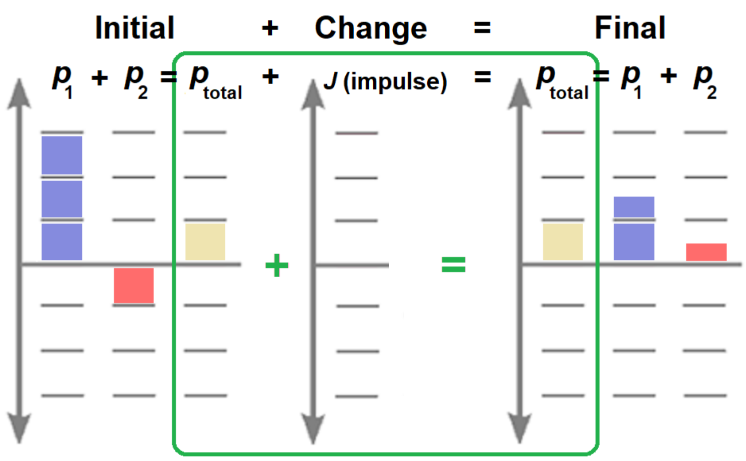
As stated above, the equation for the conservation of momentum in an elastic collision is:

$$\textbf{Before} = \textbf{After}$$
$$\sum \vec{\boldsymbol{p}}_i + \vec{\boldsymbol{J}} = \sum \vec{\boldsymbol{p}}_f$$
$$\vec{\boldsymbol{p}}_{1,i} + \vec{\boldsymbol{p}}_{2,i} + \vec{\boldsymbol{J}} = \vec{\boldsymbol{p}}_{1,f} + \vec{\boldsymbol{p}}_{2,f}$$
$$m_1\vec{\boldsymbol{v}}_{1,i} + m_2\vec{\boldsymbol{v}}_{2,i} + \vec{\boldsymbol{J}} = m_1\vec{\boldsymbol{v}}_{1,f} + m_2\vec{\boldsymbol{v}}_{2,f}$$

Notice that we have two subscripts after each "$\vec{\boldsymbol{p}}$" and each "$\vec{\boldsymbol{v}}$", because we have two separate things to keep track of. The "1" and "2" mean object #1 and object #2, and the "*i*" and "*f*" mean "initial" and "final".

Notice also that there are six variables: the two masses $(m_1$ and $m_2)$, and the four velocities $(\vec{\boldsymbol{v}}_{1,i}, \vec{\boldsymbol{v}}_{2,i}, \vec{\boldsymbol{v}}_{1,f}$ and $\vec{\boldsymbol{v}}_{2,f})$. In a typical problem, you will be given five of these six values and use algebra to solve for the remaining one.

The following momentum bar chart is for an elastic collision. Imagine that two objects are moving in opposite directions and then collide. There is no external force on the objects, so there is no impulse.



Before the collision, the first object has a momentum of +3 N·s, and the second has a momentum of −1 N·s. The total momentum is therefore +3 + (−1) = +2 N·s.

Because there are no forces changing the momentum of the system, the final momentum must also be +2 N·s. If we are told that the first object has a momentum of +1.5 N·s after the collision, we can subtract the +1.5 N·s from the total, which means the second object must have a momentum of +0.5 N·s.

Use this space for summary and/or additional notes:

Inelastic Collisions

An inelastic collision occurs either when two or more objects come together in a collision and remain together, or when one object separates into two or more objects with different velocities (*i.e.,* moving with different speeds and/or directions).

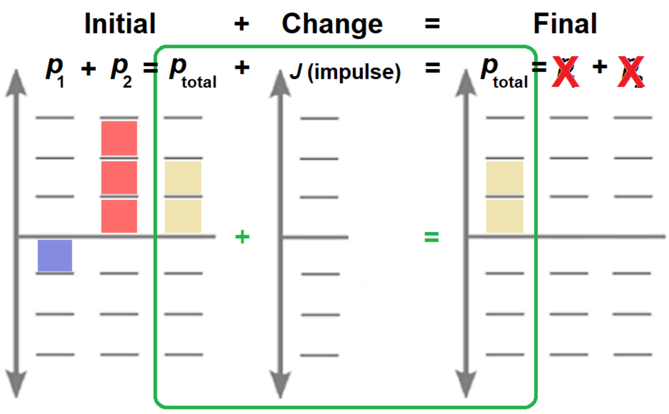
The law of conservation of momentum for an inelastic collision (with no impulse) is either:

$$\begin{gathered}\textbf{Before} = \textbf{After}\\ \sum \vec{p}_i + \vec{J} = \sum \vec{p}_f\\ \sum m\vec{v}_i + \vec{J} = \sum m\vec{v}_f\\ m_1\vec{v}_{1,i} + m_2\vec{v}_{2,i} + \vec{J} = m_T\vec{v}_f\end{gathered} \quad \text{or} \quad \begin{gathered}\textbf{Before} = \textbf{After}\\ \sum \vec{p}_i + \vec{J} = \sum \vec{p}_f\\ \sum m\vec{v}_i + \vec{J} = \sum m\vec{v}_f\\ m_T\vec{v}_i + \vec{J} = m_1\vec{v}_{1,f} + m_2\vec{v}_{2,f}\end{gathered}$$

Again we have two subscripts after each "$\vec{p}$" and each "$\vec{v}$", because we have two separate things to keep track of. The "1" and "2" mean object #1 and object #2, and "T" means total (when they are combined). The "*i*" and "*f*" mean "initial" and "final" as before

This time there are five variables: the two masses $(m_1$ and $m_2)$, and the three velocities (either $\vec{v}_{1,i}$ & $\vec{v}_{2,i}$ and $\vec{v}_f$ or $\vec{v}_i$ and $\vec{v}_{1,f}$ & $\vec{v}_{2,f}$). In a typical problem, you will be given four of these five values and use algebra to solve for the remaining one. (Remember that $m_1 + m_2 = m_T$).

The following momentum bar chart is for an inelastic collision. Two objects are moving in the same direction, and then collide.



Before the collision, the first object has a momentum of −1 N·s, and the second has a momentum of +3 N·s. The total momentum before the collision is therefore −1 + (+3) = +2 N·s.

There is no external force (*i.e.,* no impulse), so the total final momentum must still be +2 N·s. Because the objects remain together after the collision, the total momentum is the momentum of the combined objects.

Use this space for summary and/or additional notes:

Sample Problems:

Q: An object with a mass of 8.0 kg moving with a velocity of $+5.0\,\frac{\text{m}}{\text{s}}$ collides with a stationary object with a mass of 12 kg. If the two objects stick together after the collision, what is their velocity?

A: The momentum of the moving object before the collision is:

$$\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}} = (8.0)(+5.0) = +40\,\text{N}\cdot\text{s}$$

The stationary object has a momentum of zero, so the total momentum of the two objects combined is +40 N·s.

After the collision, the total mass is 8.0 kg + 12 kg = 20 kg. The momentum after the collision must still be +40 N·s, which means the velocity is:

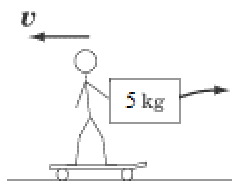
$$\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}} \qquad 40 = 20\vec{\boldsymbol{v}} \qquad \vec{\boldsymbol{v}} = +2\,\tfrac{\text{m}}{\text{s}}$$

Using the equation, we would solve this as follows:

$$\begin{aligned}
\textbf{Before} &= \textbf{After}\\
\vec{\boldsymbol{p}}_{1,i} + \vec{\boldsymbol{p}}_{2,i} &= \vec{\boldsymbol{p}}_f\\
m_1\vec{\boldsymbol{v}}_{1,i} + m_2\vec{\boldsymbol{v}}_{2,i} &= m_T\vec{\boldsymbol{v}}_f\\
(8)(5) + (12)(0) &= (8+12)\,\vec{\boldsymbol{v}}_f\\
40 &= 20\,\vec{\boldsymbol{v}}_f\\
\vec{\boldsymbol{v}}_f &= \frac{40}{20} = +2\,\tfrac{\text{m}}{\text{s}}
\end{aligned}$$

Use this space for summary and/or additional notes:

Q: Stretch* has a mass of 60. kg and is holding a 5.0 kg box as they ride on a skateboard toward the west at a speed of $3.0\,\frac{\text{m}}{\text{s}}$. (Assume the 60. kg is the mass of Stretch and the skateboard combined.) Stretch throws the box behind them, giving the box a velocity of $2.0\,\frac{\text{m}}{\text{s}}$ to the east. What is Stretch's velocity after throwing the box?



A: This problem is like an inelastic collision in reverse; Stretch and the box are together before the "collision" and apart afterwards. The equation would therefore look like this:

$$m_T\vec{\boldsymbol{v}}_i = m_s\vec{\boldsymbol{v}}_{s,f} + m_b\vec{\boldsymbol{v}}_{b,f}$$

Where the subscript "*s*" is for Stretch, and the subscript "*b*" is for the box. Note that after Stretch throws the box, they are moving one direction and the box is moving the other, which means we need to be careful about our signs. Let's choose the direction Stretch is moving (west) to be positive. Because the box is thrown to the east, this means the final velocity of the box will be:

$$\vec{\boldsymbol{v}}_{b,f} = -2.0\,\tfrac{\text{m}}{\text{s}}$$

Plugging values from the problem into the equation for the law of conservation of momentum, we get:

$$\begin{aligned}
\textbf{BEFORE} &= \textbf{AFTER}\\
\vec{\boldsymbol{p}}_i &= \vec{\boldsymbol{p}}_{s,f} + \vec{\boldsymbol{p}}_{b,f}\\
m_T\vec{\boldsymbol{v}}_i &= m_s\vec{\boldsymbol{v}}_{s,f} + m_b\vec{\boldsymbol{v}}_{b,f}\\
(60+5)(+3) &= 60\,\vec{\boldsymbol{v}}_{s,f} + (5)(-2)\\
+195 &= 60\,\vec{\boldsymbol{v}}_{s,f} + (-10)\\
+205 &= 60\,\vec{\boldsymbol{v}}_{s,f}\\
\vec{\boldsymbol{v}}_{s,f} &= \frac{+205}{60} = +3.4\,\tfrac{\text{m}}{\text{s}}
\end{aligned}$$

* The stick figure is called "Stretch" because the author is terrible at drawing, and most of his stick figures have a body part that is stretched out.

Use this space for summary and/or additional notes:

Q: A soccer ball that has a mass of 0.43 kg is rolling *east* with a velocity of $5.0\,\frac{m}{s}$. It collides with a volleyball that has a mass of 0.27 kg that is rolling *west* with a velocity of $6.5\,\frac{m}{s}$. After the collision, the soccer ball is rolling to the *west* with a velocity of $3.87\,\frac{m}{s}$. Assuming the collision is perfectly elastic and friction between both balls and the ground is negligible, what is the velocity (magnitude and direction) of the volleyball after the collision?

A: This is an elastic collision, so the soccer ball and the volleyball are separate both before and after the collision. The equation is:

$$m_s\vec{\boldsymbol{v}}_{s,i} + m_v\vec{\boldsymbol{v}}_{v,i} = m_s\vec{\boldsymbol{v}}_{s,f} + m_v\vec{\boldsymbol{v}}_{v,f}$$

Where the subscript "*s*" is for the soccer ball and the subscript "*v*" is for the volleyball. In all elastic collisions, assume we need to keep track of the directions, which means we need to be careful about our signs. We don't know which direction the volleyball will be moving after the collision (though a good guess would be that it will probably bounce off the soccer ball and move to the east). So let us arbitrarily choose east to be positive and west to be negative. This means:

quantity	**direction**	**value**
initial velocity of soccer ball	east	$+5.0\,\frac{m}{s}$
initial velocity of volleyball	west	$-6.5\,\frac{m}{s}$
final velocity of soccer ball	west	$-3.87\,\frac{m}{s}$

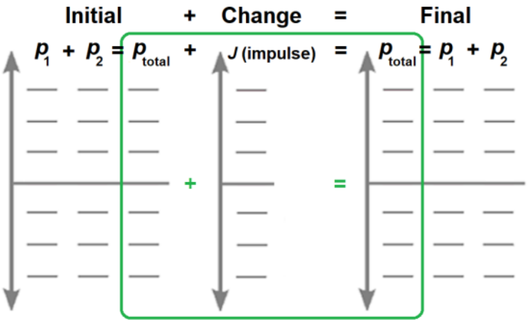
Plugging values from the problem into the equation for the law of conservation of momentum, we get:

$$\begin{aligned}
\textbf{Before} &= \textbf{After}\\
\vec{\boldsymbol{p}}_{s,i} + \vec{\boldsymbol{p}}_{v,i} &= \vec{\boldsymbol{p}}_{s,f} + \vec{\boldsymbol{p}}_{v,f}\\
m_s\vec{\boldsymbol{v}}_{s,i} + m_v\vec{\boldsymbol{v}}_{v,i} &= m_s\vec{\boldsymbol{v}}_{s,f} + m_v\vec{\boldsymbol{v}}_{v,f}\\
(0.43)(5.0)+(0.27)(-6.5) &= (0.43)(-3.87)+(0.27)\,\vec{\boldsymbol{v}}_{v,f}\\
2.15+(-1.755) &= -1.664+0.27\,\vec{\boldsymbol{v}}_{v,f}\\
0.395 &= -1.664+0.27\,\vec{\boldsymbol{v}}_{v,f}\\
2.059 &= 0.27\,\vec{\boldsymbol{v}}_{v,f}\\
\vec{\boldsymbol{v}}_{s,f} &= \frac{+2.059}{0.27} = +7.63\,\tfrac{m}{s} \text{ or } 7.63\,\tfrac{m}{s} \text{ to the east.}
\end{aligned}$$

Use this space for summary and/or additional notes:

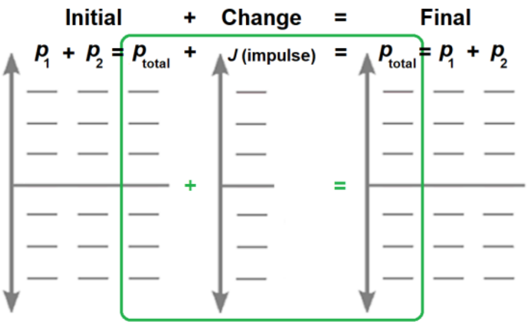
Homework Problems

1. **(M)** A turkey toss is a bizarre "sport" in which a person tries to catch a frozen turkey that is thrown through the air. A frozen turkey has a mass of 10. kg, and a 70. kg person jumps into the air to catch it. If the turkey was moving at $4.0\,\frac{\text{m}}{\text{s}}$ and the person was stationary before catching it, how fast will the person be moving after catching the frozen turkey?



Answer: $0.5\,\frac{\text{m}}{\text{s}}$

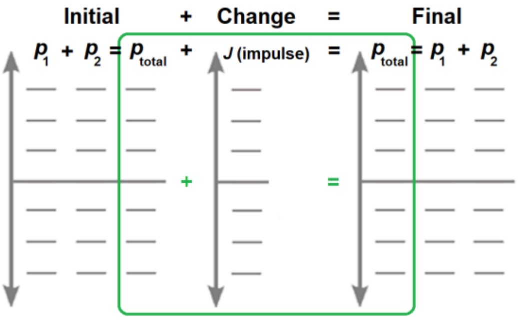
2. **(S)** A 6.0 kg bowling ball moving at $3.5\,\frac{\text{m}}{\text{s}}$ toward the back of the alley makes a collision, head-on, with a stationary 0.70 kg bowling pin. If the ball is moving $2.77\,\frac{\text{m}}{\text{s}}$ toward the back of the alley after the collision, what will be the velocity (magnitude and direction) of the pin?



Answer: $6.25\,\frac{\text{m}}{\text{s}}$ toward the back of the alley

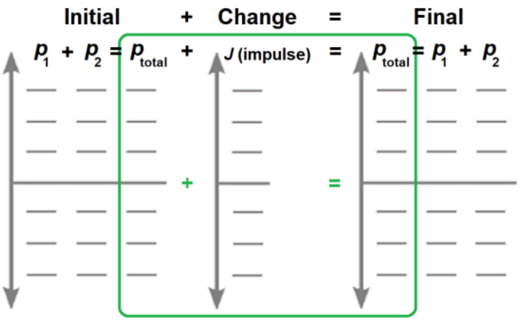
Use this space for summary and/or additional notes:

3. **(S)** An 80 kg student is standing on a cart on wheels that has a mass of 40 kg. If the student jumps off with a velocity of $+3\,\frac{\text{m}}{\text{s}}$, what will the velocity of the cart be?



Answer: $-6\,\frac{\text{m}}{\text{s}}$

4. **(M)** A 730 kg Mini (small car) runs into a stationary 2 500 kg sport utility vehicle (large car). If the Mini was moving at $10.\,\frac{\text{m}}{\text{s}}$ initially, how fast will it be moving after making a completely inelastic collision with the SUV?



Answer: $2.3\,\frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

honors & AP®

5. **(S – honors & AP®; A – CP1)** A billiard ball with mass m_b makes an elastic collision with a cue ball with mass m_c. Before the collision, the billiard ball was moving with a velocity of $\vec{v}_{b,i}$, and the cue ball was moving with a velocity of $\vec{v}_{c,i}$. After the collision, the cue ball is now moving with a velocity of $\vec{v}_{c,f}$. What is the velocity of the billiard ball after the collision?

   (*If you are not sure how to do this problem, do #6 below and use the steps to guide your algebra.*)

   Answer: $\vec{v}_{b,f} = \dfrac{m_b\vec{v}_{b,i} + m_c\vec{v}_{c,i} - m_c\vec{v}_{c,f}}{m_b}$

6. **(S)** A billiard ball with a mass of 0.16 kg makes an elastic collision with a cue ball with a mass of 0.17 kg. Before the collision, the billiard ball was moving with a velocity of $0.50\,\frac{\text{m}}{\text{s}}$ to the east, and the cue ball was moving with a velocity of $1.0\,\frac{\text{m}}{\text{s}}$ to the west. After the collision, the cue ball is now moving with a velocity of $0.515\,\frac{\text{m}}{\text{s}}$ to the east. What is the velocity (magnitude and direction) of the billiard ball after the collision?

   Hint: Remember that east and west are opposite directions; one of them will be negative.

   (*<u>You must start with the equations in your</u>* <u>Physics Reference Tables</u> *<u>and show all of the steps of GUESS.</u> You may only use the answer to question #5 above as a starting point if you have already solved that problem.*)

   Answer: $1.11\,\frac{\text{m}}{\text{s}}$ to the west

Use this space for summary and/or additional notes:

Angular Momentum

Unit: Momentum

MA Curriculum Frameworks (2016): N/A

AP Physics 1 Learning Objectives: 3.F.3.1, 3.F.3.2, 3.F.3.3, 4.D.1.1, 4.D.1.2, 4.D.2.1, 4.D.2.2, 4.D.3.1, 4.D.3.2, 5.E.1.1, 5.E.1.2, 5.E.2.1

Mastery Objective(s): (Students will be able to…)

- Explain and apply the principle of conservation of angular momentum.

Success Criteria:

- Explanation takes into account the factors affecting the angular momentum of an object before and after some change.

Language Objectives:

- Explain what happens when linear momentum is converted to angular momentum or *vice versa*.

Tier 2 Vocabulary: momentum

Labs, Activities & Demonstrations:

- Try to change the direction of rotation of a bicycle wheel.
- Spin on a turntable with weights at arm's length.
- Sit on a turntable with a spinning bicycle wheel and invert the wheel.

Notes:

angular momentum ($\vec{\boldsymbol{L}}$) : the momentum of a rotating object in the direction of rotation. Angular momentum is the property of an object that resists changes in the speed or direction of rotation. Angular momentum is measured in units of $\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$.

Just as linear momentum is the product of mass (linear inertia) and (linear) velocity, angular momentum is also the product of the moment of inertia (rotational inertia) and angular (rotational) velocity:

$\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}}$	$\vec{\boldsymbol{L}} = I\vec{\boldsymbol{\omega}}$ *
linear	**rotational**

* CP1 and honors physics students are responsible only for a qualitative understanding of angular momentum. AP® Physics students need to solve quantitative problems.

Use this space for summary and/or additional notes:

AP®

Angular momentum can also be converted to linear momentum, and *vice versa*. Angular momentum is the cross-product of radius and linear momentum:

$$\vec{L} = \vec{r} \times \vec{p} = rp\sin\theta$$

E.g., if you shoot a bullet into a door:

1. As soon as the bullet embeds itself in the door, it is constrained to move in an arc, so the linear momentum of the bullet becomes angular momentum.
2. The total angular momentum of the bullet just before impact equals the total angular momentum of the bullet and door after impact.

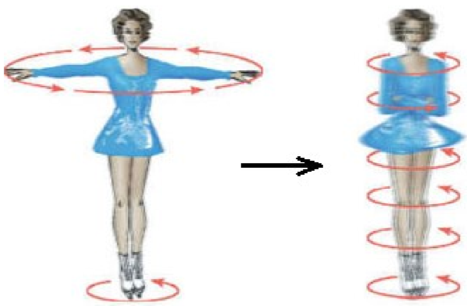
Just as a force produces a change in linear momentum, a torque produces a change in angular momentum. The net external torque on an object is its change in angular momentum with respect to time:

$$\vec{\tau}_{net} = \frac{\Delta\vec{L}}{t} = \frac{d\vec{L}}{dt} \quad \text{and} \quad \Delta\vec{L} = \vec{\tau}_{net}t$$

Conservation of Angular Momentum

Just as linear momentum is conserved unless an external force is applied, angular momentum is conserved unless an external torque is applied. This means that the total angular momentum before some change (that occurs entirely within the system) must equal the total angular momentum after the change.

An example of this occurs when a person spinning (*e.g.,* an ice skater) begins the spin with arms extended, then pulls the arms closer to the body. This causes the person to spin faster. (In physics terms, it increases the angular velocity, which means it causes angular acceleration.)



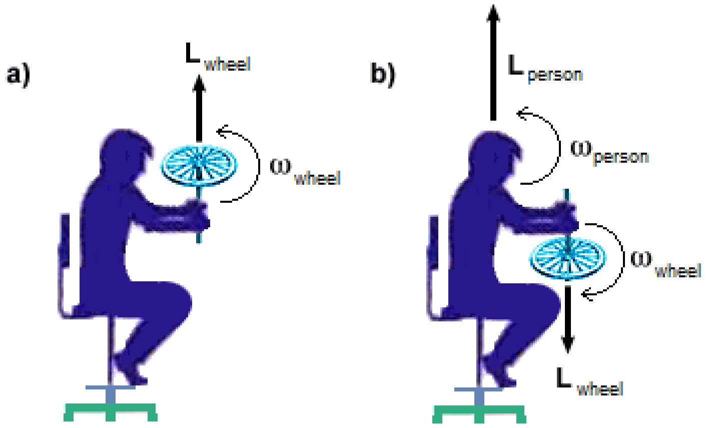
When the skater's arms are extended, the moment of inertia of the skater is greater (because there is more mass farther out) than when the arms are close to the body. Conservation of angular momentum tells us that:

$$L_i = L_f$$
$$I_i\omega_i = I_f\omega_f$$

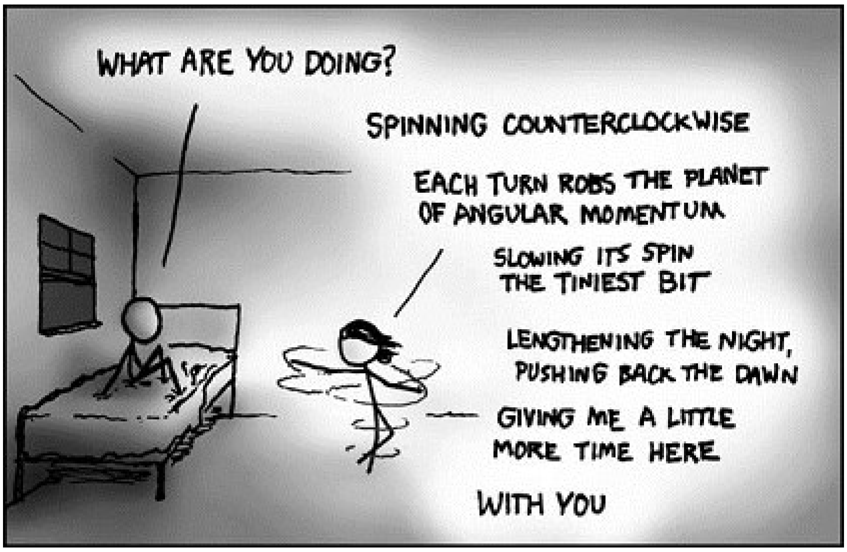
I.e., if I decreases, then ω must increase.

Use this space for summary and/or additional notes:

Another popular example, which shows the vector nature of angular momentum, is the demonstration of a person holding a spinning bicycle wheel on a rotating chair. The person then turns over the bicycle wheel, causing it to rotate in the opposite direction:



Initially, the direction of the angular momentum vector of the wheel is upwards. When the person turns over the wheel, the angular momentum of the wheel reverses direction. Because the person-wheel-chair system is an isolated system, the total angular momentum must be conserved. This means the person must rotate in the opposite direction as the wheel, so that the total angular momentum (magnitude and direction) of the person-wheel-chair system remains the same as before.



xkcd.com. Used with permission.

Use this space for summary and/or additional notes:

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Sample Problem:

Q: A "Long-Playing" (LP) phonograph record has a radius of 15 cm and a mass of 150 g. A typical phonograph could accelerate an LP from rest to its final speed in 0.35 s.

a. Calculate the angular momentum of a phonograph record (LP) rotating at $33\tfrac{1}{3}$ RPM.

b. What average torque would be exerted on the LP?

A: The angular momentum of a rotating body is $L = I\omega$. This means we need to find I (the moment of inertia) and ω (the angular velocity).

An LP is a solid disk, which means the formula for its moment of inertia is:

$$I = \tfrac{1}{2}mr^2$$

$$I = (\tfrac{1}{2})(0.15\,\text{kg})(0.15\,\text{m})^2 = 1.69\times10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$\omega = \frac{33\tfrac{1}{3}\,\text{rev}}{1\,\text{min}} \times \frac{1\,\text{min}}{60\,\text{s}} \times \frac{2\pi\,\text{rad}}{1\,\text{rev}} = 3.49\ \tfrac{\text{rad}}{\text{s}}$$

$$L = I\omega$$

$$L = (1.69\times10^{-3}\,\text{kg}\cdot\text{m}^2)(3.49\ \tfrac{\text{rad}}{\text{s}})$$

$$L = 5.89\times10^{-3}\ \tfrac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\tau = \frac{\Delta L}{\Delta t} = \frac{L - L_o}{\Delta t} = \frac{5.89\times10^{-3} - 0}{0.35} = 1.68\times10^{-2}\,\text{N}\cdot\text{m}$$

Use this space for summary and/or additional notes:

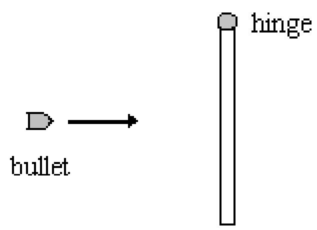
AP®

Homework Problems

1. **(M – AP®; A – honors & CP1)** A cylinder of mass 250 kg and radius 2.60 m is rotating at $4.00\,\frac{\text{rad}}{\text{s}}$ on a frictionless surface. A 500 kg cylinder of the same diameter is then placed on top of the cylinder. What is the new angular velocity?

   Answer: $1.33\,\frac{\text{rad}}{\text{s}}$

2. **(M – AP®; A – honors & CP1)** A solid oak door with a width of 0.75 m and mass of 50 kg is hinged on one side so that it can rotate freely. A bullet with a mass of 30 g is fired into the exact center of the door with a velocity of $400\,\frac{\text{m}}{\text{s}}$, as shown in the diagram at the right.

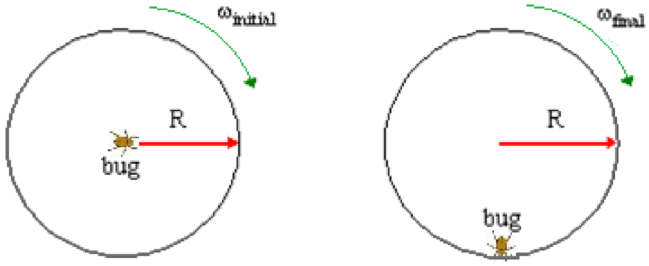
   What is the angular velocity of the door with respect to the hinge just after the bullet embeds itself in the door?
   (*Hint: Treat the bullet as a point mass. Consider the door to be a rod rotating about its end.*)

   Answer: $0.45\,\frac{\text{rad}}{\text{s}}$

Use this space for summary and/or additional notes:

AP®

Use the following diagram for questions #3 & 3 below



3. **(M – AP®; A – honors & CP1)** A bug with mass *m* crawls from the center to the outside edge of a disc of mass *M* and radius *r*, rotating with angular velocity ω, as shown in the diagram above.

   Write an expression for the angular velocity of the disc when the bug reaches the edge. You may use your work from problem #3 to guide your algebra. (*Hint: Treat the bug as a point mass.*)

   (*If you are not sure how to do this problem, do #4 below and use the steps to guide your algebra.*)

   Answer: $\omega_f = \dfrac{M\omega_i}{M+2m}$

4. **(S – AP®; A – honors & CP1)** A 12.5 g bug crawls from the center to the outside edge of a 130. g disc of radius 15.0 cm that is rotating at $11.0\,\frac{\text{rad}}{\text{s}}$, as shown in the diagram above.

   What will be the angular velocity of the disc when the bug reaches the edge? (*Hint: Treat the bug as a point mass.*)

   (*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #3 above as a starting point if you have already solved that problem.*)

   Answer: $9.23\,\frac{\text{rad}}{\text{s}}$

Use this space for summary and/or additional notes:

Introduction: Simple Harmonic Motion

Unit: Simple Harmonic Motion

Topics covered in this chapter:

This chapter discusses the physics of simple harmonic (repetitive) motion.

- *Simple Harmonic Motion* (SHM) describes the concept of repetitive back-and-forth motion and situations that apply to it.

- *Springs* and *Pendulums* describe specific examples of SHM and the specific equations relating to each.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

No MA Curriculum Frameworks are addressed in this chapter.

AP®

AP® Physics 1 Learning Objectives:

3.D.1.1: The student is able to justify the selection of data needed to determine the relationship between the direction of the force acting on an object and the change in momentum caused by that force. **[SP 4.1]**

3.B.3.1: The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. **[SP 6.4, 7.2]**

3.B.3.2: The student is able to design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force. **[SP 4.2]**

3.B.3.3: The student can analyze data to identify qualitative or quantitative relationships between given values and variables (*i.e.,* force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown. **[SP 2.2, 5.1]**

3.B.3.4: The student is able to construct a qualitative and/or a quantitative explanation of oscillatory behavior given evidence of a restoring force. **[SP 2.2, 6.2]**

Use this space for summary and/or additional notes:

AP®

5.B.2.1: The student is able to calculate the expected behavior of a system using the object model (*i.e.,* by ignoring changes in internal structure) to analyze a situation. Then, when the model fails, the student can justify the use of conservation of energy principles to calculate the change in internal energy due to changes in internal structure because the object is actually a system. **[SP 1.4, 2.1]**

5.B.3.1: The student is able to describe and make qualitative and/or quantitative predictions about everyday examples of systems with internal potential energy. **[SP 2.2, 6.4, 7.2]**

5.B.3.2: The student is able to make quantitative calculations of the internal potential energy of a system from a description or diagram of that system. **[SP 1.4, 2.2]**

5.B.3.3: The student is able to apply mathematical reasoning to create a description of the internal potential energy of a system from a description or diagram of the objects and interactions in that system. **[SP 1.4, 2.2]**

5.B.4.1: The student is able to describe and make predictions about the internal energy of systems. **[SP 6.4, 7.2]**

5.B.4.2: The student is able to calculate changes in kinetic energy and potential energy of a system, using information from representations of that system. **[SP 1.4, 2.1, 2.2]**

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Simple Harmonic Motion**, such as mass on a spring and the pendulum

1. Periodic Motion (Simple Harmonic Motion)
2. Frequency and Period
3. Springs
4. Pendulums

Skills learned & applied in this chapter:

- Understanding and representing repetitive motion.

Use this space for summary and/or additional notes:

Simple Harmonic Motion

Unit: Simple Harmonic Motion

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 3.B.3.1, 3.B.3.2, 3.B.3.3, 3.B.3.4

Mastery Objective(s): (Students will be able to…)

- Describe simple harmonic motion and explain the behaviors of oscillating systems such as springs & pendulums.

Success Criteria:

- Explanations are sufficient to predict the observed behavior.

Language Objectives:

- Explain why oscillating systems move back and forth by themselves.

Tier 2 Vocabulary: simple, harmonic

Labs, Activities & Demonstrations:

- Show & tell with springs & pendulums.

Notes:

<u>simple harmonic motion</u>: motion consisting of regular, periodic back-and-forth oscillation.

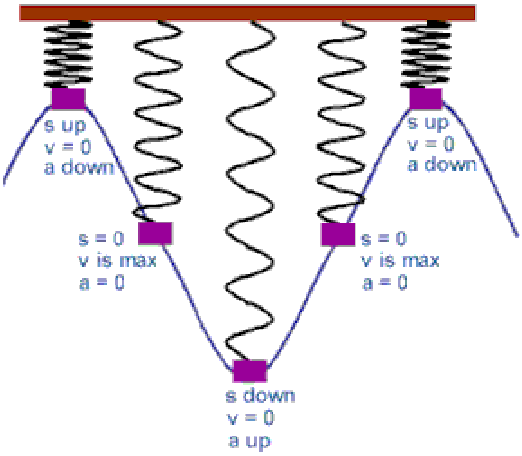
Requirements:

- The acceleration is always in the opposite direction from the displacement. This means the acceleration always slows down the motion and reverses the direction.
- In an ideal system (no friction), once simple harmonic motion is started, it would continue forever.
- A graph of displacement *vs.* time will result in the trigonometric function sine or cosine.

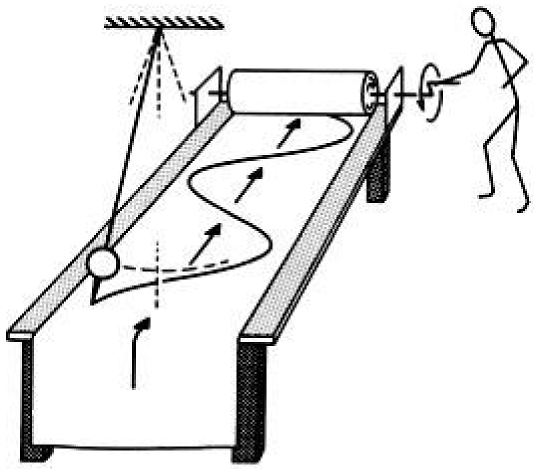
Use this space for summary and/or additional notes:

Examples of Simple Harmonic Motion

- **Springs**; as the spring compresses or stretches, the spring force accelerates it back toward its equilibrium position.



- **Pendulums**: as the pendulum swings, gravity accelerates it back toward its equilibrium position.



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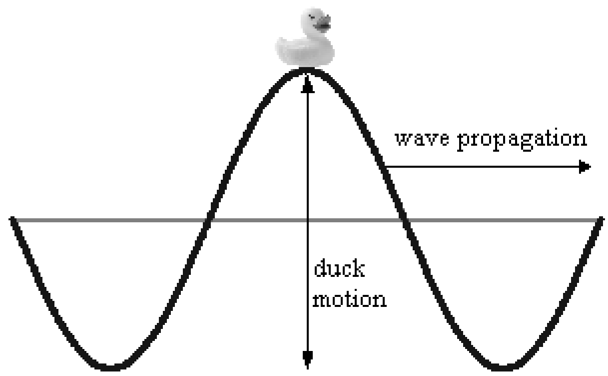
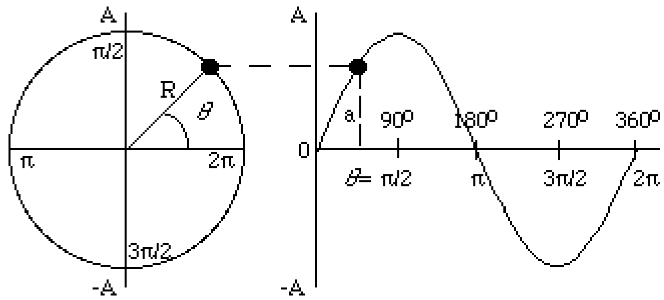
- **Waves**: waves passing through some medium (such as water or air) cause the medium to oscillate up and down, like a duck sitting on the water as waves pass by.



- **Uniform circular motion**: as an object moves around a circle, its vertical position (y-position) is continuously oscillating between $+r$ and $-r$.



Use this space for summary and/or additional notes:

AP®

Kinematics of Simple Harmonic Motion

As you can see from the uniform circular motion graph above, the *y*-position of an object in simple harmonic motion as a function of time is the function sine or cosine of the angle around the circle, depending on where you declare the starting position to be. From calculus, the general equations of periodic or oscillating motion are therefore:

position: $x = A\cos(\omega t + \phi)$

velocity: $v = -A\omega\sin(\omega t + \phi) = \frac{dx}{dt}$ *

acceleration: $a = -A\omega^2\cos(\omega t + \phi) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

where:

$x =$ displacement from the equilibrium point $x = 0$

$A =$ amplitude (maximum displacement)

$\omega =$ angular frequency

$t =$ time

$\phi =$ phase angle (offset)

The phase angle ϕ or offset is the position where the cycle starts, relative to the equilibrium (zero) point.

Because many simple harmonic motion problems (including AP® problems) are given in terms of the frequency of oscillation (number of oscillations per second), we can multiply the angular frequency by 2π to use *f* instead of ω, *i.e.*, $\omega = 2\pi f$.

On the AP® formula sheet, ϕ is assumed to be zero, resulting in the following version of the position equation:

$$x = A\cos(2\pi f t)$$

You are expected to be able to understand and use the position equation above, but simple harmonic problems that involve velocity and acceleration equations are beyond the scope of this course and have not been seen on the AP® exam.

* The derivatives $\frac{dx}{dt}$, $\frac{dv}{dt}$, and $\frac{d^2x}{dt^2}$ are from calculus. The velocity and acceleration equations are beyond the scope of the AP® Physics course.

Use this space for summary and/or additional notes:

Springs

Unit: Simple Harmonic Motion

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 3.B.3.1, 3.B.3.2, 3.B.3.3, 3.B.3.4, 5.B.2.1, 5.B.3.1, 5.B.3.2, 5.B.3.3, 5.B.4.1, 5.B.4.2

Mastery Objective(s): (Students will be able to…)

- Calculate the period of oscillation of a spring.
- Calculate the force from and potential energy stored in a spring.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what a spring constant measures.

Tier 2 Vocabulary: spring

Labs, Activities & Demonstrations:

- Spring mounted to lab stands with paper taped somewhere in the middle as an indicator.

Notes:

spring: a coiled object that resists motion parallel with the direction of propagation of the coil.

Spring Force

The equation for the force (vector) from a spring is given by Hooke's Law, named for the British physicist Robert Hooke:

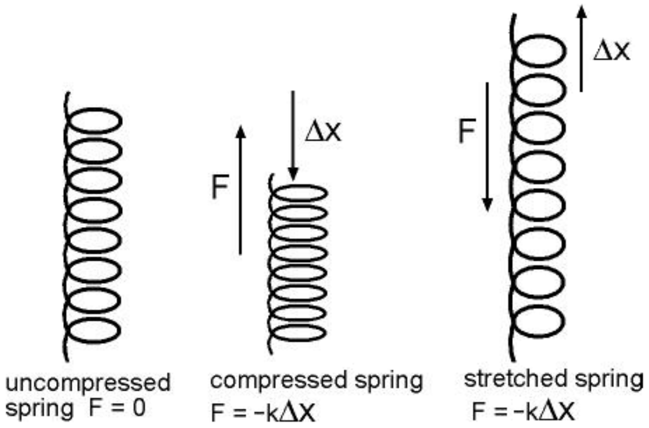
$$\vec{\boldsymbol{F}}_s = -k\vec{\boldsymbol{x}}$$

Where $\vec{\boldsymbol{F}}_s$ is the spring force (vector quantity representing the force exerted by the spring), $\vec{\boldsymbol{x}}$ is the displacement of the end of the spring (also a vector quantity), and k is the spring constant, an intrinsic property of the spring based on its mass, thickness, and the elasticity of the material that it is made of.

The negative sign in the equation is because the force is always in the opposite (negative) direction from the displacement.

Use this space for summary and/or additional notes:

A Slinky has a spring constant of $0.5\,\frac{N}{m}$, while a heavy garage door spring might have a spring constant of $500\,\frac{N}{m}$.



Potential Energy

The potential energy stored in a spring is given by the equation:

$$U = \tfrac{1}{2}kx^2$$

Where U is the potential energy (measured in joules), k is the spring constant, and x is the displacement. Note that the potential energy is always positive (or zero); this is because energy is a scalar quantity. A stretched spring and a compressed spring both have potential energy.

The Period of a Spring

<u>period</u> or <u>period of oscillation</u>: the time it takes a spring to move from its maximum displacement in one direction to its maximum displacement in the opposite direction and back again. The variable for the period is *T*, and the unit is usually seconds.

The period of a spring depends on the mass of the spring and its spring constant, and is given by the equation:

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

Use this space for summary and/or additional notes:

Frequency

frequency: the number of times something occurs in a given amount of time. Frequency is usually given by the variable f, and is measured in units of hertz (Hz). One hertz is the inverse of one second:

$$1\,\text{Hz} \equiv \frac{1}{1\,\text{s}} \equiv 1\,\text{s}^{-1}$$

Note that period and frequency are reciprocals of each other:

$$T = \frac{1}{f} \quad \text{and} \quad f = \frac{1}{T}$$

Sample Problem:

Q: A spring with a mass of 0.1 kg and a spring constant of $2.7\,\frac{\text{N}}{\text{m}}$ is compressed 0.3 m. Find the force needed to compress the spring, the potential energy stored in the spring when it is compressed, and the period of oscillation.

A: The force is given by Hooke's Law.

Substituting these values gives:

$$\vec{\boldsymbol{F}} = -k\vec{\boldsymbol{x}}$$
$$\vec{\boldsymbol{F}} = -(2.7\,\tfrac{\text{N}}{\text{m}})(+0.3\,\text{m}) = -0.81\,\text{N}$$

The potential energy is:

$$U = \tfrac{1}{2}kx^2$$
$$U = (0.5)(2.7\,\tfrac{\text{N}}{\text{m}})(0.3\,\text{m})^2 = 0.12\,\text{J}$$

The period is:

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$
$$T_s = (2)(3.14)\sqrt{\frac{0.1}{2.7}}$$
$$T_s = 6.28\sqrt{0.037} = (6.28)(0.19) = 1.2\,\text{s}$$

Use this space for summary and/or additional notes:

Homework Problems

1. **(M)** A 100.0 g mass is suspended from a spring whose constant is $50.0\,\frac{\text{N}}{\text{m}}$. The mass is then pulled down 1.0 cm and then released.

   a. **(M)** How much force was applied in order to pull the spring down the 1.0 cm?

   Answer: 0.5 N

   b. **(M)** What is the frequency of the resulting oscillation?

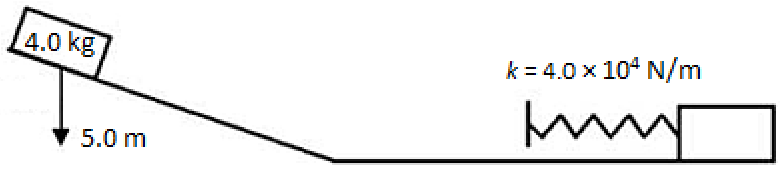
   Answer: 3.56 Hz

2. **(M)** A 1000. kg car bounces up and down on its springs once every 2.0 s. What is the spring constant of its springs?

   Answer: $9870\,\frac{\text{N}}{\text{m}}$

Use this space for summary and/or additional notes:

3. **(M – honors & AP®; S – CP1)** A 4.0 kg block is released from a height of 5.0 m on a frictionless ramp. When the block reaches the bottom of the ramp, it slides along a frictionless surface and hits a spring with a spring constant of $4.0\times10^{4}\ \frac{\text{N}}{\text{m}}$ as shown in the diagram below:



What is the maximum distance that the spring is compressed after the impact?

Answer: 0.10 m

4. **(S)** A 1.6 kg block is attached to a spring that has a spring constant of $1.0\times10^{3}\ \frac{\text{N}}{\text{m}}$. The spring is compressed a distance of 2.0 cm and the block is released from rest onto a frictionless surface. What is the speed of the block as it passes through the equilibrium position?

Answer: $0.5\ \frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

Pendulums

Unit: Simple Harmonic Motion

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: 3.B.3.1, 3.B.3.2, 3.B.3.3, 3.B.3.4, 5.B.2.1, 5.B.3.1, 5.B.3.2, 5.B.3.3, 5.B.4.1, 5.B.4.2

Mastery Objective(s): (Students will be able to…)

- Calculate the period of oscillation of a pendulum.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why the mass of the pendulum does not affect its period.

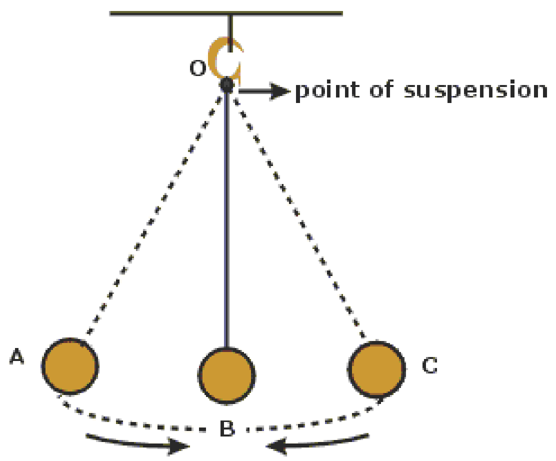
Tier 2 Vocabulary: pendulum

Labs, Activities & Demonstrations:

- Pendulum made from a mass hanging from a lab stand.

Notes:

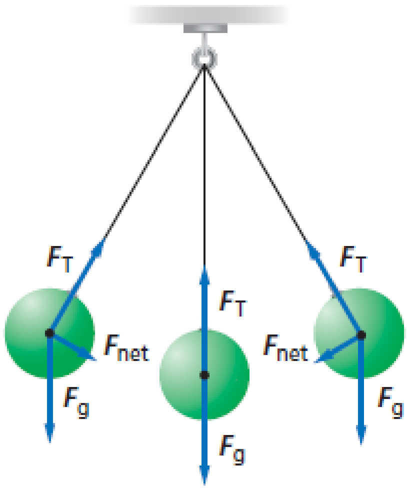
pendulum: a lever that is suspended from a point such that it can swing back and forth.



Use this space for summary and/or additional notes:

The Forces on a Pendulum

As the pendulum swings, its mass remains constant, which means the force of gravity pulling it down remains constant. The tension on the pendulum (which we can think of as a rope or string, though the pendulum can also be solid) also remains constant as it swings.



However, as the pendulum swings, the angle of the tension force changes. When the pendulum is not in the center (bottom), the vertical component of the tension is $F_T \cos\theta$, and the horizontal component is $F_T \sin\theta$. Because the angle is between 0° and 90°, $\cos\theta < 1$, which means F_g is greater than the upward component of F_T. This causes the pendulum to eventually stop. Also because the angle is between 0° and 90°, $\sin\theta > 0$, This causes the pendulum to start swinging in the opposite direction.

Use this space for summary and/or additional notes:

The Period of a Pendulum

period or period of oscillation: the time it takes a pendulum to travel from its maximum displacement in one direction to its maximum displacement in the opposite direction and back again. The variable for the period is T, and the unit is usually seconds.

Note that the time between pendulum "beats" (such as the tick-tock of a pendulum clock) are ½ of the period of the pendulum. Thus a "grandfather" clock with a pendulum that beats seconds has a period T = 2 s.

The period of a pendulum depends on the force of gravity, the length of the pendulum, and the maximum angle of displacement. For small angles (θ < 15°), the period is given by the equation:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

where T is the period of oscillation, ℓ is the length of the pendulum in meters, and g is the acceleration due to gravity (approximately $10\,\frac{\text{m}}{\text{s}^2}$ on Earth).

Note that the potential energy of a pendulum is simply the gravitational potential energy of the pendulum's center of mass.

The velocity of the pendulum at its lowest point (where the potential energy is zero and all of the energy is kinetic) can be calculated using conservation of energy.

Sample Problem:

Q: An antique clock has a pendulum that is 0.20 m long. What is its period?

A: The period is given by the equation:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$
$$T = 2(3.14)\sqrt{\frac{0.20}{10}}$$
$$T = 6.28\sqrt{0.02}$$
$$T = (6.28)(0.141)$$
$$T = 0.889\text{ s}$$

Use this space for summary and/or additional notes:

Homework Problems

1. **(S)** A 20.0 kg chandelier is suspended from a high ceiling with a cable 6.0 m long. What is its period of oscillation as it swings?

   Answer: 4.87 s

2. **(M)** What is the length of a pendulum that oscillates 24.0 times per minute?

   Answer: 1.58 m

3. **(M)** The ceiling in a physics classroom is approximately 3.6 m high. If a bowling ball pendulum reaches from the ceiling to the floor, how long does it take the bowling ball pendulum to swing across the room and back?

   Answer: 3.77 s

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Introduction: Electrostatics

Unit: Electrostatics

Topics covered in this chapter:

This chapter discusses electricity and magnetism, how they behave, and how they relate to each other.

- *Electric Charge* and *Coulomb's Law* describe the behavior of individual charged particles and their effects on each other.
- *Electric Fields* describes the behavior of an electric force field on charged particles.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-4. Use mathematical representations of ~~Newton's Law of Gravitation and~~ Coulomb's Law to describe and predict the ~~gravitational and~~ electrostatic forces between objects.

HS-PS3-1. Use algebraic expressions and the principle of energy conservation to calculate the change in energy of one component of a system when the change in energy of the other component(s) of the system, as well as the total energy of the system including any energy entering or leaving the system, is known. Identify any transformations from one form of energy to another, including thermal, kinetic, gravitational, magnetic, or electrical energy, in the system.

HS-PS3-2. Develop and use a model to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles and objects or energy stored in fields.

HS-PS3-5. Develop and use a model of magnetic or electric fields to illustrate the forces and changes in energy between two magnetically or electrically charged objects changing relative position in a magnetic or electric field, respectively.

AP® Physics 1 Learning Objectives:

This unit was removed from AP® Physics 1 starting with the 2021–22 school year. It is now part of the AP® Physics 2 curriculum.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Electric Fields, Forces, and Potentials**, such as Coulomb's law, induced charge, field and potential of groups of point charges, and charged particles in electric fields

1. Electric Charge
2. Electric Force
3. Electric Potential
4. Conductors and Insulators

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Electric Charge

Unit: Electrostatics

MA Curriculum Frameworks (2016): HS-PS3-5

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Describe properties of positive and negative electric charges.
- Describe properties of conductors and insulators.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why the mass of the pendulum does not affect its period.

Tier 2 Vocabulary: charge

Labs, Activities & Demonstrations:

- Charged balloon making hairs repel, attracting water molecules.
- Wimshurst machine.
- Van de Graaff generator.

Notes:

electric charge: a physical property of matter which causes it to experience a force when near other electrically charged matter. Electric charge is measured in coulombs (C).

positive charge: the type of charge carried by protons. Originally defined as the charge left on a piece of glass when rubbed with silk. The glass becomes positively charged because the silk pulls electrons off the glass.

negative charge: the type of charge carried by electrons. Originally defined as the charge left on a piece of amber (or rubber) when rubbed with fur (or wool). The amber becomes negatively charged because the amber pulls the electrons off the fur.

elementary charge: the magnitude (amount) of charge on one proton or one electron. One elementary charge equals 1.60×10^{-19} C. Because ordinary matter is made of protons and electrons, the amount of charge carried by any object must be an integer multiple of the elementary charge.

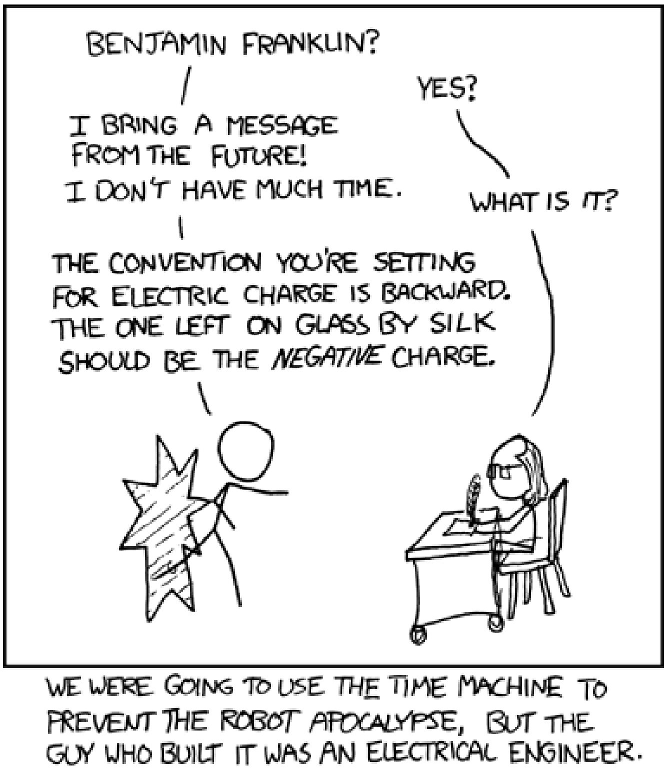
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Note however, that the quarks that protons and neutrons are made of carry fractional charges; up-type quarks carry a charge of +⅔ of an elementary charge, and down-type quarks carry a charge of −⅓ of an elementary charge. A proton is made of two up quarks and one down quark and carries a charge of +1 elementary charge. A neutron is made of one up quark and two down quarks and carries no charge.

static electricity: stationary electric charge, such as the charge left on silk or amber in the above definitions.

electric current (sometimes called electricity): the movement of electrons through a medium (substance) from one location to another. Note, however, that electric current is defined as the direction a positively charged particle would move. Thus electric current "flows" in the opposite direction from the actual electrons.



xkcd.com. Used with permission.

Some Devices that Produce, Use or Store Charge

capacitor: a device that stores electric charge.

battery: a device that uses chemical reactions to produce an electric current.

generator: a device that converts mechanical energy (motion) into an electric current.

motor: a device that converts an electric current into mechanical energy.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Conductors *vs.* Insulators

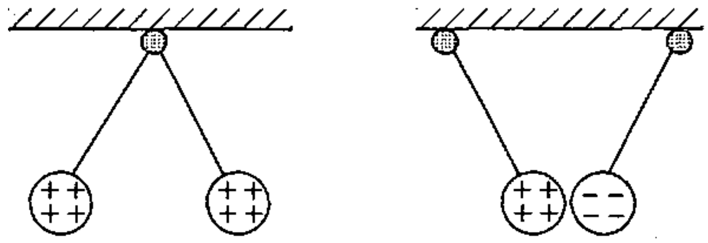
conductor: a material that allows charges to move freely through it. Examples of conductors include metals and liquids with positive and negative ions dissolved in them (such as salt water). When charges are transferred to a conductor, the charges distribute themselves evenly throughout the substance.

insulator: a material that does not allow charges to move freely through it. Examples of insulators include nonmetals and most pure chemical compounds (such as glass or plastic). When charges are transferred to an insulator, they cannot move, and remain where they are placed.

Behavior of Charged Particles

- **Like charges repel.** A pair of the same type of charge (two positive charges or two negative charges) exert a force that pushes the charges away from each other.

- **Opposite charges attract.** A pair of opposite types of charge (a positive charge and a negative charge) exert a force that pulls the charges toward each other.



- **Charge is conserved.** Electric charges cannot be created or destroyed, but can be transferred from one location or medium to another. (This is analogous to the laws of conservation of mass and energy.)

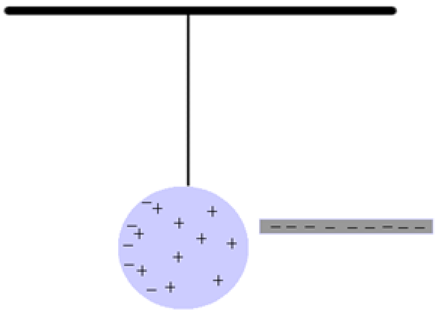
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Charging by Induction

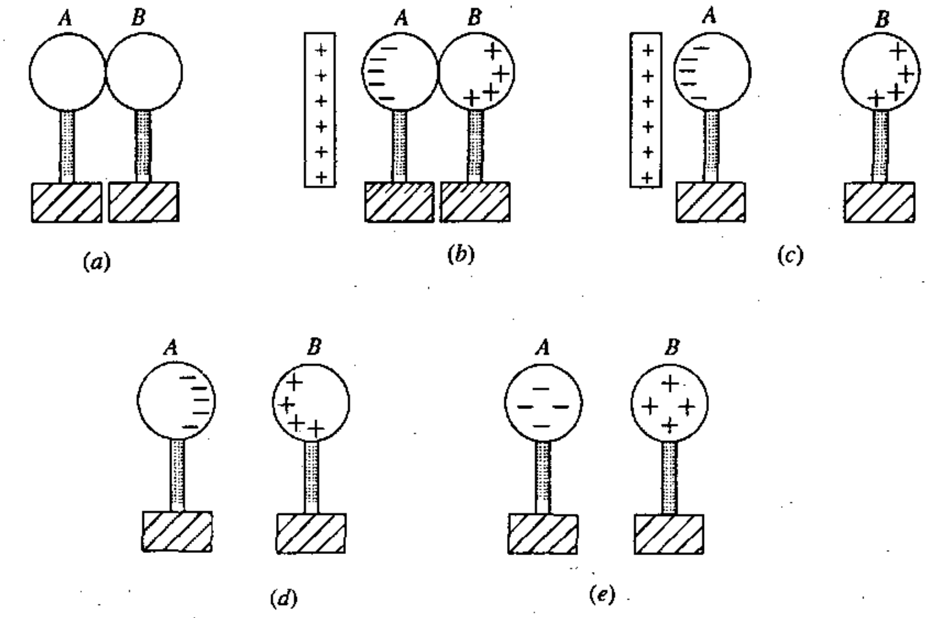
<u>induction</u>: when an electrical charge on one object causes a charge in a second object.

When a charged rod is brought near a neutral object, the charge on the rod attracts opposite charges and repels like charges that are near it. The diagram below shows a negatively-charged rod repelling negative charges.



If the negatively-charged rod above were touched to the sphere, some of the charges from the rod would be transferred to the sphere at the point of contact, and the sphere would acquire an overall negative charge.

A process for inducing charges in a pair of metal spheres is shown below:



a. Metal spheres *A* and *B* are brought into contact.

b. A positively charged object is placed near (but not in contact with) sphere *A*. This induces a negative charge in sphere *A*, which in turn induces a positive charge in sphere *B*.

c. Sphere *B* (which is now positively charged) is moved away.

d. The positively charged object is removed.

e. The charges distribute themselves throughout the metal spheres.

Use this space for summary and/or additional notes:

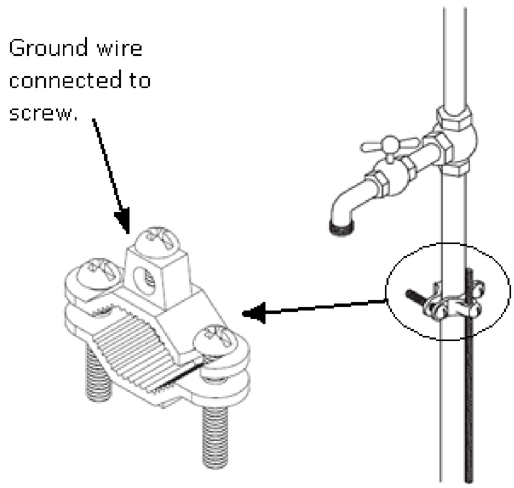
CP1 & honors (not AP®)

Grounding

For the purposes of our use of electric charges, the ground (Earth) is effectively an endless supply of both positive and negative charges. Under normal circumstances, if a charged object is touched to the ground, electrons will move to neutralize the charge, either by flowing from the object to the ground or from the ground to the object.

Grounding a charged object or circuit means neutralizing the electrical charge on an object or portion of the circuit. *The charge of any object that is connected to ground is zero, by definition.*

In buildings, the metal pipes that bring water into the building are often used to ground the electrical circuits. The metal pipe is a good conductor of electricity, and carries the unwanted charge out of the building and into the ground outside.



Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Coulomb's Law

Unit: Electrostatics

MA Curriculum Frameworks (2016): HS-PS2-4

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Solve problems using Coulomb's Law
- Quantitatively predict the effects on the electrostatic force when one of the variables (amount of electric charge or distance) in Coulomb's Law is changed.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how force and distance both affect the amount of force between two charged objects.

Tier 2 Vocabulary: charge

Labs, Activities & Demonstrations:

- Charged balloon or Styrofoam sticking to wall.
- Charged balloon pushing meter stick.
- Van de Graaff generator with negative electrode attached to inertia balance pan.

Notes:

Electric charge is measured in Coulombs (abbreviation "C"). One Coulomb is the amount of electric charge transferred by a current of 1 ampere for a duration of 1 second.

+1 C is the charge of 6.2415×10^{18} protons.
−1 C is the charge of 6.2415×10^{18} electrons.

A single proton or electron therefore has a charge of $\pm 1.6022 \times 10^{-19}$ C. This amount of charge is called the elementary charge, because it is the charge of one elementary particle.

An object can only have an integer multiple of this amount of charge, because it is impossible* to have a charge that is a fraction of a proton or electron.

* This is true for macroscopic objects. Certain quarks, which are the particles that protons and neutrons are made of, have charges of $\frac{1}{3}$ or $\frac{2}{3}$ of an elementary charge.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Because charged particles attract or repel each other, that attraction or repulsion must be a force, which can be measured and quantified. The force is directly proportional to the strengths of the charges, and inversely proportional to the square of the distance. The formula is:

$$F_e = \frac{kq_1q_2}{r^2}$$

where:

F_e = electrostatic force of repulsion between electric charges. A positive value of F_e denotes that the charges are repelling (pushing away from) each other; a negative value of F_e denotes that the charges are attracting (pulling towards) each other.

k = electrostatic constant = $8.9876 \times 10^9 \frac{\text{N·m}^2}{\text{C}^2}$, which we usually approximate as $9.0 \times 10^9 \frac{\text{N·m}^2}{\text{C}^2}$.

q_1 and q_2 = the charges on objects #1 and #2 respectively

r = distance (radius, because it goes outward in every direction) between the centers of the two charges

This formula is Coulomb's Law, named for its discoverer, the French physicist Charles-Augustin de Coulomb.

Sample problems:

Q: Find the force of electrostatic attraction between the proton and electron in a hydrogen atom if the radius of the atom is 37.1 pm

A: The charge of a single proton is 1.60×10^{-19} C, and the charge of a single electron is -1.60×10^{-19} C.

$$37.1\,\text{pm} = 3.71 \times 10^{-11}\,\text{m}$$

$$F_e = \frac{kq_1q_2}{r^2}$$

$$F_e = \frac{(8.99 \times 10^9)(1.60 \times 10^{-19})(-1.60 \times 10^{-19})}{(3.71 \times 10^{-11})^2}$$

$$F_e = -1.67 \times 10^{-7}\,\text{N}$$

The value of the force is negative, which signifies that the force is attractive.

Use this space for summary and/or additional notes:

honors (not AP®)

Q: Two charged particles, each with charge $+q$ (which means $q_1 = q_2 = q$) are separated by distance d. If the amount of charge on one of the particles is halved and the distance is doubled, what will be the effect on the force between them?

A: To solve this problem, we first set up Coulomb's Law:

$$F_e = \frac{kq_1q_2}{r^2}$$

Now, we replace one of the charges with half of itself—let's say q_1 will become $(0.5\,q_1)$. Similarly, we replace the distance r with $(2r)$. This gives:

$$F_e = \frac{k(0.5q_1)q_2}{(2r)^2}$$

Simplifying and rearranging this expression gives:

$$F_e = \frac{0.5kq_1q_2}{4r^2} = \frac{0.5}{4}\cdot\frac{kq_1q_2}{r^2} = \frac{1}{8}\cdot\frac{kq_1q_2}{r^2}$$

Therefore, the new F_e will be $\frac{1}{8}$ of the old F_e.

As was discussed on page 381, a useful shortcut for these kinds of problems is to set them up as "before and after" problems, using the number 1 for every quantity on the "before" side, and replacing the ones that change with their new values on the "after" side:

Before	**After**
$F_e = \frac{1\cdot 1\cdot 1}{1^2} = 1$	$F'_e = \frac{1\cdot 1\cdot 0.5}{2^2} = \frac{0.5}{4} = \frac{1}{8}$

Thus F'_e is $\frac{1}{8}$ of the original F_e.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Homework Problems

1. **(M)** What is the magnitude of the electric force between two objects, each with a charge of $+2.00\times10^{-6}$ C, which are separated by a distance of 1.50 m? Is the force attractive or repulsive?

   Answer: 0.016 N, repulsive

honors (not AP®)

2. **(M – honors & AP®; S – CP1)** An object with a charge of $+q_1$ is separated from a second object with an unknown charge by a distance *d*. If the objects attract each other with a force *F*, what is the charge on the second object? (*If you are not sure how to do this problem, do #3 below and use the steps to guide your algebra.*)

   Answer: $q_2 = -\dfrac{Fd^2}{kq_1}$

CP1 & honors (not AP®)

3. **(S – honors & AP®; M – CP1)** An object with a charge of $+1.50\times10^{-2}$ C is separated from a second object with an unknown charge by a distance of 0.500 m. If the objects attract each other with a force of 1.35×10^{6} N, what is the charge on the second object?
   (*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #2 above as a starting point if you have already solved that problem.*)

   Answer: -2.50×10^{-3} C

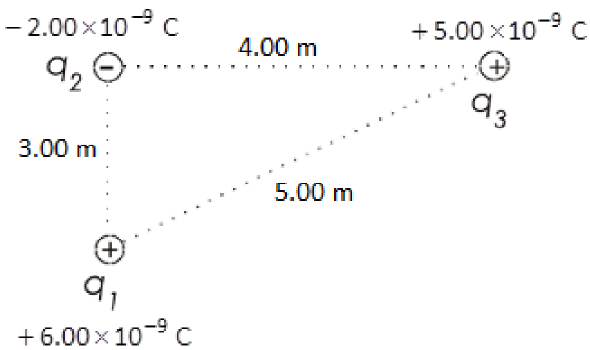
Use this space for summary and/or additional notes:

honors (not AP®)

4. **(M – honors & AP®; S – CP1)** The distance between an alpha particle (+2 elementary charges) and an electron (−1 elementary charge) is 2.00×10^{-25} m. If that distance is tripled, what will be the effect on the force between the charges?

Answer: The new F_e will be $\frac{1}{9}$ of the old F_e.

5. **(A)** Three elementary charges, particle q_1 with a charge of $+6.00\times10^{-9}$ C, particle q_2 with a charge of -2.00×10^{-9} C, and particle q_3 with a charge of $+5.00\times10^{-9}$ C, are arranged as shown in the diagram below.



What is the net force (magnitude and direction) on particle q_3?

Answer: 7.16×10^{-9} N at an angle of 65.2° above the *x*-axis.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Electric Fields

Unit: Electrostatics

MA Curriculum Frameworks (2016): HS-PS3-1, HS-PS3-2, HS-PS3-5

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Sketch electric field lines and vectors around charged particles or objects.
- Solve problems involving the forces on a charge due to an electric field.

Success Criteria:

- Sketches show arrows pointing from positive charges to negative charges.
- Electric field vectors show longer arrows where charges are larger and shorter arrows where charges are smaller.
- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how the electric force on a charged particle changes as you get closer to or farther away from another charged object.

Tier 2 Vocabulary: charge, field

Labs, Activities & Demonstrations:

- students holding copper pipe in one hand and zinc-coated steel pipe in other—measure with voltmeter. (Can chain students together.)

Notes:

<u>force field</u>: a region in which an object experiences a force because of some intrinsic property of the object that enables the force to act on it. Force fields are vectors, which means they have both a magnitude and a direction.

Recall that a gravitational field applies a force to an object based on its mass. $\vec{F}_g = m\vec{g}$, where $\vec{g}$ represents the magnitude and direction of the gravitational field.

<u>electric field</u> $(\vec{E})$: an electrically charged region (force field) that exerts a force on any charged particle within the region.

An electric field applies a force to an object based on its electrical charge. $\vec{F}_e = q\vec{E}$, where $\vec{E}$ represents the magnitude and direction of the electric field.

Use this space for summary and/or additional notes:

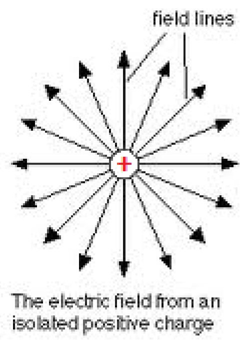
CP1 & honors (not AP®)

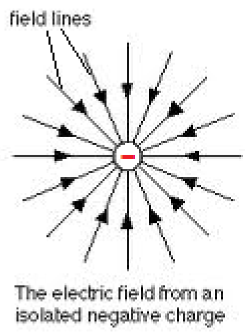
<u>field lines</u>: lines with arrows that show the direction of an electric field.

Field lines are lines that show the directions of force on an object. In an electric field, the object is assumed to be a positively-charged particle. This means that *the direction of the electric field is from positive to negative*, *i.e.,* field lines go outward in all directions from a positively-charged particle, and inward from all directions toward a negatively-charged particle.

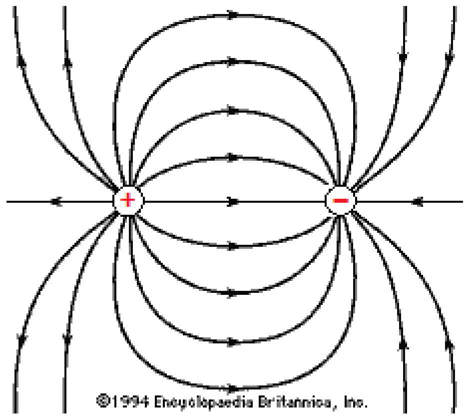
This means that a positively charged particle (such as a proton) would move in the direction of the arrows, and a negatively charged particle (such as an electron) would move in the opposite direction.

The simplest electric field is the region around a single charged particle:





If a positive and a negative charge are near each other, the field lines go from the positive charge toward the negative charge:

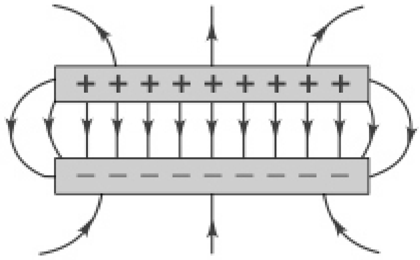


(Note that even though this is a two-dimensional drawing, the field itself is three-dimensional. Some field lines come out of the paper from the positive charge and go into the paper toward the negative charge, and some go behind the paper from the positive charge and come back into the paper from behind toward the negative charge.)

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

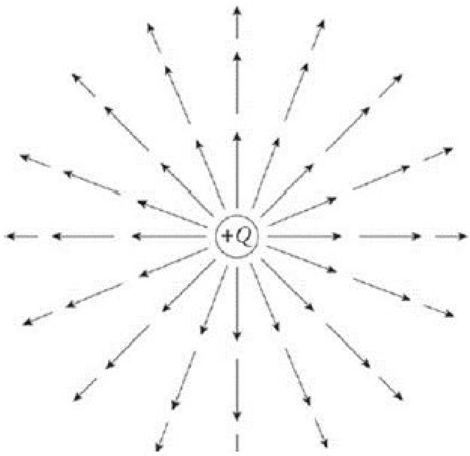
In the case of two charged plates (flat surfaces), the field lines would look like the following:



Electric Field Vectors

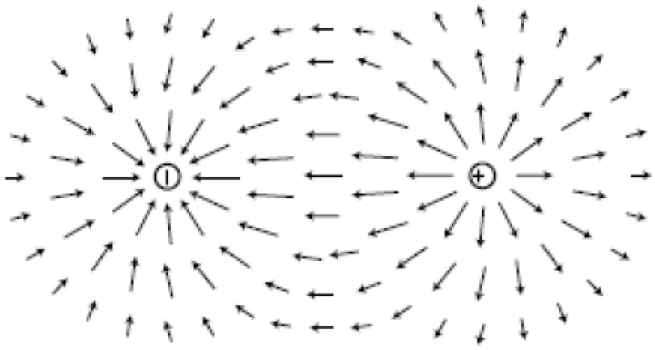
<u>electric field vector</u>: an arrow representing the strength and direction of an electric field at a point represented on a map.

A map of an electric field can be drawn using field vectors instead of field lines. Electric field vectors are preferred, because in addition to showing the direction of the electric field, they also show the relative strength. For example, this diagram shows the electric field around a positive charge. Notice that:



- The field vectors point in the direction of the electric field (from positive to negative).
- The field vectors are longer where the electric field is stronger and shorter where the electric field is weaker.

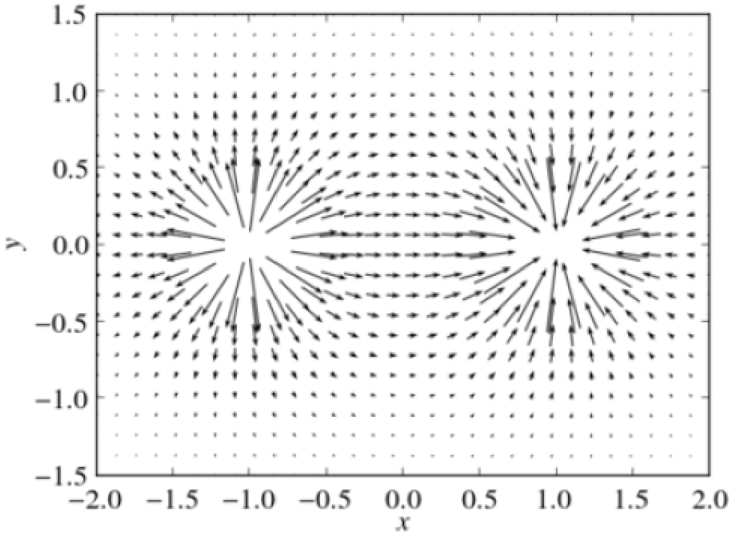
The electric field vectors around a pair of point charges, one positive and one negative, would look like the following:



Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

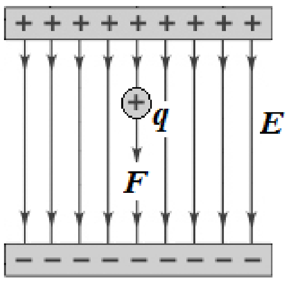
If the point charges were not shown, you could use a field vector diagram to determine their locations:



In the above example, there must be a positive point charge at coördinates (−1.0, 0) and a negative point charge at coördinates (+1.0, 0)

Electric Field Strength

We can measure the strength of an electric field by placing a particle with a positive charge (q) in the field, and measuring the force ($\vec{F}$) on the particle.



Coulomb's Law tells us that the force on the charge is due to the charges from the electric field:

$$F_e = \frac{kq_1q_2}{d^2}$$

If the positive and negative charges on the two surfaces that make the electric field are equal, *the force is the same everywhere in between the two surfaces*. (This is because as the particle gets farther from one surface, it gets closer to the other.) This means that the force on the particle is related only to the charges that make up the electric field and the charge of the particle.

We can therefore describe the electric field $(\vec{E})$ as the force between the electric field and our particle, divided by the charge of our particle:

$$\vec{E} = \frac{\vec{F}}{q} \quad \text{or} \quad \vec{F} = q\vec{E}$$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

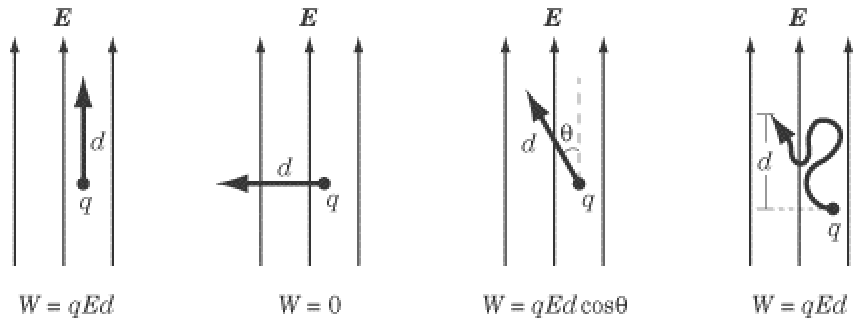
Work Done on a Charge by an Electric Field

Recall that work is the dot product of force and displacement:

$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

Because $\vec{F} = q\vec{E}$, we can substitute:

$$W = q\vec{E} \cdot \vec{d} = qEd\cos\theta$$



Electric Potential

Recall from the work-energy theorem that work equals a change in energy. Because an electric field can do work on a charged particle, an electric field must therefore apply energy to the particle.

<u>electric potential</u> (V): the electric potential energy of a charged particle in an electric field.

Electric potential is measured in volts (V).

Electric potential is exactly analogous to gravitational potential energy. In a gravitational field, a particle has potential energy because gravity can make it move. In an electric field, a particle has potential energy because the electric field can make it move.

However, one important difference is that electric potential is *per unit of charge*, whereas gravitational potential energy is not per unit of mass.

$$\frac{U_g}{m} = \vec{g} \bullet \vec{h}$$

gravitational potential energy *per unit of mass*

$$V = \frac{W}{q} = \vec{E} \bullet \vec{d}$$

electric potential
(already per unit of charge)

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Homework Problems

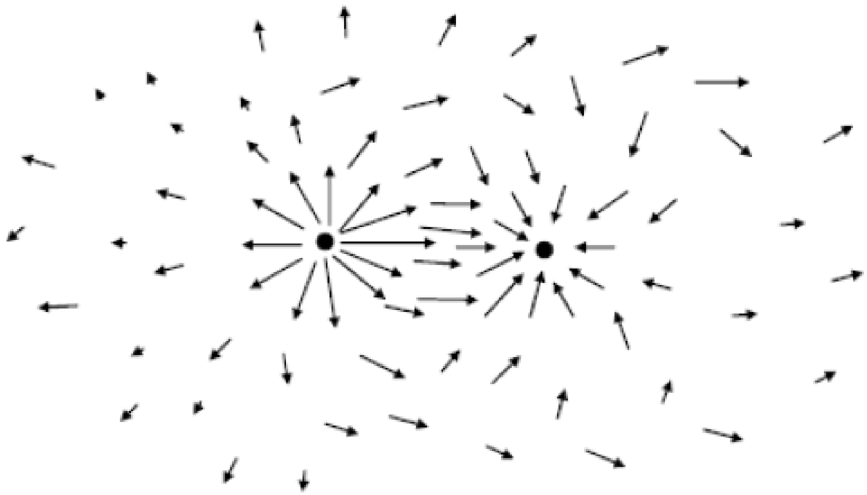
1. Sketch the electric field around each of the following charged particles:

   a. 

   b. 

   c. 

2. In The following electric field vector diagram:



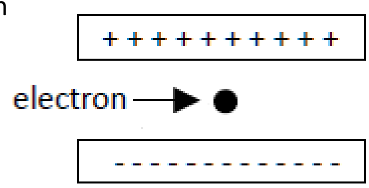
   a. Label the point charges (the black dots) with the sign of their respective charges (positive or negative).

   b. Which of the two charges is stronger? Explain how you can tell.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

2. An electron is placed exactly halfway between two charged parallel plates, as shown in the diagram at the right.



   a. Which direction does the electron move?

   b. As the electron moves, does the force acting on it increase, decrease, or remain the same?

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Introduction: DC Circuits

Unit: DC Circuits

Topics covered in this chapter:

This chapter discusses electricity and magnetism, how they behave, and how they relate to each other.

- *Electric Current & Ohm's Law* describes basic circuits, equations and calculations involving the flow of charged particles (electric current).
- *Electrical Components* illustrates and describes the behaviors of specific components of electric circuits.
- *Circuits*, *Series Circuits*, *Parallel Circuits*, and *Mixed Series & Parallel Circuits* describe the behavior of electrical components in specific arrangements of circuits and how to calculate quantities relating to the individual components and the entire circuit, based on the way the components are arranged.
- *Measuring Voltage, Current & Resistance* describes correct use of a voltmeter, ammeter and ohmmeter to measure quantities of interest.

One of the new challenges encountered in this chapter is interpreting and simplifying circuit diagrams, in which different equations may apply to different parts of the circuit.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-9(MA). Evaluate simple series and parallel circuits to predict changes to voltage, current, or resistance when simple changes are made to a circuit.

AP® Physics 1 Learning Objectives:

This unit was removed from AP® Physics 1 starting with the 2021–22 school year. It is now part of the AP® Physics 2 curriculum.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Circuit Elements and DC Circuits**, such as resistors, light bulbs, series and parallel networks, Ohm's law, and Joule's law

1. Voltage
2. Current
3. Resistance
4. Energy, Power, ~~and Heat~~
5. Circuits

Skills learned & applied in this chapter:

- Working with material-specific constants from a table.
- Identifying electric circuit components.
- Simplifying circuit diagrams.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Electric Current & Ohm's Law

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Solve problems involving relationships between voltage, current, resistance and power.

Success Criteria:

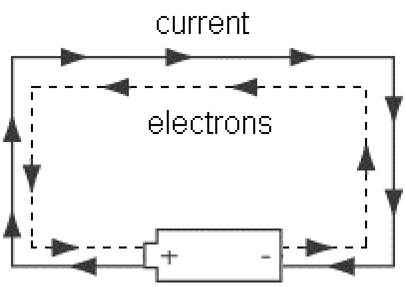
- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Describe the relationships between voltage, current, resistance, and power.

Tier 2 Vocabulary: current, resistance, power

Notes:



electric current: the flow of charged particles from one place to another, caused by a difference in electric potential.

The direction of electric current is defined as the direction that a positively-charged particle would move. However, the particles that are actually moving are electrons, which are negatively charged.

This means that electric current as we define it "travels" in the *opposite* direction from the actual electrons.

Electric current ($\vec{I}$) is a vector quantity and is measured in amperes (A), often abbreviated as "amps". One ampere equals one coulomb per second.

$$I = \frac{\Delta q}{t}$$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

<u>voltage (potential difference)</u> (ΔV)*: the difference in electric potential energy between two locations, per unit of charge.

$$\Delta V = \frac{W}{q}$$

Potential difference is the work (W) done on a charge per unit of charge (q). Potential difference (ΔV) is a scalar quantity (in DC circuits) and is measured in volts (V), which are equal to joules per coulomb.

The total voltage in a circuit is usually determined by the power supply that is used for the circuit (usually a battery in DC circuits).

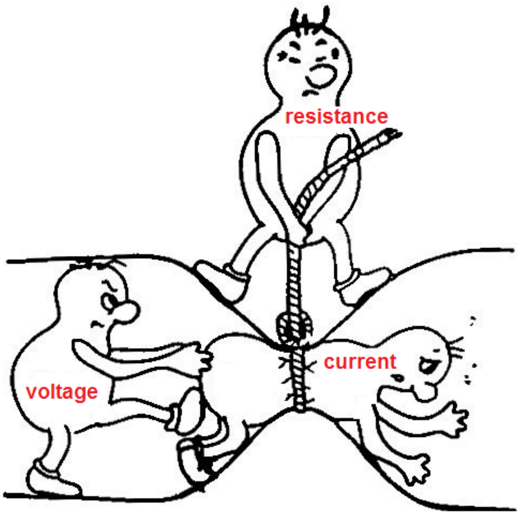
<u>resistance</u>: the amount of electromotive force (electric potential) needed to force a given amount of current through an object.

$$R = \frac{\Delta V}{I}$$

Resistance (R) is a scalar quantity and is measured in ohms (Ω). One ohm is one volt per ampere.

This relationship is Ohm's Law, named for the German physicist Georg Ohm. Ohm's Law is more commonly written:

$$I = \frac{\Delta V}{R} \quad \text{or} \quad \Delta V = IR$$



Simply put, Ohm's Law states that an object has an ability to resist electric current flowing through it. The more resistance an object has, the more voltage you need to force electric current through it. Or, for a given voltage, the more resistance an object has, the less current will flow through it.

Resistance is an intrinsic property of a substance. In this course, we will limit problems that involve calculations to ohmic resistors, which means their resistance does not change with temperature.

Choosing the voltage and the arrangement of objects in the circuit (which determines the resistance) is what determines how much current will flow.

Electrical engineers use arrangements of resistors in circuits in order to adjust the amount of current that flows through the components.

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* Note that most physics texts (and most physicists and electricians) use V for both potential and voltage, and students have to rely on context to tell the difference. In these notes, to make the distinction clear, we will use the variable V for absolute electric potential, and ΔV for voltage (potential difference).

Use this space for summary and/or additional notes:

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resistivity: the innate ability of a substance to offer electrical resistance. The resistance of an object is therefore a function of the resistivity of the substance (ρ), and of the length (L) and cross-sectional area (A) of the object. In MKS units, resistivity is measured in ohm-meters (Ω·m).

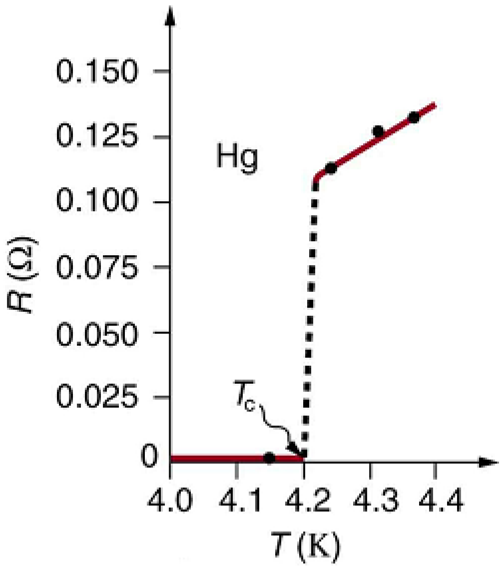
$$R = \frac{\rho L}{A}$$

Resistivity changes with temperature. For small temperature differences (less than 100 °C), resistivity is given by:

$$\rho = \rho_o (1 + \alpha \Delta T)$$

where ρ_o is the resistivity at some reference temperature and α is a coëfficient that describes how much that substance's resistivity changes with temperature. For conductors, α is positive (which means their resistivity increases with temperature). For metals at room temperature, resistivity typically varies from +0.003 to +0.006 K^{-1}.

Some materials become superconductors (meaning that they have essentially no resistance) at very low temperatures. The temperature below which a material becomes a superconductor is called the critical temperature (T_c). For example, the critical temperature for mercury is 4.2 K, as shown in the graph to the right.



conductivity: the innate ability of a substance to conduct electricity. Conductivity (σ) is the inverse of resistivity, and is measured in siemens (S). Siemens used to be called mhos (symbol ℧). ("Mho" is "ohm" spelled backwards.)

$$\sigma = \frac{1}{\rho}$$

ohmic resistor: a resistor whose resistance is the same regardless of voltage and current. The filament of an incandescent light bulb is an example of a non-ohmic resistor, because the current heats up the filament, which increases its resistance. (This is necessary in order for the filament to also produce light.)

capacitance: the ability of an object to hold an electric charge. Capacitance (C) is a scalar quantity and is measured in farads (F). One farad equals one coulomb per volt.

$$C = \frac{Q}{\Delta V}$$

Note: capacitance is covered in AP® Physics 2.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

power: as discussed in the mechanics section of this course, power (P) is the work done per unit of time and is measured in watts (W).

In electric circuits:

$$P = \frac{W}{t} = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

work: recall from mechanics that work (W) equals the energy transferred into or out of a system, and that work equals power times time. Work is measured in either joules (J) or newton-meters (N·m). The equations for work are the equations for power (above) multiplied by time:

$$W = Pt = I(\Delta V)t = I^2 Rt = \frac{(\Delta V)^2 t}{R} = (\Delta V)q$$

Electrical work or energy is often measured in kilowatt-hours (kW·h).

$$1\,\text{kW}\cdot\text{h} = 3.6\times10^6\ \text{J} = 3.6\,\text{MJ}$$

Summary of Terms and Variables

Term	Variable	Unit	Term	Variable	Unit
charge	q or Q	coulomb (C)	resistance	R	ohm (Ω)
current	I	ampere (A)	power	P	watt (W)
voltage	V	volt (V)	work	W	joule (J)

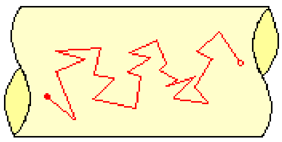
Notice that some of the variables use the same letters as some of the units. For example, the variable "*W*" is work, but the unit "W" is watts, which measures power. This means you need to be especially careful to keep track of which is which!

Alternating Current vs. Direct Current

Electric current can move in two ways.

direct current: electric current flows through the circuit, starting at the positive terminal of the battery or power supply, and ending at the negative terminal. Batteries supply direct current. A typical AAA, AA, C, or D battery supplies 1.5 volts DC.

However, the net flow of charged particles through a wire is very slow. Electrons continually collide with one another in all directions as they drift slowly through the circuit. Electrons in a DC circuit have a net velocity of about one meter per hour.



Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

<u>alternating current</u>: electric current flows back and forth, in one direction and then the other like a sine wave. The current alternates at a particular frequency. In the U.S., household current is 110 volts AC with a frequency of 60 Hz.

Alternating current requires higher voltages in order to operate devices, but has the advantage that the voltage drop is much less over a length of wire than with direct current.

Sample Problems:

Q: A simple electrical device uses 1.5 A of current when plugged into a 110 V household electrical outlet. How much current would the same device draw if it were plugged into a 12 V outlet in a car?

A: Resistance is a property of a specific object. Because we are not told otherwise, we assume the device is ohmic and the resistance is the same regardless of the current.

Therefore, our strategy is to use the information about the device plugged into a household outlet to determine the device's resistance, then use the resistance to determine how much current it draws in the car.

In the household outlet:

$$R = \frac{\Delta V}{I} = \frac{110}{1.5} = 73.\bar{3}\,\Omega$$

In the car:

$$I = \frac{\Delta V}{R} = \frac{12}{73.\bar{3}} = 0.163\text{ A}$$

Q: A laptop computer uses 10 W of power. The laptop's power supply adjusts the current so that the power is the same regardless of the voltage supplied. How much current would the computer draw from a 110 V household outlet? How much current would the same laptop computer need to draw from a 12 V car outlet?

A: The strategy for this problem is the same as the previous one.

Household outlet:

$$P = I\Delta V$$

$$I = \frac{P}{\Delta V} = \frac{10}{110} = 0.091\text{ A}$$

Car outlet:

$$I = \frac{P}{\Delta V} = \frac{10}{12} = 0.8\bar{3}\text{ A}$$

Use this space for summary and/or additional notes:

honors (not AP®)

Q: A 100 Ω resistor is 0.70 mm in diameter and 6.0 mm long. If you wanted to make a 470 Ω resistor out of the same material (with the same diameter), what would the length need to be? If, instead, you wanted to make a resistor the same length, what would the new diameter need to be?

A: In both cases, $R = \dfrac{\rho L}{A}$.

For a resistor of the same diameter (same cross-sectional area), ρ and A are the same, which means:

$$\frac{R'}{R} = \frac{L'}{L}$$
$$L' = \frac{R'L}{R} = \frac{(470)(6.0)}{100} = 28.2\,\text{mm}$$

For a resistor of the same length, ρ and L are the same, which means:

$$\frac{R'}{R} = \frac{A}{A'} = \frac{\pi r^2}{\pi (r')^2} = \frac{\pi (d/2)^2}{\pi (d'/2)^2} = \frac{d^2}{(d')^2}$$
$$d' = \sqrt{\frac{Rd^2}{R'}} = d\sqrt{\frac{R}{R'}} = 0.70\sqrt{\frac{100}{470}} = 0.70\sqrt{0.213} = 0.323\,\text{mm}$$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Homework Problems

1. An MP3 player uses a standard 1.5 V battery. How much resistance is in the circuit if it uses a current of 0.010 A?

   Answer: 150 Ω

2. How much current flows through a hair dryer plugged into a 110 V circuit if it has a resistance of 25 Ω?

   Answer: 4.4 A

3. A battery pushes 1.2 A of charge through the headlights in a car, which has a resistance of 10 Ω. What is the potential difference across the headlights?

   Answer: 12 V

4. A circuit used for electroplating copper applies a current of 3.0 A for 16 hours. How much charge is transferred?

   Answer: 172 800 C

5. What is the power when a voltage of 120 V drives a 2.0 A current through a device?

   Answer: 240W

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

6. What is the resistance of a 40. W light bulb connected to a 120 V circuit?

   Answer: 360 Ω

7. If a component in an electric circuit dissipates 6.0 W of power when it draws a current of 3.0 A, what is the resistance of the component?

   Answer: 0.67 Ω

8. A 0.7 mm diameter by 60 mm long pencil "lead" is made of graphite, which has a resistivity of approximately $1.0\times10^{-4}\ \Omega\cdot\text{m}$. What is its resistance?

   Hints:

   - *You will need to convert mm to m.*
   - *You will need to convert the diameter to a radius before using $A=\pi r^2$ to find the area.*

   Answer: 15.6 Ω

honors (not AP®)

9. A cylindrical object has radius *r* and length *L* and is made from a substance with resistivity *ρ*. A potential difference of *V* is applied to the object. Derive an expression for the current that flows through it.

   Hint: this is a two-step problem.

   Answer: $I=\dfrac{VA}{\rho L}$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

10. Some children are afraid of the dark and ask their parents to leave the hall light on all night. Suppose the hall light in a child's house has two 75. W incandescent light bulbs (150 W total), the voltage is 120 V, and the light is left on for 8.0 hours.

    a. How much current flows through the light fixture?

       Answer: 1.25 A

    b. How many kilowatt-hours of energy would be used in one night?

       Answer: 1.2 kW·h

    c. If the power company charges 22 ¢ per kilowatt-hour, how much does it cost to leave the light on overnight?

       Answer: 26.4 ¢

    d. If the two incandescent bulbs are replaced by LED bulbs that use 12.2 W each (24.4 W total) how much would it cost to leave the light on overnight?

       Answer: 4.3 ¢

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Electrical Components

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Identify electrical components using the components themselves and/or the symbols used in circuit diagrams.
- Describe the purpose of various electrical components and how they are used in circuits.

Success Criteria:

- Descriptions correctly identify the component.
- Purpose and use of component is correct.

Language Objectives:

- Explain the components in an actual circuit or a circuit diagram, and describe what each one does.

Tier 2 Vocabulary: component, resistor, fuse

Labs, Activities & Demonstrations:

- Show & tell with actual components.

Notes:

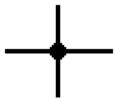
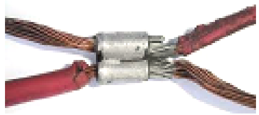
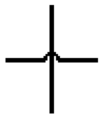
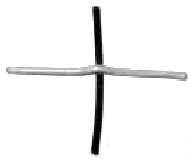
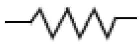
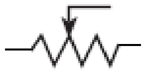
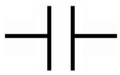
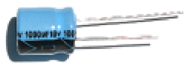
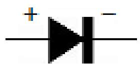
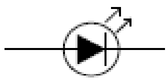
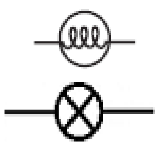
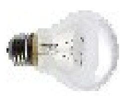
electrical component: an object that performs a specific task in an electric circuit. A circuit is a collection of components connected together so that the tasks performed by the individual components combine in some useful way.

circuit diagram: a picture that represents a circuit, with different symbols representing the different components.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

The following table describes some common components of electrical circuits, what they do, and the symbols that are used to represent them in circuit diagrams.

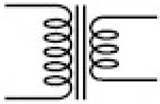
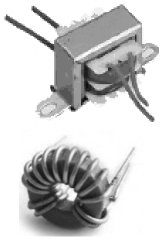
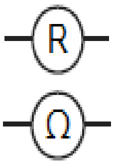
Component	Symbol	Picture	Description
wire			Carries current in a circuit.
junction			Connection between multiple wires.
unconnected wires			Wires pass by each other but are not connected.
battery			Supplies current at a fixed voltage.
resistor			Resists flow of current.
potentiometer (rheostat, dimmer)			Provides variable (adjustable) resistance.
capacitor			Stores charge.
diode			Allows current to flow in only one direction (from + to −).
light-emitting diode (LED)			Diode that gives off light when current flows through it.
switch			Opens / closes circuit.
incandescent lamp (light)			Provides light (and resistance).

Use this space for summary and/or additional notes:

Electrical Components

Big Ideas | Details

CP1 & honors (not AP®)

Component	Symbol	Picture	Description
transformer			Increases or decreases voltage.
voltmeter			Measures voltage (volts).
ammeter			Measures current (amperes).
ohmmeter			Measures resistance (ohms).
fuse			Opens circuit if too much current flows through it.
ground		 (clamps to water pipe)	Neutralizes charge.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

DC Circuits

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Identify electrical circuits or sections of circuits as series or parallel.

Success Criteria:

- Descriptions correctly identify the component.
- Descriptions correctly describe which type of circuit (series or parallel) the component is in.

Language Objectives:

- Identify which components are in series *vs.* parallel in a mixed circuit.

Tier 2 Vocabulary: series, parallel

Labs, Activities & Demonstrations:

- Example circuit with light bulbs & switches.
- Fuse demo using a single strand from a multi-strand wire.

Notes:

circuit: an arrangement of electrical components that allows electric current to pass through them so that the tasks performed by the individual components combine in some useful way.

closed circuit: a circuit that has a complete path for current to flow from the positive terminal of the battery or power supply through the components and back to the negative terminal.

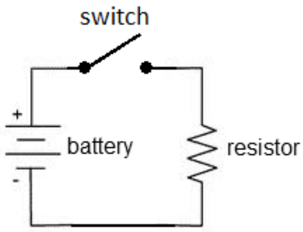
open circuit: a circuit that has a gap such that current cannot flow from the positive terminal to the negative terminal.

short circuit: a circuit in which the positive terminal is connected directly to the negative terminal with no load (resistance) in between.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

A diagram of a simple electric circuit might look like the diagram to the right.



When the switch is closed, the electric current flows from the positive terminal of the battery through the switch, through the resistor, and back to the negative terminal of the battery.

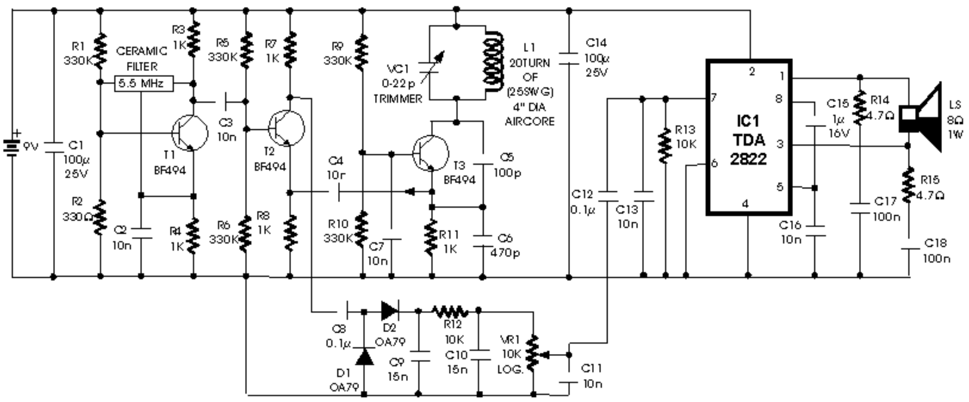
An electric circuit needs a power supply (often a battery) that provides current at a specific voltage (electric potential difference), and one or more components that use the energy provided.

The battery or power supply continues to supply current, provided that:

1. There is a path for the current to flow from the positive terminal to the negative terminal, and
2. The total resistance of the circuit is small enough to allow the current to flow.

If the circuit is broken, current cannot flow and the chemical reactions inside the battery stop.

As circuits become more complex, the diagrams reflect this increasing complexity. The following is a circuit diagram for a metal detector:



Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance contributed by each component of a circuit.

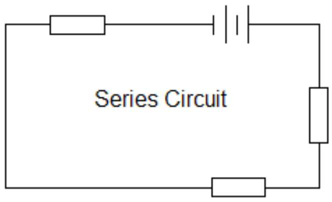
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

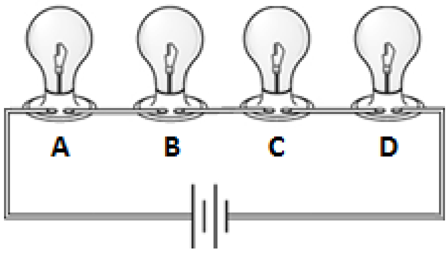
Series vs. Parallel Circuits

If a circuit has multiple components, they can be arranged in series or parallel.

series: Components in **series** lie along the same path, one after the other.



In a series circuit, all of the current flows through every component, one after another. If the current is interrupted anywhere in the circuit, no current will flow. For example, in the following series circuit, if any of light bulbs A, B, C, or D is removed, no current can flow and none of the light bulbs will be illuminated.



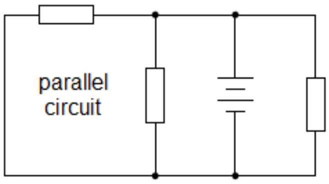
Because some of the electric potential energy (voltage) is "used up" by each bulb in the circuit, each additional bulb means the voltage is divided among more bulbs and is therefore less for each bulb. This is why light bulbs get dimmer as you add more bulbs in series.

Christmas tree lights used to be wired in series. This caused a lot of frustration, because if one bulb burned out, the entire string went out, and it could take several tries to find which bulb was burned out.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

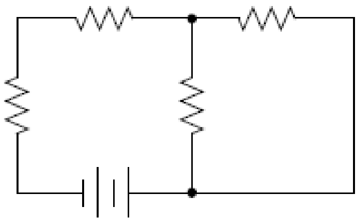
<u>parallel</u>: Components in **<u>parallel</u>** lie in separate paths.



In a parallel circuit, the current divides at each junction, with some of the current flowing through each path. If the current is interrupted in one path, current can still flow through the other paths. For example, in the parallel circuit to the right, if any of light bulbs A, B, C, or D is removed, current still flows through the remaining bulbs.

Because the voltage across each branch is equal to the total voltage, all of the bulbs will light up with full brightness, regardless of how many bulbs are in the circuit. (However, each separate light bulb draws the same amount of current as if it were the only thing in the circuit, so the total current in the circuit increases with each new branch. This is why you trip a circuit breaker or blow a fuse if you have too many high-power components plugged into the same circuit.)

Note that complex circuits may have some components that are in series with each other and other components that are in parallel.



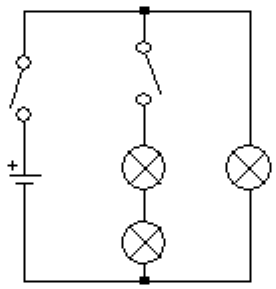
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Sample Problem:

Q: A circuit consists of a battery, two switches, and three light bulbs. Two of the bulbs are in series with each other, and the third bulb is in parallel with the others. One of the switches turns off the two light bulbs that are in series with each other, and the other switch turns off the entire circuit. Draw a schematic diagram of the circuit, using the correct symbol for each component.

A:



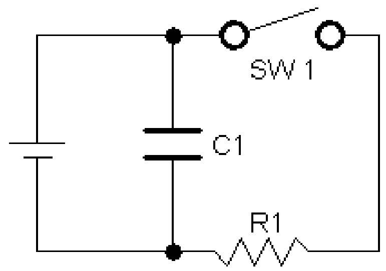
Note that no sensible person would intentionally wire a circuit this way. It would make much more sense to have the second switch on the branch with the one light bulb, so you could turn off either branch separately or both branches by opening both switches. This is an example of a strange circuit that a physics teacher would use to make sure you really can follow exactly what the question is asking!

Use this space for summary and/or additional notes:

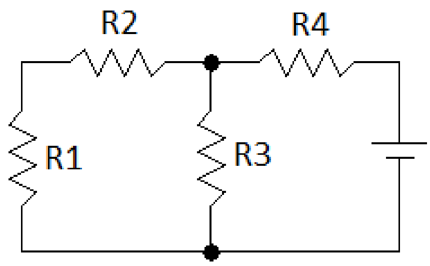
CP1 & honors (not AP®)

Homework Problems

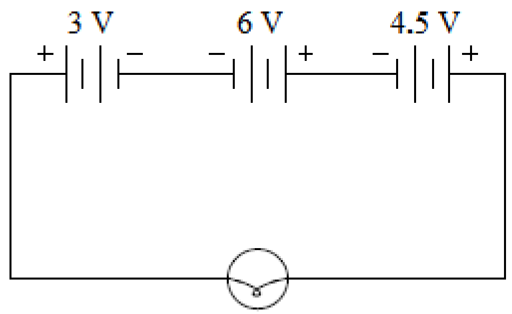
1. The circuit shown to the right contains a battery, switch (SW1), capacitor (C1), and resistor (R1). Which of components C1 and SW1 are in series with R1? Which are in parallel with R1?



2. The circuit shown to the right contains a battery and four resistors (R1, R2, R3, and R4). Which resistors are in series with R1? Which are in parallel with R1?



3. The following bizarre circuit contains three batteries and a light bulb. What is the potential difference across the light bulb? (*Hint: remember to check the +/− orientation of the batteries.*)



Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Kirchhoff's Rules

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Apply Kirchhoff's junction and loop rules to determine voltages and currents in circuits.

Success Criteria:

- Loop rule correctly applied (electric potential differences add to zero).
- Junction rule correctly applied (total current into a junction equals total current out).

Language Objectives:

- Explain why electric potential has to add to zero around a loop and why current into a junction has to add up to current out.

Tier 2 Vocabulary: loop, junction

Notes:

In 1845, the German physicist Gustav Kirchhoff came up with two simple rules that describe the behavior of current in complex circuits. Those rules are:

Kirchhoff's junction rule: the total current coming into any junction must equal the total current coming out of the junction.

The junction rule is based on the concept that electric charge cannot be created or destroyed. Current is simply the flow of electric charge, so any charges that come into a junction must also come out of it.

Kirchhoff's loop rule: the sum of the voltages around any closed loop must add up to zero.

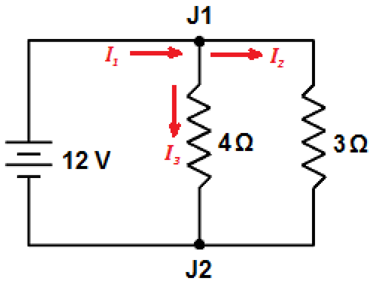
The loop rule is based on the concept that voltage is the difference in electric potential between one location in the circuit and another. If you come back to the same point in the circuit, the difference in electric potential between where you started and where you ended (the same place) must be zero. Therefore, any increases and decreases in voltage around the loop must cancel.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Junction Rule Example:

As an example of the junction rule, consider the following circuit:



The junction rule tells us that the current flowing into junction J1 must equal the current flowing out. If we assume current I_1 flows into the junction, and currents I_2 and I_3 flow out of it, then $I_1 = I_2 + I_3$.

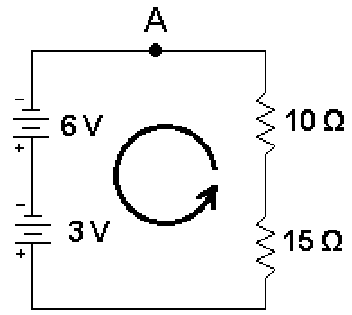
We know that the voltage across both resistors is 12 V. From Ohm's Law we can determine that the current through the 3 Ω resistor is $I_2 = 4\text{ A}$, and the current through the 4 Ω resistor is $I_3 = 3\text{ A}$. The junction rule tells us that the total current must therefore be $I_1 = I_2 + I_3 = 4\text{A} + 3\text{A} = 7\text{A}$.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Loop Rule Example:

For the loop rule, consider the following circuit:



If we start at point A and move counterclockwise around the loop (in the direction of the arrow), the voltage should be zero when we get back to point A.

For this example, we are moving around the circuit in the same direction that the current flows, because that makes the most intuitive sense. However, it wouldn't matter if we moved clockwise instead—just as with vector quantities, we choose a positive direction and assign each quantity to a positive or negative number accordingly, and the math tells us what is actually happening.

Let us arbitrarily define the absolute electric potential at point A to be zero. Starting from point A, we first move through the 6 V battery. We are moving from the negative pole to the positive pole of the battery. Batteries add electric potential, so the voltage increases by +6 V. When we move through the second battery, the voltage increases by +3 V. Now, the electric potential is +9 V.

Next, we move through the 15 Ω resistor. When we move through a resistor in the positive direction (of current flow), the voltage drops, so we assign the resistor a voltage of $-15I$ (based on $\Delta V = IR$, where I is the current through the resistor). Similarly, the voltage across the 10 Ω resistor is $-10I$. Applying the loop rule gives:

$$\begin{aligned} 6+3+(-15I)+(-10I) &= 0 \\ 9-25I &= 0 \\ 9 &= 25I \\ I = \frac{9}{25} &= 0.36\text{ A} \end{aligned}$$

Now that we know the total current, we can use it to find the voltage drop (potential difference) across the two resistors.

$$\Delta V_{10\,\Omega} = IR = (0.36)(10) = 3.6\text{ V} \qquad \Delta V_{15\,\Omega} = IR = (0.36)(15) = 5.4\text{ V}$$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Series Circuits

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Calculate voltage, current, resistance and power in series circuits.

Success Criteria:

- Correct relationships are applied for voltage, current, resistance and power in series circuits.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the relationships for voltages, current, resistance and power in series circuits.

Tier 2 Vocabulary: series

Labs, Activities & Demonstrations:

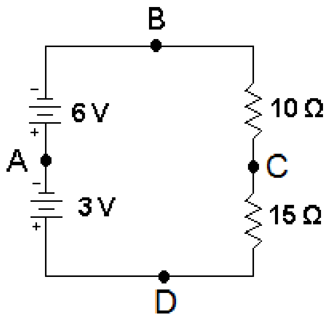
- Circuit with light bulbs wired in series.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Notes:

The diagram to the right shows two batteries and two resistors in series.



Current

Because there is only one path, all of the current flows through every component. This means the current is the same through every component in the circuit:

$$I_{total} = I_1 = I_2 = I_3 = \ldots$$

Voltage

In a series circuit, if there are multiple voltage sources (*e.g.,* batteries), the voltages add:

$$\Delta V_{total} = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots$$

In the above circuit, there are two batteries, one that supplies 6 V and one that supplies 3 V. The voltage from A to B is +6 V, the voltage from A to D is −3 V (note that A to D means measuring from negative to positive), and the voltage from D to B is (+3 V) + (+6 V) = +9 V.

Resistance

If there are multiple resistors, each one contributes to the total resistance and the resistances add:

$$R_{total} = R_1 + R_2 + R_3 + \ldots$$

In the above circuit, the resistance between points B and D is 10Ω + 15Ω = 25Ω.

Power

In all circuits (series and parallel), any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \ldots$$

Calculations

You can calculate the voltage, current, resistance, and power of each component separately, any subset of the circuit, or entire circuit, using the equations:

$$\Delta V = IR \qquad\qquad P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

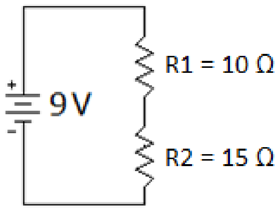
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

"Solving" the circuit for these quantities is much like solving a Sudoku puzzle. You systematically decide which variables (for each component and/or the entire circuit) you have enough information to solve for. Each result enables you to determine more and more of the, until you have found all of the quantities you need.

Sample Problem:

Suppose we are given the following series circuit:



and we are asked to fill in the following table:

	R_1	R_2	Total
Voltage (ΔV)			9 V
Current (I)			
Resistance (R)	10 Ω	15 Ω	
Power (P)			

First, we recognize that resistances in series add, which gives us:

	R_1	R_2	Total
Voltage (ΔV)			9 V
Current (I)			
Resistance (R)	10 Ω	15 Ω	***25 Ω***
Power (P)			

Now, we know two variables in the "Total" column, so we use $\Delta V = IR$ to find the current.

$$\Delta V = IR$$
$$9 = (I)(25)$$
$$I = \frac{9}{25} = 0.36\text{ A}$$

Because this is a series circuit, the total current is also the current through R_1 and R_2.

	R_1	R_2	Total
Voltage (ΔV)			9 V
Current (I)	***0.36 A***	***0.36 A***	***0.36 A***
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)			

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

As soon as we know the current, we can find the voltage across R_1 and R_2, again using $\Delta V = IR$.

	R_1	R_2	Total
Voltage (ΔV)	***3.6 V***	***5.4 V***	9 V
Current (I)	0.36 A	0.36 A	0.36 A
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)			

Finally, we can fill in the power, using $P = I\,\Delta V$:

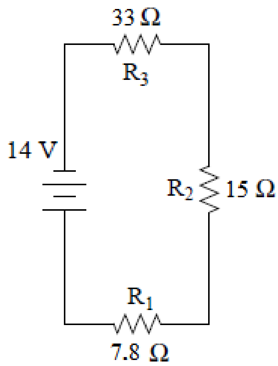
	R_1	R_2	Total
Voltage (ΔV)	3.6 V	5.4 V	9 V
Current (I)	0.36 A	0.36 A	0.36 A
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)	***1.30 W***	***1.94 W***	***3.24 W***

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Homework Problems

1. Fill in the table for the following circuit:



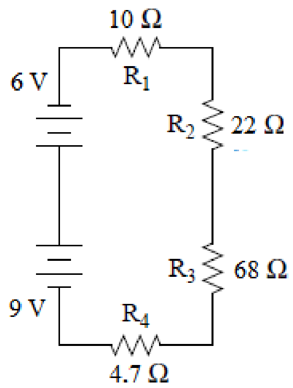
	R_1	R_2	R_3	Total
Voltage (ΔV)				14 V
Current (I)				
Resist. (R)	7.8 Ω	15 Ω	33 Ω	
Power (P)				

(The space below is intentionally left blank for calculations.)

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

2. Fill in the table for the following circuit.
(*Hint: Notice that the batteries are oriented in opposite directions.*)



	R_1	R_2	R_3	R_4	Total
Voltage (ΔV)					
Current (I)					
Resist. (R)	10 Ω	22 Ω	68 Ω	4.7 Ω	
Power (P)					

(The space below is intentionally left blank for calculations.)

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Parallel Circuits

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Calculate voltage, current, resistance and power in parallel circuits.

Success Criteria:

- Correct relationships are applied for voltage, current, resistance and power in parallel circuits.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the relationships for voltages, current, resistance and power in parallel circuits.

Tier 2 Vocabulary: parallel

Labs, Activities & Demonstrations:

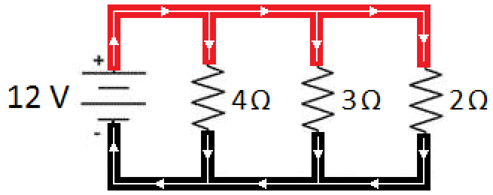
- Circuit with light bulbs wired in parallel.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Notes:

The following circuit shows a battery and three resistors in parallel:



Current

The current divides at each junction (as indicated by the arrows). This means the current through each path must add up to the total current:

$$I_{total} = I_1 + I_2 + I_3 + \ldots$$

Voltage

In a parallel circuit, the potential difference (voltage) across the battery is always the same (12 V in the above example). Therefore, the potential difference between *any point* on the top wire and *any point* on the bottom wire must be the same. This means the voltage is the same across each path:

$$\Delta V_{total} = \Delta V_1 = \Delta V_2 = \Delta V_3 = \ldots$$

Power

Just as with series circuits, in a parallel circuit, any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \ldots$$

Resistance

If there are multiple resistors, the effective resistance of each path becomes less as there are more paths for the current to flow through. The total resistance is given by the formula:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Some students find it confusing that the combined resistance of a group of resistors in parallel is always less than any single resistor by itself.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Electric current is analogous to water in a pipe:

- The current corresponds to the flow rate.
- The voltage corresponds to the pressure between one side and the other.
- The resistance would correspond to how small the pipe is (*i.e.,* how hard it is to push water through the pipes). A smaller pipe has more resistance; a larger pipe will let water flow through more easily than a smaller pipe.

The voltage (pressure) drop is the same between one side and the other because less water flows through the smaller pipes and more water flows through the larger ones until the pressure is completely balanced. The same is true for electrons in a parallel circuit.

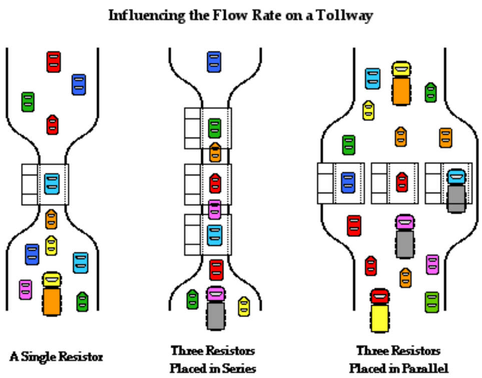
The water will flow through the *set* of pipes more easily than it would through any one pipe by itself. The same is true for resistors. As you add more resistors, you add more pathways for the current, which means less total resistance.

Another common analogy is to compare resistors with toll booths on a highway.

One toll booth slows cars down while the drivers pay the toll.

Multiple toll booths in series would slow traffic down more.

Multiple toll booths in parallel make traffic flow faster because there are more paths for the cars to follow. Each additional toll booth further reduces the resistance to the flow of traffic.



Calculations

Just as with series circuits, you can calculate the voltage, current, resistance, and power of each component and the entire circuit using the equations:

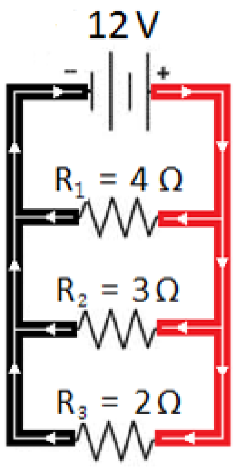
$$\Delta V = IR \qquad\qquad P = I\Delta V = I^2R = \frac{(\Delta V)^2}{R}$$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Sample Problem

Suppose we are given the following parallel circuit:



and we are asked to fill in the following table:

	R_1	R_2	R_3	Total
Voltage (ΔV)				12 V
Current (I)				
Resistance (R)	4 Ω	3 Ω	2 Ω	
Power (P)				

Because this is a parallel circuit, the total voltage equals the voltage across all three branches, so we can fill in 12 V for each resistor.

The next thing we can do is use $\Delta V = IR$ to find the current through each resistor:

	R_1	R_2	R_3	Total
Voltage (ΔV)	***12 V***	***12 V***	***12 V***	***12 V***
Current (I)	***3 A***	***4 A***	***6 A***	***13 A***
Resistance (R)	4 Ω	3 Ω	2 Ω	
Power (P)				

In a parallel circuit, the current adds, so the total current is 3 + 4 + 6 = 13 A.

Now, we have two ways of finding the total resistance. We can use $\Delta V = IR$ with the total voltage and current, or we can use the formula for resistances in parallel:

$$\Delta V = IR$$
$$12 = 13R$$
$$R = \frac{12}{13} = 0.923\,\Omega$$

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$\frac{1}{R_{total}} = \frac{1}{4} + \frac{1}{3} + \frac{1}{2}$$
$$\frac{1}{R_{total}} = \frac{3}{12} + \frac{4}{12} + \frac{6}{12} = \frac{13}{12}$$
$$R_{total} = \frac{12}{13} = 0.923\,\Omega$$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Now we have:

	R_1	R_2	R_3	Total
Voltage (ΔV)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	***0.923 Ω***
Power (P)				

As we did with series circuits, we can calculate the power, using $P = I\,\Delta V$:

	R_1	R_2	R_3	Total
Voltage (V)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	0.923 Ω
Power (P)	***36 W***	***48 W***	***72 W***	***156 W***

Batteries in Parallel

One question that has not been answered yet is what happens when batteries are connected in parallel.

If the batteries have the same voltage, the potential difference (voltage) remains the same, but the total current is the combined current from the two batteries.

However, if the batteries have different voltages there is a problem, because each battery attempts to maintain a constant potential difference (voltage) between its terminals. This results in the higher voltage battery overcharging the lower voltage battery.

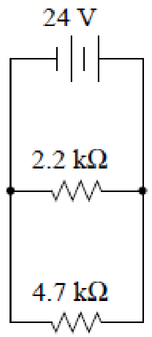
Remember that physically, batteries are electrochemical cells—small solid-state chemical reactors with redox reactions taking place in each cell. If one battery overcharges the other, material is deposited on the cathode (positive terminal) until the cathode becomes physically too large for its compartment, at which point the battery bursts and the chemicals leak out.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Homework Problems

1. Fill in the table for the following circuit:



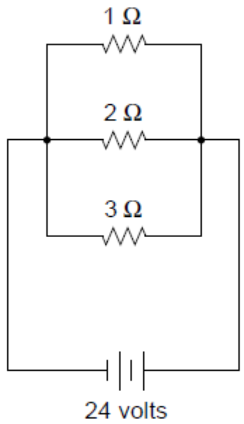
	R_1	R_2	Total
Voltage (ΔV)			24 V
Current (I)			
Resist. (R)	2 200 Ω	4 700 Ω	
Power (P)			

(The space below is intentionally left blank for calculations.)

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

2. Fill in the table for the following circuit:



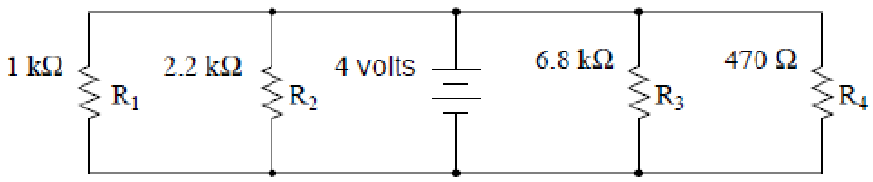
	R_1	R_2	R_3	Total
Voltage (ΔV)				24 V
Current (I)				
Resist. (R)	1 Ω	2 Ω	3 Ω	
Power (P)				

(The space below is intentionally left blank for calculations.)

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

3. Fill in the table for the following circuit:



	R_1	R_2	R_3	R_4	Total
Voltage (ΔV)					4 V
Current (I)					
Resistance (R)	1 000 Ω	2 200 Ω	6 800 Ω	470 Ω	
Power (P)					

(The space below is intentionally left blank for calculations.)

Use this space for summary and/or additional notes:

honors (not AP®)

Mixed Series & Parallel Circuits

Unit: DC Circuits

MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Calculate voltage, current, resistance and power in mixed series & parallel circuits.

Success Criteria:

- Correct relationships are applied for voltage, current, resistance and power in mixed series & parallel circuits.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the relationships for voltages, current, resistance and power in mixed series & parallel circuits.

Tier 2 Vocabulary: series, parallel

Labs, Activities & Demonstrations:

- Light bulb mystery circuits.

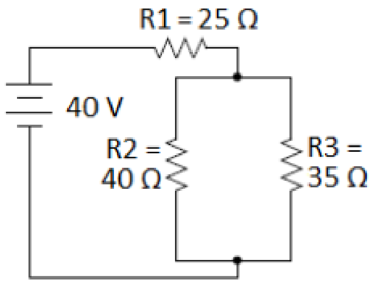
Notes:

If a circuit has mixed series and parallel sections, you can determine the various voltages, currents and resistances by applying Kirchhoff's Rules and/or by "simplifying the circuit." Simplifying the circuit, in this case, means replacing resistors in series or parallel with a single resistor of equivalent resistance.

Use this space for summary and/or additional notes:

honors (not AP®)

For example, suppose we need to solve the following mixed series & parallel circuit for voltage, current, resistance and power for each resistor:



Because the circuit has series and parallel sections, we cannot simply use the series and parallel rules across the entire table.

	R_1	R_2	R_3	Total
Voltage (ΔV)				40 V
Current (I)				
Resistance (R)	25 Ω	40 Ω	35 Ω	
Power (P)				

We can use Ohm's Law $(\Delta V = IR)$ and the power equation $(P = I\Delta V)$ on each individual resistor and the totals for the circuit (columns), but we need two pieces of information for each resistor in order to do this.

Our strategy will be:

1. Simplify the resistor network until all resistances are combined into one equivalent resistor to find the total resistance.
2. Use $\Delta V = IR$ to find the total current.
3. Work backwards through your simplification, using the equations for series and parallel circuits in the appropriate sections of the circuit until you have all of the information.

Step 1: If we follow the current through the circuit, we see that it goes through resistor R1 first. Then it splits into two parallel pathways. One path goes through R2 and R3, and the other goes through R4 and R5.

There is no universal shorthand for representing series and parallel components, so let's define the symbols "—" to show resistors in series, and "||" to show resistors in parallel. The above network of resistors could be represented as:

R1 — (R2 || R3)

Now, we simplify the network just like a math problem—start with the innermost parentheses and work your way out.

Use this space for summary and/or additional notes:

honors (not AP®)

Step 2: Combine the parallel 40Ω and 35Ω resistors into a single equivalent resistance:

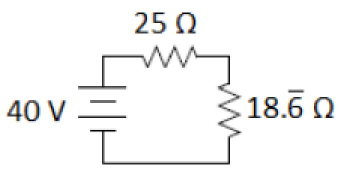
$$25\,\Omega - (40\,\Omega \parallel 35\,\Omega) \longrightarrow 25\,\Omega - (R_{eq.,||})$$

$$\frac{1}{R_{total}} = \frac{1}{40} + \frac{1}{35}$$

$$\frac{1}{R_{total}} = 0.0250 + 0.0286 = 0.0536$$

$$R_{total} = \frac{1}{0.0536} = 18.\bar{6}\,\Omega$$

Now our circuit is equivalent to:



Step 3: Add the two resistances in series to get the total combined resistance of the circuit:

$$25\,\Omega - 18.\bar{6}\,\Omega \longrightarrow R_{total}$$

$$18.\bar{6} + 25 = 43.\bar{6}\,\Omega$$

This gives:

	R_1	R_2	R_3	Total
Voltage (ΔV)				40 V
Current (I)				
Resistance (R)	25 Ω	40 Ω	35 Ω	***43.6̄ Ω***
Power (P)				

Use this space for summary and/or additional notes:

honors (not AP®)

Step 4: Now that we know the total voltage and resistance, we can use Ohm's Law to find the total current:

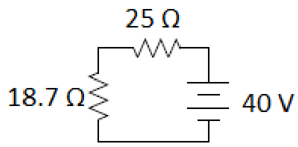
$$\Delta V = IR$$
$$40 = I(43.\overline{6})$$
$$I = \frac{40}{43.\overline{6}} = 0.916\text{ A}$$

While we're at it, let's use $P = I\Delta V = (0.916)(40) = 36.6\text{ W}$ to find the total power.

	R_1	R_2	R_3	Total
Voltage (ΔV)				40 V
Current (I)				***0.916 A***
Resistance (R)	25 Ω	40 Ω	35 Ω	$43.\overline{6}\,\Omega$
Power (P)				***36.6 W***

Now we work backwards.

The next-to-last simplification step was:



The 25 Ω resistor is R1. All of the current goes through it, so the current through R1 must be 0.916 A. Using Ohm's Law, this means the voltage drop across R1 must be:

$$\Delta V = IR = (0.916)(25) = 22.9\text{ V}$$

and the power must be:

$$P = I\Delta V = (0.916)(22.9) = 21.0\text{ W}$$

This means that the voltage across the parallel portion of the circuit (R2 ‖ R3) must be 40 – 22.9 = 17.1 V. Therefore, the voltage is 17.1 V across *both* parallel branches (because voltage is the same across parallel branches).

	R_1	R_2	R_3	Total
Voltage (ΔV)	***22.9 V***	***17.1 V***	***17.1 V***	40 V
Current (I)	***0.916 A***			0.916 A
Resistance (R)	25 Ω	40 Ω	35 Ω	$43.\overline{6}\,\Omega$
Power (P)	***21.0 W***			36.6 W

Use this space for summary and/or additional notes:

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We can use this and Ohm's Law to find the current through one branch:

$$\Delta V_{40\,\Omega} = \Delta V_{35\,\Omega} = 40 - \Delta V_1 = 40 - 22.9 = 17.1V$$

$$\Delta V_{40\,\Omega} = I_{40\,\Omega} R_{40\,\Omega}$$

$$I_{40\,\Omega} = \frac{\Delta V_{40\,\Omega}}{R_{40\,\Omega}} = \frac{17.1}{40} = 0.428\ A$$

We can use Kirchhoff's Junction Rule to find the current through the other branch:

$$I_{total} = I_{40\,\Omega} + I_{35\,\Omega}$$

$$0.916 = 0.428 + I_{35\,\Omega}$$

$$I_{35\,\Omega} = 0.488\ \text{A}$$

This gives us:

	R_1	R_2	R_3	Total
Voltage (Δ*V*)	22.9 V	17.1 V	17.1 V	40 V
Current (*I*)	0.916 A	***0.428 A***	***0.488 A***	0.916 A
Resistance (*R*)	25 Ω	40 Ω	35 Ω	$43.\overline{6}\ \Omega$
Power (*P*)	21.0 W			36.6 W

Finally, because we now have current and resistance for each of the resistors R_2 and R_3, we can use $P = I\Delta V$ to find the power:

$$P_2 = I_2 \Delta V_2 = (0.428)(17.1) = 7.32\ \text{W}$$

$$P_3 = I_3 \Delta V_3 = (0.488)(17.1) = 8.34\ \text{W}$$

	R_1	R_2	R_3	Total
Voltage (Δ*V*)	22.9 V	17.1 V	17.1 V	40 V
Current (*I*)	0.916 A	0.428 A	0.488 A	0.916 A
Resistance (*R*)	25 Ω	40 Ω	35 Ω	$43.\overline{6}\ \Omega$
Power (*P*)	21.0 W	***7.32 W***	***8.34 W***	36.6 W

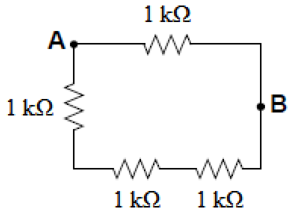
Alternately, because the total power is the sum of the power of each component, once we had the power in all but one resistor, we could have subtracted from the total to find the last one.

Use this space for summary and/or additional notes:

honors (not AP®)

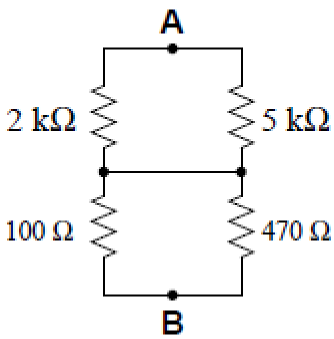
Homework Problems

1. What is the equivalent resistance between points **A** and **B**?



Answer: 750 Ω

2. What is the equivalent resistance between points **A** and **B**?



Hints:

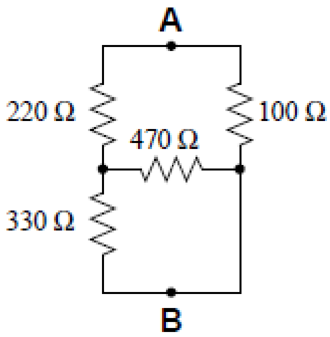
- *Convert the resistances from* kΩ *to* Ω*.*
- *Redrawing the circuit to separate the top and bottom halves may make it easier to understand what is going on.*

Answer: 1511 Ω or 1.511 kΩ

Use this space for summary and/or additional notes:

honors (not AP®)

3. What is the equivalent resistance between points **A** and **B**?



Hint:

- *It may be easier to see what is going on if you move point B to the junction on the right side of the circuit diagram.*

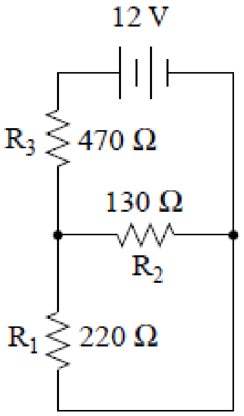
(The space below is intentionally left blank for calculations.)

Answer: 80.5 Ω

Use this space for summary and/or additional notes:

honors (not AP®)

4. Fill in the table for the circuit below:



	R_1	R_2	R_3	Total
Voltage (ΔV)				12 V
Current (I)				
Resistance (R)	220 Ω	130 Ω	470 Ω	
Power (P)				

(The space below is intentionally left blank for calculations.)

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Measuring Voltage, Current & Resistance

Unit: DC Circuits

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Accurately measure voltage and current in a DC circuit.

Success Criteria:

- Multimeter wires are plugged in to the correct jacks and dial is set to the correct quantity.
- Measurements are taken at appropriate points in the circuit. (Voltage is measured in parallel and current is measured in series.)

Language Objectives:

- Explain how to set up the multimeter correctly.
- Explain where to take the measurements and why.

Tier 2 Vocabulary: meter

Labs, Activities & Demonstrations:

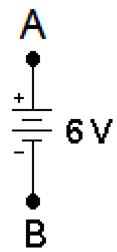
- Show & tell with digital multi-meter.
- Measurement of voltages and currents in a live DC circuit.

Notes:

Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance in each component of a circuit. In order to analyze actual circuits, it is necessary to be able to measure these quantities.

Measuring Voltage

Suppose we want to measure the electric potential (voltage) across the terminals of a 6 V battery. The diagram would look like this:



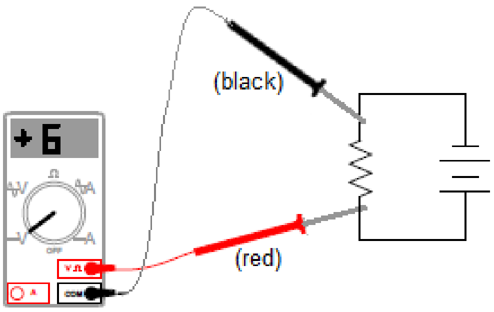
The potential difference (voltage) between points A and B is either +6 V or −6 V, depending on the direction. The voltage from A to B (positive to negative) is +6 V, and the voltage from B to A (negative to positive) is −6 V.

When measuring voltage, the circuit needs to be powered up with current flowing through it. Make sure that the voltmeter is set for volts (DC or AC, as appropriate) and that the red lead is plugged into the V Ω socket (for measuring volts or ohms). Then touch the two leads *in parallel* with the two points you want to measure the voltage across. (*Remember that voltage is the same across all branches of a **parallel** circuit. You want the voltmeter in parallel so the voltmeter reads the same voltage as the voltage across the component that you are measuring.*)

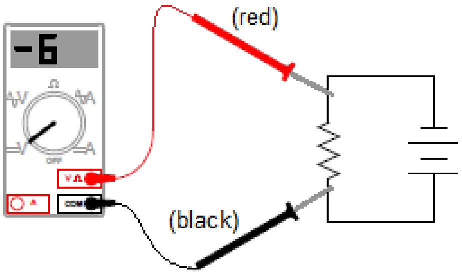
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

On a voltmeter (a meter that measures volts or voltage), the voltage is measured assuming the current is going from the red (+) lead to the black (−) lead. In the following circuit, if you put the red (+) lead on the more positive end of a resistor and the black (−) lead on the more negative end, the voltage reading would be positive. In this circuit, the voltmeter reads a potential difference of +6 V:



If you switch the leads, so the black (−) lead is on the more positive end and the red (+) lead is on the more positive end, the voltage reading would be negative. In this circuit, the voltmeter reads −6 V:



The reading of −6 V indicates that the potential difference is 6 V, but the red lead has a lower electric potential than the black lead. This means that current is actually flowing in the opposite direction from the way the voltmeter is measuring—from the black (−) lead to the red (+) lead.

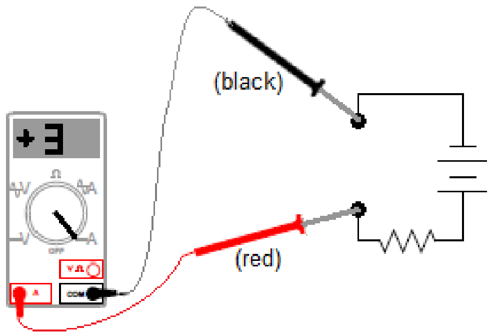
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Measuring Current

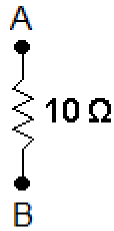
When measuring current, the circuit needs to be open between two points. Make sure the ammeter is set for amperes (A), milliamperes (mA) or microamperes (µA) AC or DC, depending on what you expect the current in the circuit to be. Make sure the red lead is plugged into appropriate socket (A if the current is expected to be 0.5 A or greater, or mA/µA if the current is expected to be less than 0.5 A). Then touch one lead to each of the two contact points, so that the ammeter is *in series* with the rest of the circuit. (*Remember that current is the same through all components in a* ***series*** *circuit. You want the ammeter in series so that all of the current flows through it.*)

On an ammeter (a meter that measures current), the current is measured assuming that it is flowing from the red (+) lead to the black (−) lead. In the following circuit, if you put the red (+) lead on the side that is connected to the positive terminal and the black (−) lead on the end that is connected to the negative terminal, the current reading would be positive. In this circuit, the current is +3 A:



As with the voltage example above, if you switched the leads, the reading would be −3 A instead of +3 A.

Measuring Resistance



Resistance does not have a direction. If you placed an ohmmeter (a meter that measures resistance) across points A and B, it would read 10 Ω regardless of which lead is on which point.

However, because an ohmmeter needs to supply a small amount of current across the component and measure the resistance, the reading is more susceptible to measurement problems, such as the resistance of the wire itself, how well the probes are making contact with the circuit, *etc.* It is often more reliable to measure the voltage and current and calculate resistance using Ohm's Law ($\Delta V = IR$).

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Introduction: Magnetism & Electromagnetism

Unit: Magnetism & Electromagnetism

Topics covered in this chapter:

This chapter discusses electricity and magnetism, how they behave, and how they relate to each other.

- *Magnetism* describes properties of magnets and what causes objects to be magnetic.
- *Magnetic Fields* describes the behavior of objects that are susceptible to magnetism in a magnetic force field.
- *Electricity & Magnetism* describes how electric fields and magnetic fields affect each other.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-5. Provide evidence that an electric current can produce a magnetic field and that a changing magnetic field can produce an electric current.

HS-PS3-5. Develop and use a model of magnetic or electric fields to illustrate the forces and changes in energy between two magnetically or electrically charged objects changing relative position in a magnetic or electric field, respectively.

AP® Physics 1 Learning Objectives:

This unit was never part of the AP® Physics 1 curriculum. It is part of the AP® Physics 2 curriculum.

Topics from this chapter assessed on the SAT Physics Subject Test:

Electromagnetism is not covered on the SAT Physics Subject Test.

Skills learned & applied in this chapter:

- Relating one topic to another.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Magnetism

Unit: Magnetism & Electromagnetism

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- List and explain properties of magnets.

Success Criteria:

- Explanations account for observed behavior.

Language Objectives:

- Explain why we call the ends of a magnet "north" and "south".

Tier 2 Vocabulary: magnet

Labs, Activities & Demonstrations:

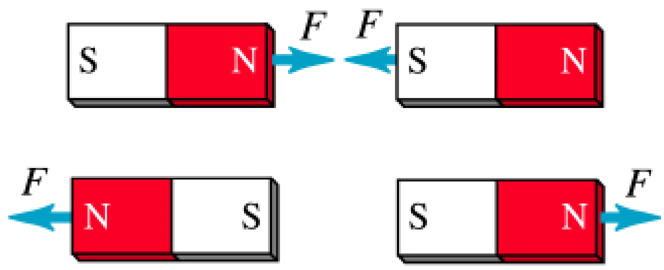
- neodymium magnets
- ring magnets repelling each other on a dowel
- magnets attracting each other across a gap

Notes:

magnet: a material with electrons that can align in a manner that attracts or repels other magnets.

A magnet has two ends or "poles", called "north" and "south". If a magnet is allowed to spin freely, the end that points toward the north on Earth is called the north end of the magnet. The end that points toward the south on Earth is called the south end of the magnet. (The Earth's magnetic poles are near, but not in exactly the same place as its geographic poles.)

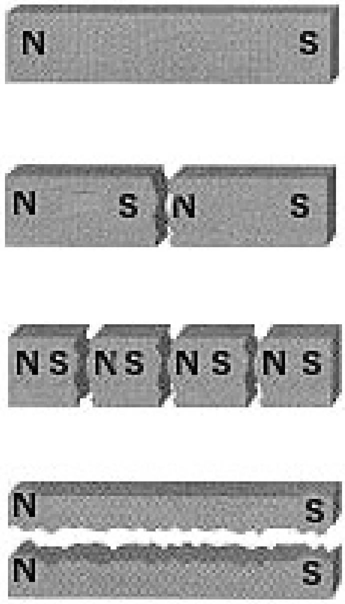
All magnets have a north and south pole. As with charges, opposite poles attract, and like poles repel.



Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

If you were to cut a magnet in half, each piece would be a magnet with its own north and south pole:



Electrons and Magnetism

Magnetism is caused by unpaired electrons in atoms. As you may remember from chemistry, electrons within atoms reside in energy regions called "orbitals". Each orbital can hold up to two electrons.

If two electrons share an orbital, they have opposite spins. (Note that the electrons are not actually spinning. "Spin" is the term for the intrinsic property of certain subatomic particles that is believed to be responsible for magnetism.) This means that if one electron aligns itself with a magnetic field, the other electron in the same orbital becomes aligned to oppose the magnetic field, and there is no net force.

However, if an orbital has only one electron, that electron is free to align with the magnetic field, which causes an attractive force between the magnet and the magnetic material. For example, as you may have learned in chemistry, the electron configuration for iron is:

unpaired electrons

1s	2s	2p	3s	3p	4s	3d
↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓	↑↓ ↑ ↑ ↑ ↑

The inner electrons are paired up, but four of the electrons in the 3d sublevel are unpaired, and are free to align with an external magnetic field.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Magnetic measurements and calculations involve fields that act over 3-dimensional space and change continuously with position. This means that most calculations relating to magnetic fields need to be represented using multivariable calculus, which is beyond the scope of this course.

magnetic permeability (magnetic permittivity): the ability of a material to support the formation of a magnetic field. Magnetic permeability is represented by the variable μ. The magnetic permeability of empty space is $\mu_0 = 4\pi \times 10^{-7}\ \frac{\text{N}}{\text{A}^2}$.

diamagnetic: a material whose electrons cannot align with a magnetic field. Diamagnetic materials have very low magnetic permeabilities.

paramagnetic: a material that has electrons that can align with a magnetic field. Paramagnetic materials have relatively high magnetic permeabilities.

ferromagnetic: a material that can form crystals with permanently-aligned electrons, resulting in a permanent magnet. Ferromagnetic materials can have very high magnetic permeabilities. Some naturally-occurring materials that exhibit ferromagnetism include iron, cobalt, nickel, gadolinium, dysprosium, and magnetite (Fe_3O_4).

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Magnetic Fields

Unit: Magnetism & Electromagnetism

MA Curriculum Frameworks (2016): HS-PS3-5

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Describe and draw magnetic fields.

Success Criteria:

- Magnetic field lines connect north and south poles of the magnet.
- Arrows on field lines point from north to south.

Language Objectives:

- Explain how a compass works.

Tier 2 Vocabulary: field, north pole, south pole

Labs, Activities & Demonstrations:

- magnetic field demonstrator plate
- placing various objects into the gap between two magnets
- ferrofluid

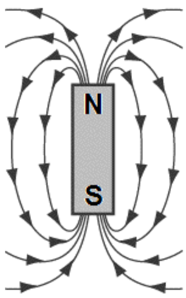
Notes:

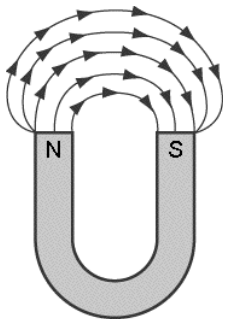
magnetic field: a region in which magnetic attraction and repulsion are occurring.

Similar to gravitational fields and electric fields, a magnetic field is a region in which there is a force on objects that have unpaired electrons that can respond to a magnetic field.

magnetic susceptibility: a measure of the degree of magnetization of a material when it is placed into a magnetic field.

Similar to an electric field, we represent a magnetic field by drawing field lines. Magnetic field lines point from the north pole of a magnet toward the south pole, and they show the direction that the north end of a compass or magnet would be deflected if it was placed in the field:



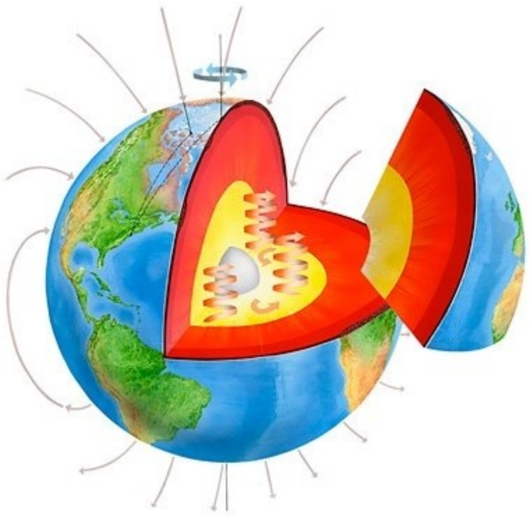


Use this space for summary and/or additional notes:

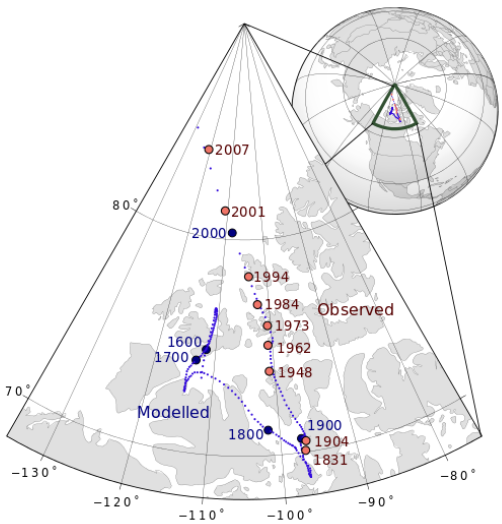
CP1 & honors (not AP®)

The Earth's Magnetic Field

The molten iron outer core and very hot inner core of the Earth causes a magnetic field over the entire planet:



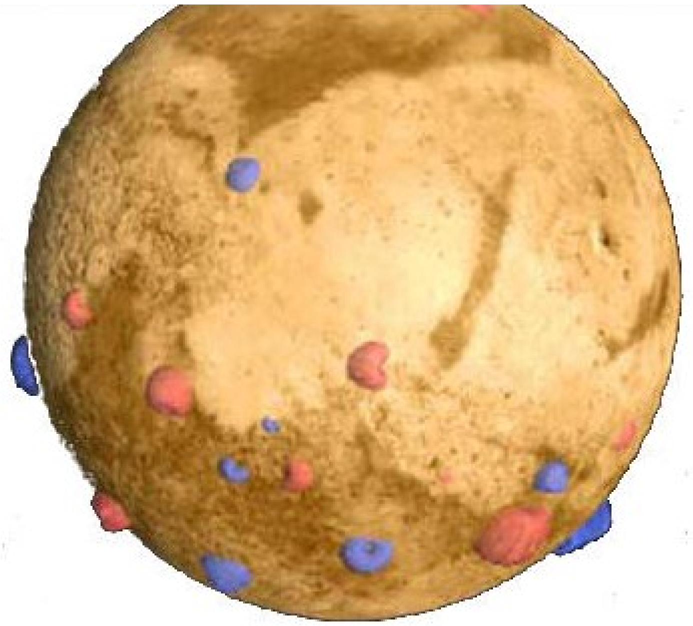
Because the core of the Earth is in constant motion, the Earth's magnetic field is constantly changing. The exact location of the Earth's magnetic north and south poles varies by about 80 km over the course of a day because of the rotation of the Earth. Its average location (shown on the map of Northern Canada below) drifts by about 50 km each year:



Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Not all planets have a planetary magnetic field. Mars, for example, is believed to have once had a planetary magnetic field, but the planet cooled off enough to disrupt the processes that caused it. Instead, Mars has some very strong localized magnetic fields that were formed when minerals cooled down in the presence of the planetary magnetic field:



In this picture, the blue and red areas represent regions with strong localized magnetic fields. On Mars, a compass could not be used in the ways that we use a compass on Earth; if you took a compass to Mars, the needle would point either toward or away from each these regions.

Jupiter, on the other hand, has a planetary magnetic field that is twenty times as strong as that of Earth. This field may be caused by water with dissolved electrolytes or by liquid hydrogen.

Use this space for summary and/or additional notes:

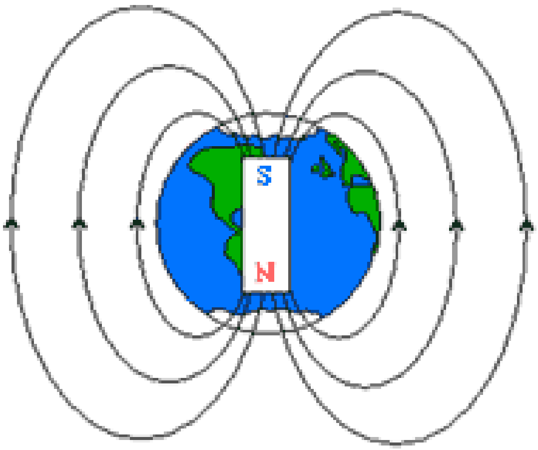
CP1 & honors (not AP®)

Recall that the north pole of a magnet is the end that points toward the north end of the Earth. This must mean that if the Earth is a giant magnet, one of its magnetic poles must be near the geographic north pole, and the other magnetic pole must be near the geographic south pole.

For obvious reasons, the Earth's magnetic pole near the north pole is called the Earth's "north magnetic pole" or "magnetic north pole". Similarly, the Earth's magnetic pole near the south pole is called the Earth's "south magnetic pole" or "magnetic south pole".

However, because the north pole of a magnet points toward the north, the Earth's north magnetic pole (meaning its location) must therefore be the *south* pole of the giant magnet that is the Earth.

Similarly, because the south pole of a magnet points toward the south, the Earth's south magnetic pole (meaning its location) must therefore be the *north* pole of the giant Earth-magnet.



Unfortunately, the term "magnetic north pole," "north magnetic pole" or any other similar term almost always means the magnetic pole that is in the north part of the Earth. There is no universally-accepted way to name the poles of the Earth-magnet.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Electromagnetism

Unit: Magnetism & Electromagnetism

MA Curriculum Frameworks (2016): HS-PS2-5

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Describe and explain ways that electric and magnetic fields affect each other.
- Calculate the voltage and current changes in a step-up or step-down transformer.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Voltage and current changes are described accurately.

Language Objectives:

- Explain how various devices work including solenoids, electromagnets and electric motors.

Tier 2 Vocabulary: electromagnet, transformer

Labs, Activities & Demonstrations:

- current-carrying wire in a magnetic field
- electromagnet
- electric motor
- Lenz's Law (magnets through copper pipe)
- wire & galvanometer jump rope
- neodymium magnet & CRT screen

Notes:

Magnetic Fields and Moving Charges

Like gravitational and electric fields, a magnetic field is a force field. (Recall that force fields are vector quantities, meaning that they have both magnitude and direction.) The strength of a magnetic field is measured in teslas (T), named after the Serbian-American physicist Nikola Tesla.

$$1\,\text{T} = 1\,\frac{\text{N}}{\text{A·m}}$$

In the 1830s, physicists Michael Faraday and Joseph Henry each independently discovered that an electric current could be produced by moving a magnet through a coil of wire, or by moving a wire through a magnetic field. This process is called electromagnetic induction.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

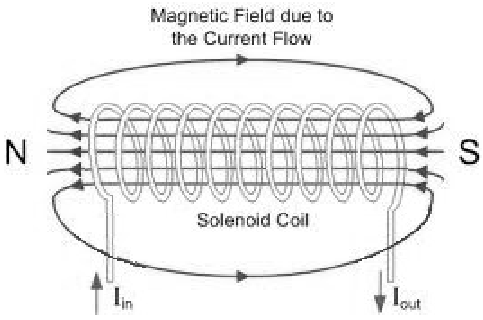
A quantitative study of electromagnetic induction is addressed in Physics 2. In this course, it is sufficient to understand that:

1. Moving charges produce a magnetic field.
2. A changing magnetic field creates an electric force, which causes electric charges to move. (Recall that current is defined as the movement of electric charges.)

Devices that Use Electromagnetism

Solenoid

A solenoid is a coil made of fine wire. When a current is passed through the wire, it produces a magnetic field through the center of the coil.



When a current is applied, a permanent magnet placed in the center of the solenoid will be attracted or repelled and will move.

One of the most common uses of a solenoid is for electric door locks.

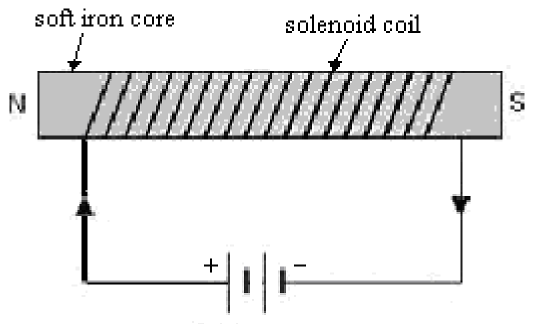
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Electromagnet

An electromagnet is a device that acts as magnet only when electric current is flowing through it.

An electromagnet is made by placing a soft iron core in the center of a solenoid. The high magnetic permeability of iron causes the resulting magnetic field to become thousands of times stronger:



Because the iron core is not a permanent magnet, the electromagnet only works when current is flowing through the circuit. When the current is switched off, the electromagnet stops acting like a magnet and releases whatever ferromagnetic objects might have been attracted to it.

Of course, the above description is a simplification. Real ferromagnetic materials such as iron usually experience magnetic remanence, meaning that some of the electrons in the material remain aligned, and the material is weakly magnetized.

While magnetic remanence is undesirable in an electromagnet, it is the basis for magnetic computer storage media, such as audio and computer tapes and floppy and hard computer disks. To write information onto a disk, a disk head (an electromagnetic that can be moved radially) is pulsed in specific patterns as the disk spins. The patterns are encoded on the disk as locally magnetized regions.

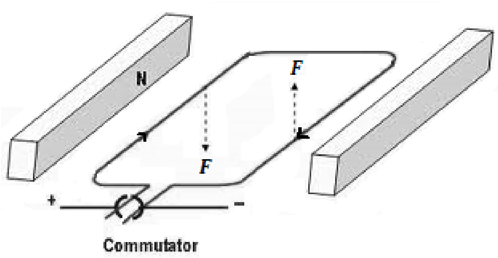
When encoded information is read from the disk, the moving magnetic regions produce a changing electric field that causes an electric current in the disk head.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

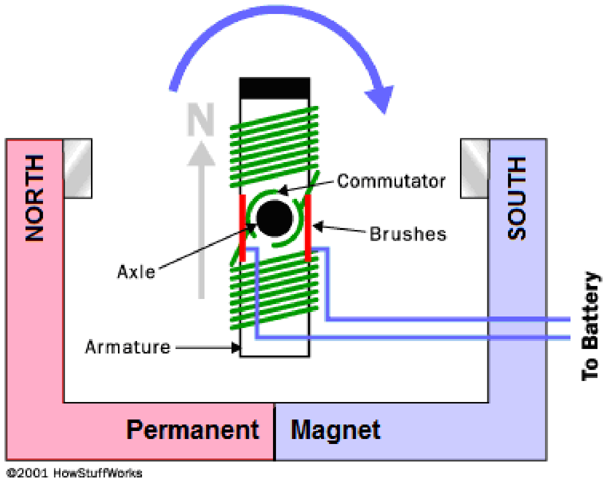
Motor

The force produced by a moving current in a magnetic field can be used to cause a loop of wire to spin:



A *commutator* is used to reverse the direction of the current as the loop turns, so that the combination of attraction and repulsion always applies force in the same direction.

If we replace the loop of wire with an electromagnet (a coil of wire wrapped around a material such as iron that has both a high electrical conductivity and a high magnetic permeability), the electromagnet will spin with a strong force.



An electromagnet that spins because of its continuously switching attraction and repulsion to the magnetic field produced by a separate set of permanent magnets is called a *motor*. A motor turns electric current into rotational motion, which can be used to do work.

Use this space for summary and/or additional notes:

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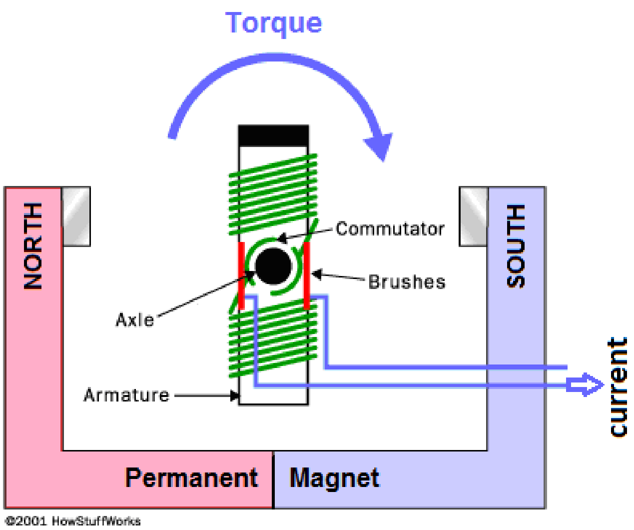
Generator

A *generator* uses the same components as a motor and operates under the same principle, except that a mechanical force is used to spin the coil. When the coil moves through the magnetic field, it produces an electric current. Thus a generator is a device that turns rotational motion (work) into an electric current.



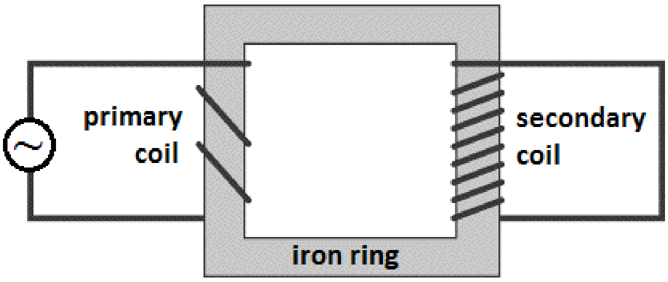
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Inductor (Transformer)

Because electric current produces a magnetic field, a ring made of a ferromagnetic material can be used to move an electric current. An inductor (transformer) is a device that takes advantage of this phenomenon in order to increase or decrease the voltage in an AC circuit.

The diagram below shows an inductor or transformer.



The current on the input side (primary) generates a magnetic field in the iron ring. The magnetic field in the ring generates a current on the output side (secondary).

In this particular transformer, the coil wraps around the output side more times than the input. This means that each time the current goes through the coil, the magnetic field adds to the electromotive force (voltage). This means the voltage will increase in proportion to the increased number of coils on the output side. However, the magnetic field on the output side will produce less current with each turn, which means the current will decrease in the same proportion:

$$\frac{\#turns_{in}}{\#turns_{out}} = \frac{\Delta V_{in}}{\Delta V_{out}} = \frac{I_{out}}{I_{in}}$$

$$P_{in} = P_{out}$$

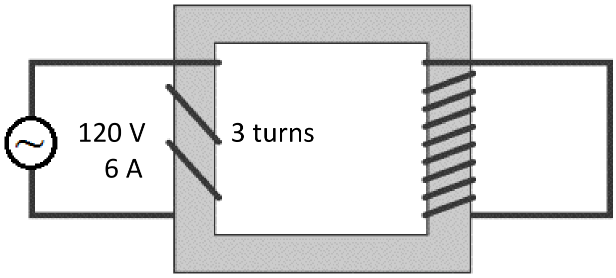
A transformer like this one, which produces an increase in voltage, is called a *step-up transformer*; a transformer that produces a decrease in voltage is called a *step-down transformer*.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Sample Problem:

Q: If the input voltage to the following transformer is 120 V, and the input current is 6 A, what are the output voltage and current?



A: The voltage on either side of a transformer is proportional to the number of turns in the coil on that side. In the above transformer, the primary has 3 turns, and the secondary coil has 9 turns. This means the voltage on the right side will be $\frac{9}{3} = 3$ times as much as the voltage on the left, or 360 V. The current will be $\frac{3}{9} = \frac{1}{3}$ as much, or 2 A.

We can also use:

$$\frac{\#turns_{in}}{\#turns_{out}} = \frac{\Delta V_{in}}{\Delta V_{out}} \qquad \frac{\#turns_{in}}{\#turns_{out}} = \frac{I_{out}}{I_{in}}$$

$$\frac{3}{9} = \frac{120\text{ V}}{\Delta V_{out}} \qquad \frac{3}{9} = \frac{I_{out}}{6\text{ A}}$$

$$\Delta V_{out} = 360\text{ V} \qquad I_{out} = 2\text{ A}$$

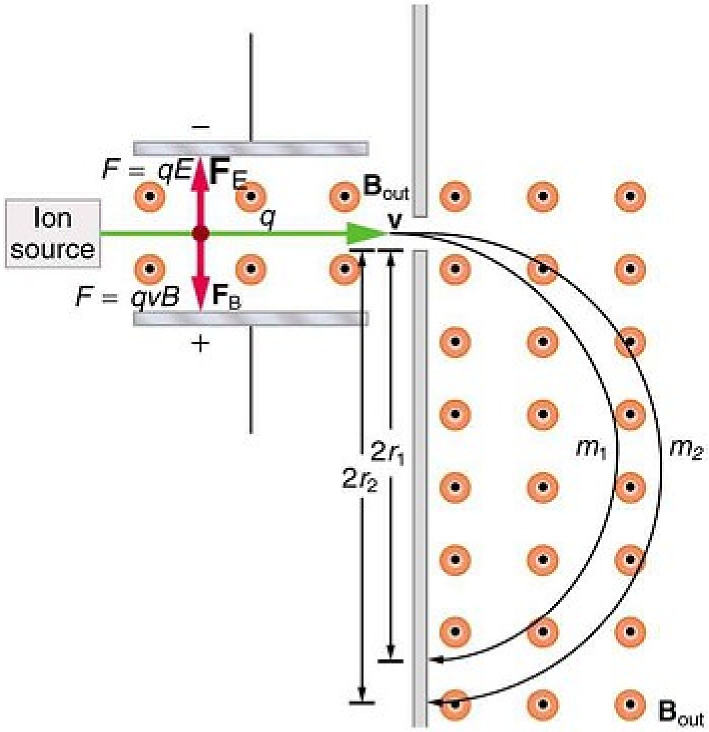
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Mass Spectrometer

A mass spectrometer is a device uses the path of a charged particle in a magnetic field to determine its mass.

The particle is first selected for the desired velocity by a combination of externally-applied magnetic and electric fields. Then the particle enters a chamber with only a magnetic field. (In the example below, the magnetic field, which is represented by the bullseyes, is directed out of the page.)



The magnetic field applies a force on the particle perpendicular to its path (downward in this example). As the particle's direction changes, the direction of the applied force changes with it, causing the particle to move in a circular path until it hits the detector.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Introduction: Waves

Unit: Waves

Topics covered in this chapter:

This chapter discusses properties of waves that travel through a medium (mechanical waves).

- *Waves* gives general information about waves, including vocabulary and equations. *Reflection and Superposition* describes what happens when two waves share space within a medium.
- *Sound & Music* describes the properties and equations of waves that relate to music and musical instruments.
- *Sound Level* describes the decibel scale and how loudness is measured.
- *The Doppler Effect* describes the change in pitch due to motion of the source or receiver (listener).
- *Exceeding the Speed of Sound* describes the Mach scale and sonic booms.
- *Electromagnetic Waves* describes special properties of electromagnetic waves, such as light and radio waves.
- *Color* describes visible light and our perceptions of color.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling within various media. Recognize that electromagnetic waves can travel through empty space (without a medium) as compared to mechanical waves that require a medium.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

AP® Physics 1 Learning Objectives:

This unit was removed from AP® Physics 1 starting with the 2021–22 school year. It is now part of the AP® Physics 2 curriculum.

Topics from this chapter assessed on the SAT Physics Subject Test:

- **General Wave Properties**, such as wave speed, frequency, wavelength, superposition, standing wave diffraction, and the Doppler effect.

  1. Wave Motion
  2. Transverse Waves and Longitudinal Waves
  3. Superposition
  4. Standing Waves and Resonance
  5. The Doppler Effect

Skills learned & applied in this chapter:

- Visualizing wave motion.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Waves

Unit: Waves

MA Curriculum Frameworks (2016): HS-PS4-1

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Describe and explain properties of waves (frequency, wavelength, etc.)
- Distinguish between transverse, longitudinal and torsional waves.
- Calculate wavelength, frequency, period, and velocity of a wave.

Success Criteria:

- Parts of a wave are identified correctly.
- Descriptions & explanations account for observed behavior.

Language Objectives:

- Describe how waves propagate.

Tier 2 Vocabulary: wave, crest, trough, frequency, wavelength

Labs, Activities & Demonstrations:

- Show & tell: transverse waves in a string tied at one end, longitudinal waves in a spring, torsional waves.
- Buzzer in a vacuum.
- Tacoma Narrows Bridge collapse movie.
- Japan tsunami TV footage.

Notes:

wave: a disturbance that travels from one place to another,* carrying energy.

medium: a substance that a wave travels through.

propagation: the process of a wave traveling through a medium or empty space.

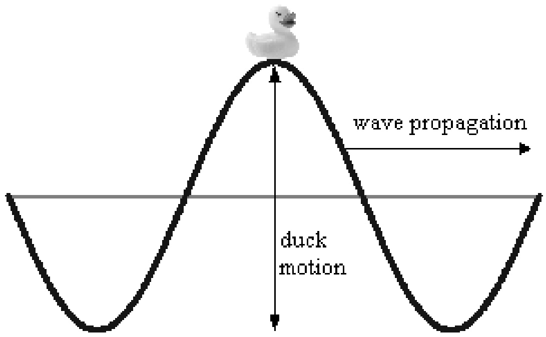
* This is my favorite definition in these notes. I jokingly suggest that I nickname some of my students "wave" based on this definition.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

mechanical wave: a wave that propagates through a medium via contact between particles of the medium. Some examples of mechanical waves include ocean waves and sound waves.

1. The energy of the wave is transmitted via the particles of the medium as the wave passes through it.
2. The wave travels through the medium. The particles of the medium are moved by the wave passing through, and then return to their original position. (The duck sitting on top of the wave below is an example.)



3. Waves generally move fastest in solids and slowest in liquids. The velocity of a mechanical wave is dependent on characteristics of the medium:

state	relevant factors	example: medium	example: velocity of sound
gas	density, pressure	air (20 °C and 1 atm)	$343\,\frac{m}{s}$ $(768\,\frac{mi}{hr})$
liquid	density, compressibility	water (20 °C)	$1\,481\,\frac{m}{s}$ $(3\,317\,\frac{mi}{hr})$
solid	stiffness	steel (longitudinal wave)	$6\,000\,\frac{m}{s}$ $(13\,000\,\frac{mi}{hr})$

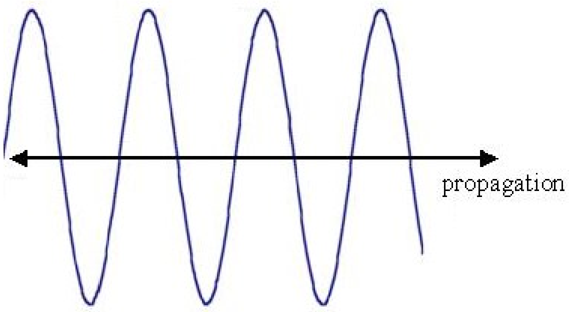
electromagnetic wave: a wave of electricity and magnetism interacting with each other. Electromagnetic waves can propagate through empty space, and are slowed down by interactions with a medium.

Use this space for summary and/or additional notes:

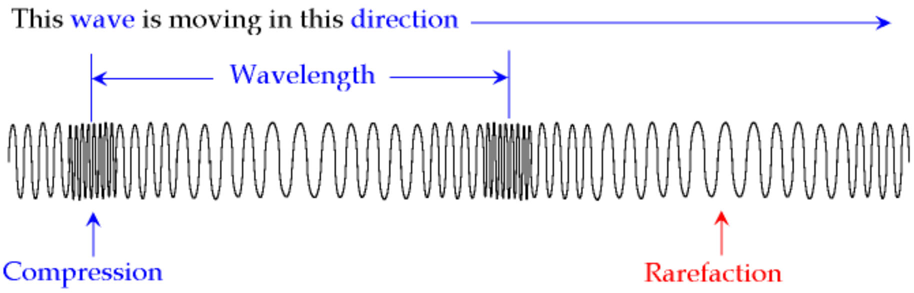
CP1 & honors (not AP®)

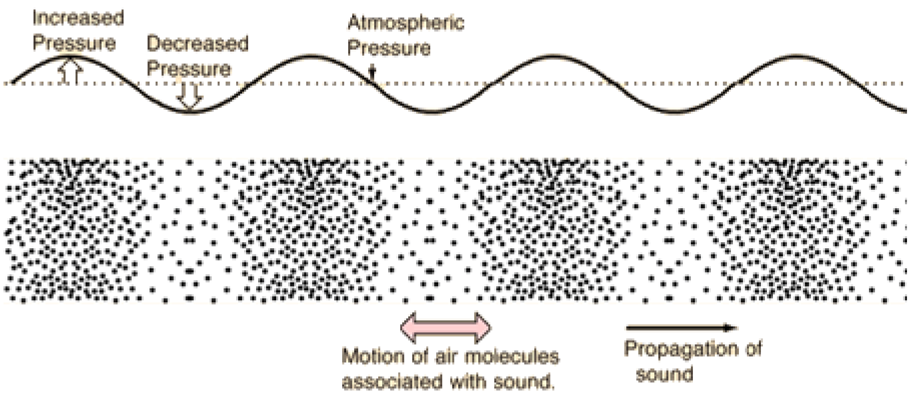
Types of Waves

<u>transverse wave</u>: moves its medium up & down (or back & forth) as it travels through. Examples: light, ocean waves



<u>longitudinal wave</u> (or compressional wave): compresses and decompresses the medium as it travels through. Examples: compression of a spring, sound.

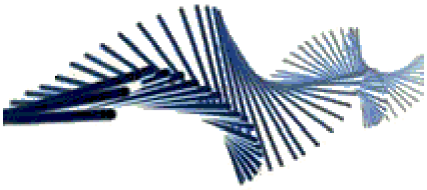




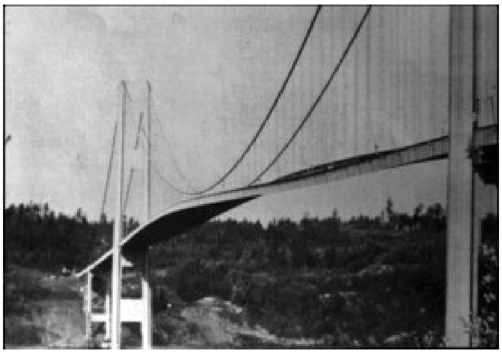
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

<u>torsional wave</u>: a type of transverse wave that propagates by twisting about its direction of propagation.



The most famous example of the destructive power of a torsional wave was the Tacoma Narrows Bridge, which collapsed on November 7, 1940. On that day, strong winds caused the bridge to vibrate torsionally. At first, the edges of the bridge swayed about eighteen inches. (This behavior had been observed previously, earning the bridge the nickname "Galloping Gertie".) However, after a support cable snapped, the vibration increased significantly, with the edges of the bridge being displaced up to 28 feet! Eventually, the bridge started twisting in two halves, one half twisting clockwise and the other half twisting counterclockwise, and then back again. This opposing torsional motion eventually caused the bridge to twist apart and collapse.



The bridge's collapse was captured on film. Video clips of the bridge twisting and collapsing are available on the internet. There is a detailed analysis of the bridge's collapse at http://www.vibrationdata.com/Tacoma.htm

Use this space for summary and/or additional notes:

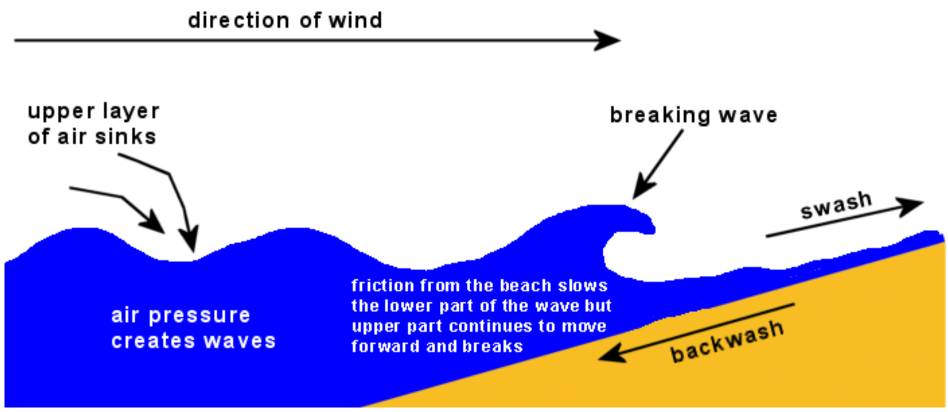
CP1 & honors (not AP®)

Ocean Waves

Surface Waves

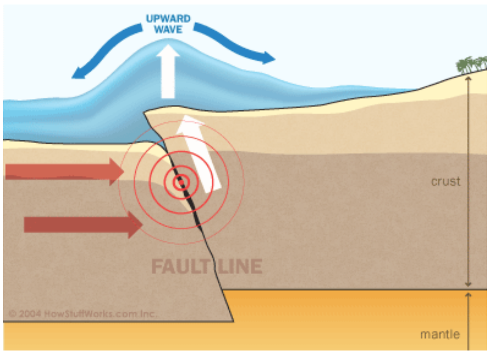
<u>surface wave</u>: a transverse wave that travels at the interface between two mediums.

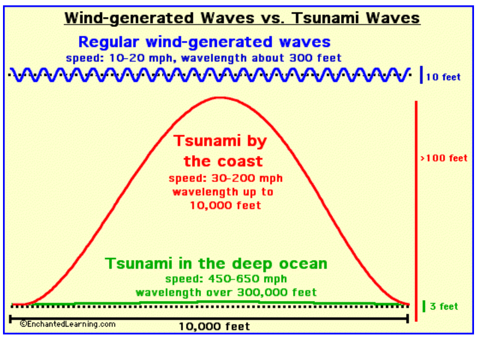
Ocean waves are an example of surface waves, because they travel at the interface between the air and the water. Surface waves on the ocean are caused by wind disturbing the surface of the water. Until the wave gets to the shore, surface waves have no effect on water molecules far below the surface.



Tsunamis

The reason tsunamis are much more dangerous than regular ocean waves is because tsunamis are created by earthquakes on the ocean floor. The tsunami wave propagates through the entire depth of the water, which means tsunamis carry many times more energy than surface waves.



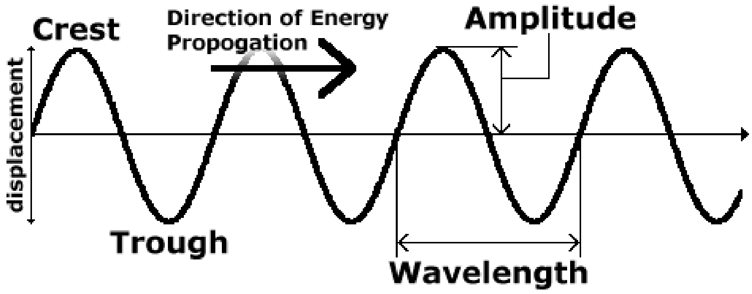


This is why a 6–12 foot high surface wave breaks harmlessly on the beach; however, a tsunami that extends 6–12 feet above the surface of the water includes a significant amount of energy throughout the entire depth of the water, and can destroy an entire city.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Properties of Waves



crest: the point of maximum positive displacement of a transverse wave. (The highest point.)

trough: the point of maximum negative displacement of a transverse wave. (The lowest point.)

amplitude (*A*): the distance of maximum displacement of a point in the medium as the wave passes through it. (The maximum height or depth.)

wavelength (λ): the length of the wave, measured from a specific point in the wave to the same point in the next wave. Unit = distance (m, cm, nm, *etc.*)

frequency (*f* or ν): the number of waves that travel past a point in a given time. Unit = $^1/_{\text{time}}$ (Hz = $^1/_s$)

Note that while high school physics courses generally use the variable *f* for frequency, college courses usually use *ν* (the Greek letter "nu", which is different from but easy to confuse with the Roman letter "v").

period or time period (*T*): the amount of time between two adjacent waves. Unit = time (usually seconds)

$$T = {}^1/_f \quad \text{and} \quad f = {}^1/_T$$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

<u>velocity</u>: the velocity of a wave depends on its frequency (f) and its wavelength (λ):

$$v = \lambda f$$

The velocity of electromagnetic waves (such as light, radio waves, microwaves, X-rays, *etc.*) is called the <u>speed of light</u>, which is $3.00\times10^8\ \frac{m}{s}$ in a vacuum. The speed of light is slower in a medium that has an index of refraction* greater than 1.

The velocity of a wave traveling through a string under tension (such as a piece of string, a rubber band, a violin/guitar string, *etc.*) depends on the tension and the ratio of the mass of the string to its length:

$$v_{string} = \sqrt{\frac{F_T L}{m}}$$

where F_T is the tension in the string, L is the length, and m is the mass. This will be discussed further in Sound & Music, on page 583.

Sample Problem:

Q: The Boston radio station WZLX broadcasts waves with a frequency of 100.7 MHz. If the waves travel at the speed of light, what is the wavelength?

A: $f = 100.7\,\text{MHz} = 100\,700\,000\,\text{Hz} = 1.007\times10^8\ \text{Hz}$

$v = c = 3.00\times10^8\ \frac{m}{s}$

$v = \lambda f$

$3.00\times10^8 = \lambda\,(1.007\times10^8)$

$\lambda = \dfrac{3.00\times10^8}{1.007\times10^8} = 2.98\,\text{m}$

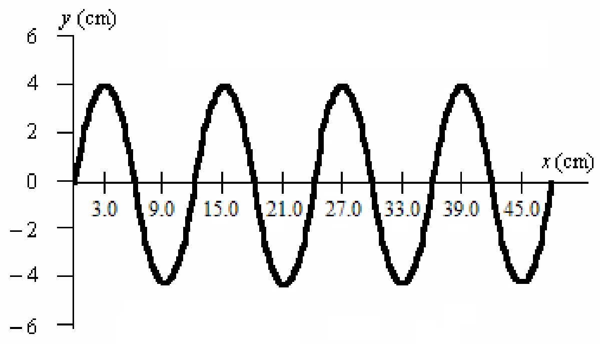
* The index of refraction is a measure of how much light bends when it moves between one medium and another. The sine of the angle of refraction is proportional to the speed of light in that medium. Index of refraction is studied as part of optics in Physics 2.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Homework Problems

1. Consider the following wave:



   a. What is the amplitude of this wave?

   b. What is its wavelength? (Give your answer in both cm and m.)

   c. If the velocity of this wave is $30\,\frac{m}{s}$, what is its period?

   Answer: 0.004 s

2. What is the speed of wave with a wavelength of 0.25 m and a frequency of 5.5 Hz?

   Answer: $1.375\,\frac{m}{s}$

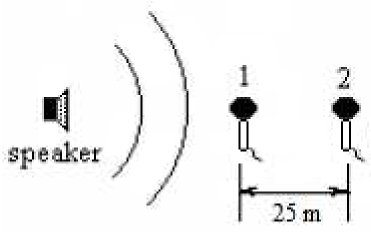
3. A sound wave traveling in water at 10 °C has a wavelength of 0.65 m. What is the frequency of the wave.
   (*Note: you will need to look up the speed of sound in water at 10 °C in* Table W. Properties of Water and Air *on page 692 of your Physics Reference Tables*.)

   Answer: 2 226Hz

Use this space for summary and/or additional notes:

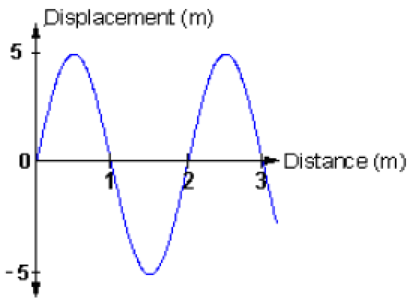
CP1 & honors (not AP®)

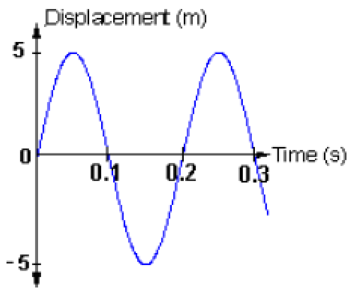
4. Two microphones are placed in front of a speaker as shown in the diagram to the right. If the air temperature is 30 °C, what is the time delay between the two microphones?



Answer: 0.0716 s

5. The following are two graphs of the same wave. The first graph shows the displacement *vs.* distance, and the second shows displacement *vs.* time.





a. What is the wavelength of this wave?

b. What is its amplitude?

c. What is its frequency?

Answer: 5 Hz

d. What is its velocity?

Answer: $10\,\frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Reflection and Superposition

Unit: Waves

MA Curriculum Frameworks (2016): HS-PS4-1

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain the behavior of waves when they pass each other in the same medium and when they reflect off something.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain what happens when two waves pass through each other.

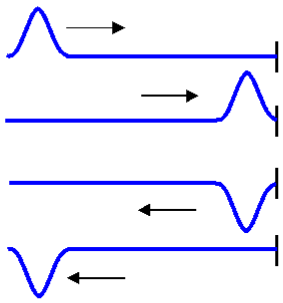
Tier 2 Vocabulary: reflection

Labs, Activities & Demonstrations:

- waves on a string or spring anchored at one end
- large Slinky with longitudinal and transverse waves passing each other

Notes:

Reflection of Waves



reflection: when a wave hits a fixed (stationary) point and "bounces" back.

Notice that when the end of the rope is fixed, the reflected wave is inverted. (If the end of the rope were free, the wave would not invert.)

Use this space for summary and/or additional notes:

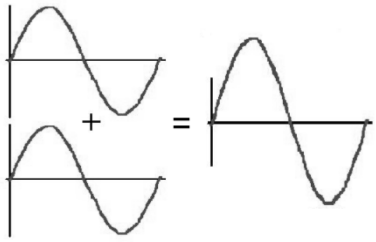
CP1 & honors (not AP®)

Superposition of Waves

When waves are superimposed (occupy the same space), their amplitudes add.

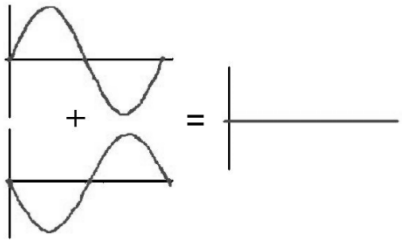
<u>constructive interference</u>: when waves add in a way that the amplitude of the resulting wave is larger than the amplitudes of the component waves.

Because the wavelengths are the same and the maximum, minimum, and zero points all coïncide (line up), the two component waves are said to be "in phase" with each other.



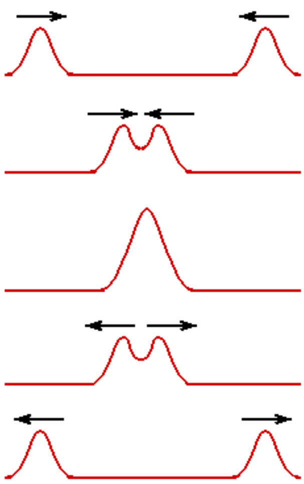
<u>destructive interference</u>: when waves add in a way that the amplitude of the resulting wave is smaller than the amplitudes of the component waves. (Sometimes we say that the waves "cancel" each other.)

Because the wavelengths are the same but the maximum, minimum, and zero points do not coïncide, the waves are said to be "out of phase" with each other.

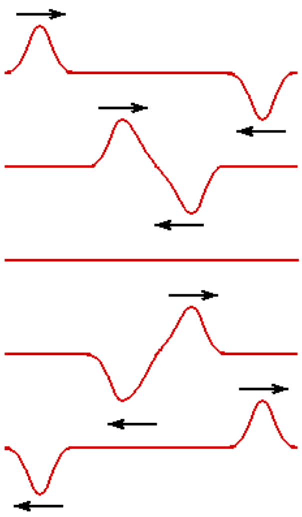


Note that waves can travel in two opposing directions at the same time. When this happens, the waves pass through each other, exhibiting constructive and/or destructive interference as they pass:

Constructive Interference



Destructive Interference



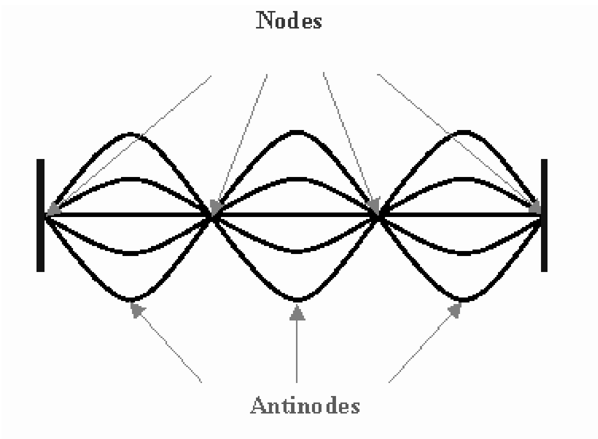
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

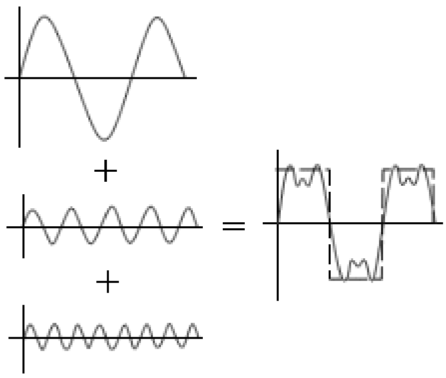
Standing Waves

standing wave: when half of the wavelength is an exact fraction of the length of a medium that is vibrating, the wave reflects back and the reflected wave interferes constructively with itself. This causes the wave to appear stationary.

Points along the wave that are not moving are called "nodes". Points of maximum displacement are called "antinodes".



When we add waves with different wavelengths and amplitudes, the result can be complex:



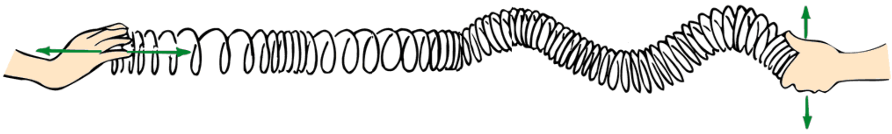
This is how radio waves encode a signal on top of a "carrier" wave. Your radio's antenna receives ("picks up") radio waves within a certain range of frequencies. Imagine that the bottom wave (the one with the shortest wavelength and highest frequency) is the "carrier" wave. If you tune your radio to its frequency, the radio will filter out other waves that don't include the carrier frequency. Then your radio subtracts the carrier wave, and everything that is left is sent to the speakers.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Homework Problem

1. A Slinky is held at both ends. The person on the left creates a longitudinal wave, while at same time the person on the right creates a transverse wave with the same frequency. Both people stop moving their ends of the Slinky just as the waves are about to meet.



   a. Draw a picture of what the Slinky will look like when the waves completely overlap.

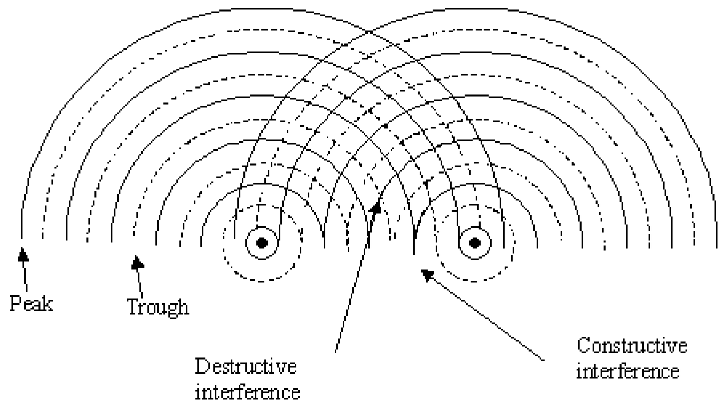
   b. Draw a picture of what the Slinky will look like just after the waves no longer overlap.

Use this space for summary and/or additional notes:

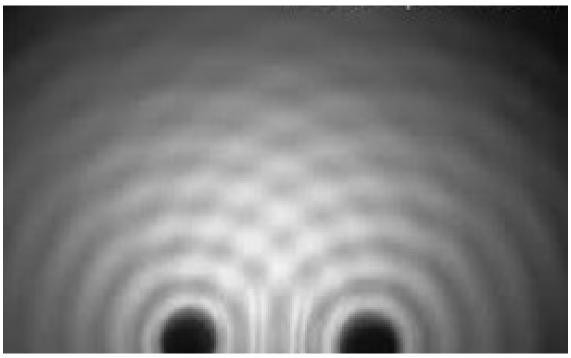
CP1 & honors (not AP®)

Two-Dimensional Interference Patterns

When two progressive waves propagate into each other's space, the waves produce interference patterns. This diagram shows how interference patterns form:



The resulting interference pattern looks like the following picture:



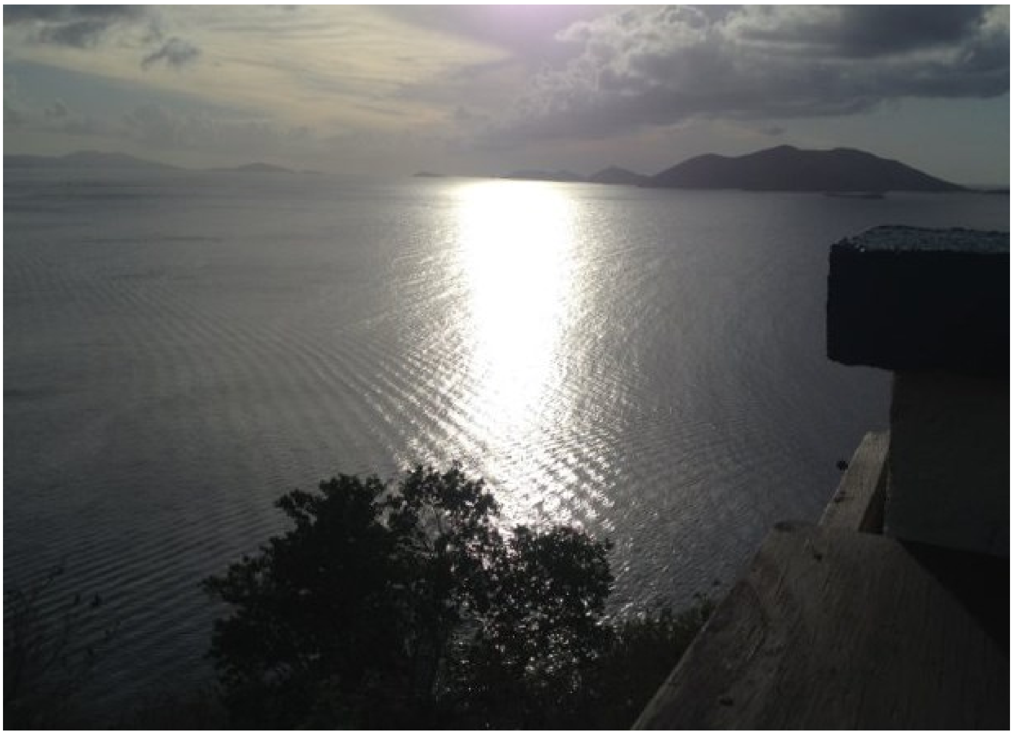
In this picture, the bright regions are wave peaks, and the dark regions are troughs. The brightest intersections are regions where the peaks interfere constructively, and the darkest intersections are regions where the troughs interfere constructively.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

The following picture* shows an interference pattern created by ocean waves, one of which has been reflected off a point on the shore. The wave at the left side of the picture is traveling toward the right, and the reflected wave at the bottom right of the picture is traveling toward the top of the picture.

Because the sun is low in the sky (the picture was taken just before sunset), the light is reflected off the water, and the crests of the waves produce shadows behind them.



* The picture is taken from Tortola in the British Virgin Islands, looking west toward Jost Van Dyke.

Use this space for summary and/or additional notes:

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Sound & Music

Unit: Waves

MA Curriculum Frameworks (2016): HS-PS4-1

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Describe how musical instruments produce sounds.
- Describe how musical instruments vary pitch.
- Calculate frequencies of pitches produced by a vibrating string or in a pipe.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what produces the vibrations in various types of musical instruments.

Tier 2 Vocabulary: pitch

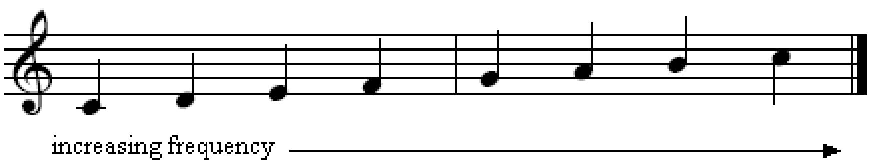
Labs, Activities & Demonstrations:

- Show & tell: violin, penny whistle, harmonica, boomwhackers.
- Helmholtz resonators—bottles of different sizes/air volumes, slapping your cheek with your mouth open.
- Frequency generator & speaker.
- Rubens tube ("sonic flame tube").
- Measure the speed of sound in air using a resonance tube.

Notes:

Sound waves are caused by vibrations that create longitudinal (compressional) waves in the medium they travel through (such as air).

<u>pitch</u>: how "high" or "low" a musical note is. The pitch is determined by the frequency of the sound wave.



Use this space for summary and/or additional notes:

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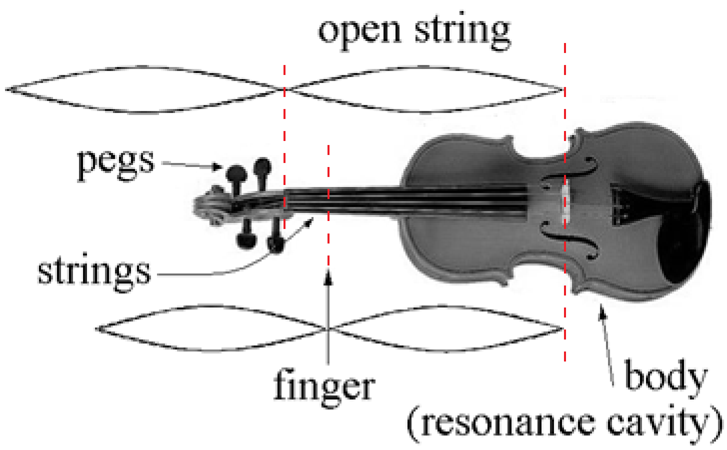
resonance: when the wavelength of a half-wave (or an integer number of half-waves) coincides with one of the dimensions of an object. This creates standing waves that reinforce and amplify each other. The body of a musical instrument is an example of an object that is designed to use resonance to amplify the sounds that the instrument produces.

String Instruments

A string instrument (such as a violin or guitar) typically has four or more strings. The lower strings (strings that sound with lower pitches) are thicker, and higher strings are thinner. Pegs are used to tune the instrument by increasing (tightening) or decreasing (loosening) the tension on each string.



The vibration of the string creates a half-wave, *i.e.,* λ = 2L. The musician changes the half-wavelength by using a finger to shorten the part of the string that vibrates. (A shorter wavelength produces a higher frequency = higher pitch.)

The velocity of the wave produced on a string depends on the tension and the length and mass of the vibrating portion. The velocity is given by the equation:

$$v_{string} = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{F_T L}{m}}$$

where:

v = velocity (Hz)

m = mass of string (kg)

F_T = tension (N)

L = length of string (m) $= \frac{\lambda}{2}$

μ = mass per unit length of string $\left(\frac{\text{kg}}{\text{m}}\right)$

Given the velocity and wavelength, we can calculate the frequency (pitch):

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F_T L}{m}} = \sqrt{\frac{F_T}{4mL}}$$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Pipes and Wind Instruments

A pipe (in the musical instrument sense) is a tube filled with air. The design of the mouthpiece (or air inlet) causes the air to oscillate as it enters the pipe. This causes the air molecules to compress and spread out at regular intervals based on the dimensions of the closed section of the instrument, which determines the wavelength. The wavelength and speed of sound determine the frequency.

Most wind instruments use one of three ways of causing the air to oscillate:

Brass Instruments

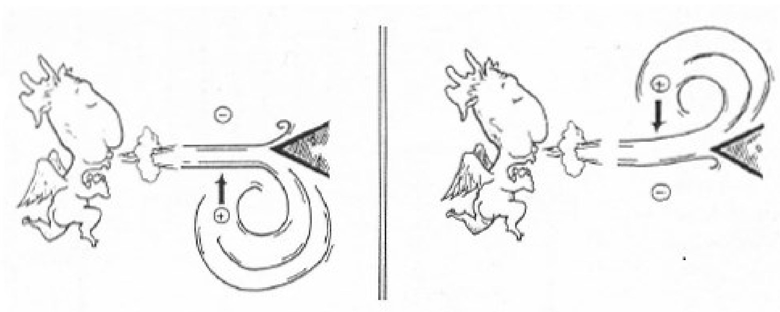
With brass instruments like trumpets, trombones, French horns, *etc.*, the player presses his/her lips tightly against the mouthpiece, and the player's lips vibrate at the appropriate frequency.

Reed Instruments

With reed instruments, air is blown past a reed (a semi-stiff object) that vibrates back and forth. Clarinets and saxophones use a single reed made from a piece of cane (a semi-stiff plant similar to bamboo). Oboes and bassoons ("double-reed instruments") use two pieces of cane that vibrate against each other. Harmonicas and accordions use reeds made from a thin piece of metal.

Whistles (Instruments with Fipples)

Instruments with fipples include recorders, whistles and flutes. A fipple is a sharp edge that air is blown past. The separation of the air going past the fipple results in a pressure difference on one side *vs.* the other. Air moves toward the lower pressure side, causing air to build up and the pressure to increase. When the pressure becomes greater than the other side, the air switches abruptly to the other side of the fipple. Then the pressure builds on the other side until the air switches back:



The frequency of this back-and-forth motion is what determines the pitch.

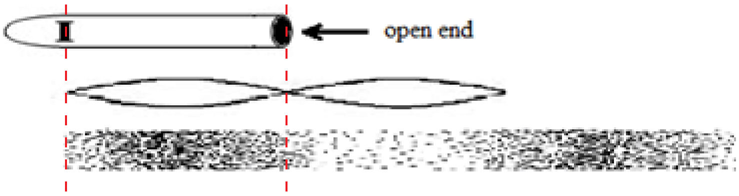
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Open vs. Closed-Pipe Instruments

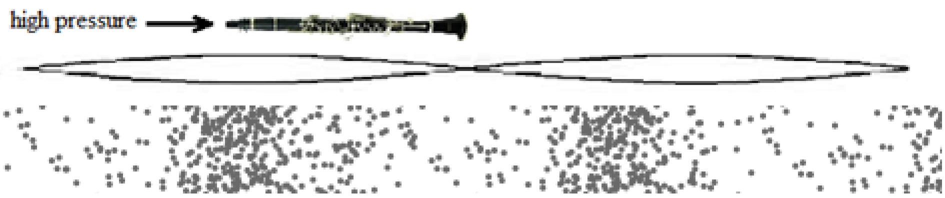
An open-pipe instrument has an opening at each end. A closed-pipe instrument has an opening at one end, and is closed at the other.

Examples of open-pipe instruments include uncapped organ pipes, whistles, recorders and flutes.



Notice that the two openings determine where the air pressure *must* be equal to atmospheric pressure (*i.e.,* the air is neither compressed nor expanded). This means that the length of the body of the instrument (L) is a half-wave, and that the wavelength (λ) of the sound produced must therefore be twice as long, *i.e.,* $\lambda = 2L$. (This is similar to string instruments, in which the length of the vibrating string is a half-wave.)

Examples of closed-pipe instruments include clarinets and all brass instruments. Air is blown in at high pressure via the mouthpiece, which means the mouthpiece is an antinode—a region of maximum displacement of the individual air molecules. This means that the body of the instrument is the distance from the antinode to a region of atmospheric pressure, *i.e.,* one-fourth of a wave. This means that for closed-pipe instruments, $\lambda = 4L$.



The difference in the resonant wavelength ($4L$ *vs.* $2L$) is why a closed-pipe instrument (*e.g.,* a clarinet) sounds an octave lower than an open-pipe instrument of similar length (*e.g.,* a flute)—twice the wavelength results in half the frequency.

Use this space for summary and/or additional notes:

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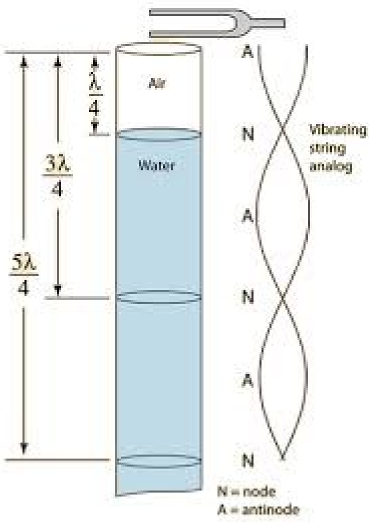
The principle of a closed-pipe instrument can be used in a lab experiment to determine the frequency of a tuning fork (or the speed of sound) using a resonance tube—an open tube filled with water to a specific depth.



A tuning fork generates an oscillation of a precise frequency at the top of the tube. Because this is a closed pipe, the source (just above the tube) is an antinode (maximum amplitude).

When the height of air above the water is exactly ¼ of a wavelength $\left(\frac{\lambda}{4}\right)$, the waves that are reflected back have maximum constructive interference with the source wave, which causes the sound to be significantly amplified. This phenomenon is called resonance.

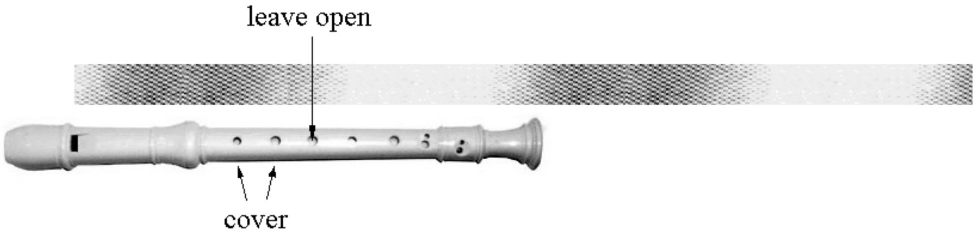
Resonance will occur at every antinode—*i.e.,* any integer plus ¼ of a wave ($\frac{\lambda}{4}$, $\frac{3\lambda}{4}$, $\frac{5\lambda}{4}$, *etc.*)

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Playing Different Pitches (Frequencies)

For an instrument with holes, like a flute or recorder, the first open hole is the first place in the pipe where the pressure is equal to atmospheric pressure, which determines the half-wavelength (or quarter-wavelength):



The speed of sound in air is v_s ($343\,\frac{\text{m}}{\text{s}}$ at 20 °C and 1 atm), which means the frequency of the note (from the formula $v_s = \lambda f$) will be:

$f = \dfrac{v_s}{2L}$ for an open-pipe instrument (*e.g.,* flute, recorder, whistle)

$f = \dfrac{v_s}{4L}$ for an closed-pipe instrument (*e.g.,* clarinet, brass instrument).

Note that the frequency is directly proportional to the speed of sound in air. The speed of sound increases as the temperature increases, which means that as the air gets colder, the frequency gets lower, and as the air gets warmer, the frequency gets higher. This is why wind instruments go flat at colder temperatures and sharp at warmer temperatures. Musicians claim that the instrument is going out of tune, but actually it's not the instrument that is out of tune, but the speed of sound!

Note however, that the frequency is inversely proportional to the wavelength (which depends largely on the length of the instrument). This means that the extent to which the frequency changes with temperature will be different for different-sized instruments, which means the band will become more and more out of tune with itself as the temperature changes.

Use this space for summary and/or additional notes:

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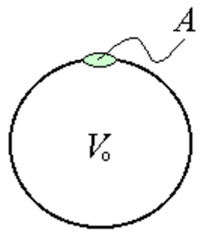
Helmholtz Resonators

The resonant frequency of a bottle or similar container (called a Helmholtz resonator, named after the German physicist Hermann von Helmholtz) is more complicated to calculate.

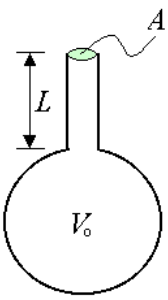
For an enclosed volume of air with a single opening, the resonant frequency depends on the resonant frequency of the air in the large cavity, and the cross-sectional area of the opening.



Resonant frequency:

$$f = \frac{v_s}{2\pi} \cdot \frac{A}{V_o}$$

For a bottle with a neck, the air in the neck behaves like a spring, with a spring constant that is proportional to the volume of air in the neck:



Resonant frequency:

$$f = \frac{v_s}{2\pi} \sqrt{\frac{A}{V_o L}}$$

where:

- f = resonant frequency
- v_s = speed of sound in air ($343\,\frac{\text{m}}{\text{s}}$ at 20 °C and 1 atm)
- A = cross-sectional area of the neck of the bottle (m^2)
- V_o = volume of the main cavity of the bottle (m^3)
- L = length of the neck of the bottle (m)

(Note that it may be more convenient to use measurements in cm, cm^2, and cm^3, and use $v_s = 34\,300\,\frac{\text{cm}}{\text{s}}$.)

Blowing across the top of an open bottle is an example of a Helmholtz resonator.

You can make your mouth into a Helmholtz resonator by tapping on your cheek with your mouth open. You can change the pitch by opening or closing your mouth a little, which changes the area of the opening (A).

Use this space for summary and/or additional notes:

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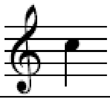
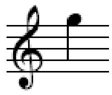
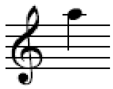
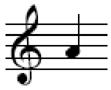
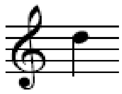
Frequencies of Music Notes

The frequencies that correspond with the pitches of the Western equal temperament scale are:

pitch		frequency (Hz)	pitch		frequency (Hz)
	A	440.0		E	659.3
	B	493.9		F	698.5
	C	523.3		G	784.0
	D	587.3		A	880.0

A note that is an octave above another note has exactly twice the frequency of the lower note. For example, the A in on the second line of the treble clef staff has a frequency of 440 Hz.[*] The A an octave above it (one ledger line above the staff) has a frequency of 440 × 2 = 880 Hz.

Harmonic Series

harmonic series: the additional, shorter standing waves that are generated by a vibrating string or column of air that correspond with integer numbers of half-waves.

fundamental frequency: the natural resonant frequency of a particular pitch.

harmonic: a resonant frequency produced by vibrations that contain an integer number of half-waves that add up to the half-wavelength of the fundamental.

The harmonics are numbered based on their pitch relative to the fundamental frequency. The harmonic that is closest in pitch is the 1st harmonic, the next closest is the 2nd harmonic, *etc.*

Any sound wave that is produced in a resonance chamber (such as a musical instrument) will produce the fundamental frequency plus all of the other waves of the harmonic series. The fundamental is the loudest, and each harmonic gets more quiet as you go up the harmonic series.

[*] Most bands and orchestras define the note "A" to be exactly 440 Hz, and use it for tuning.

Use this space for summary and/or additional notes:

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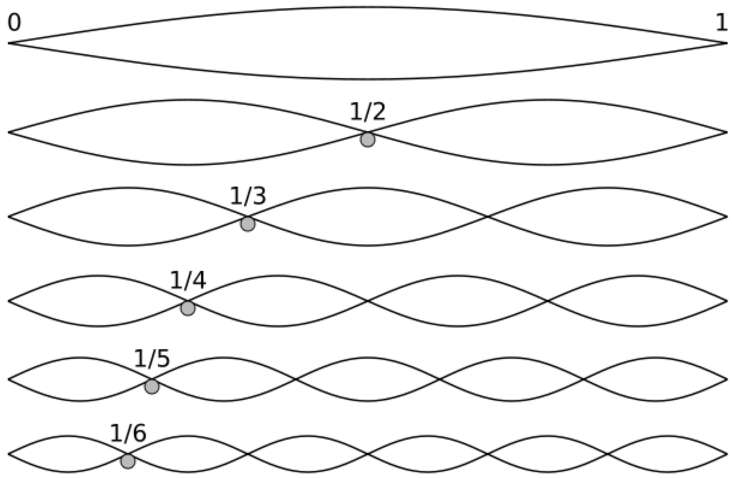
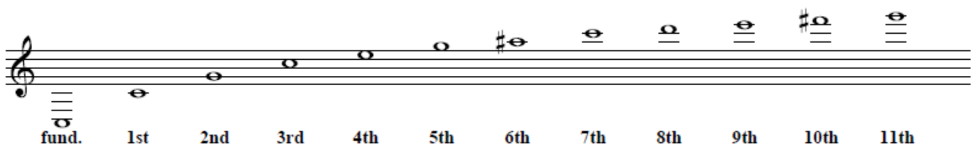
The following diagram shows the waves of the fundamental frequency and the first five harmonics in a pipe or a vibrating string:



Fraction of String	Wave-length	Harmonic	Frequency	Pitch (relative to fundamental)
1	$2L$	—	f_o	Fundamental.
$1/2$	$2L/2$	1st	$2f_o$	One octave above.
$1/3$	$2L/3$	2nd	$3f_o$	One octave + a fifth above.
$1/4$	$2L/4$	3rd	$4f_o$	Two octaves above.
$1/5$	$2L/5$	4th	$5f_o$	Two octaves + approximately a major third above.
$1/6$	$2L/6$	5th	$6f_o$	Two octaves + a fifth above.
$1/n$	$2L/n$	(n-1)th	nf_o	*etc.*



Use this space for summary and/or additional notes:

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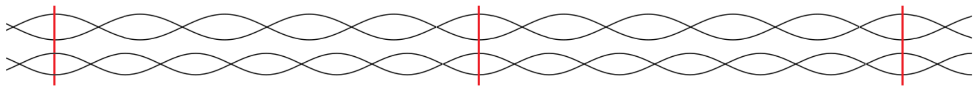
Beats

When two or more waves are close but not identical in frequency, their amplitudes reinforce each other at regular intervals.

For example, when the following pair of waves travels through the same medium, the amplitudes of the two waves have maximum constructive interference every five half-waves (2½ full waves) of the top wave and every six half-waves (3 full waves) of the bottom wave.



If this happens with sound waves, you will hear a pulse or "beat" every time the two maxima coïncide.

The closer the two wavelengths (and therefore also the two frequencies) are to each other, the more half-waves it takes before the amplitudes coïncide. This means that as the frequencies get closer, the time between beats gets longer.

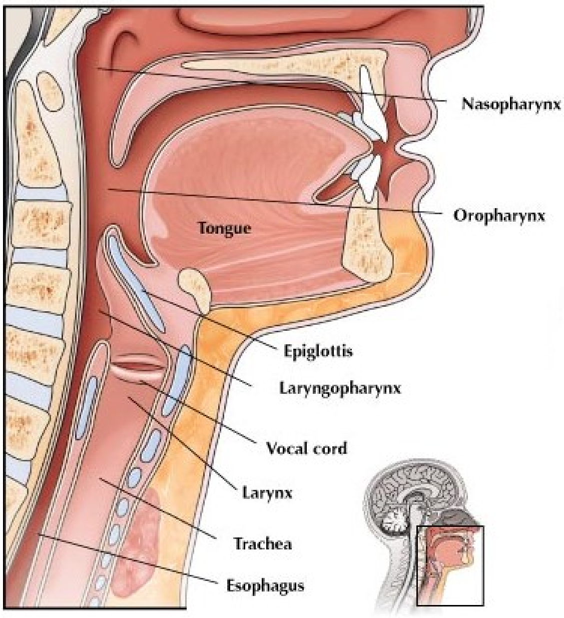
Piano tuners listen for these beats, and adjust the tension of the string they are tuning until the time between beats gets longer and longer and finally disappears.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

The Biophysics of Sound

When a person speaks, abdominal muscles force air from the lungs through the larynx.



The vocal cord vibrates, and this vibration creates sound waves. Muscles tighten or loosen the vocal cord, which changes the frequency at which it vibrates. Just like in a string instrument, the change in tension changes the pitch. Tightening the vocal cord increases the tension and produces a higher pitch, and relaxing the vocal cord decreases the tension and produces a lower pitch.

This process happens when you sing. Amateur musicians who sing a lot of high notes can develop laryngitis from tightening their laryngeal muscles too much for too long. Professional musicians need to train themselves to keep their larynx muscles relaxed and use other techniques (such as air pressure, which comes from breath support via the abdominal muscles) to adjust their pitch.

Use this space for summary and/or additional notes:

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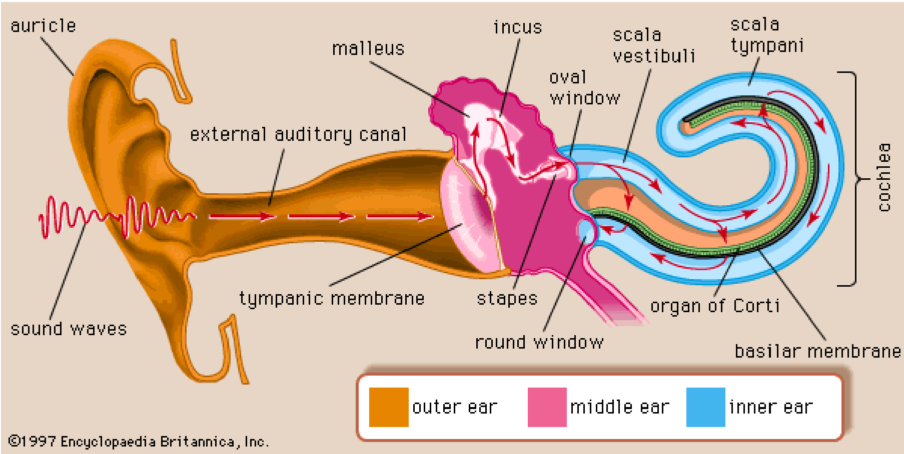
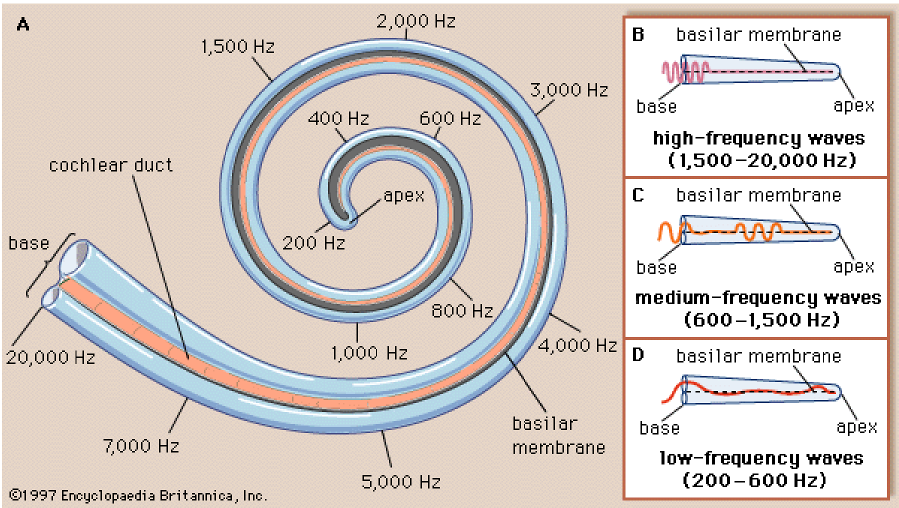
When the sound reaches the ears, it travels through the auditory canal and causes the tympanic membrane (eardrum) to vibrate. The vibrations of the tympanic membrane cause pressure waves to travel through the middle ear and through the oval window into the cochlea.



The basilar membrane in the cochlea is a membrane with cilia (small hairs) connected to it, which can detect very small movements of the membrane. As with a resonance tube, the wavelength determines exactly where the sound waves will vibrate the basilar membrane the most strongly, and the brain determines the pitch (frequency) of a sound based on the precise locations excited by these frequencies.

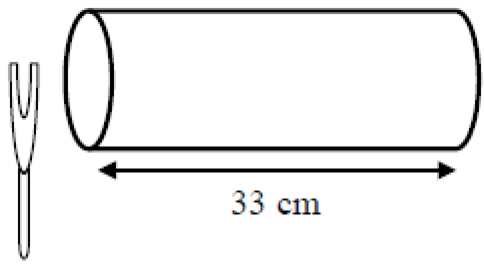


Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Homework Problems

A tuning fork is used to establish a standing wave in an open ended pipe filled with air at a temperature of 20°C, where the speed of sound is $343\,\frac{\text{m}}{\text{s}}$, as shown below:



The sound wave resonates at the 3nd harmonic frequency of the pipe. The length of the pipe is 33 cm.

1. Sketch the standing wave inside of the pipe. (For simplicity, you may sketch a transverse wave to represent the standing wave.)

2. Determine the wavelength of the resonating sound wave.

   Answer: 22 cm

3. Determine the frequency of the tuning fork.

   Answer: 1 559 Hz

4. What is the next higher frequency that will resonate in this pipe?

   Answer: 2 079 Hz

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Sound Level (Loudness)

Unit: Waves

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain sound levels in decibels.
- Explain the Lombard Effect.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain how loudness is measured.

Tier 2 Vocabulary: level

Labs, Activities & Demonstrations:

- VU meter.

Notes:

sound level: the perceived intensity of a sound. Usually called "volume".

Sound level is usually measured in decibels (dB). One decibel is one tenth of one bel.

Sound level is calculated based on the logarithm of the ratio of the power (energy per unit time) causing a sound vibration to the power that causes some reference sound level.

You will not be asked to calculate decibels from an equation, but you should understand that because the scale is logarithmic, a difference of one bel (10 dB) represents a tenfold increase or decrease in sound level.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

The following table lists the approximate sound levels of various sounds:

sound level (dB)	**Description**
0	threshold of human hearing at 1 kHz
10	a single leaf falling to the ground
20	background in TV studio
30	quiet bedroom at night
36	whispering
40	quiet library or classroom
42	quiet voice
40–55	typical dishwasher
50–55	normal voice
60	TV from 1 m away
	normal conversation from 1 m away
60–65	raised voice
60–80	passenger car from 10 m away
70	typical vacuum cleaner from 1 m away
75	crowded restaurant at lunchtime
72–78	loud voice
85	hearing damage (long-term exposure)
84–90	shouting
80–90	busy traffic from 10 m away
100–110	rock concert, 1 m from speaker
110	chainsaw from 1 m away
110–140	jet engine from 100 m away
120	threshold of discomfort
	hearing damage (single exposure)
130	threshold of pain
140	jet engine from 50 m away
194	sound waves become shock waves

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Adjusting Sound Level in Conversation

In crowds, people unconsciously adjust the sound levels of their speech in order to be heard above the ambient noise. This behavior is called the Lombard effect, named for Étienne Lombard, the French doctor who first described it.

The Lombard coëfficient is the ratio of the increase in sound level of the speaker to the increase in sound level of the background noise:

$$L = \frac{\text{increase in speech level (dB)}}{\text{increase in background noise (dB)}}$$

Researchers have observed values of the Lombard coëfficient ranging from 0.2 to 1.0, depending on the circumstances.

When you are working in groups in a classroom, as the noise level gets louder, each person has to talk louder to be heard, which in turn makes the noise level louder. The Lombard effect creates a feedback loop in which the sound gets progressively louder and louder until your teacher complains and everyone resets to a quieter volume.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Doppler Effect

Unit: Waves

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain the Doppler Effect and give examples.
- Calculate the apparent shift in wavelength/frequency due to a difference in velocity between the source and receiver.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how loudness is measured.

Tier 2 Vocabulary: shift

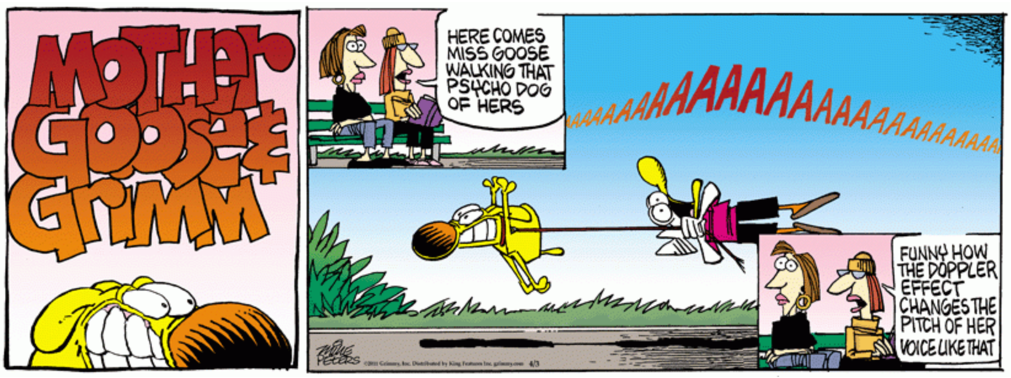
Labs, Activities & Demonstrations:

- Buzzer on a string.

Notes:

<u>Doppler effect</u> or <u>Doppler shift</u>: the apparent change in frequency/wavelength of a wave due to a difference in velocity between the source of the wave and the observer. The effect is named for the Austrian physicist Christian Doppler.

You have probably noticed the Doppler effect when an emergency vehicle with a siren drives by.



Use this space for summary and/or additional notes:

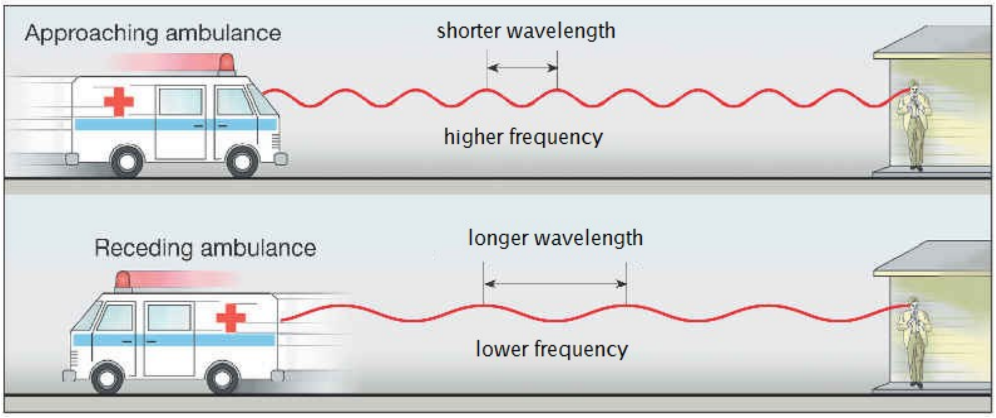
CP1 & honors (not AP®)

Why the Doppler Shift Happens

The Doppler shift occurs because a wave is created by a series of pulses at regular intervals, and the wave moves at a particular speed.

If the source is approaching, each pulse arrives sooner than it would have if the source had been stationary. Because frequency is the number of pulses that arrive in one second, the moving source results in an increase in the frequency observed by the receiver.

Similarly, if the source is moving away from the observer, each pulse arrives later, and the observed frequency is lower.



Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Calculating the Doppler Shift

The change in frequency is given by the equation:

$$f = f_o \left(\frac{v_w \pm v_r}{v_w \pm v_s} \right)$$

where:

f = observed frequency

f_o = frequency of the original wave

v_w = velocity of the wave

v_r = velocity of the receiver (you)

v_s = velocity of the source

The rule for adding or subtracting velocities is:

- The receiver's (your) velocity is in the numerator. If you are moving toward the sound, this makes the pulses arrive sooner, which makes the frequency higher. So if you are moving ***toward*** the sound, ***add*** your velocity. If you are moving ***away*** from the sound, ***subtract*** your velocity.
- The source's velocity is in the denominator. If the source is moving toward you, this makes the frequency higher, which means the denominator needs to be smaller. This means that if the source is moving ***toward*** you, ***subtract*** its velocity. If the source is moving ***away*** from you, ***add*** its velocity.

Don't try to memorize a rule for this—you will just confuse yourself. It's safer to reason through the equation. If something that's moving would make the frequency higher, that means you need to make the numerator larger or the denominator smaller. If it would make the frequency lower, that means you need to make the numerator smaller or the denominator larger.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Sample Problem:

Q: The horn on a fire truck sounds at a pitch of 350 Hz. What is the perceived frequency when the fire truck is moving toward you at $20\,\frac{\text{m}}{\text{s}}$? What is the perceived frequency when the fire truck is moving away from you at $20\,\frac{\text{m}}{\text{s}}$? Assume the speed of sound in air is $343\,\frac{\text{m}}{\text{s}}$.

A: The observer is not moving, so $v_r = 0$.

The fire truck is the source, so its velocity appears in the denominator.

When the fire truck is moving toward you, that makes the frequency higher. This means we need to make the denominator smaller, which means we need to ***subtract*** v_s:

$$f = f_o\left(\frac{v_w}{v_w - v_s}\right) = 350\left(\frac{343}{343-20}\right) = 350(1.062) = 372\,\text{Hz}$$

When the fire truck is moving away, the frequency will be lower, which mean we need to make the denominator larger. This means we need to ***add*** v_s:

$$f = f_o\left(\frac{v_w}{v_w + v_s}\right) = 350\left(\frac{343}{343+20}\right) = 350(0.9449) = 331\,\text{Hz}$$

Note that the pitch shift in each direction corresponds with about one half-step on the musical scale.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Exceeding the Speed of Sound

Unit: Waves

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain the what a "sonic boom" is.
- Calculate Mach numbers.

Success Criteria:

- Explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how a sonic boom is produced.

Tier 2 Vocabulary: sonic boom

Labs, Activities & Demonstrations:

- Crack a bullwhip.

Notes:

The speed of an object relative to the speed of sound in the same medium is called the Mach number (abbreviation Ma), named after the Austrian physicist Ernst Mach.

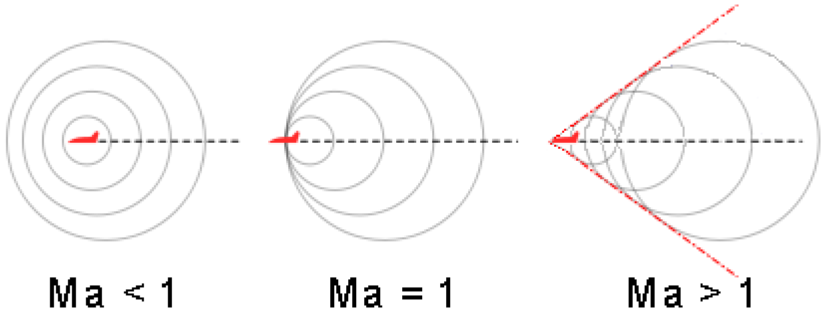
$$Ma = \frac{v_{object}}{v_{sound}}$$

Thus "Mach 1" or a speed of Ma = 1 is the speed of sound. An object such as an airplane that is moving at 1.5 times the speed of sound would be traveling at "Mach 1.5" or Ma = 1.5.

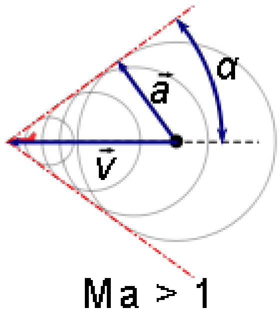
Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

When an object such as an airplane is traveling slower than the speed of sound (Ma < 1) , the jet engine noise is Doppler shifted just like any other sound wave. When the airplane's velocity reaches the speed of sound (Ma = 1), the leading edge of all of the sound waves produced by the plane coincides. These waves amplify each other, producing a loud shock wave called a "sonic boom".



When an airplane is traveling faster than sound, the sound waves coincide at points behind the airplane at a specific angle, α:



The angle α is given by the equation:

$$\sin(\alpha) = \frac{1}{Ma}$$

Note that the airplane cannot be heard at points outside of the region defined by the angle α. Note also that the faster the airplane is traveling, the smaller the angle α, and the narrower the cone.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

The shock wave temporarily increases the temperature of the air affected by it. If the air is humid enough, when it cools by returning to its normal pressure, the water vapor condenses and forms a cloud, called a vapor cone:

The "crack" of a bullwhip is also a sonic boom—when a bullwhip is snapped sharply, the end of the bullwhip travels faster than sound and creates a miniature shock wave.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Electromagnetic Waves

Unit: Waves

MA Curriculum Frameworks (2016): HS-PS4-1, HS-PS4-3, HS-PS4-5

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Describe the regions of the electromagnetic spectrum.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain why ultraviolet waves are more dangerous than infrared.

Tier 2 Vocabulary: wave, light, spectrum

Labs, Activities & Demonstrations:

- red *vs.* green *vs.* blue lasers on phosphorescent surface
- blue laser & tonic water
- wintergreen Life Savers™ (triboluminescence)

Notes:

electromagnetic wave: a transverse, traveling wave that is caused by oscillating electric and magnetic fields.

Electromagnetic waves travel through space and do not require a medium. The electric field creates a magnetic field, which creates an electric field, which creates another magnetic field, and so on. The repulsion from these induced fields causes the wave to propagate.

Electromagnetic waves (such as light, radio waves, etc.) travel at a constant speed—the speed of light. The speed of light depends on the medium it is traveling through, but it is a constant within its medium (or lack thereof), and is denoted by the letter "*c*" in equations. In a vacuum, the speed of light is:

$$c = 3.00 \times 10^{8}\ {}^{m}/_{s} = 186{,}000 \text{ miles per second}$$

Recall that the speed of a wave equals its frequency times its wavelength:

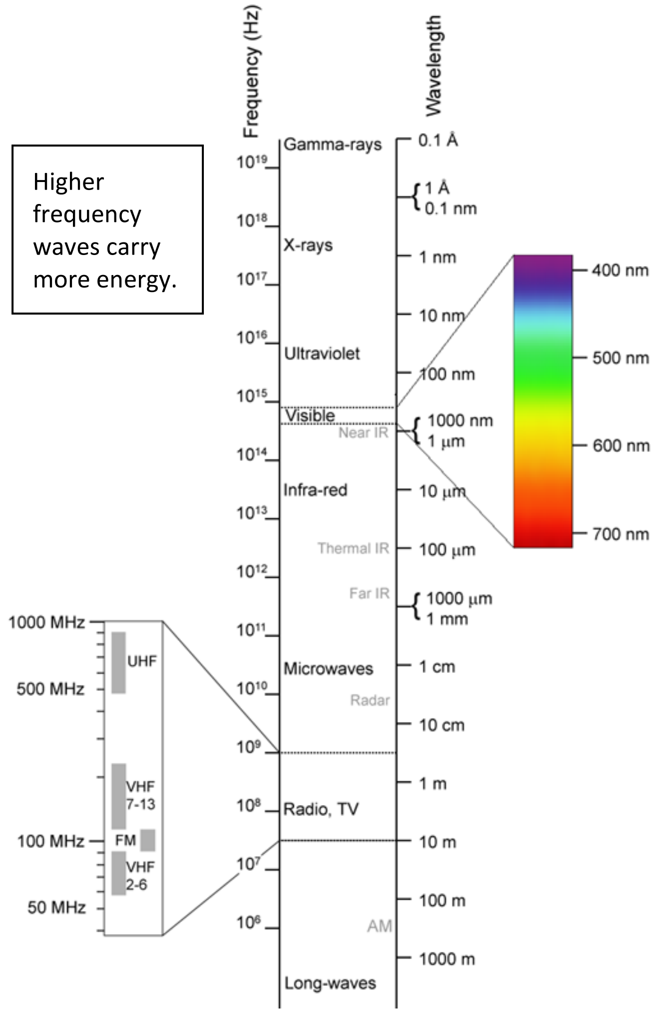
$$c = \lambda f$$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

electromagnetic spectrum: the entire range of possible frequencies and wavelengths for electromagnetic waves. The waves that make up the electromagnetic spectrum are shown in the diagram below:

Higher frequency waves carry more energy.



visible spectrum: wavelengths of light that can be detected by the human eye. These are wavelengths from roughly 400 nm to 700 nm.

The energy (E) that a wave carries is proportional to the frequency. (Think of it as the number of bursts of energy that travel through the wave every second.) For electromagnetic waves (including light), the constant of proportionality is Planck's constant (named after the physicist Max Planck), which is denoted by a script h in equations.

The energy of a wave is given by the Planck-Einstein equation:

$$E = hf = \frac{hc}{\lambda}$$

where E is the energy of the wave in Joules, f is the frequency in Hz, h is Planck's constant, which is equal to $6.626\times10^{-34}\ \text{J}\cdot\text{s}$, c is the speed of light, and λ is the wavelength in meters.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Antennas

An antenna is a piece of metal that is affected by electromagnetic waves and is used to amplify waves of specific wavelengths. The optimum length for an antenna is either the desired wavelength, or some fraction of the wavelength such that one wave is an exact multiple of the length of the antenna. (*E.g.,* good lengths for an antenna could be $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, *etc.* of the wavelength.)

Sample problem:

Q: What is the wavelength of a radio station that broadcasts at 98.5 MHz?

A:
$$c = \lambda f$$
$$3.00\times10^{8} = \lambda(9.85\times10^{7})$$
$$\lambda = \frac{3.00\times10^{8}}{9.85\times10^{7}} = 3.05\,\text{m}$$

Q: What would be a good length for an antenna that might be used to receive this radio station?

A: 3.05 m (about 10 feet) is too long to be practical for an antenna. Somewhere between half a meter and a meter is a good range.

$\frac{1}{4}$ wave would be 0.76 m (76 cm), which would be a good choice.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Color

Unit: Waves

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain how colors are produced and mixed.
- Explain why we see colors the way we do.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain how someone who is red-green color blind might see a green object.

Tier 2 Vocabulary: color, mixing

Labs, Activities & Demonstrations:

- colored light box

Notes:

Light with frequencies/wavelengths in the part of the spectrum that the eye can detect is called visible light.

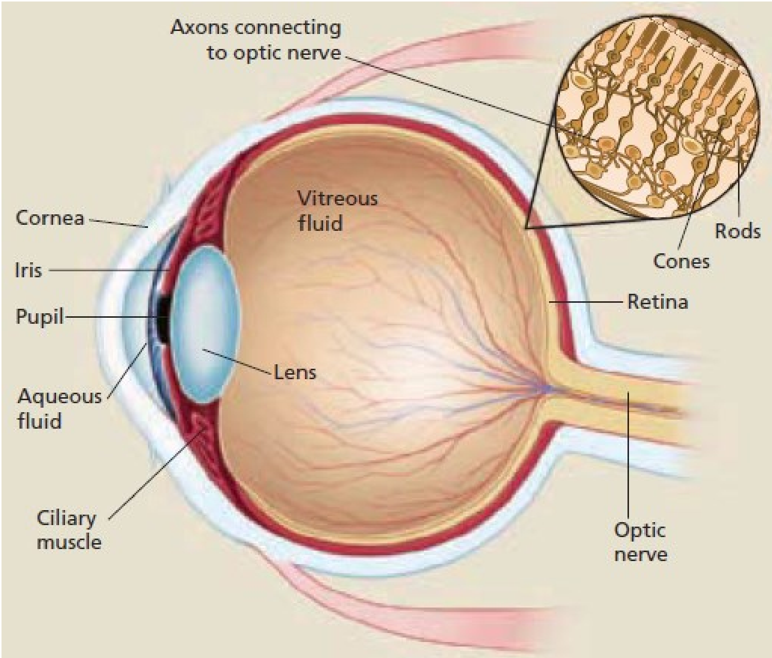
color: the perception by the human eye of how a light wave appears, based on its wavelength/frequency.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

How We See Color

Humans (and other animals) have two types of cells in our retina that respond to light:

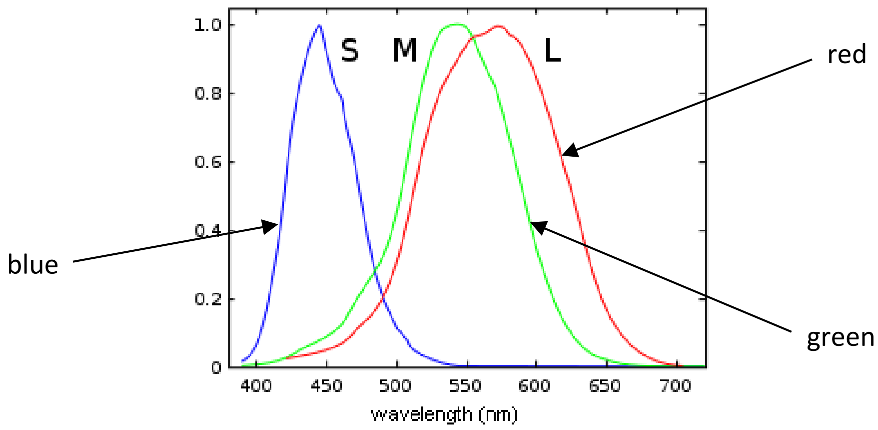


Rod cells resolve the physical details of images. Cone cells are responsible for distinguishing colors. Rod cells can operate in low light, but cone cells need much more light; this is why we cannot see colors in low light.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

There are three different types of cone cells in our eyes, called "S", "M", and "L", which stand for "short," "medium," and "long." Each type of cone cells responds to different wavelengths of light, having a peak (maximum) absorbance in a different part of the visible spectrum:



For example, light with a wavelength of 400–450 nm appears blue to us, because most of the response to this light is from the S cells, and our brains are wired to perceive this response as blue color. Light with a wavelength of around 500 nm would stimulate mostly the M cells and would appear green. Light with a wavelength of around 570 nm would stimulate the M and L cells approximately equally. When green and red receptors both respond, our brains perceive the color as yellow.

Colorblindness occurs when a genetic mutation causes a deficiency or absence of one or more types of cone cells. Most common is a deficiency in the expression of M cone cells, which causes red-green colorblindness. This means that a person with red-green colorblindness would see both colors as red.

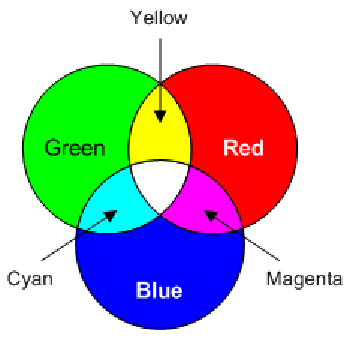
Because colorblindness is recessive and the relevant gene is on the X-chromosome, red-green colorblindness is much more common in men than in women.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Direct Light: Additive Mixing

Because our cone cells respond to red, green, and blue light, we call these colors the primary colors of light. Other colors can be made by mixing different amounts of these colors, thereby stimulating the different types of cone cells to different degrees. When all colors are mixed, the light appears white.



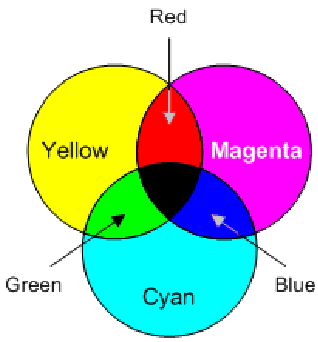
Additive (light)

primary color: light that excites only one type of cone cell. The primary colors of light are red, green, and blue.

secondary color: light that is a combination of exactly two primary colors. The secondary colors of light are cyan, magenta, and yellow.

Reflected Light: Subtractive Mixing

When light shines on an object, properties of that object cause it to absorb certain wavelengths of light and reflect others. The wavelengths that are reflected are the ones that make it to our eyes, causing the object to appear that color.



Subtractive (paint)

pigment: a material that changes the color of reflected light by absorbing light with specific wavelengths.

primary pigment: a material that absorbs light of only one primary color (and reflects the other two primary colors). The primary pigments are cyan, magenta, and yellow. Note that these are the secondary colors of light.

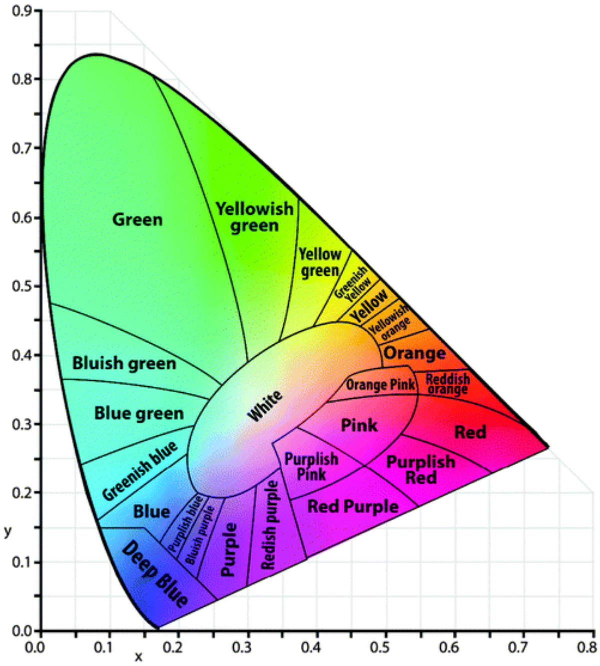
secondary pigment: a pigment that absorbs two primary colors (and reflects the other). The secondary pigments are red, green, and blue. Note that these are the primary colors of light.

Use this space for summary and/or additional notes:

Big Ideas | Details

CP1 & honors (not AP®)

Of course, our perception of color is biological, so mixing primary colors is not a simple matter of taking a weighted average of positions on the color wheel. The relationship between the fractions of primary colors used to produce a color and the color perceived is called chromaticity. The following diagram shows the colors that would be produced by varying the intensities of red, green, and blue light.



On this graph, the *x*-axis is the fraction (from 0 – 1) of red light, the *y*-axis is the fraction of green light, and the fraction of blue is implicit [1 – (red + green)].

Notice that equal fractions (0.33) of red, green and blue light would produce white light.

To show the effects of mixing two colors, plot each color's position on the graph and connect them with a line. The linear distance along that line shows the proportional effects of mixing. (*E.g.,* the midpoint would represent the color generated by 50% of each of the source colors.) This method is how fireworks manufacturers determine the mixtures of different compounds that will produce the desired colors.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Introduction: Thermal Physics (Heat)

Unit: Thermal Physics (Heat)

Topics covered in this chapter:

This chapter is about heat as a form of energy and the ways in which heat affects objects, including how it is stored and how it is transferred from one object to another.

- *Heat & Temperature* describes the concept of heat as a form of energy and how heat energy is different from temperature.
- *Heat Transfer* describes conduction, radiation and convection how to calculate the rate of the transfer of heat energy via conduction and radiation.
- *Specific Heat Capacity & Calorimetry* describes different substances' and objects' abilities to store heat energy. *Phase Changes & Heating Curves* addresses the additional calculations that apply when a substance goes through a phase change (such as melting or boiling).
- *Thermal Expansion* describes the calculation of the change in size of an object caused by heating or cooling.

New challenges specific to this chapter include looking up and working with constants that are different for different substances.

Standards addressed in this chapter:

MA Curriculum Frameworks (2016):

HS-PS2-6. Communicate scientific and technical information about why the molecular-level structure is important in the functioning of designed materials.

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.

HS-PS3-2. Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

HS-PS3-4. Plan and conduct an investigation to provide evidence that the transfer of thermal energy when two components of different temperature are combined within a closed system results in a more uniform energy distribution among the components in the system (second law of thermodynamics).

AP® Physics 1 Learning Objectives:

This unit was never part of the AP® Physics 1 curriculum. It is part of the AP® Physics 2 curriculum.

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Thermal Properties**, such as temperature, heat transfer, specific and latent heats, and thermal expansion.
- **Laws of Thermodynamics**, such as first and second laws, internal energy, entropy, and heat engine efficiency.

1. Heat and Temperature
2. The Kinetic Theory of Gases & the Ideal Gas Law
3. The Laws of Thermodynamics
4. Heat Engines

Skills learned & applied in this chapter:

- Working with material-specific constants from a table.
- Working with more than one instance of the same quantity in a problem.
- Combining equations and graphs.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Heat & Temperature

Unit: Thermal Physics (Heat)

MA Curriculum Frameworks (2016): HS-PS3-1, HS-PS3-2, HS-PS4-3a

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain heat energy in macroscopic and microscopic terms.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain the difference between heat and temperature.

Tier 2 Vocabulary: heat, temperature

Labs, Activities & Demonstrations:

- Heat a small weight and large weight to slightly different temperatures.
- Fire syringe.
- Steam engine.
- Incandescent light bulb in water.
- Mixing (via molecular motion/convection) of hot vs. cold water with food coloring)

Vocabulary:

<u>heat</u>: energy that can be transferred by moving atoms or molecules via transfer of momentum.

<u>temperature</u>: a measure of the average kinetic energy of the particles (atoms or molecules) of a system.

<u>thermometer</u>: a device that measures temperature, most often via thermal expansion and contraction of a liquid or solid.

Notes:

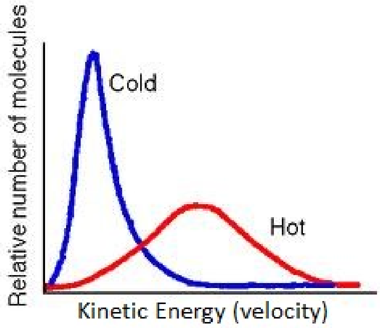
Heat is energy that is stored as the translational kinetic energy of the particles that make up an object or substance.

You may remember from chemistry that particles (atoms or molecules) are always moving (even at absolute zero), and that energy can transfer via elastic collisions between the particles of one object or substance and the particles of another.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Note that heat is the energy itself, whereas temperature is a measure of the quality of the heat—the average of the kinetic energies of the individual molecules:



Note that the particles of a substance have a range of kinetic energies, and the temperature is the average. Notice that when a substance is heated, the particles acquire a wider range of kinetic energies, with a higher average.

When objects are placed in contact, heat is transferred from each object to the other via the transfer of momentum that occurs when the individual molecules collide. Molecules that have more energy transfer more energy than they receive. Molecules that have less energy receive more energy than they transfer. This means three things:

1. Individual collisions transfer energy in both directions. The particles of a hot substance transfer energy to the cold substance, but the particles of the cold substance also transfer energy to the hot substance.

2. The net (overall) flow of energy is from objects with a higher temperature (more kinetic energy) to objects with a lower temperature (less kinetic energy). *I.e.,* more energy is transferred from the hot substance to the cold substance than *vice versa*.

3. If you wait long enough, all of the molecules will have the same temperature (*i.e.,* the same average kinetic energy).

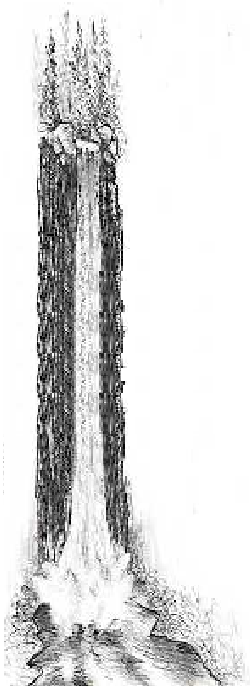
Use this space for summary and/or additional notes:

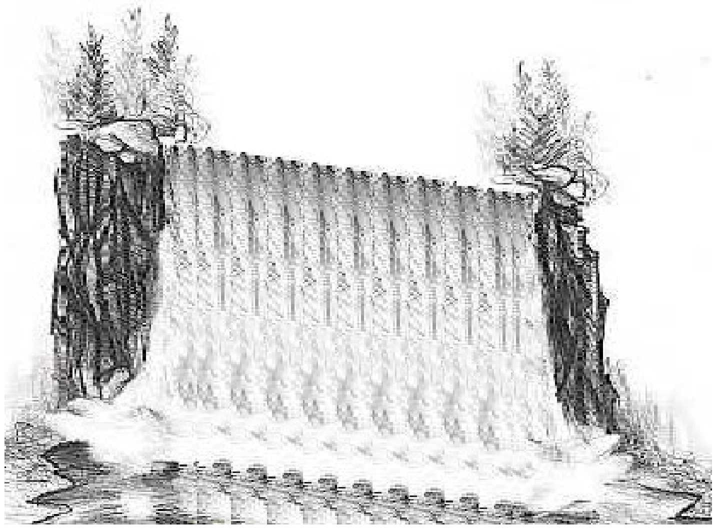
CP1 & honors (not AP®)

This means that the ***temperature*** of one object relative to another determines which direction the heat will flow, much like the way the elevation (vertical position) of one location relative to another determines which direction water will flow.

However, the total heat (energy) contained in an object depends on the mass as well as the temperature, in the same way that the total change in energy of the water going over a waterfall depends on the mass of the water as well as the height.

Consider two waterfalls, one of which is twice the height of the second, but the second of which has ten times as much water going over it as the first:





$\Delta U = mg(2h)$ $\Delta U = (10m)gh$

In the above pictures, each drop of water falling from the waterfall on the left has more gravitational potential energy, but more total energy goes over the waterfall on the right.

Similarly, each particle in an object at a higher temperature has more thermal energy than each particle in another object at a lower temperature.

If we built a waterway between the two falls, water could flow from the top of the first waterfall to the top of the second, but not *vice versa*.

Similarly, the net flow of heat is from a smaller object with higher temperature to a larger object with a lower temperature, but not *vice versa*.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Heat Flow

system: the region or collection of objects under being considered in a problem.

surroundings: everything that is outside of the system.

E.g., if a metal block is heated, we would most likely define the system to be the block, and the surroundings to be everything else.

We generally use the variable Q to represent heat in physics equations.

Heat flow is always represented in relation to the system.

Heat Flow	**Sign of *Q***	**System**	**Surroundings**
from the surroundings into the system	+ (positive)	gains heat (gets warmer)	lose heat (get colder)
from the system out to the surroundings	− (negative)	loses heat (gets colder)	gain heat (get hotter)

A positive value of Q means heat is flowing *into* the system. Because the heat is transferred from the molecules outside the system to the molecules in the system, the energy of the system increases, and the energy of the surroundings decreases.

A negative value of Q means heat is flowing *out of* the system. Because the heat is transferred from the molecules in the system to the molecules outside the system, the energy of the system decreases, and the energy of the surroundings increases.

This can be confusing. Suppose you set a glass of ice water on a table. When you pick up the glass, your hand gets colder because heat is flowing from your hand (which is part of the surroundings) into the system (the glass of ice water). This means the system (the glass of ice water) is gaining heat, and the surroundings (your hand, the table, *etc.*) are losing heat. The value of Q would be positive in this example.

In simple terms, you need to remember that your hand is part of the *surroundings*, not part of the system.

thermal equilibrium: when all of the particles in a system have the same average kinetic energy (temperature). When a system is at thermal equilibrium, no net heat is transferred. (*I.e.,* collisions between particles may still transfer energy, but the average temperature of the particles in the system—what we measure with a thermometer—is not changing.)

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Heat Transfer

Unit: Thermal Physics (Heat)

MA Curriculum Frameworks (2016): HS-PS3-4a

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain heat transfer by conduction, convection and radiation.
- Calculate heat transfer using Fourier's Law of Heat Conduction.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the mechanisms by which heat is transferred.

Tier 2 Vocabulary: conduction, radiation

Labs, Activities & Demonstrations:

- Heat a piece of sheet metal against a block of wood.
- Radiometer & heat lamp.
- Almond & cheese stick.
- Flammable soap bubbles.
- Ice cubes melting in fresh vs. salt water (with food coloring).

Notes:

Heat transfer is the flow of heat energy from one object to another. Heat transfer usually occurs through three distinct mechanisms: conduction, radiation, and convection.

conduction: transfer of heat through collisions of particles by objects that are *in direct contact* with each other. Conduction occurs when there is a net transfer of momentum from the molecules of an object with a higher temperature transfer to the molecules of an object with a lower temperature.

thermal conductivity (k): a measure of the amount of heat that a given length of a substance can conduct in a specific amount of time. Thermal conductivity is measured in units of $\frac{\text{J}}{\text{m·s·°C}}$ or $\frac{\text{W}}{\text{m·°C}}$.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

<u>conductor</u>: an object that allows heat to pass through itself easily; an object with high thermal conductivity.

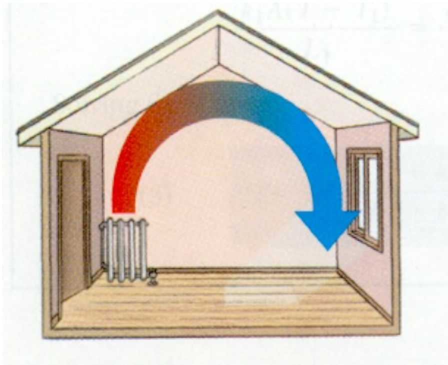
<u>insulator</u>: an object that does not allow heat to pass through itself easily; a poor conductor of heat; an object with low thermal conductivity.

<u>radiation</u>: transfer of heat *<u>through space</u>* via electromagnetic waves (light, microwaves, *etc.*)

<u>convection</u>: transfer of heat *<u>by motion of particles</u>* that have a higher temperature exchanging places with particles that have a lower temperature. Convection usually occurs when air moves around a room.

<u>Natural convection</u> occurs when particles move because of differences in density. In a heated room, because cool air is more dense than warm air, the force of gravity is stronger on the cool air, and it is pulled harder toward the ground than the warm air. The cool air displaces the warm air, pushing it upwards out of the way.

In a room with a radiator, the radiator heats the air, which causes it to expand and be displaced upward by the cool air nearby. When the (less dense) warm air reaches the ceiling, it spreads out, and it continues to cool as it spreads. When the air reaches the opposite wall, it is forced downward toward the floor, across the floor, and back to the radiator.



<u>Forced convection</u> can be achieved by moving heated or cooled air using a fan. Examples of this include ceiling fans and convection ovens. If your radiator does not warm your room enough in winter, you can use a fan to speed up the process of convection. (Make sure the fan is moving the air in the same direction that would happen from natural convection. Otherwise, the fan will be fighting against physics!)

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Specific Heat Capacity & Calorimetry*

Unit: Thermal Physics (Heat)

MA Curriculum Frameworks (2016): HS-PS3-1, HS-PS4-3a

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Calculate the heat transferred when an object with a known specific heat capacity is heated.
- Perform calculations related to calorimetry.
- Describe what is happening at the molecular level when a system is in thermal equilibrium.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what the specific heat capacity of a substance measures.
- Explain how heat is transferred between one substance and another.

Tier 2 Vocabulary: heat, specific heat capacity, coffee cup calorimeter

Labs, Activities & Demonstrations:

- Calorimetry lab.

Notes:

Different objects have different abilities to hold heat. For example, if you enjoy pizza, you may have noticed that the sauce holds much more heat (and burns your mouth much more readily) than the cheese or the crust.

The amount of heat that a given mass of a substance can hold is based on its specific heat capacity.

* Calorimetry is usually taught in chemistry classes. However, the topic was moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016.

Use this space for summary and/or additional notes:

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<u>specific heat capacity</u> (*C*): a measure of the amount of heat required per gram of a substance to produce a specific temperature change in the substance.

C_p: specific heat capacity, measured at constant pressure. For gases, this means the measurement was taken allowing the gas to expand as it was heated.

C_v: specific heat capacity, measured at constant volume. For gases, this means the measurement was made in a sealed container, allowing the pressure to rise as the gas was heated.

For solids and liquids, $C_p \approx C_v$ because the pressure and volume change very little as they are heated. For gases, $C_p > C_v$ (always). For ideal gases, $C_p - C_v = R$, where *R* is a constant known as "the gas constant."

When there is a choice, C_p is more commonly used than C_v because it is easier to measure. When dealing with solids and liquids, most physicists just use C for specific heat capacity and don't worry about the distinction.

Calculating Heat from a Temperature Change

The amount of heat gained or lost when an object changes temperature is given by the equation:

$$Q = mC\Delta T$$

where:

Q = heat (J or kJ)
m = mass (g or kg)
C = specific heat capacity ($\frac{\text{kJ}}{\text{kg·K}}$)
ΔT = temperature change (K or °C)

Because problems involving heat often involve large amounts of energy, specific heat capacity is often given in kilojoules per kilogram per degree Celsius.

Note that $1\frac{\text{kJ}}{\text{kg·K}} \equiv 1\frac{\text{kJ}}{\text{kg·°C}} \equiv 1\frac{\text{J}}{\text{g·°C}}$ and $1\frac{\text{cal}}{\text{g·°C}} \equiv 1\frac{\text{kcal}}{\text{kg·°C}} = 4.18\frac{\text{J}}{\text{g·°C}} = 4.18\frac{\text{kJ}}{\text{kg·K}}$

You need to be careful with the units. If the mass is given in kilograms (kg), your specific heat capacity will have units of $\frac{\text{kJ}}{\text{kg·°C}}$ and the heat energy will come out in kilojoules (kJ). If mass is given in grams, you will use units of $\frac{\text{J}}{\text{g·°C}}$ and the heat energy will come out in joules (J).

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Specific Heat Capacities of Some Substances

Substance	Specific Heat Capacity ($\frac{kJ}{kg \cdot K}$)	Substance	Specific Heat Capacity ($\frac{kJ}{kg \cdot K}$)
water at 20 °C	4.181	aluminum	0.897
ethylene glycol (anti-freeze)	2.460	glass	0.84
		iron	0.450
ice at −10 °C	2.080	copper	0.385
steam at 100 °C	2.11	brass	0.380
steam at 130 °C	1.99	silver	0.233
vegetable oil	2.00	lead	0.160
air	1.012	gold	0.129

Calorimetry

calorimetry: the measurement of heat flow

In a calorimetry experiment, heat flow is calculated by measuring the mass and temperature change of an object and applying the specific heat capacity equation.

calorimeter: an insulated container for performing calorimetry experiments.

coffee cup calorimeter: a calorimeter that is only an insulated container—it does not include a thermal mass (such as a mass of water). It is usually made of Styrofoam, and is often nothing more than a Styrofoam coffee cup.

bomb calorimeter: a calorimeter for measuring the heat produced by a chemical reaction. A bomb calorimeter is a double-wall metal container with water between the layers of metal. The heat from the chemical reaction makes the temperature of the water increase. Because the mass and specific heat of the calorimeter (water and metal) are known, the heat produced by the reaction can be calculated from the increase in temperature of the water.

It has a great name, but a bomb calorimeter doesn't involve actually blowing anything up. ☺

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Solving Coffee Cup Calorimetry Problems

Most coffee cup calorimetry problems involve placing a hot object in contact with a colder one. Many of them involve placing a hot piece of metal into cold water.

To solve the problems, assume that both objects end up at the same temperature.

If we decide that heat gained (going into a substance) by each object that is getting hotter is positive, and heat lost (coming out of a substance) by every substance that is getting colder is negative, then the basic equation is:

$$\text{Heat Lost} + \text{Heat Gained} = \text{Change in Thermal Energy}$$
$$\sum Q_{lost} + \sum Q_{gained} = \Delta Q$$

If the calorimeter is insulated, then no heat is gained or lost by the entire system (which means $\Delta Q = 0$).

If we have two substances (#1 and #2), one of which is getting hotter and the other of which is getting colder, then our equation becomes:

$$\text{Heat Lost} + \text{Heat Gained} = \text{Change in Thermal Energy}$$
$$\sum Q_{lost} + \sum Q_{gained} = \Delta Q = 0$$
$$m_1 C_1 \Delta T_1 + m_2 C_2 \Delta T_2 = 0$$

In this example, ΔT_1 would be negative and ΔT_2 would be positive.

To solve a calorimetry problem, there are six quantities that you need: the two masses, the two specific heat capacities, and the two temperature changes. (You might be given initial and final temperatures for either or both, in which case you'll need to subtract. Remember that if the temperature increases, ΔT is positive, and if the temperature decreases, ΔT is negative.) The problem will usually give you all but one of these and you will need to find the missing one.

If you need to find the final temperature, use $\Delta T = T_f - T_i$ on each side. You will have both T_i numbers, so the only variable left will be T_f. (The algebra is straightforward, but ugly.)

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Sample Problems:

Q: An 0.050 kg block of aluminum is heated and placed in a calorimeter containing 0.100 kg of water at 20. °C. If the final temperature of the water was 30. °C, to what temperature was the aluminum heated?

A: To solve the problem, we need to look up the specific heat capacities for aluminum and water in *Table K. Thermal Properties of Selected Materials* on page 687 of your Physics Reference Tables. The specific heat capacity of aluminum is $0.898 \frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$, and the specific heat capacity for water is $4.181 \frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$.

We also need to realize that we are looking for the initial temperature of the aluminum. ΔT is always ***final – initial***, which means $\Delta T_{\text{Al}} = 30 - T_{i,\text{Al}}$. (Because the aluminum starts out at a higher temperature, this will give us a negative number, which is what we want.)

$$m_{\text{Al}}C_{\text{Al}}\Delta T_{\text{Al}} + m_{\text{w}}C_{\text{w}}\Delta T_{\text{w}} = 0$$
$$(0.050)(0.897)(30 - T_i) + (0.100)(4.181)(30 - 20) = 0$$
$$0.0449(30 - T_i) + 4.181 = 0$$
$$1.3455 - 0.0449T_i + 4.181 = 0$$
$$5.5265 = 0.0449T_i$$
$$T_i = \frac{5.5265}{0.0449} = 123.2\,^\circ\text{C}$$

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Q: An 0.025 kg block of copper at 95°C is dropped into a calorimeter containing 0.075 kg of water at 25°C. What is the final temperature?

A: We solve this problem the same way. The specific heat capacity for copper is $0.385 \frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$, and $\Delta T_{\text{Cu}} = T_f - 95$ and $\Delta T_{\text{w}} = T_f - 25$. This means T_f will appear in two places. The algebra will be even uglier, but it's still a straightforward Algebra 1 problem:

$$m_{\text{Cu}}C_{\text{Cu}}\Delta T_{\text{Cu}} + m_{\text{w}}C_{\text{w}}\Delta T_{\text{w}} = 0$$
$$(0.025)(0.385)(T_f - 95_i) + (0.075)(4.181)(T_f - 25) = 0$$
$$0.009625(T_f - 95) + 0.3138(T_f - 25) = 0$$
$$0.009625\,T_f - (0.009625)(95) + 0.3136\,T_f - (0.3138)(25) = 0$$
$$0.009625\,T_f - 0.9144 + 0.3138\,T_f - 7.845 = 0$$
$$0.3234\,T_f = 8.759$$
$$T_f = \frac{8.759}{0.3234} = 27\,^\circ\text{C}$$

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Homework Problems

You will need to look up specific heat capacities in *Table K. Thermal Properties of Selected Materials* on page 687 of your Physics Reference Tables.

1. 375 kJ of heat is added to a 25.0 kg granite rock. If the temperature increases by 19.0 °C, what is the specific heat capacity of granite?

   Answer: $0.790\,\frac{\text{J}}{\text{g}\cdot{}^{\circ}\text{C}}$

2. A 0.040 kg block of copper at 95 °C is placed in 0.105 kg of water at an unknown temperature. After equilibrium is reached, the final temperature is 24 °C. What was the initial temperature of the water?

   Answer: 21.5 °C

3. A sample of metal with a specific heat capacity of $0.50\,\frac{\text{J}}{\text{g}\cdot{}^{\circ}\text{C}}$ is heated to 98 °C and then placed in an 0.055 kg sample of water at 22 °C. When equilibrium is reached, the final temperature is 35 °C. What was the mass of the metal?

   Answer: 0.0948 kg

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4. A 0.280 kg sample of a metal with a specific heat capacity of $0.430\,\frac{\text{J}}{\text{g}\cdot{}^{\circ}\text{C}}$ is heated to 97.5 °C then placed in an 0.0452 kg sample of water at 31.2 °C. What is the final temperature of the metal and the water?

Answer: 57 °C

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Phase Changes & Heating Curves*

Unit: Thermal Physics (Heat)

MA Curriculum Frameworks (2016): HS-PS3-1, HS-PS3-2, HS-PS3-4a

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Determine the amount of heat required for all of the phase changes that occur over a given temperature range.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what the heat is used for in each step of a heating curve.

Tier 2 Vocabulary: specific heat capacity, heating curve

Labs, Activities & Demonstrations:

- Evaporation from washcloth.
- Fire & ice (latent heat of paraffin).

Notes:

phase: a term that relates to how rigidly the atoms or molecules in a substance are connected.

solid: molecules are rigidly connected. A solid has a definite shape and volume.

liquid: molecules are loosely connected—bonds are continuously forming and breaking. A liquid has a definite volume, but not a definite shape.

gas: molecules are not connected. A gas has neither a definite shape nor a definite volume. Gases will expand to fill whatever space they occupy.

plasma: the system has enough heat to remove electrons from atoms, which means the system is comprised of particles with rapidly changing charges.

phase change: when an object or substance changes from one phase to another through gaining or losing heat.

* Heating curves are usually taught in chemistry classes. However, the topic was moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016.

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Breaking bonds requires energy. Forming bonds releases energy. This is true for the bonds that hold a solid or liquid together as well as for chemical bonds (regardless of what previous teachers may have told you!)

I.e., you need to add energy to turn a solid to a liquid (melt it), or to turn a liquid to a gas (boil it). Energy is released when a gas condenses or a liquid freezes. (*E.g.,* ice in your ice tray needs to give off heat in order to freeze. Your freezer needs to remove that heat in order to make this happen.)

The reason evaporation causes cooling is because the system (the water) needs to absorb heat from its surroundings (*e.g.,* your body) in order to make the change from a liquid to a gas (vapor). When the water absorbs heat from you and evaporates, you have less heat, which means you have cooled off.

Calculating the Heat of Phase Changes

heat of fusion (ΔH_{fus}) (sometimes called "latent heat" or "latent heat of fusion"): the amount of heat required to melt one kilogram of a substance. This is also the heat released when one kilogram of a liquid substance freezes. For example, the heat of fusion of water is $334\,\frac{\text{J}}{\text{g}}$. The heat required to melt a sample of water is therefore:

$$Q = m\Delta H_{fus} = m(334\,\tfrac{\text{J}}{\text{g}})$$

heat of vaporization (ΔH_{vap}): the amount of heat required to vaporize (boil) one kilogram of a substance. This is also the heat released when one kilogram of a gas condenses. For example, the heat of vaporization of water is $2260\,\frac{\text{J}}{\text{g}}$. The heat required to boil a sample of water is therefore:

$$Q = m\Delta H_{vap} = m(2260\,\tfrac{\text{J}}{\text{g}})$$

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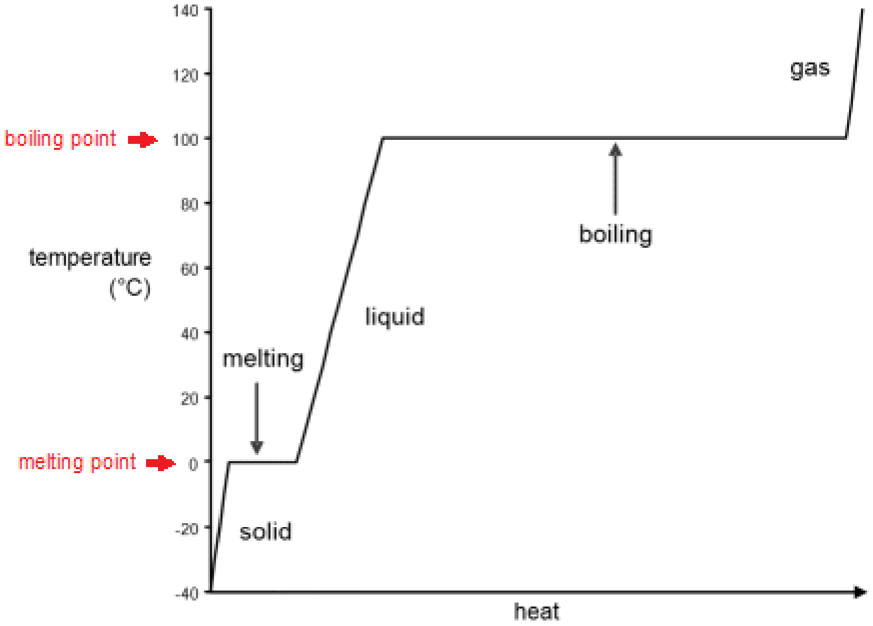
heating curve: a graph of temperature *vs.* heat added. The following is a heating curve for water:



In the "solid" portion of the curve, the sample is solid water (ice). As heat is added, the temperature increases. The specific heat capacity of ice is $2.11\,\frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$, so the heat required is:

$$Q_{solid} = mC\Delta T = m(2.11\,\tfrac{\text{J}}{\text{g}^\circ\text{C}})\Delta T$$

In the "melting" portion of the curve, the sample is a mixture of ice and water. As heat is added, the ice melts, but the temperature remains at 0°C until all of the ice is melted. The heat of fusion of ice is $334\,\frac{\text{J}}{\text{g}}$, so the heat required is:

$$Q_{melt} = m\Delta H_{fus} = m(334\,\tfrac{\text{J}}{\text{g}})$$

In the "liquid" portion of the curve, the sample is liquid water. As heat is added, the temperature increases. The specific heat capacity of liquid water is $4.181\,\frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$, so the heat required is:

$$Q_{liquid} = mC\Delta T = m(4.181\,\tfrac{\text{J}}{\text{g}^\circ\text{C}})\Delta T$$

In the "boiling" portion of the curve, the sample is a mixture of water and water vapor (steam). As heat is added, the water boils, but the temperature remains at 100°C until all of the water has boiled. The heat of vaporization of water is $2260\,\frac{\text{J}}{\text{g}}$, so the heat required is:

$$Q_{melt} = m\Delta H_{vap} = m(2260\,\tfrac{\text{J}}{\text{g}})$$

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In the "gas" portion of the curve, the sample is water vapor (steam). As heat is added, the temperature increases. The specific heat capacity of steam is approximately $2.08\,\frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$. (This is at 100 °C; the specific heat capacity of steam decreases as the temperature increases.) The heat required is:

$$Q_{gas} = mC\Delta T = m(2.08\,\tfrac{\text{J}}{\text{g}^\circ\text{C}})\Delta T$$

Steps for Solving Heating Curve Problems

A heating curve problem is a problem in which a substance is heated across a temperature range that passes through the melting and/or boiling point of the substance, which means the problem includes heating or cooling steps and melting/freezing or boiling/condensing steps.

1. Sketch the heating curve for the substance over the temperature range in question. Be sure to include the melting and boiling steps as well as the heating steps.

2. From your sketch, determine whether the temperature range in the problem passes through the melting and/or boiling point of the substance.

3. Split the problem into:
    a. Heating (or cooling) steps within each temperature range.
    b. Melting or boiling (or freezing or condensing) steps.

4. Find the heat required for each step.
    a. For the heating/cooling steps, use the equation $Q = mC\Delta T$.
    b. For melting/freezing steps, use the equation $Q = m\Delta H_{fus}$.
    c. For boiling/condensing steps, use the equation $Q = m\Delta H_{vap}$.

5. Add the values of *Q* from each step to find the total.

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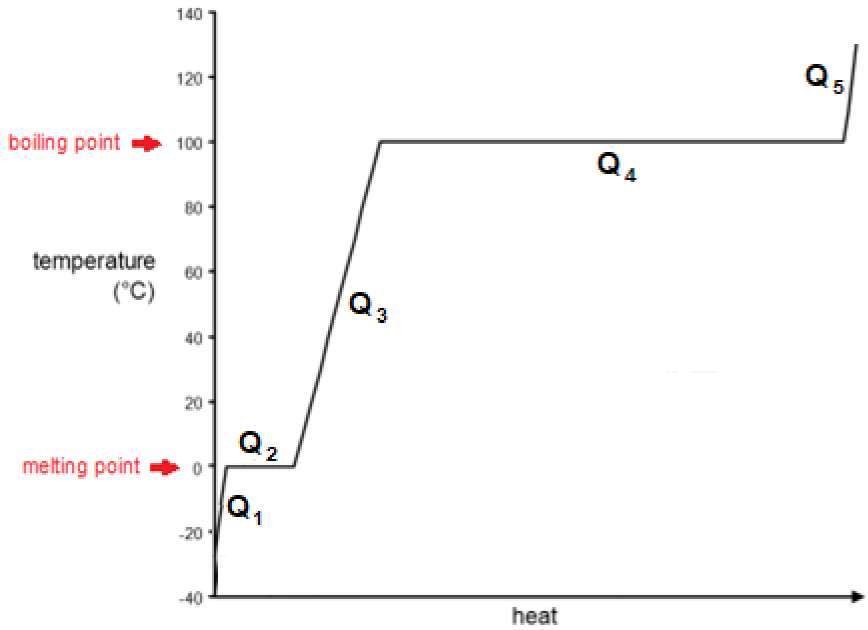
Sample Problem

Q: How much heat would it take to raise the temperature of 15.0 g of H_2O from −25.0 °C to +130.0 °C?

A: The H_2O starts out as ice. We need to:

1. Heat the ice from −25.0 °C to its melting point (0 °C).
2. Melt the ice.
3. Heat the water up to its boiling point (from 0 °C to 100 °C).
4. Boil the water.
5. Heat the steam from 100 °C to 130 °C.
6. Add up the heat for each step to find the total.



1. heat solid: $Q_1 = mC\Delta T = (15)(2.11)(25) = 791.25\text{ J}$

2. melt ice: $Q_2 = m\Delta H_{fus} = (15)(334) = 5\,010\text{ J}$

3. heat liquid: $Q_3 = mC\Delta T = (15)(4.181)(100) = 6\,270\text{ J}$

4. boil water: $Q_4 = m\Delta H_{vap} = (15)(2260) = 33\,900\text{ J}$

5. heat gas: $Q_5 = mC\Delta T = (15)(2.08)(30) = 936\text{ J}$

6. $$Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$$
$$Q = 791 + 5\,010 + 6\,270 + 33\,900 + 936 = 46\,910\text{ J}$$

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Homework Problems

For the following problems, use data from the following table:

	C (sol.) $\left(\frac{kJ}{kg\cdot °C}\right)$	**M.P.** (°C)	**Δ*H*fus** $\left(\frac{kJ}{kg}\right)$	**C (liq)** $\left(\frac{kJ}{kg\cdot °C}\right)$	**B.P.** (°C)	**Δ*H*vap** $\left(\frac{kJ}{kg}\right)$	**C_p (gas)** $\left(\frac{kJ}{kg\cdot °C}\right)$
water	2.11	0	334	4.18	100	2260	2.08*
potassium	0.560	62	61.4	1.070	760	2025	0.671
mercury	0.142	−39	11.3	0.140	357	293	0.104
silver	0.217	962	111	0.318	2212	2360	—

*Note that because of the volume change from heating, the specific heat capacity of gases, C_p, increases with increasing temperature.

1. A 0.0250 kg sample of water is heated from −40.0°C to 150. °C.

   a. Sketch the heating curve for the above process. Label the starting temperature, melting point, boiling point, and final temperature on the *y*-axis.

   b. Calculate the heat required for each step of the heating curve, and the total heat required.

   Answer: 80.01 kJ

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2. A 0.085 kg sample of mercury is heated from 25°C to 500. °C.

   a. Sketch the heating curve for the above process. Label the starting temperature, melting point, boiling point, and final temperature on the *y*-axis.

   b. Calculate the heat required for each step of the heating curve, and the total heat required.

      Answer: 30.12 kJ

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Thermal Expansion

Unit: Thermal Physics (Heat)

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Calculate changes in length & volume for solids, liquids and gases that are undergoing thermal expansion or contraction.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what the heat is used for in each step of a heating curve.

Tier 2 Vocabulary: expand, contract

Labs, Activities & Demonstrations:

- Balloon with string & heat gun.
- Brass ball & ring.
- Bi-metal strip.

Notes:

expand: to become larger

contract: to become smaller

thermal expansion: an increase in the length and/or volume of an object caused by a change in temperature.

When a substance is heated, the particles it is made of move farther and faster. This causes the particles to move farther apart, which causes the substance to expand.

Solids tend to keep their shape when they expand. (Liquids and gases do not have a definite shape to begin with.)

A few materials are known to contract with increasing temperature over specific temperature ranges. One well-known example is liquid water, which contracts as it heats from 0 °C to 4 °C. (Water expands as the temperature increases above 4 °C.)

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Thermal Expansion of Solids and Liquids

Thermal expansion is quantified in solids and liquids by defining a coëfficient of thermal expansion. The changes in length and volume are given by the equation:

Length: $\Delta L = \alpha L_i \Delta T$

Volume: $\Delta V = \beta V_i \Delta T$

where:

ΔL = change in length (m) L_i = initial length (m)
α = linear coëfficient of thermal expansion ($°C^{-1}$)

ΔV = change in volume (m^3) V_i = initial volume (m^3)
β = volumetric coëfficient of thermal expansion ($°C^{-1}$)

ΔT = temperature change (°C)

Values of α and β at 20°C for some solids and liquids:

Substance	α ($°C^{-1}$)	β ($°C^{-1}$)	Substance	α ($°C^{-1}$)	β ($°C^{-1}$)
aluminum	2.3×10^{-5}	6.9×10^{-5}	gold	1.4×10^{-5}	4.2×10^{-5}
copper	1.7×10^{-5}	5.1×10^{-5}	iron	1.18×10^{-5}	3.33×10^{-5}
brass	1.9×10^{-5}	5.6×10^{-5}	lead	2.9×10^{-5}	8.7×10^{-5}
diamond	1×10^{-6}	3×10^{-6}	mercury	6.1×10^{-5}	1.82×10^{-4}
ethanol	2.5×10^{-4}	7.5×10^{-4}	silver	1.8×10^{-5}	5.4×10^{-5}
glass	8.5×10^{-6}	2.55×10^{-6}	water (liq.)	6.9×10^{-5}	2.07×10^{-4}

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<u>expansion joint</u>: a space deliberately placed between two objects to allow room for the objects to expand without coming into contact with each other.

Bridges often have expansion joints in order to leave room for sections of the bridge to expand or contract without damaging the bridge or the roadway.

Railroad rails are sometimes welded together in order to create a smoother ride, which enables high-speed trains to use them. Unfortunately, if expansion joints are not placed at frequent enough intervals, thermal expansion can cause the rails to bend and buckle, resulting in derailments:

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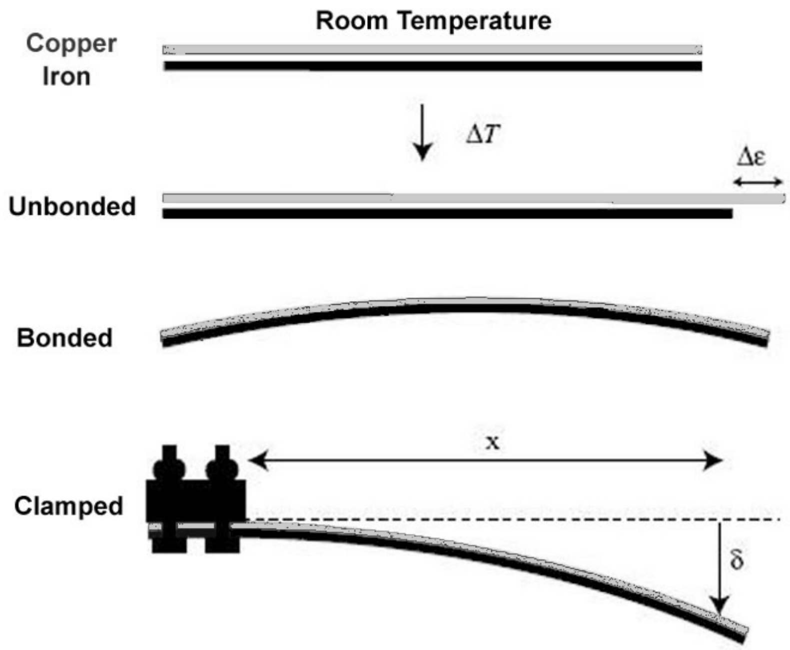
bimetal strip: a strip made from two metals with different coëfficients of thermal expansion that are bonded together. When the strip is heated or cooled, the two metals expand or contract different amounts, which causes the strip to bend. When the strip is returned to room temperature, the metals revert back to their original lengths.



Sample Problems:

Q: Find the change in length of an 0.40 m brass rod that is heated from 25 °C to 980 °C.

A: For brass, $\alpha = 1.9\times10^{-5}\ {}^{\circ}\mathrm{C}^{-1}$.

$$\Delta L = \alpha L_i \Delta T$$
$$\Delta L = (1.9\times10^{-5})(0.40)(955)$$
$$\Delta L = 0.0073\text{ m}$$

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Q: A typical mercury thermometer contains about 0.22 cm^3 (about 3.0 g) of mercury. Find the change in volume of the mercury in a thermometer when it is heated from 25 °C to 50. °C.

A: For mercury, $\beta = 1.82\times10^{-4}\ °\text{C}^{-1}$.

$$\Delta V = \beta V_i \Delta T$$
$$\Delta V = (1.82\times10^{-4})(0.22)(25)$$
$$\Delta V = 0.00091\ \text{cm}^3$$

If the distance from the 25 °C to the 50 °C mark is about 3.0 cm, we could use this information to figure out the bore (diameter of the column of mercury) of the thermometer:

$$V = \pi r^2 h$$
$$0.00091 = (3.14)r^2(3.0)$$
$$r^2 = \frac{0.00091}{(3.14)(3.0)} = 9.66\times10^{-5}$$
$$r = \sqrt{9.66\times10^{-5}} = 0.0098\ \text{cm}$$

The bore is the diameter, which is twice the radius, so the bore of the thermometer is (2)(0.0098) = 0.0197 cm, which is about 0.20 mm.

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Homework Problems

You will need to look up coëfficients of thermal expansion in *Table K. Thermal Properties of Selected Materials* on page 687 of your Physics Reference Tables.

1. A brass rod is 27.50 cm long at 25°C. How long would the rod be if it were heated to 750.°C in a flame?

   Answer: 27.88 cm

2. A steel bridge is 625 m long when the temperature is 0°C.

   a. If the bridge did not have any expansion joints, how much longer would the bridge be on a hot summer day when the temperature is 35°C?
   (Use the linear coëfficient of expansion for iron.)

      Answer: 0.258m

   b. Why do bridges need expansion joints?

3. A 15.00 cm long bimetal strip is aluminum on one side and copper on the other. If the two metals are the same length at 20.0°C, how long will each be at 800.°C?

   Answers: aluminum: 15.269 cm; copper: 15.199 cm

4. A glass volumetric flask is filled with water exactly to the 250.00 mL line at 50.°C. What volume will the water occupy after it cools down to 20.°C?

   Answer: 248.45 mL

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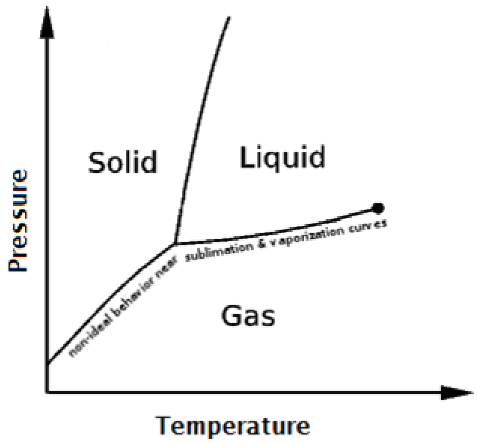
Thermal Expansion of Gases



<u>ideal gas</u>: a gas that behaves as if each molecule acts independently, according to kinetic-molecular theory. Most gases behave ideally except at temperatures and pressures near the vaporization curve on a phase diagram. (*I.e.,* gases stop behaving ideally when conditions are close to those that would cause the gas to condense to a liquid or solid.)

For an ideal gas, the change in volume for a change in temperature (provided that the pressure and number of molecules are kept constant) is given by Charles' Law:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where V_1 and T_1 are the initial volume and temperature, and V_2 and T_2 are the final volume and temperature, respectively. Volume can be any volume unit (as long as it is the same on both sides), but temperature *<u>must</u>* *<u>be</u>* in Kelvin.

Sample Problem:

Q: If a 250 mL container of air is heated from 25°C to 95°C, what is the new volume?

A: Temperatures must be in Kelvin, so we need to convert first.

T_1 = 25°C + 273 = 298 K T_2 = 95°C + 273 = 368 K

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{250}{298} = \frac{V_2}{368}$$

$$V_f = 308.7 \approx 310 \text{ mL}$$

Because we used mL for V_1, the value of V_2 is therefore also in mL.

Use this space for summary and/or additional notes:

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Homework Problems

1. A sample of argon gas was cooled, and its volume went from 380. mL to 250. mL. If its final temperature was −45.0 °C, what was its original temperature?

   Answer: 347 K or 74 °C

2. A balloon contains 250. mL of air at 50 °C. If the air in the balloon is cooled to 20.0 °C, what will be the new volume of the air?

   Answer: 226.8mL

Use this space for summary and/or additional notes:

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Introduction: Atomic, Particle, and Nuclear Physics

Unit: Atomic, Particle, and Nuclear Physics

Topics covered in this chapter:

This chapter discusses the particles that atoms and other matter are made of, how those particles interact, and the process by which radioactive decay can change the composition of a substance from one element into another.

- *The Bohr Model of the Hydrogen Atom* describes the first attempts to use quantum mechanics to describe the behavior of the electrons in an atom. *The Quantum Mechanical Model of the Atom* describes the evolution of atomic theory from the Bohr model to the present day.
- *Fundamental Forces* describes the four natural forces that affect everything in the universe. The strong nuclear force and the weak nuclear force are particularly relevant to this chapter.
- *The Standard Model* and *Particle Interactions* describe properties of and interactions between the particles that all matter is made of.
- *Radioactive Decay*, *Nuclear Equations*, *Mass Defect & Binding Energy*, *Half-Life*, and *Nuclear Fission & Fusion* describe and give equations for the nuclear changes that radioactive elements undergo.

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One of the challenging aspects of this chapter is that it describes process that happen on a scale that is much too small to observe directly. Another challenge is the fact that the Standard Model continues to evolve. Many of the connections between concepts that make other topics easier to understand have yet to be made in the realm of atomic & particle physics.

Standards addressed in this chapter:

Massachusetts Curriculum Frameworks (2016):

HS-PS1-8: Develop a model to illustrate the energy released or absorbed during the processes of fission, fusion, and radioactive decay.

HS-PS4-3. Evaluate the claims, evidence, and reasoning behind the idea that electromagnetic radiation can be described by either a wave model or a particle model, and that for some situations involving resonance, interference, diffraction, refraction, or the photoelectric effect, one model is more useful than the other.

AP® Physics 1 Learning Objectives:

This unit was never part of the AP® Physics 1 curriculum. It is part of the AP® Physics 2 curriculum.

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Quantum Phenomena**, such as photons and the photoelectric effect.
- **Atomic Physics**, such as the Rutherford and Bohr models, atomic energy levels, and atomic spectra.
- **Nuclear and Particle Physics**, such as radioactivity, nuclear reactions, and fundamental particles.

1. The Discovery of the Atom
2. Quantum Physics
3. Nuclear Physics

Use this space for summary and/or additional notes:

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Photoelectric Effect

Unit: Atomic, Particle, and Nuclear Physics

MA Curriculum Frameworks (2016): HS-PS4-3

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain the photoelectric effect.
- Calculate the work function of an atom and the kinetic energy of electrons emitted.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct with correct units and reasonable rounding.

Language Objectives:

- Explain why a minimum amount of energy is needed in order to emit an electron.

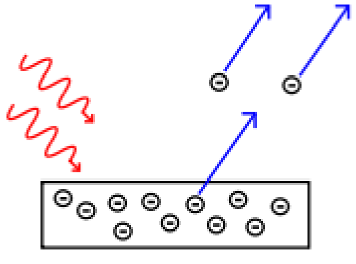
Tier 2 Vocabulary: work function

Labs, Activities & Demonstrations:

- threshold voltage to light an LED
- glow-in-the-dark substance and red *vs.* green *vs.* blue laser pointer

Notes:

The photoelectric effect was discovered in 1887 when Heinrich Hertz discovered that electrodes emitted sparks more effectively when ultraviolet light was shone on them. We now know that the particles are electrons, and that ultraviolet light of sufficiently high frequency (which varies from element to element) causes the electrons to be emitted from the surface of the element:



The photoelectric effect requires light with a sufficiently high frequency, because the frequency of the light is related to the amount of energy it carries. The energy of the photons needs to be above a certain threshold frequency in order to have enough energy to ionize the atom.

Use this space for summary and/or additional notes:

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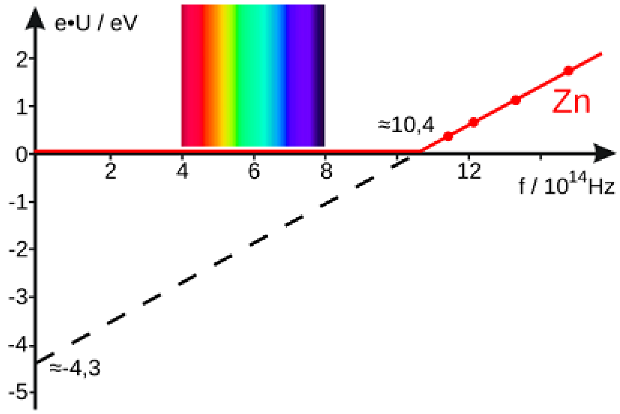
For example, a minimum frequency of 10.4×10^{14} Hz is needed to dislodge electrons from a zinc atom:



The maximum kinetic energy of the emitted electron is equal to Planck's constant times the difference between the frequency of incident light (f) and the minimum threshold frequency of the element (f_o):

$$K_{max} = h(f - f_o)$$

The quantity hf_o is called the "work function" of the atom, and is denoted by the variable ϕ. Thus the kinetic energy equation can be rewritten as:

$$K_{max} = hf - \phi$$

Values of the work function for different elements range from about 2.3–6 eV. $(1\text{ eV} = 1.6\times10^{-19}\text{ J})$

In 1905, Albert Einstein published a paper explaining that the photoelectric effect was evidence that energy from light was carried in discrete, quantized packets. This discovery, for which Einstein was awarded the Nobel prize in physics in 1921, led to the birth of the field of quantum physics.

Use this space for summary and/or additional notes:

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Bohr Model of the Hydrogen Atom

Unit: Atomic, Particle, and Nuclear Physics

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain how Bohr's model unified recent developments in the fields of spectroscopy, atomic theory and early quantum theory.
- Calculate the frequency/wavelength of light emitted using the Rydberg equation.
- Calculate the energy associated with a quantum number using Bohr's equation.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct with correct rounding and reasonable units.

Language Objectives:

- Explain why the Bohr Model was such a big deal.

Tier 2 Vocabulary: model, quantum

Notes:

Significant Developments Prior to 1913

Atomic Theory

Discovery of the Electron (1897): J.J. Thompson determined that cathode rays were actually particles emitted from atoms that the cathode was made of. These particles had an electrical charge, so they were named "electrons".

Discovery of the Nucleus (1909): Ernest Rutherford's famous "gold foil experiment" determined that atoms contained a dense, positively-charged region that comprised most of the atom's mass. This region was named the "nucleus", after the nucleus of a cell.

Rutherford ("Planetary") Model of the Atom (1911): The atom was believed to be like a miniature solar system, with electrons orbiting the nucleus in much the same way as planets orbit the sun.

Use this space for summary and/or additional notes:

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Early Quantum Theory

"Old" Quantum Theory (ca. 1900): sub-atomic particles obey the laws of classical mechanics, but that only certain "allowed" states are possible.

Spectroscopy

Balmer Formula (1885): Johann Balmer devised an empirical equation to relate the emission lines in the visible spectrum for the hydrogen atom.

Rydberg Formula (1888): Johannes Rydberg developed a generalized formula that could describe the wave numbers of all of the spectral lines in hydrogen (and similar elements).

There are several series of spectral lines for hydrogen, each of which converge at different wavelengths. Rydberg described the Balmer series in terms of a pair of integers (n_1 and n_2, where $n_1 < n_2$), and devised a single formula with a single constant (now called the Rydberg constant) that relates them.

$$\frac{1}{\lambda_{vac}} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

The value of Rydberg's constant is $\dfrac{m_e e^4}{8\varepsilon_o^2 h^3 c} = 10\,973\,731.6\text{ m}^{-1} \approx 1.1\times10^7\text{ m}^{-1}$

where m_e is the rest mass of the electron, e is the elementary charge, ε_o is the permittivity of free space, h is Planck's constant, and c is the speed of light in a vacuum.

Rydberg's equation was later found to be consistent with other series discovered later, including the Lyman series (in the ultraviolet region; first discovered in 1906) and the Paschen series (in the infrared region; first discovered in 1908).

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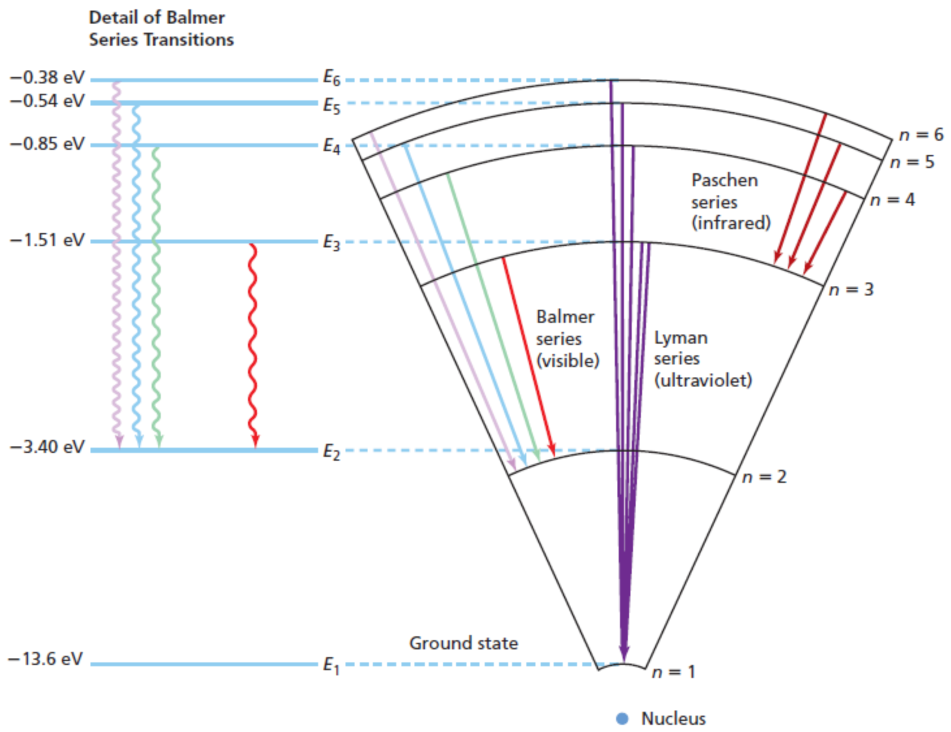
Those series and their converging wavelengths are:

Series	Wavelength	n_1	n_2
Lyman	91 nm	1	$2 \rightarrow \infty$
Balmer	365 nm	2	$3 \rightarrow \infty$
Paschen	820 nm	3	$4 \rightarrow \infty$

The following diagram shows Lyman, Balmer and Paschen series transitions form higher energy levels (n_2) back to lower ones (n_1).



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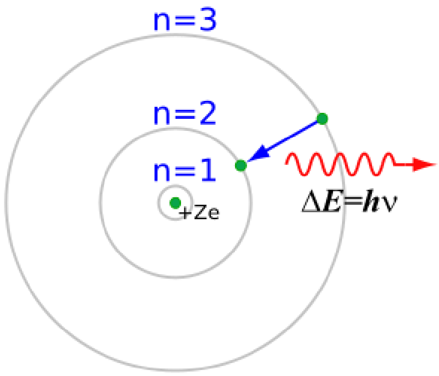
Bohr's Model of the Atom (1913)

In 1913, Niels Bohr combined atomic, quantum and spectroscopy theories into a single unified theory. Bohr hypothesized that electrons moved around the nucleus as in Rutherford's model, but that these electrons had only certain allowed quantum values of energy, which could be described by a quantum number (n). The value of that quantum number was the same n as in Rydberg's equation, and that using quantum numbers in Rydberg's equation could predict the wavelengths of light emitted when the electrons gained or lost energy by moved from one quantum level to another.



Bohr's model gained wide acceptance, because it related several prominent theories of the time. The theory worked well for hydrogen, giving a theoretical basis for Rydberg's equation. Bohr defined the energy associated with a quantum number (n) in terms of Rydberg's constant:

$$E_n = -\frac{R_H}{n^2}$$

Although the Bohr model worked well for hydrogen, the equations could not be solved exactly for atoms with more than one electron, because of the additional effects that electrons exert on each other (*e.g.,* the Coulomb force, $F_e = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$).

Use this space for summary and/or additional notes:

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Quantum Mechanical Model of the Atom

Unit: Atomic, Particle, and Nuclear Physics

MA Curriculum Frameworks (2016): HS-PS4-3

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Explain the de Broglie model of the atom.
- Explain the Schrödinger model of the atom.
- Explain the wave-particle duality of nature.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

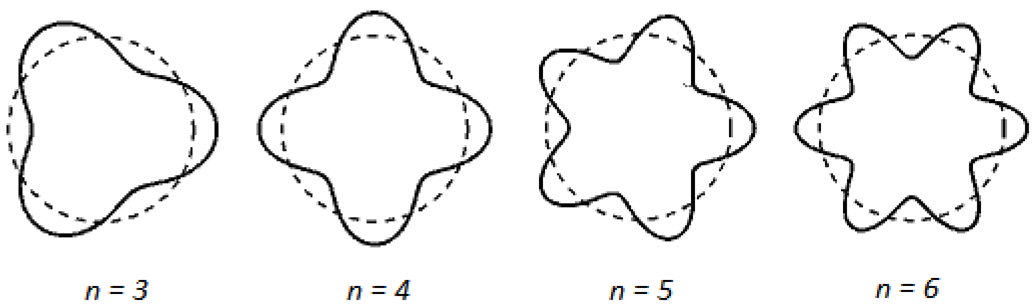
- Explain the important features of each model of the atom.

Tier 2 Vocabulary: model, quantum

Notes:

Louis de Broglie

In 1924, French physicist Louis de Broglie suggested that, like light, electrons can act as both a particle and a wave. He theorized that the reason that only integer values for quantum numbers were possible was because as the electron orbits the nucleus, its path must be an integer multiple of the wavelength:



Use this space for summary and/or additional notes:

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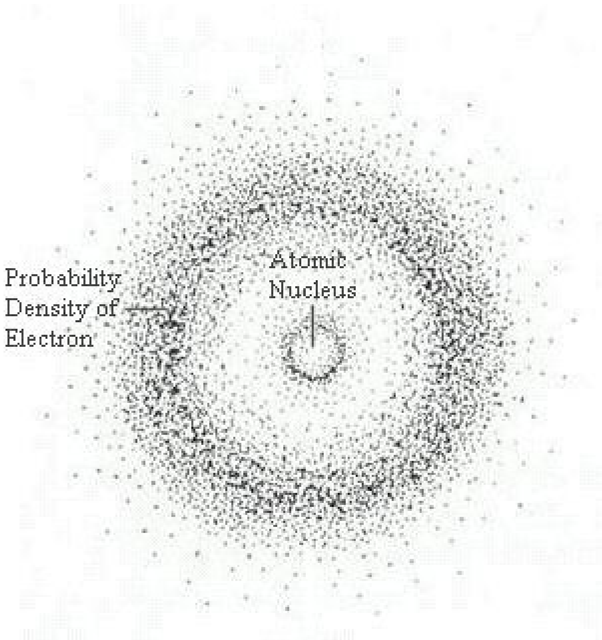
Erwin Schrödinger

In 1925, Austrian physicist Erwin Schrödinger found that by treating each electron as a unique wave function, the energies of the electrons could be predicted by the mathematical solutions to a wave equation. The use of Schrödinger's wave equation to construct a probability map for where the electrons can be found in an atom is the basis for the modern quantum-mechanical model of the atom.

To understand the probability map, it is important to realize that because the electron acts as a wave, it is detectable when the amplitude of the wave is nonzero, but not detectable when the amplitude is zero. This makes it appear as if the electron is teleporting from place to place around the atom. If you were somehow able to take a time-lapse picture of an electron as it moves around the nucleus, the picture might look something like the diagram to the right.



Notice that there is a region close to the nucleus where the electron is unlikely to be found, and a ring a little farther out where there is a much higher probability of finding the electron.

As you get farther and farther from the nucleus, Schrödinger's equation predicts different shapes for these probability distributions. These regions of high probability are called "orbitals," because of their relation to the orbits originally suggested by the planetary model.

Schrödinger was awarded the Nobel prize in physics in 1933 for this discovery.

Wave-Particle Duality

While de Broglie's and Schrödinger's theories of the wave nature of the electron explain several observed behaviors and allow for mathematical predictions, this does not negate previous observations that the electron also has mass and obeys laws of classical mechanics and special relativity.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Fundamental Forces

Unit: Atomic, Particle, and Nuclear Physics

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Name, describe, and give relative magnitudes of the four fundamental forces of nature.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

- Explain why the gravitational force is more relevant than the electromagnetic force in astrophysics.

Tier 2 Vocabulary: model, quantum

Notes:

All forces in nature ultimately come from one of the following four forces:

strong force (or "strong nuclear force" or "strong interaction"): an attractive force between quarks. The strong force holds the nuclei of atoms together. The energy comes from converting mass to energy.
Effective range: about the size of the nucleus of an average-size atom.

weak force (or "weak nuclear force" or "weak interaction"): the force that causes protons and/or neutrons in the nucleus to become unstable and leads to beta nuclear decay. This happens because the weak force causes an up or down quark to change its flavor. (This process is described in more detail in the section on Standard Model of Particle Physics, starting on page 654.)
Relative Strength: 10^{-6} to 10^{-7} times the strength of the strong force.
Effective range: about $^{1}/_{3}$ the diameter of an average nucleus.

electromagnetic force: the force between electrical charges. If the charges are the same ("like charges")—both positive or both negative—the particles repel each other. If the charges are different ("opposite charges")—one positive and one negative—the particles attract each other.
Relative Strength: about $^{1}/_{137}$ as strong as the strong force.
Effective range: ∞, but gets smaller as $(\text{distance})^2$.

gravitational force: the force that causes masses to attract each other. Usually only observable if one of the masses is very large (like a planet).
Relative Strength: only 10^{-39} times as strong as the strong force.
Effective range: ∞, but gets smaller as $(\text{distance})^2$.

Use this space for summary and/or additional notes:

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Standard Model

Unit: Atomic, Particle, and Nuclear Physics

MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives: N/A

Mastery Objective(s): (Students will be able to…)

- Name and describe the particles of the Standard Model.
- Describe interactions between particles, according to the Standard Model.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

- Explain the important features of each model of the atom.

Tier 2 Vocabulary: model, quantum

Notes:

The Standard Model is a theory of particle physics that:

- identifies the particles that matter is ultimately comprised of
- describes properties of these particles, including their mass, charge, and spin
- describes interactions between these particles

The Standard Model dates to the mid-1970s, when the existence of quarks was first experimentally confirmed. Physicists are still discovering new particles and relationships between particles, so the model and the ways it is represented are evolving, much like atomic theory and the Periodic Table of the Elements was evolving at the turn of the twentieth century. The table and the model described in these notes represent our understanding, as of 2024. By the middle of this century, the Standard Model may evolve to a form that is substantially different from the way we represent it today.

The Standard Model in its present form does not incorporate dark matter, dark energy, or gravitational attraction.

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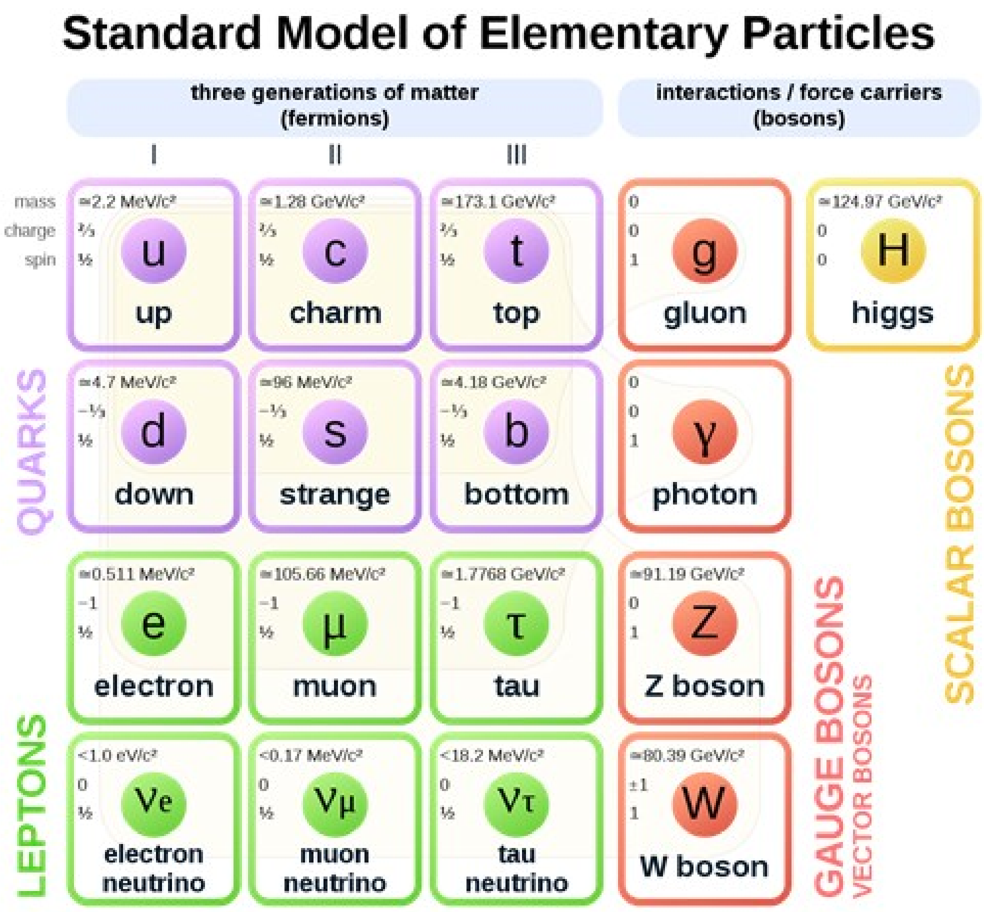
The Standard Model is often presented in a table, with rows, columns, and color-coded sections used to group subsets of particles according to their properties.

As of 2021, the Standard Model is represented by a table similar to this one:



Properties shown in the table include mass, charge, and spin.

- **Mass** is shown in units of electron volts divided by the speed of light squared $\left(\frac{\text{eV}}{\text{c}^2}\right)$. An electron volt (eV) is the energy acquired by an electric potential difference of one volt applied to one electron. (Recall that the metric prefix "M" stands for mega (10^6) and the metric prefix "G" stands for giga (10^9).) The c^2 in the denominator comes from Einstein's equation, $E = mc^2$, solved for m.
- **Charge** is the same property that we studied in the electricity unit. The magnitude and sign of charge is relative to the charge of an electron, which is defined to be −1.
- **Spin** is the property that is believed to be responsible for magnetism. (The name is because magnetism was previously thought to come from a magnetic field produced by electrons spinning within their orbitals.)

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Fundamental Particles

Quarks

Quarks are particles that participate in strong interactions (sometimes called the "strong force") through the action of "color charge" (which will be described later). Because protons and neutrons (which make up most of the mass of an atom) are made of three quarks each, quarks are the subatomic particles that make up most of the ordinary matter* in the universe.

- quarks have color charge (*i.e.,* they interact via the strong force)
- quarks have spin of $\pm \frac{1}{2}$
- "up-type" quarks carry a charge of $+\frac{2}{3}$; "down-type" quarks carry a charge of $-\frac{1}{3}$.

There are six flavors† of quarks: up and down, charm and strange, and top and bottom. (Originally, top and bottom quarks were called truth and beauty.)

Leptons

Leptons are the smaller particles that make up most matter. The most familiar lepton is the electron. Leptons participate in "electroweak" interactions, meaning combinations of the electromagnetic and weak forces.

- leptons do not have color charge (*i.e.,* they do not interact via the strong force)
- leptons have spins of $+\frac{1}{2}$
- electron-type leptons have a charge of −1; neutrinos do not have a charge.
- neutrinos oscillate, which makes their mass indefinite.

Gauge Bosons

Gauge bosons are the particles that carry force—their interactions are responsible for the fundamental forces of nature: the strong force, the weak force, the electromagnetic force and the gravitational force. The hypothetical particle responsible for the gravitational force is the graviton, which has not yet been detected (as of 2024).

- photons are responsible for the electromagnetic force.
- gluons are responsible for the strong interaction (strong force)
- W and Z bosons are responsible for the weak interaction (weak force)

* Matter that is not "ordinary matter" is called "dark matter", whose existence is theorized but not yet proven.

† Yes, "flavors" really is the correct term.

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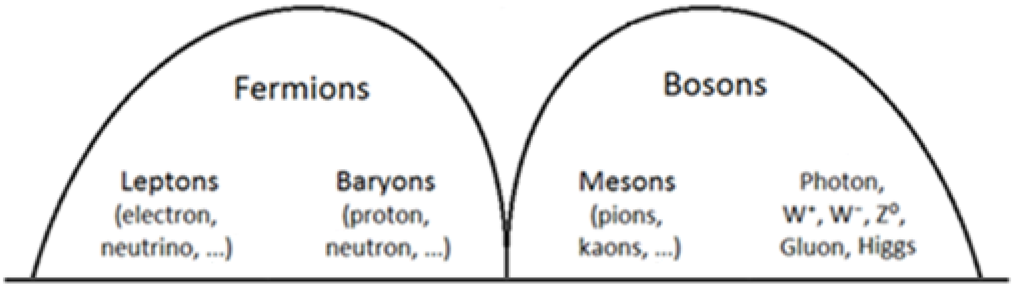
Scalar Bosons

At present, the only scalar boson we know of is the Higgs boson, discovered in 2012, which is responsible for mass.

Classes of Particles



Fermions

Quarks and leptons (the left columns in the table of the Standard Model) are fermions. Fermions are described by Fermi-Dirac statistics and obey the Pauli exclusion principle (which states that no two particles in an atom may have the same exact set of quantum numbers—numbers that describe the energy states of the particle).

Fermions are the building blocks of matter. They have a spin of ½, and each fermion has its own antiparticle (see below).

Bosons

Bosons (the right columns in the table of the Standard Model) are described by Bose-Einstein statistics, have integer spins and do not obey the Pauli Exclusion Principle. Interactions between boson are responsible for forces and mass.

Antiparticles

Each particle in the Standard Model has a corresponding antiparticle. Like chemical elements in the Periodic Table of the Elements, fundamental particles are designated by their symbols in the table of the Standard Model. Antiparticles are designated by the same letter, but with a line over it. For example, an up quark would be designated "u", and an antiup quark would be designated "$\overline{\text{u}}$".

The antiparticle of a fermion has the same name as the corresponding particle, with the prefix "anti-", and has the opposite charge. *E.g.,* the antiparticle of a tau neutrino is a tau antineutrino. (However, for historical reasons an antielectron is usually called a positron.) *E.g.,* up quark carries a charge of $+\frac{2}{3}$, which means an antiup quark carries a charge of $-\frac{2}{3}$.

Each of the fundamental bosons ***is*** its own antiparticle, except for the W^- boson, whose antiparticle is the W^+ boson.

Use this space for summary and/or additional notes:

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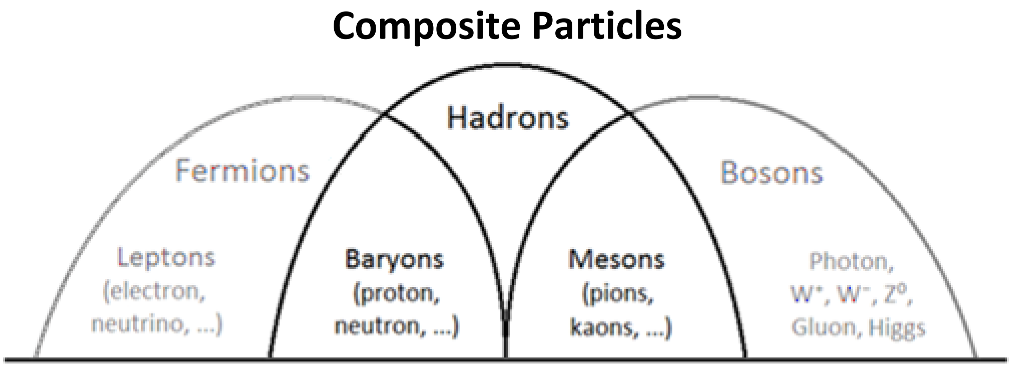
When a particle collides with its antiparticle, the particles annihilate each other, and their mass is converted to energy ($E = mc^2$) and released.



Hadrons

Hadrons are a special class of strongly-interacting composite particles (meaning that they are comprised of multiple individual particles). Hadrons can be bosons or fermions. Hadrons composed of strongly-interacting fermions are called baryons; hadrons composed of strongly-interacting bosons are called mesons.

Baryons

The most well-known baryons are protons and neutrons, which each comprised of three quarks. Protons are made of two up quarks and one down quark ("uud"), and carry a charge of +1. Neutrons are made of one up quark and two down quarks ("udd"), and carry a charge of zero.

Some of the better-known baryons include:

- nucleons (protons & neutrons).
- hyperons, *e.g.,* the Λ, Σ, Ξ, and Ω particles. These contain one or more strange quarks, and are much heavier than nucleons.
- various charmed and bottom baryons.
- pentaquarks, which contain four quarks and an antiquark.

Mesons

Ordinary mesons are comprised of a quark plus an antiquark. Examples include the pion, kaon, and the J/Ψ. Mesons mediate the residual strong force between nucleons.

Some of the exotic mesons include:

- tetraquarks, which contain two quarks and two antiquarks.
- glueball, a bound set of gluons with no quarks.
- hybrid mesons, which contain one or more quark/antiquark pairs and one or more gluons.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

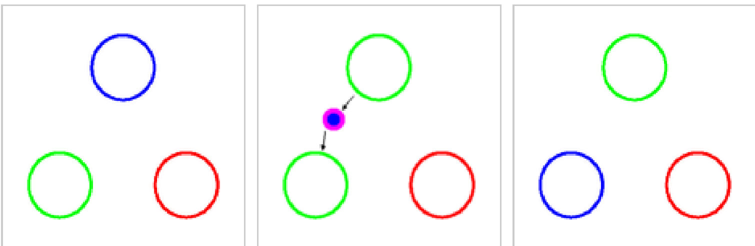
Color Charge

Color charge is the property that is responsible for the strong nuclear interaction. All electrons and fermions (particles that have half-integer spin quantum numbers) must obey the Pauli Exclusion Principle, which states that no two particles within the same larger particle (such as a hadron or atom) can have identical sets of quantum numbers.

For electrons, (as you learned in chemistry), if two electrons share the same orbital, they need to have opposite spins. In the case of quarks, all quarks have a spin of $+\frac{1}{2}$, so in order to satisfy the Pauli Exclusion Principle, if a proton or neutron contains three quarks, there has to be some other quantum property that has different values for each of those quarks. This property is called "color charge" (or sometimes just "color*").

The "color" property has three values, which are called "red," "green," and "blue" (named after the primary colors of light). When there are three quarks in a subatomic particle, the colors have to be different, and have to add up to "colorless". (Recall that combining each of the primary colors of light produces white light, which is colorless.)

Quarks can exchange color charge by emitting a gluon that contains one color and one anticolor. Another quark absorbs the gluon, and both quarks undergo color change. For example, suppose a blue quark emits a blue antigreen gluon:



You can imagine that the quark sent away its own blue color (the "blue" in the "blue antigreen" gluon). Because it also sent out antigreen, it was left with green so it became a green quark. Meanwhile, the antigreen part of the gluon finds the green quark and cancels its color. The blue from the blue antigreen gluon causes the receiving quark to become blue. After the interaction, the particle once again has one red, one green, and one blue quark, which means color charge is conserved.

* Just like "spin" is the name of a property of energy that has nothing to do with actual spinning, "color" is a property that has nothing to do with actual color. In fact, quarks couldn't possibly have actual color—the wavelengths of visible light are thousands of times larger than quarks!

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Radioactive Decay*

Unit: Atomic, Particle, and Nuclear Physics

MA Curriculum Frameworks (2016): HS-PS1-8

MA Curriculum Frameworks (2006): N/A

Mastery Objective(s): (Students will be able to…)

- Explain the causes of nuclear instability.
- Explain the processes of α, $\beta-$, and $\beta+$ decay and electron capture.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

- Explain what happens in each of the four types of radioactive decay.

Tier 2 Vocabulary: decay, capture

Labs, Activities & Demonstrations:

- (old) smoke detector & Geiger counter

Notes:

nuclear instability: When something is unstable, it is likely to change. If the nucleus of an atom is unstable, changes can occur that affect the number of protons and neutrons in the atom.

Note that when this happens, the nucleus ends up with a different number of protons. This causes the atom to literally turn into an atom of a different element. When this happens, the physical and chemical properties instantaneously change into the properties of the new element!

radioactive decay: the process by which the nucleus of an atom changes, transforming the element into a different element or isotope.

nuclear equation: an equation describing (through chemical symbols) what happens to an atom as it undergoes radioactive decay.

* Radioactive decay is usually taught in chemistry classes. However, the topic was moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016.

Use this space for summary and/or additional notes:

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Causes of Nuclear Instability

Two of the causes of nuclear instability are:

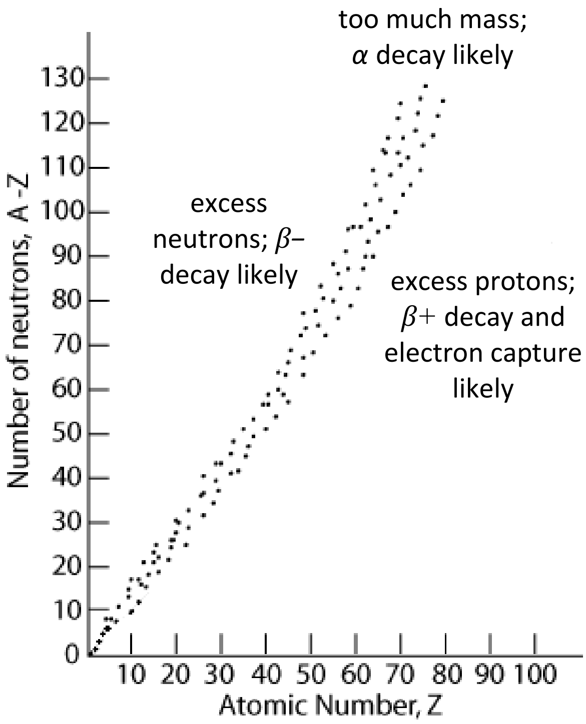
Size

Because the strong force acts over a limited distance, when nuclei get too large (more than 82 protons), it is no longer possible for the strong force to keep the nucleus together indefinitely. The form of decay that results from an atom exceeding its stable size is called alpha (α) decay.

The Weak Nuclear Force

The weak force is caused by the exchange (absorption and/or emission) of W and Z bosons. This causes a down quark to change to an up quark or vice-versa. The change of quark flavor has the effect of changing a proton to a neutron, or a neutron to a proton. (Note that the action of the weak force is the only known way of changing the flavor of a quark.) The form of decay that results from the action of the weak force is called beta (β) decay.

<u>band of stability</u>: isotopes with a ratio of protons to neutrons that results in a stable nucleus (one that does not spontaneously undergo radioactive decay). This observation suggests that the ratio of up to down quarks within the nucleus is somehow involved in preventing the weak force from causing quarks to change flavor.



Use this space for summary and/or additional notes:

Big Ideas | Details

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alpha (α) decay: a type of radioactive decay in which the nucleus loses two protons and two neutrons (an alpha particle). An alpha particle is a ${}^{4}_{2}He^{2+}$ ion (the nucleus of a helium-4 atom), with two protons, a mass of 4 amu, and a charge of +2. For example:

$$^{238}_{92}U \rightarrow\ ^{234}_{90}Th +\ ^{4}_{2}He$$

Atoms are most likely to undergo alpha decay if they have an otherwise stable proton/neutron ratio but a large atomic number.

Alpha decay has never been observed in atoms with an atomic number less than 52 (tellurium), and is rare in elements with an atomic number less than 73 (tantalum).

Net effects of α decay:

- Atom loses 2 protons and 2 neutrons (atomic number goes down by 2 and mass number goes down by 4)
- An α particle (a ${}^{4}_{2}He^{+2}$ ion) is ejected from the nucleus at high speed.

beta minus ($\beta-$) decay: a type of radioactive decay in which a neutron is converted to a proton and the nucleus ejects a high speed electron (${}^{0}_{-1}e$).

Note that a neutron consists of one up quark and two down quarks (udd), and a proton consists of two up quarks and one down quark (uud). When $\beta-$ decay occurs, the weak force causes one of the quarks changes its flavor from down to up, which causes the neutron (uud) to change into a proton (udd). Because a proton was gained, the atomic number increases by one. However, because the proton used to be a neutron, the mass number does not change. For example:

$$^{32}_{15}P \rightarrow\ ^{32}_{16}S +\ ^{0}_{-1}e$$

Atoms are likely to undergo $\beta-$ decay if they have too many neutrons and not enough protons to achieve a stable neutron/proton ratio. Almost all isotopes that are heavier than isotopes of the same element within the band of stability (because of the "extra" neutrons) undergo $\beta-$ decay.

Net effects of $\beta-$ decay:

- Atom loses 1 neutron and gains 1 proton (atomic number goes up by 1; mass number does not change)
- A $\beta-$ particle (an electron) is ejected from the nucleus at high speed.

Note that a $\beta-$ particle is assigned an atomic number of −1. *This does not mean an electron is some sort of "anti-proton".* The −1 is just used to make the equation for the number of protons work out in the nuclear equation.

Use this space for summary and/or additional notes:

Big Ideas	Details

CP1 & honors (not AP®)

<u>beta plus ($\beta+$) decay</u>: a type of radioactive decay in which a proton is converted to a neutron and the nucleus ejects a high speed antielectron (positron, ${}^{0}_{+1}e$).

With respect to the quarks, $\beta+$ decay is the opposite of $\beta-$ decay When $\beta+$ decay occurs, one of the quarks changes its flavor from up to down, which changes the proton (uud) into a neutron (udd). Because a proton was lost, the atomic number decreases by one. However, because the neutron used to be a proton, the mass number does not change. For example:

$$^{23}_{12}\text{Mg} \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{+1}e$$

Atoms are likely to undergo $\beta+$ decay if they have too many protons and not enough neutrons to achieve a stable neutron/proton ratio. Almost all isotopes that are lighter than the isotopes of the same element that fall within the band of stability ("not enough neutrons") undergo $\beta+$ decay.

<u>Net effects of $\beta+$ decay:</u>

- Atom loses 1 proton and gains 1 neutron (atomic number goes down by 1; mass number does not change)
- A $\beta+$ particle (an antielectron or positron) is ejected from the nucleus at high speed.

<u>electron capture</u> (sometimes called "K-capture"): when the nucleus of the atom "captures" an electron from the innermost shell (the K-shell) and incorporates it into the nucleus. This process is exactly the reverse of $\beta-$ decay; during electron capture, a quark changes flavor from up to down, which changes a proton (uud) into a neutron (udd):

$$^{23}_{12}\text{Mg} + {}^{0}_{-1}e \rightarrow {}^{23}_{11}\text{Na}$$

Note that $\beta+$ decay and electron capture produce the same products. Electron capture can sometimes (but not often) occur without $\beta+$ decay. However, $\beta+$ decay is <u>*always*</u> accompanied by electron capture.

Atoms are likely to undergo electron capture (and usually also $\beta+$ decay) if they have too many protons and not enough neutrons to achieve a stable neutron/proton ratio. Almost all isotopes that are lighter than the isotopes of the same element that fall within the band of stability undergo electron capture, and usually also $\beta+$ decay.

<u>Net effects of electron capture:</u>

- An electron is absorbed by the nucleus.
- Atom loses 1 proton and gains 1 neutron (atomic number goes down by 1; mass number does not change)

Use this space for summary and/or additional notes:

Big Ideas | Details

CP1 & honors (not AP®)

gamma (γ) rays: most radioactive decay produces energy. Some of that energy is emitted in the form of gamma rays, which are very high energy photons of electromagnetic radiation. (Radio waves, visible light, and X-rays are other types of electromagnetic radiation.) Because gamma rays are waves (which have no mass), they can penetrate far into substances and can do a lot of damage. Because gamma rays are not particles, emission of gamma rays does not change the composition of the nucleus.

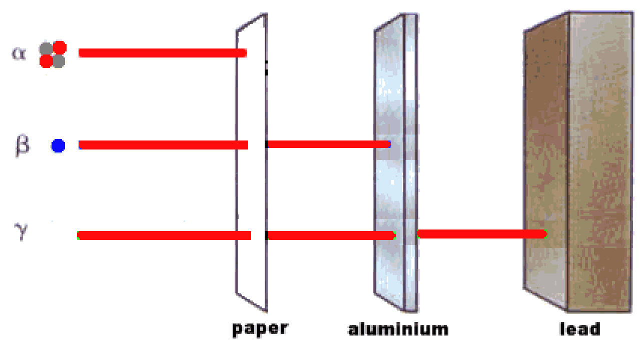
All of the types of radioactive decay mentioned in these notes also produce γ rays. This means to be complete, we would add gamma radiation to each of the radioactive decay equations described above:

$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He} + {}^{0}_{0}\gamma \qquad {}^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + {}^{0}_{-1}e + {}^{0}_{0}\gamma$$

$$^{23}_{12}\text{Mg} \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{+1}e + {}^{0}_{0}\gamma \qquad {}^{23}_{12}\text{Mg} + {}^{0}_{-1}e \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{0}\gamma$$

penetrating power: the distance that radioactive particles can penetrate into/through another substance is directly related to the velocity of the emission (faster = more penetrating) and inversely related to the mass of the emission (heaver = less penetrating):

gamma rays	**beta particles**	**alpha particles**
lightest	⟷	heaviest
fastest	⟷	slowest
most penetrating power	⟷	least penetrating power



Note also that denser substances (such as lead) do a better job of blocking and absorbing radioactive emissions. This is why lead is commonly used as shielding for experiments involving radioactive substances.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Nuclear Equations*

Unit: Atomic, Particle, and Nuclear Physics

MA Curriculum Frameworks (2016): HS-PS1-8

MA Curriculum Frameworks (2006): N/A

Mastery Objective(s): (Students will be able to…)

- Determine the products of α, $\beta-$, and $\beta+$ decay and electron capture.

Success Criteria:

- Equations give the correct starting material and products.

Language Objectives:

- Describe the changes to the nucleus during radioactive decay.

Tier 2 Vocabulary: decay, capture

Notes:

nuclear equation: a chemical equation describing the process of an isotope undergoing radioactive decay. For example:

$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$$

In a nuclear equation, the number of protons (atomic number) and the total mass (mass number) are conserved on both sides of the arrow. If you look at the bottom (atomic) numbers, and replace the arrow with an = sign, you would have the following:

$$92 = 90 + 2$$

Similarly, if you look at the top (mass) numbers, and replace the arrow with an = sign, you would have:

$$238 = 234 + 4$$

* Nuclear equations are usually taught in chemistry classes. However, the topic was moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Sample problems:

Q: What are the products of beta-minus ($\beta-$) decay of ^{131}I?

A: A $\beta-$ particle is an electron, which we write as $^{0}_{-1}e$ in a nuclear equation. This means ^{131}I decays into some unknown particle plus $^{0}_{-1}e$. The equation is:

$$^{131}_{53}\mathrm{I} \rightarrow {}^{m}_{p}X + {}^{0}_{-1}e$$

We can write the following equations for the atomic and mass numbers:

Atomic #s: $53 = p + -1 \rightarrow p = 54$; therefore X is Xe

Mass #s: $131 = m + 0 \rightarrow m = 131$

Therefore, particle X is $^{131}_{54}\mathrm{Xe}$ So our final answer is:

The two products of decay in this reaction are $^{131}_{54}\mathrm{Xe}$ and $^{0}_{-1}e$.

Q: Which particle was produced in the following radioactive decay reaction:

$$^{212}_{86}\mathrm{Rn} \rightarrow {}^{208}_{84}\mathrm{Po} + {}^{m}_{p}X$$

A: The two equations are:

Atomic #s: $86 = 84 + p \rightarrow p = 2$; therefore X is He

Mass #s: $212 = 208 + m \rightarrow m = 4$

Therefore, particle X is $^{4}_{2}\mathrm{He}$, which means it is an α particle.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Homework Problems

For these problems, you will need to use a periodic table and radioactive decay information from *Table EE. Selected Radioisotopes* on page 696 of your Physics Reference Tables.

Give the nuclear equation(s) for radioactive decay of the following:

1. ^{222}Rn
2. ^{85}Kr
3. ^{220}Fr
4. ^{37}K
5. ^{3}H

Give the starting material for the following materials produced by radioactive decay:

6. Alpha (α) decay resulting in $^{267}_{108}Hs$
7. Beta-minus ($\beta-$) decay resulting in $^{185}_{75}Re$

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Mass Defect & Binding Energy

Unit: Atomic, Particle, and Nuclear Physics

MA Curriculum Frameworks (2016): HS-PS1-8

MA Curriculum Frameworks (2006): N/A

Mastery Objective(s): (Students will be able to…)

- Calculate the binding energy of an atom.
- Calculate the energy given off by a radioactive decay based on the binding energies before and after.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain where the energy behind the strong force (which holds the nucleus together) comes from.

Tier 2 Vocabulary: defect

Notes:

binding energy: the energy that holds the nucleus of an atom together through the strong nuclear force

The binding energy comes from the small amount of mass (the mass defect) that was turned into a large amount of energy, given by the equation:

$$E = mc^2$$

where E is the binding energy, m is the mass defect, and c is the speed of light ($3\times10^8\ \frac{\text{m}}{\text{s}}$, which means c^2 is a very large number).

mass defect: the difference between the actual mass of an atom, and the sum of the masses of the protons, neutrons, and electrons that it contains. The mass defect is the amount of mass that was turned into binding energy.

- A proton has a mass of 1.6726×10^{-27} kg = 1.0073 amu
- A neutron has a mass of 1.6749×10^{-27} kg = 1.0087 amu
- An electron has a mass of 9.1094×10^{-31} kg = 0.0005486 amu

To calculate the mass defect, total up the masses of each of the protons, neutrons, and electrons in an atom. The actual (observed) atomic mass of the atom is always *less* than this number. The "missing mass" is called the mass defect.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Sample problem:

Q: Calculate the mass defect of 1 mole of uranium-238.

A: $^{238}_{92}U$ has 92 protons, 146 neutrons, and 92 electrons. This means the total mass of one atom of $^{238}_{92}U$ should theoretically be:

92 protons × 1.0073 amu = 92.6704 amu
146 neutrons × 1.0087 amu = 147.2661 amu
92 electrons × 0.000 5486 amu = 0.0505 amu
92.6704 + 147.2661 + 0.0505 = 239.9870 amu

The actual observed mass of one atom of $^{238}_{92}U$ is 238.0003 amu.

The mass defect of one atom of $^{238}_{92}U$ is therefore
239.9870 − 238.0003 = 1.9867 amu.

One mole of $^{238}_{92}U$ would have a mass of 238.0003 g, and therefore a total mass defect of 1.9867 g, or 0.001 9867 kg.

Because $E = mc^2$, that means the binding energy of one mole of $^{238}_{92}U$ is:

$$0.001\,9867\,\text{kg} \times (3.00 \times 10^8)^2 = 1.79 \times 10^{14}\ \text{J}$$

In case you don't realize just how large that number is, the binding energy of just 238 g (1 mole) of $^{238}_{92}U$ would be enough energy to heat every house on Earth for an entire winter!

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Half-Life*

Unit: Atomic, Particle, and Nuclear Physics

MA Curriculum Frameworks (2016): N/A

MA Curriculum Frameworks (2006): N/A

Mastery Objective(s): (Students will be able to…)

- Calculate the amount of material remaining after an integer number of half-lives.
- Calculate the elapsed time (integer half-lives) based on the amount of material remaining.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why the *mass* of material that decays keeps decreasing.

Tier 2 Vocabulary: life, decay

Labs, Activities & Demonstrations:

- half-life of dice or M & M candies

Notes:

The atoms of radioactive elements are unstable, and they spontaneously decay (change) into atoms of other elements.

For any given atom, there is a certain probability, *P*, that it will undergo radioactive decay in a given amount of time. The half-life, τ, is how much time it would take to have a 50 % probability of the atom decaying. If you start with *n* atoms, after one half-life, half of them (0.5*n*) will have decayed.

Amount of Material Remaining

If we start with 32 g of ^{53}Fe, which has a half-life (τ) of 8.5 minutes, we would observe the following:

# minutes	0	8.5	17	25.5	34
# half lives	0	1	2	3	4
amount left	32 g	16 g	8 g	4 g	2 g

* Half-life is usually taught in chemistry classes. However, the topic was moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Finding the Time that has Passed (integer number of half-lives)

If the amount you started with divided by the amount left is an exact power of two, you have an integer number of half-lives and you can make a table.

Sample problem:

Q: If you started with 64 g of ^{131}I, how long would it take until there was only 4 g remaining? The half-life (τ) of ^{131}I is 8.07 days.

A: $\frac{64}{4} = 16$ which is a power of 2, so we can simply make a table:

# half lives	0	1	2	3	4
amount remaining	64 g	32 g	16 g	8 g	4 g

From the table, after 4 half-lives, we have 4 g remaining.

The half-life (τ) of ^{131}I is 8.07 days.

$$8.07 \times 4 = 32.3 \text{ days}$$

Finding the amount remaining and time that has passed for a non-integer number of half-lives requires logarithms, and is beyond the scope of this course.

Homework Problems

For these problems, you will need to use half-life information from *Table EE. Selected Radioisotopes* on page 696 of your physics reference tables.

1. If a lab had 128 g of ^{3}H waste 49 years ago, how much of it would be left today? (*Note: you may round off to a whole number of half-lives.*)

   Answer: 8 g

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

2. Suppose you set aside a 20. g sample of ^{42}K at 8:30am on Friday for an experiment, but you get called into an all-day meeting. When you are finally able to perform the experiment at 10:54am on Monday, how much of the ^{42}K will be left?

   Answer: 0.31 g

3. If a school wants to dispose of small amounts of radioactive waste, they can store the materials for ten half-lives, and then dispose of the materials as regular trash.

   a. If we had a sample of ^{32}P, how long would we need to store it before disposing of it?

      Answer: 143 days

   b. If we had started with 64 g of ^{32}P, how much ^{32}P would be left after ten half-lives? Approximately what fraction of the original amount would be left?

      Answer: 0.063 g; approximately $\frac{1}{1000}$ of the original amount.

4. If the carbon in a sample of human bone contained only one-fourth of the expected amount of ^{14}C, how old is the sample?

   (*Hint: pretend you started with* 1 g *of* ^{14}C *and you have* 0.25 g *remaining.*)

   Answer: 11 460 years

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Nuclear Fission & Fusion*

Unit: Atomic, Particle, and Nuclear Physics

MA Curriculum Frameworks (2016): HS-PS1-8

MA Curriculum Frameworks (2006): N/A

Mastery Objective(s): (Students will be able to…)

- Identify nuclear processes as "fission" or "fusion".
- Describe the basic construction and operation of fission-based and fusion-based nuclear reactors.

Success Criteria:

- Descriptions account for how the energy is produced and how the radiation is contained.

Language Objectives:

- Explain how fission-based and fusion-based nuclear reactors work.

Tier 2 Vocabulary: fusion, nuclear

Notes:

Fission

fission: splitting of the nucleus of an atom, usually by bombarding it with a high-speed neutron.

When atoms are split by bombardment with neutrons, they can divide in hundreds of ways. For example, when ^{235}U is hit by a neutron, it can split more than 200 ways. Three examples that have been observed are:

$$\begin{aligned}
{}^{1}_{0}n + {}^{235}_{92}\mathrm{U} &\rightarrow {}^{90}_{37}\mathrm{Rb} + {}^{144}_{55}\mathrm{Cs} + 2\ {}^{1}_{0}n \\
{}^{1}_{0}n + {}^{235}_{92}\mathrm{U} &\rightarrow {}^{87}_{35}\mathrm{Br} + {}^{146}_{57}\mathrm{La} + 3\ {}^{1}_{0}n \\
{}^{1}_{0}n + {}^{235}_{92}\mathrm{U} &\rightarrow {}^{72}_{30}\mathrm{Br} + {}^{160}_{62}\mathrm{Sm} + 4\ {}^{1}_{0}n
\end{aligned}$$

Note that each of these bombardments produces more neutrons. A reaction that produces more fuel (in this case, neutrons) than it consumes will accelerate. This self-propagation is called a chain reaction.

Note also that the neutron/proton ratio of ^{235}U is about 1.5. The stable neutron/proton ratio of each of the products would be approximately 1.2. This means that almost all of the products of fission reactions have too many neutrons to be stable, which means they will themselves undergo $\beta-$ decay.

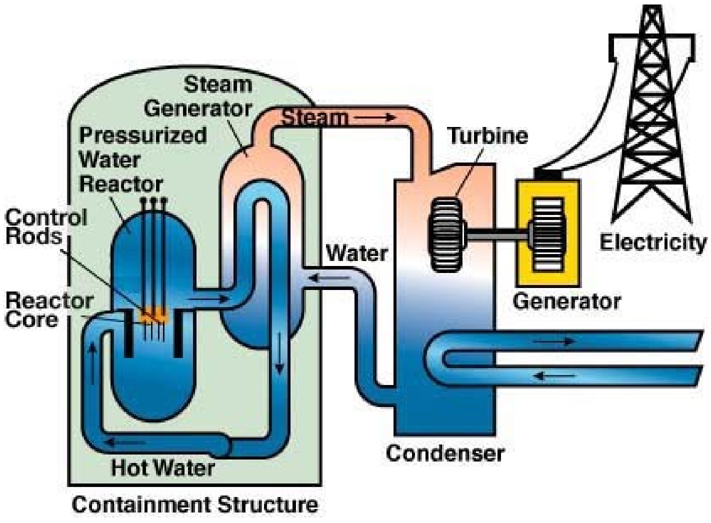
* The nuclear fission & fusion topic is usually taught in chemistry classes. However, the topic was moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016.

Use this space for summary and/or additional notes:

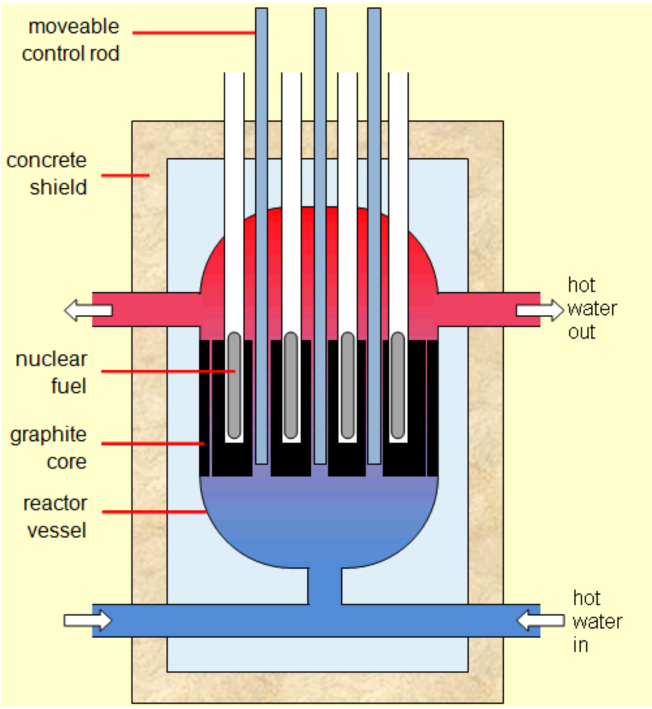
CP1 & honors (not AP®)

Nuclear Fission Reactors

In a nuclear reactor, the heat from a fission reaction is used to heat water. The radioactive hot water from the reactor (under pressure, so it can be heated well above 100 °C without boiling) is used to boil clean (non-radioactive) water. The clean steam is used to turn a turbine, which generates electricity.



The inside of the reactor looks like this:



The fuel is the radioactive material (such as ^{235}U) that is undergoing fission. The graphite in the core of the reactor is used to absorb some of the neutrons. The moveable control rods are adjusted so they can absorb some or all of the remaining neutrons as desired. If the control rods are all the way down, all of the neutrons are absorbed and no heating occurs. When the reactor is in operation, the control rods are raised just enough to make the reaction proceed at the desired rate.

Use this space for summary and/or additional notes:

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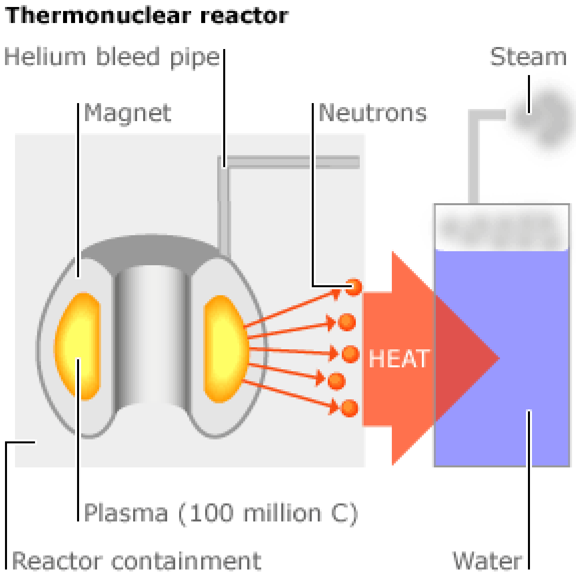
Fusion

<u>fusion</u>: the joining together of the nuclei of two atoms, accomplished by colliding them at high speeds.

Nuclear fusion reactions occur naturally on stars (such as the sun), and are the source of the heat and energy that stars produce.

On the sun, fusion occurs between atoms of deuterium (^{2}H) to produce helium:

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{4}_{2}He$$



The major challenge in building nuclear fusion reactors is the high temperatures produced—on the order of 10^{6}–10^{9} °C. In a tokamak fusion reactor, the starting materials are heated until they become plasma—a sea of highly charged ions and electrons. The highly charged plasma is kept away from the sides by powerful electromagnets.

At the left is a schematic of the ITER tokamak reactor currently under construction in southern France.

MIT has a smaller tokamak reactor at its Plasma Science & Fusion Center. The MIT reactor is able to conduct fusion reactions lasting for only a few seconds; if the reaction continued beyond this point, the current in the electromagnets that is necessary to generate the high magnetic fields required to confine the reaction would become hot enough to melt the copper wire and fuse the coils of the electromagnet together.

After each "burst" (short fusion reaction), the electromagnets in the MIT reactor need to be cooled in a liquid nitrogen bath (−196 °C) for fifteen minutes before the reactor is ready for the next burst.

Use this space for summary and/or additional notes:

CP1 & honors (not AP®)

Practical Uses for Nuclear Radiation

Unit: Atomic, Particle, and Nuclear Physics

MA Curriculum Frameworks (2016): HS-PS1-8

MA Curriculum Frameworks (2006): N/A

Mastery Objective(s): (Students will be able to…)

- Identify & describe practical (peaceful) uses for nuclear radiation.

Success Criteria:

- Descriptions give examples and explain how radiation is essential to the particular use.

Language Objectives:

- Explain how radiation makes certain scientific procedures possible.

Tier 2 Vocabulary: radiation

Notes:

While most people think of the dangers and destructive power of nuclear radiation, there are a lot of other uses of radioactive materials:

Power Plants: nuclear reactors can generate electricity in a manner that does not produce CO_2 and other greenhouse gases.

Cancer Therapy: nuclear radiation can be focused in order to kill cancer cells in patients with certain forms of cancer. Radioprotective drugs are now available that can help shield non-cancerous cells from the high-energy gamma rays.

Radioactive Tracers: chemicals made with radioactive isotopes can be easily detected in complex mixtures or even in humans. This enables doctors to give a patient a chemical with a small amount of radioactive material and track the progress of the material through the body and determine where it ends up. It also enables biologists to grow bacteria with radioactive isotopes and follow where those isotopes end up in subsequent experiments.

Use this space for summary and/or additional notes:

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Irradiation of Food: food can be exposed to high-energy gamma rays in order to kill germs. These gamma rays kill all of the bacteria in the food, but do not make the food itself radioactive. (Gamma rays cannot build up inside a substance.) This provides a way to create food that will not spoil for months on a shelf in a store. There is a lot of irrational fear of irradiated food in the United States, but irradiation is commonly used in Europe. For example, irradiated milk will keep for months on a shelf at room temperature without spoiling.

Carbon Dating: Because ^{14}C is a long-lived isotope (with a half-life of 5 700 years), the amount of ^{14}C in archeological samples can give an accurate estimate of their age. One famous use of carbon dating was its use to prove that the Shroud of Turin (the supposed burial shroud of Jesus Christ) was fake, because it was actually made between 1260 C.E. and 1390 C.E.

Smoke Detectors: In a smoke detector, ^{241}Am emits positively-charged alpha particles, which are directed towards a metal plate. This steady flow of positive charges completes an electrical circuit. If there is a fire, smoke particles neutralize positive charges. This makes the flow of charges through the electrical circuit stop, which is used to trigger the alarm.

Use this space for summary and/or additional notes:

Appendix: AP® Physics 1 Equation Tables

ADVANCED PLACEMENT PHYSICS 1 EQUATIONS, EFFECTIVE 2021

CONSTANTS AND CONVERSION FACTORS			
Proton mass,	$m_p = 1.67\times10^{-27}$ kg	Electron charge magnitude,	$e = 1.60\times10^{-19}$ C
Neutron mass,	$m_n = 1.67\times10^{-27}$ kg	Coulomb's law constant,	$k = 1/4\pi\varepsilon_0 = 9.0\times10^9\ \mathrm{N\cdot m^2/C^2}$
Electron mass,	$m_e = 9.11\times10^{-31}$ kg	Universal gravitational constant,	$G = 6.67\times10^{-11}\ \mathrm{m^3/kg\cdot s^2}$
Speed of light,	$c = 3.00\times10^8$ m/s	Acceleration due to gravity at Earth's surface,	$g = 9.8\ \mathrm{m/s^2}$

UNIT SYMBOLS								
	meter,	m	kelvin,	K	watt,	W	degree Celsius,	°C
	kilogram,	k	hertz,	Hz	coulomb,	C		
	second,	s	newton,	N	volt,	V		
	ampere,	A	joule,	J	ohm,	Ω		

PREFIXES		
Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	1/2	3/5	$\sqrt{2}/2$	4/5	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	4/5	$\sqrt{2}/2$	3/5	½	0
$\tan\theta$	0	$\sqrt{3}/3$	3/4	1	4/3	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. The frame of reference of any problem is assumed to be inertial unless otherwise stated.

II. Assume air resistance is negligible unless otherwise stated.

III. In all situations, positive work is defined as work done on a system.

IV. The direction of current is conventional current: the direction in which positive charge would drift.

V. Assume all batteries and meters are ideal unless otherwise stated.

ADVANCED PLACEMENT PHYSICS 1 EQUATIONS, EFFECTIVE 2015

MECHANICS

$v_x = v_{xo} + a_x t$

$x = x_o + v_{xo}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{xo}^2 + 2a_x(x - x_o)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\left|\vec{F}_f\right| \le \mu \left|\vec{F}_n\right|$

$a_c = \frac{v^2}{r}$

$\vec{p} = m\vec{v}$

$\Delta\vec{p} = \vec{F}\Delta t$

$K = \frac{1}{2}mv^2$

$\Delta E = W = F_{||}d = Fd\cos\theta$

$P = \frac{\Delta E}{\Delta t}$

$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$

$\omega = \omega_o + \alpha t$

$x = A\cos(2\pi ft)$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$\tau = r_{\perp}F = rF\sin\theta$

$L = I\omega$

$\Delta L = \tau\Delta t$

$\left|\vec{F}_s\right| = k\left|\vec{x}\right|$

$U_s = \frac{1}{2}kx^2$

$\rho = \frac{m}{V}$

a = acceleration
A = amplitude
d = distance
E = energy
f = frequency
F = force
I = rotational inertia
K = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
r = radius or separation
T = period
t = time
U = potential energy
V = volume
v = speed
W = work done on a system
x = position
y = height
α = angular acceleration
μ = coefficient of friction
θ = angle
ρ = density
τ = torque
ω = angular speed

$\Delta U_g = mg\Delta y$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$

$\vec{g} = \frac{\vec{F}_g}{m}$

$U_g = G\frac{m_1 m_2}{r}$

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = bh$

Circle
$A = \frac{1}{2}bh$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

Rectangular solid
$V = \ell wh$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + 2\pi r^2$

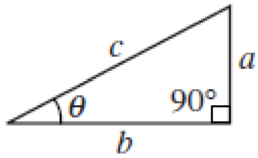
Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

Right triangle
$c^2 = a^2 + b^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



Appendix: Physics Reference Tables*

Contents:

Table A. Metric Prefixes

Factor		Prefix	Symbol
1 000 000 000 000 000 000 000 000 000 000	10^{30}	quetta	Q
1 000 000 000 000 000 000 000 000 000	10^{27}	ronna	R
1 000 000 000 000 000 000 000 000	10^{24}	yotta	Y
1 000 000 000 000 000 000 000	10^{21}	zeta	Z
1 000 000 000 000 000 000	10^{18}	exa	E
1 000 000 000 000 000	10^{15}	peta	P
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto	h
10	10^{1}	deca	da
1	10^{0}	—	—
0.1	10^{-1}	deci	d
0.01	10^{-2}	centi	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	µ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p
0.000 000 000 000 001	10^{-15}	femto	f
0.000 000 000 000 000 001	10^{-18}	atto	a
0.000 000 000 000 000 000 001	10^{-21}	zepto	z
0.000 000 000 000 000 000 000 001	10^{-24}	yocto	y
0.000 000 000 000 000 000 000 000 001	10^{-27}	ronto	r
0.000 000 000 000 000 000 000 000 000 001	10^{-30}	quecto	q

Move Decimal Point to the Left

Move Decimal Point to the Right

* Data from various sources, including: The University of the State of New York, The State Education Department. Albany, NY, *Reference Tables for Physical Setting/Physics, 2006 Edition*. http://www.p12.nysed.gov/apda/reftable/physics-rt/physics06tbl.pdf,
SparkNotes: SAT Physics website. http://www.sparknotes.com/testprep/books/sat2/physics/,
The Engineering Toolbox: https://www.engineeringtoolbox.com,
and The College Board: *Equations and Constants for AP® Physics 1 and AP® Physics 2.*

Table B. Physical Constants

Description	Symbol	Precise Value	Common Approximation
acceleration due to gravity on Earth strength of gravity field on Earth	g	$9.7639\,\frac{m}{s^2}$ to $9.8337\,\frac{m}{s^2}$ average value at sea level is $9.806\,65\,\frac{m}{s^2}$	$9.8\,\frac{m}{s^2} \equiv 9.8\,\frac{N}{kg}$ or $10\,\frac{m}{s^2} \equiv 10\,\frac{N}{kg}$
universal gravitational constant	G	$6.673\,84(80)\times10^{-11}\,\frac{N\cdot m^2}{kg^2}$	$6.67\times10^{-11}\,\frac{N\cdot m^2}{kg^2}$
speed of light in a vacuum	c	$299\,792\,458\,\frac{m}{s}$ *	$3.00\times10^{8}\,\frac{m}{s}$
elementary charge (proton or electron)	e	$\pm1.602\,176\,634\times10^{-19}$ C*	$\pm1.60\times10^{-19}$ C
1 coulomb (C)		$6.241\,509\,074\times10^{18}$ elementary charges	6.24×10^{18} elementary charges
(electric) permittivity of a vacuum	ε_o	$8.854\,187\,82\times10^{-12}\,\frac{A^2\cdot s^4}{kg\cdot m^3}$	$8.85\times10^{-12}\,\frac{A^2\cdot s^4}{kg\cdot m^3}$
(magnetic) permeability of a vacuum	μ_o	$4\pi\times10^{-7} = 1.256\,637\,06\times10^{-6}\,\frac{T\cdot m}{A}$	$1.26\times10^{-6}\,\frac{T\cdot m}{A}$
electrostatic constant	k	$\frac{1}{4\pi\varepsilon_o} = 8.987\,551\,787\,368\,176\,4\times10^{9}\,\frac{N\cdot m^2}{C^2}$ *	$8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}$
1 electron volt (eV)		$1.602\,176\,565(35)\times10^{-19}$ J	1.60×10^{-19} J
Planck's constant	h	$6.626\,070\,15\times10^{-34}$ J·s*	6.63×10^{-34} J·s
1 universal (atomic) mass unit (u)		$931.494\,061(21)$ MeV/ c^2 $1.660\,538\,921(73)\times10^{-27}$ kg	931 MeV/c^2 1.66×10^{-27} kg
Avogadro's constant	N_A	$6.022\,140\,76\times10^{23}$ mol^{-1} *	6.02×10^{23} mol^{-1}
Boltzmann constant	k_B	$1.380\,649\times10^{-23}\,\frac{J}{K}$ *	$1.38\times10^{-23}\,\frac{J}{K}$
universal gas constant	R	$8.314\,4621(75)\,\frac{J}{mol\cdot K}$	$8.31\,\frac{J}{mol\cdot K}$
Rydberg constant	R_H	$\frac{m_e e^4}{8\varepsilon_o^2 h^3 c} = 10\,973\,731.6\,\frac{1}{m}$	1.10×10^{7} m^{-1}
Stefan-Boltzmann constant	σ	$\frac{2\pi^5 R^4}{15h^3c^2} = 5.670\,374\,419\times10^{-8}\,\frac{J}{m^2\cdot s\cdot K^4}$	$5.67\times10^{-8}\,\frac{J}{m^2\cdot s\cdot K^4}$
standard atmospheric pressure at sea level		101 325 Pa ≡ 1.01325 bar*	100 000 Pa ≡ 1.0 bar
rest mass of an electron	m_e	$9.109\,382\,15(45)\times10^{-31}$ kg	9.11×10^{-31} kg
mass of a proton	m_p	$1.672\,621\,777(74)\times10^{-27}$ kg	1.67×10^{-27} kg
mass of a neutron	m_n	$1.674\,927\,351(74)\times10^{-27}$ kg	1.67×10^{-27} kg

*denotes an exact value (by definition)

Table C. Quantities, Variables and Units

Quantity	Variable	MKS Unit Name	MKS Unit Symbol	S.I. Base Unit
position	$\vec{\boldsymbol{x}}$	meter*	m	m
distance/displacement, (length, height)	$d, \vec{\boldsymbol{d}}, (L, h)$	meter*	m	m
angle	θ	radian, degree	—, °	—
area	A	square meter	m^2	m^2
volume	V	cubic meter, liter	m^3	m^3
time	t	second*	s	s
velocity	$\vec{\boldsymbol{v}}$	meter/second	$\frac{m}{s}$	$\frac{m}{s}$
speed of light	c			
angular velocity	$\vec{\boldsymbol{\omega}}$	radians/second	$\frac{1}{s^2}$, s^{-1}	$\frac{1}{s^2}$, s^{-1}
acceleration	$\vec{\boldsymbol{a}}$	meter/second²	$\frac{m}{s^2}$	$\frac{m}{s^2}$
acceleration due to gravity	$\vec{\boldsymbol{g}}$			
angular acceleration	$\vec{\boldsymbol{\alpha}}$	radians/second²	$\frac{1}{s^2}$, s^{-2}	$\frac{1}{s^2}$, s^{-2}
mass	m	kilogram*	kg	kg
force	$\vec{\boldsymbol{F}}$	newton	N	$\frac{kg \cdot m}{s^2}$
gravitational field	$\vec{\boldsymbol{g}}$	newton/kilogram	$\frac{N}{kg}$	$\frac{m}{s^2}$
pressure	P	pascal	Pa	$\frac{kg}{m \cdot s^2}$
energy (generic)	E	joule	J	$\frac{kg \cdot m^2}{s^2}$
potential energy	U			
kinetic energy	K, E_k			
heat	Q			
work	W	joule , newton-meter	J , N·m	$\frac{kg \cdot m^2}{s^2}$
torque	$\vec{\boldsymbol{\tau}}$	newton-meter	N·m	$\frac{kg \cdot m^2}{s^2}$
power	P	watt	W	$\frac{kg \cdot m^2}{s^3}$
momentum	$\vec{\boldsymbol{p}}$	newton-second	N·s	$\frac{kg \cdot m}{s}$
impulse	$\vec{\boldsymbol{J}}$			
moment of inertia	I	kilogram-meter²	$kg \cdot m^2$	$kg \cdot m^2$
angular momentum	$\vec{\boldsymbol{L}}$	newton-meter-second	N·m·s	$\frac{kg \cdot m^2}{s}$
frequency	f	hertz	Hz	s^{-1}
wavelength	λ	meter	m	m
period	T	second	s	s
index of refraction	n	—	—	—
electric current	$\vec{\boldsymbol{I}}$	ampere*	A	A
electric charge	q	coulomb	C	A·s
electric potential potential difference (voltage) electromotive force (emf)	V ΔV ε	volt	V	$\frac{kg \cdot m^2}{A \cdot s^3}$
electrical resistance	R	ohm	Ω	$\frac{kg \cdot m^2}{A^2 \cdot s^3}$
capacitance	C	farad	F	$\frac{A^2 \cdot s^4}{m^2 \cdot kg}$
electric field	$\vec{\boldsymbol{E}}$	netwon/coulomb volt/meter	$\frac{N}{C}$, $\frac{V}{m}$	$\frac{kg \cdot m}{A \cdot s^3}$
magnetic field	$\vec{\boldsymbol{B}}$	tesla	T	$\frac{kg}{A \cdot s^2}$
temperature	T	kelvin*	K	K
amount of substance	n	mole*	mol	mol
luminous intensity	I_v	candela*	cd	cd

Variables representing vector quantities are typeset in ***bold italics*** with ***arrows***. * = S.I. base unit

Table D. Mechanics Formulas and Equations

Topic	Equations
Kinematics (Distance, Velocity & Acceleration)	$\vec{d} = \Delta\vec{x} = \vec{x} - \vec{x}_o$ $\frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2} (= \vec{v}_{ave.})$ $\vec{v} - \vec{v}_o = \vec{a}t$ $\vec{d} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2$ $\vec{v}^2 - \vec{v}_o^2 = 2\vec{a}\vec{d}$
Forces & Dynamics	$\sum \vec{F} = \vec{F}_{net} = m\vec{a}$ $F_f \le \mu_s F_N \qquad F_f = \mu_k F_N$ $\vec{F}_g = m\vec{g} = \frac{Gm_1m_2}{r^2}$
Circular/ Centripetal Motion & Force	$a_c = \frac{v^2}{r}$ $F_c = ma_c$
Simple Harmonic Motion	$T = \frac{1}{f}$ $T_s = 2\pi\sqrt{\frac{m}{k}} \qquad T_p = 2\pi\sqrt{\frac{L}{g}}$ $\vec{F}_s = -k\vec{x}$ $U_s = \frac{1}{2}kx^2$
Energy, Work & Power	$U_g = mgh = \frac{Gm_1m_2}{r}$ $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ $W = \Delta E = \Delta(U_g + K)$ $W = F_{\parallel}d = \vec{F}_{net} \bullet \vec{d} = Fd\cos\theta$ $TME = U_g + K$ $TME_i + W = TME_f$ $P = \frac{W}{t} = \vec{F} \bullet \vec{v} = Fv\cos\theta$
Momentum	$\vec{p} = \sum m\vec{v}$ $\Sigma m_i\vec{v}_i + \vec{J} = \Sigma m_f\vec{v}_f$ $\vec{J} = \Delta\vec{p} = \vec{F}_{net}t$

var. = name of quantity (unit)

- Δ = change in something (*E.g.,* Δx means change in x)
- Σ = sum
- d = distance (m)
- $\vec{d}$ = displacement (m)
- $\vec{x}$ = position (m)
- t = time (s)
- $\vec{v}$ = velocity $\left(\frac{\text{m}}{\text{s}}\right)$
- $\vec{v}_{ave.}$ = average velocity $\left(\frac{\text{m}}{\text{s}}\right)$
- $\vec{a}$ = acceleration $\left(\frac{\text{m}}{\text{s}^2}\right)$
- f = frequency $\left(\text{Hz} = \frac{1}{\text{s}}\right)$
- $\vec{F}$ = force (N)
- $\vec{F}_{net}$ = net force (N)
- F_f = force due to friction (N)
- $\vec{F}_g$ = force due to gravity (N)
- $\vec{F}_N$ = normal force (N)
- m = mass (kg)
- $\vec{g}$ = strength of gravity field = aceleration due to gravity $= 10\,\frac{\text{N}}{\text{kg}} = 10\,\frac{\text{m}}{\text{s}^2}$ on Earth
- G = gravitational constant $= 6.67 \times 10^{-11}\,\frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}$
- r = radius (m)
- μ = coëfficient of friction* (*dimensionless*)
- θ = angle (°, radians)
- k = spring constant $\left(\frac{\text{N}}{\text{m}}\right)$
- $\vec{x}$ = displacement of spring (m)
- L = length of pendulum (m)
- E = energy (J)
- $K = E_k$ = kinetic energy (J)
- U = potential energy (J)
- TME = total mechanical energy (J)
- h = height (m)
- Q = heat (J)
- P = power (W)
- W = work (J, N·m)
- T = (time) period (Hz)
- $\vec{p}$ = momentum (N·s)
- $\vec{J}$ = impulse (N·s)
- π = pi (mathematical constant) = 3.14159 26535 89793...

*characteristic property of a substance (to be looked up)

Table E. Approximate Coëfficients of Friction

Substance	Static (μ_s)	Kinetic (μ_k)	Substance	Static (μ_s)	Kinetic (μ_k)
rubber on concrete (dry)	0.90	0.68	wood on wood (dry)	0.42	0.30
rubber on concrete (wet)		0.58	wood on wood (wet)	0.2	
rubber on asphalt (dry)	0.85	0.67	wood on metal	0.3	
rubber on asphalt (wet)		0.53	wood on brick	0.6	
rubber on ice		0.15	wood on concrete	0.62	
steel on ice	0.03	0.01	Teflon on Teflon	0.04	0.04
waxed ski on snow	0.14	0.05	Teflon on steel	0.04	0.04
aluminum on aluminum	1.2	1.4	graphite on steel	0.1	
cast iron on cast iron	1.1	0.15	leather on wood	0.3–0.4	
steel on steel	0.74	0.57	leather on metal (dry)	0.6	
copper on steel	0.53	0.36	leather on metal (wet)	0.4	
diamond on diamond	0.1		glass on glass	0.9–1.0	0.4
diamond on metal	0.1–0.15		metal on glass	0.5–0.7	

Table F. Angular/Rotational Mechanics Formulas and Equations

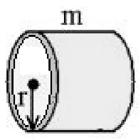
Angular Kinematics (Distance, Velocity & Acceleration)	$\Delta\vec{\boldsymbol{\theta}} = \vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\theta}}_o$ $\dfrac{\Delta\vec{\boldsymbol{\theta}}}{t} = \dfrac{\vec{\boldsymbol{\omega}}_o + \vec{\boldsymbol{\omega}}}{2} (= \vec{\boldsymbol{\omega}}_{ave.})$ $\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\omega}}_o = \vec{\boldsymbol{\alpha}} t$ $\Delta\vec{\boldsymbol{\theta}} = \vec{\boldsymbol{\omega}}_o t + \frac{1}{2}\vec{\boldsymbol{\alpha}} t^2$ $\vec{\boldsymbol{\omega}}^2 - \vec{\boldsymbol{\omega}}_o^2 = 2\vec{\boldsymbol{\alpha}}(\Delta\vec{\boldsymbol{\theta}})$
Circular/ Centripetal Motion	$s = r\Delta\theta \quad v_T = r\omega \quad a_T = r\alpha$ $a_c = \dfrac{v^2}{r} = \omega^2 r$
Rotational Dynamics	$x_{cm} = \dfrac{\sum m_i x_i}{\sum m_i}$ $I = \_\_mr^2 = \int_0^m r^2 dm$ $F_c = ma_c = mr\omega^2$ $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} \qquad \tau = rF\sin\theta = r_\perp F$ $\sum \vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\tau}}_{net} = I\vec{\boldsymbol{\alpha}}$
Simple Harmonic Motion	$T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$ $x = A\cos(2\pi ft) + \phi$
Angular Momentum	$\vec{\boldsymbol{L}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{p}} = I\vec{\boldsymbol{\omega}} \qquad L = rp\sin\theta = I\omega$ $\Delta\vec{\boldsymbol{L}} = \vec{\boldsymbol{\tau}}\Delta t$
Angular/ Rotational Energy, Work & Power	$K_r = \frac{1}{2}I\omega^2$ $K = K_t + K_r = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ $W_r = \tau\Delta\theta$ $P = \dfrac{W}{t} = \tau\omega$

var. = name of quantity (unit)

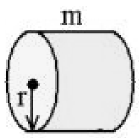
- Δ = change in something (*E.g.*, Δx = change in x)
- Σ = sum
- s = arc length (m)
- t = time (s)
- a_c = centripetal acceleration $\left(\frac{\text{m}}{\text{s}^2}\right)$
- F_c = centripetal force (N)
- m = mass (kg)
- r = radius (m)
- $\vec{\boldsymbol{r}}$ = radius (vector)
- θ = angle (°, radians)
- $\vec{\boldsymbol{\omega}}$ = angular velocity $\left(\frac{\text{rad}}{\text{s}}\right)$
- $\vec{\boldsymbol{\alpha}}$ = angular velocity $\left(\frac{\text{rad}}{\text{s}^2}\right)$
- $\vec{\boldsymbol{\tau}}$ = torque (N·m)
- x = position (m)
- f = frequency (Hz)
- A = amplitude (m)
- ϕ = phase offset (°, rad)
- E = energy (J)
- $K = E_k$ = kinetic energy (J)
- K_t = translational kinetic energy (J)
- K_r = rotational kinetic energy (J)
- P = power (W)
- W = work (J, N·m)
- $\vec{\boldsymbol{p}}$ = momentum (N·s)
- $\vec{\boldsymbol{L}}$ = angular momentum (N·m·s)

Table G. Moments of Inertia

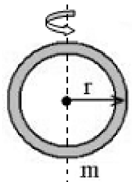
Point Mass: $I = mr^2$



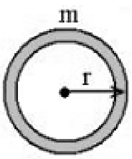
Hollow Cylinder: $I = mr^2$



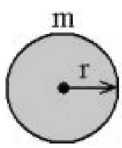
Solid Cylinder: $I = \frac{1}{2}mr^2$



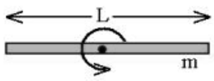
Hoop About Diameter: $I = \frac{1}{2}mr^2$



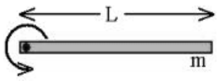
Hollow Sphere: $I = \frac{2}{3}mr^2$



Solid Sphere: $I = \frac{2}{5}mr^2$



Rod About the Middle: $I = \frac{1}{12}mL^2$



Rod About the End: $I = \frac{1}{3}mL^2$

Table H. Heat and Thermal Physics Formulas and Equations

Temperature	$T_{°F} = 1.8(T_{°C}) + 32$ $T_K = T_{°C} + 273.15$
Heat	$Q = mC\Delta T$ $Q_{melt} = m\Delta H_{fus}$ $Q_{boil} = m\Delta H_{vap}$ $C_p - C_v = R$ $\Delta L = \alpha L_i \Delta T$ $\Delta V = \beta V_i \Delta T$ $P = \frac{Q}{t} = (\pm) kA\frac{\Delta T}{L}$ $P = \frac{Q}{t} = \varepsilon\sigma A T^4$ (*in this section, P = power*)
Thermo-dynamics	$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ $PV = nRT = Nk_BT$ $P\Delta V = nR\Delta T = Nk_B\Delta T$ $\Delta U = Q + W$ $U = \frac{3}{2}nRT \quad \Delta U = \frac{3}{2}nR\Delta T$ $W = -P\Delta V = -\int_{V_1}^{V_2} P\,dV$ $K_{(molecular)} = \frac{3}{2}RT$ $U = \frac{3}{2}nRT = \frac{3}{2}Nk_BT$ $\Delta U = \frac{3}{2}nR\Delta T = \frac{3}{2}Nk_B\Delta T$ $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_BT}{\mu}}$ $\Delta S = \frac{Q_{rev}}{T} \geq \frac{Q}{T}$ $A = U - TS$ $\Delta A = \Delta U - T\Delta S$ (*in this section, P = pressure*)

var. = name of quantity (unit)

- Δ = change in something (*E.g.,* Δx = change in x)
- $T = T_K$ = Kelvin temperature (K)
- $T_{°F}$ = Fahrenheit temperature (°F)
- $T_{°C}$ = Celsius temperature (°C)
- Q = heat (J, kJ)
- m = mass (kg)
- C = specific heat capacity* $\left(\frac{kJ}{kg\cdot°C}, \frac{J}{g\cdot°C}\right)$
- t = time (s)
- L = length (m)
- k = coëfficient of thermal conductivity* $\left(\frac{J}{m\cdot s\cdot°C}, \frac{W}{m\cdot°C}\right)$
- ε = emissivity* (*dimensionless*)
- H_{fus} = latent heat of fusion $\left(\frac{kJ}{kg}, \frac{J}{g}\right)$
- H_{vap} = heat of vaporization $\left(\frac{kJ}{kg}, \frac{J}{g}\right)$
- σ = Stefan-Boltzmann constant $= 5.67\times10^{-8}\ \frac{J}{m^2\cdot s\cdot K^4}$
- V = volume (m^3)
- α = linear coëfficient of thermal expansion* ($°C^{-1}$)
- β = volumetric coëfficient of thermal expansion* ($°C^{-1}$)
- P = power (W)
- P = pressure (Pa)
- n = number of moles (mol)
- N = number of molecules
- R = gas constant $= 8.31\ \frac{J}{mol\cdot K}$
- k_B = Boltzmann constant $= 1.38\times10^{-23}\ \frac{J}{K}$
- U = internal energy (J)
- W = work (J, N·m)
- v_{rms} = root mean square speed $\left(\frac{m}{s}\right)$
- μ = molecular mass* (kg)
- M = molar mass* $\left(\frac{kg}{mol}\right)$
- K = kinetic energy (J)
- Q_{rev} = "reversible" heat (J)
- S = entropy $\left(\frac{J}{K}\right)$
- A = Helmholtz free energy (J)

*characteristic property of a substance (to be looked up)

Table I. Thermodynamics Equation Map

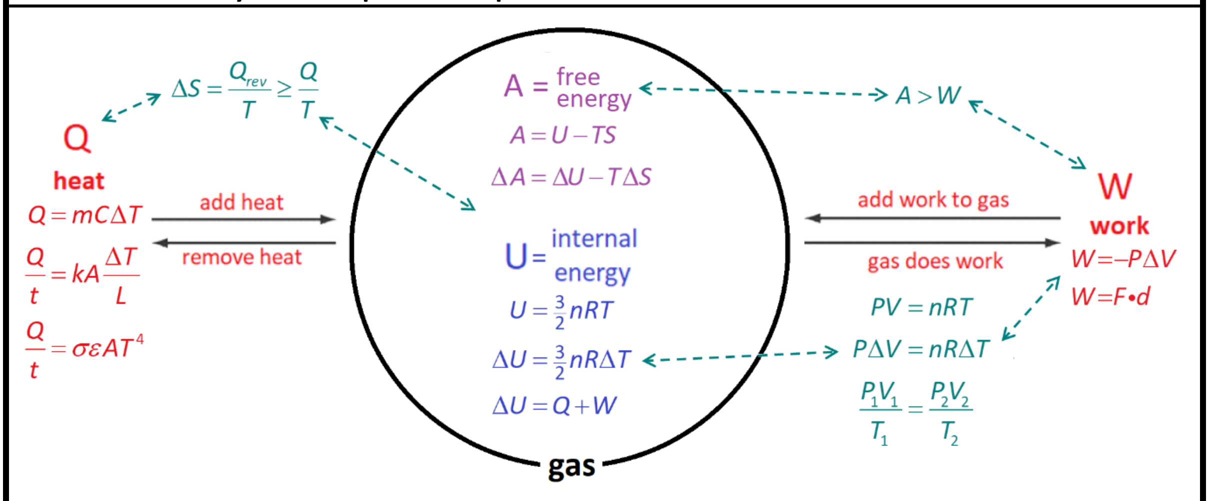


Table J. Pressure-Volume Curves and Equations

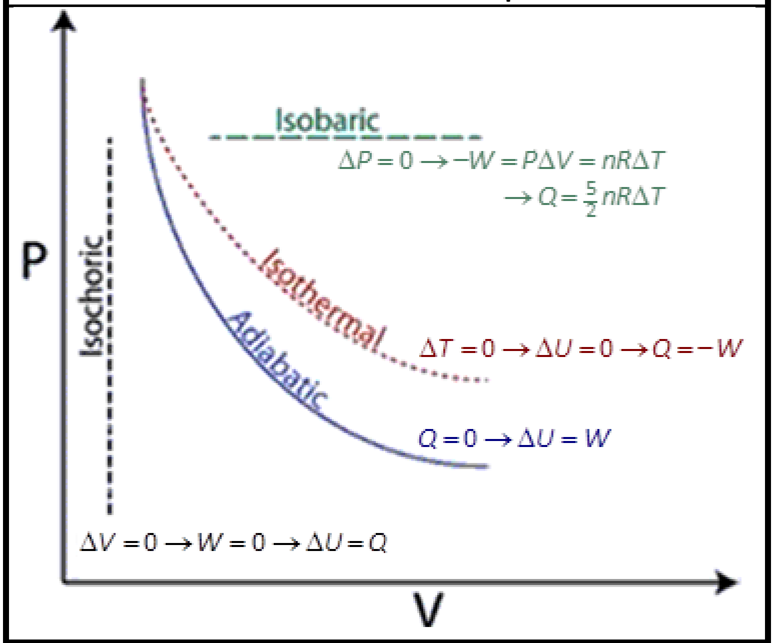


Table K. Thermal Properties of Selected Materials

Substance	Melting Point (°C)	Boiling Point (°C)	Heat of Fusion ΔH_{fus} $\left(\frac{kJ}{kg}, \frac{J}{g}\right)$	Heat of Vaporiz-ation ΔH_{vap} $\left(\frac{kJ}{kg}, \frac{J}{g}\right)$	Specific Heat Capacity C $\left(\frac{kJ}{kg \cdot °C}\right)$ at 25°C	Thermal Conduct-ivity k $\left(\frac{J}{m \cdot s \cdot °C}\right)$ at 25°C	Emissivity ε black body = 1	Coefficients of Expansion at 20°C: Linear α (°C^{-1})	Coefficients of Expansion at 20°C: Volumetric β (°C^{-1})
air (gas)	—	—	—	—	1.012	0.024	—	—	—
aluminum (solid)	659	2467	395	10460	0.897	250	0.09*	2.3×10^{-5}	6.9×10^{-5}
ammonia (gas)	−75	−33.3	339	1369	4.7	0.024	—	—	—
argon (gas)	−189	−186	29.5	161	0.520	0.016	—	—	—
carbon dioxide (gas)	−78		574		0.839	0.0146	—	—	—
copper (solid)	1086	1187	134	5063	0.385	401	0.03*	1.7×10^{-5}	5.1×10^{-5}
brass (solid)	—	—	—	—	0.380	120	0.03*	1.9×10^{-5}	5.6×10^{-5}
diamond (solid)	3550	4827	10 000	30 000	0.509	2200	—	1×10^{-6}	3×10^{-6}
ethanol (liquid)	−117	78	104	858	2.44	0.171	—	2.5×10^{-4}	7.5×10^{-4}
glass (solid)	—	—	—	—	0.84	0.96–1.05	0.92	8.5×10^{-6}	2.55×10^{-5}
gold (solid)	1063	2660	64.4	1577	0.129	310	0.025*	1.4×10^{-5}	4.2×10^{-5}
granite (solid)	1240	—	—	—	0.790	1.7–4.0	0.96	—	—
helium (gas)	—	−269	—	21	5.193	0.142	—	—	—
hydrogen (gas)	−259	−253	58.6	452	14.30	0.168	—	—	—
iron (solid)	1535	2750	289	6360	0.450	80	0.31	1.18×10^{-5}	3.33×10^{-5}
lead (solid)	327	1750	24.7	870	0.160	35	0.06	2.9×10^{-5}	8.7×10^{-5}
mercury (liquid)	−39	357	11.3	293	0.140	8	—	6.1×10^{-5}	1.82×10^{-4}
paraffin wax (solid)	46–68	~300	~210	—	2.5	0.25	—	—	—
silver (solid)	962	2212	111	2360	0.233	429	0.025*	1.8×10^{-5}	5.4×10^{-5}
zinc (solid)	420	906	112	1760	0.387	120	0.05*	$\sim 3\times10^{-5}$	8.9×10^{-5}
steam (gas) @ 100°C			—	2260	2.080	0.016	—	—	—
water (liq.) @ 25°C	0	100	334		4.181	0.58	0.95	6.9×10^{-5}	2.07×10^{-4}
ice (solid) @ −10°C				—	2.11	2.18	0.97	—	—

*polished surface

Table L. Electricity Formulas & Equations

Electrostatic Charges & Electric Fields	$\vec{F}_e = \frac{kq_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}$ $\vec{E} = \frac{\vec{F}_e}{q} = \frac{Q}{\varepsilon_o A}$ $\quad \vec{E} = \frac{kq}{r^2} = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{r^2} = \frac{\Delta V}{\Delta r}$ $W = q\vec{E}\bullet\vec{d} = qEd_{\parallel} = qEd\cos\theta$ $\Delta V = \frac{W}{q} = \vec{E}\bullet\vec{d} = Ed_{\parallel} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$ $\Delta U_E = q\Delta V \qquad U_E = \frac{kq_1q_2}{r}$
Circuits and Electrical Components	$\Delta V = IR \qquad I = \frac{\Delta Q}{\Delta t} = \frac{\Delta V}{R}$ $\mathcal{E} = IR$ $P = I\Delta V = I^2R = \frac{(\Delta V)^2}{R}$ $W = Pt = I\Delta Vt$ $R = \frac{\rho L}{A}$ $C = \kappa\varepsilon_o\frac{A}{d}$ $Q = C\Delta V$ $U_{capacitor} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$ $P_{total} = P_1 + P_2 + P_3 + \ldots = \sum P_i$ $U_{total} = U_1 + U_2 + U_3 + \ldots = \sum U_i$
Series Circuits (or Series Sections of Circuits)	$I_{total} = I_1 = I_2 = I_3 = \ldots$ $\Delta V_{total} = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots = \sum \Delta V_i$ $R_{equiv.} = R_1 + R_2 + R_3 + \ldots = \sum R_i$ $Q_{total} = Q_1 = Q_2 = Q_3 = \ldots$ $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots = \sum \frac{1}{C_i}$
Parallel Circuits (or Parallel Sections of Circuits)	$I_{total} = I_1 + I_2 + I_3 + \ldots = \sum I_i$ $\Delta V_{total} = \Delta V_1 = \Delta V_2 = \Delta V_3 = \ldots$ $\frac{1}{R_{equiv.}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots = \sum \frac{1}{R_i}$ $Q_{total} = Q_1 + Q_2 + Q_3 + \ldots = \sum Q_i$ $C_{total} = C_1 + C_2 + C_3 + \ldots = \sum C_i$
Resistor-Capacitor (RC) Circuits	charging: $\frac{I}{I_o} = e^{-t/RC}$ charging: $\frac{Q}{Q_{max}} = 1 - e^{-t/RC}$ discharging: $\frac{I}{I_o} = \frac{V}{V_o} = \frac{Q}{Q_{max}} = e^{-t/RC}$

var. = name of quantity (unit)

Δ = change in something. (*E.g.,* Δx = change in x)
$\vec{F}_e$ = force due to electric field (N)
ε_o = electric permittivity of a vacuum
$= 8.85\times10^{-12}\ \frac{\text{A}^2\cdot\text{s}^4}{\text{kg}\cdot\text{m}^3}$
k = electrostatic constant
$= \frac{1}{4\pi\varepsilon_o} = 9.0\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$
q = point charge (C)
Q = charge (C)
$\vec{E}$ = electric field $\left(\frac{\text{N}}{\text{C}}, \frac{\text{V}}{\text{m}}\right)$
V = electric potential (V)
ΔV = voltage = electric potential difference (V)
$\mathcal{E}$ = emf = electromotive force (V)
W = work (J, N·m)
$\kappa = \varepsilon_r$ = relative permittivity* (*dimensionless*)
d = distance (m)
r = radius (m)
$\vec{I}$ = current (A)
t = time (s)
R = resistance (Ω)
P = power (W)
ρ = resistivity ($\Omega\cdot$m)
L = length (m)
A = cross-sectional area (m^2)
C = capacitance (F)
U = potential energy (J)
π = pi (mathematical constant)
= 3.14159 26535 89793...
e = Euler's number (mathematical constant)
= 2.78182 81812 84590...

*characteristic property of a substance (to be looked up)

Table M. Electricity & Magnetism Formulas & Equations

Magnetism and Electro-magnetism	$\vec{F}_M = q(\vec{v}\times\vec{B})$ $F_M = qvB\sin\theta$ $\vec{F}_M = \ell(\vec{I}\times\vec{B})$ $F_M = \ell\, I\, B\sin\theta$ $\Delta V = \ell(\vec{v}\times\vec{B})$ $\Delta V = \ell vB\sin\theta$ $B = \dfrac{\mu_o\, I}{2\pi\, r}$ $\Phi_B = \vec{B}\bullet\vec{A} = BA\cos\theta$ $\mathcal{E} = \dfrac{\Delta\Phi_B}{\Delta t} = BLv$	$var.$ = name of quantity (unit) Δ = change in something. (*E.g.,* Δx = change in x) $\vec{F}_e$ = force due to electric field (N) $\vec{v}$ = velocity (of moving charge or wire) $\left(\frac{\text{m}}{\text{s}}\right)$ q = point charge (C) ΔV = voltage = electric potential difference (V) $\mathcal{E}$ = emf = electromotive force (V) r = radius (m) = distance from wire $\vec{I}$ = current (A) L = length (m) t = time (s) A = cross-sectional area (m^2) $\vec{B}$ = magnetic field (T) μ_o = magnetic permeability of a vacuum $= 4\pi\times10^{-7}\ \frac{\text{T}\bullet\text{m}}{\text{A}}$ Φ_B = magnetic flux ($\text{T}\cdot\text{m}^2$)
Electro-magnetic Induction	$\dfrac{\#turns_{in}}{\#turns_{out}} = \dfrac{V_{in}}{V_{out}} = \dfrac{I_{out}}{I_{in}}$ $P_{in} = P_{out}$	

Table N. Resistor Color Code

Color	Digit	Multiplier
black	0	$\times 10^0$
brown	1	$\times 10^1$
red	2	$\times 10^2$
orange	3	$\times 10^3$
yellow	4	$\times 10^4$
green	5	$\times 10^5$
blue	6	$\times 10^6$
violet	7	$\times 10^7$
gray	8	$\times 10^8$
white	9	$\times 10^9$
gold		± 5%
silver		± 10%

Table O. Symbols Used in Electrical Circuit Diagrams

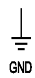
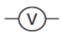
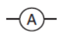
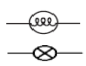
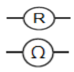
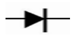
Component	Symbol	Component	Symbol
wire		battery	
switch		ground	
fuse		resistor	
voltmeter		variable resistor (rheostat, potentiometer, dimmer)	
ammeter		lamp (light bulb)	
ohmmeter		capacitor	
		diode	

Table P. Resistivities at 20°C

Conductors		Semiconductors		Insulators	
Substance	**Resistivity $(\Omega\cdot\text{m})$**	**Substance**	**Resistivity $(\Omega\cdot\text{m})$**	**Substance**	**Resistivity $(\Omega\cdot\text{m})$**
silver	1.59×10^{-8}	germanium	0.001 to 0.5	deionized water	1.8×10^{5}
copper	1.72×10^{-8}	silicon	0.1 to 60	glass	1×10^{9} to 1×10^{13}
gold	2.44×10^{-8}	sea water	0.2	rubber, hard	1×10^{13} to 1×10^{13}
aluminum	2.82×10^{-8}	drinking water	20 to 2 000	paraffin (wax)	1×10^{13} to 1×10^{17}
tungsten	5.60×10^{-8}			air	1.3×10^{16} to 3.3×10^{16}
iron	9.71×10^{-8}				
nichrome	1.50×10^{-6}			quartz, fused	7.5×10^{17}
graphite	3×10^{-5} to 6×10^{-4}				

Table Q. Waves & Optics Formulas & Equations

Waves	$v = \lambda f$ $f = \frac{1}{T}$ $v_{\text{wave on a string}} = \sqrt{\frac{F_T}{\mu}}$ $f_{\text{doppler shifted}} = f\left(\frac{\vec{\boldsymbol{v}}_{\text{wave}} + \vec{\boldsymbol{v}}_{\text{detector}}}{\vec{\boldsymbol{v}}_{\text{wave}} + \vec{\boldsymbol{v}}_{\text{source}}}\right)$ $x = A\cos(2\pi f t + \phi)$
Reflection, Refraction & Diffraction	$\theta_i = \theta_r$ $n = \frac{c}{v}$ $n_1 \sin\theta_1 = n_2 \sin\theta_2$ $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ $\frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$ $\Delta L = m\lambda = d\sin\theta$
Mirrors & Lenses	$f = \frac{r_c}{2}$ $\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$ $M = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$

var. = name of quantity (unit)

- $\Delta =$ change in something (*E.g.,* Δx = change in x)
- $v =$ velocity of wave $\left(\frac{\text{m}}{\text{s}}\right)$
- $\vec{\boldsymbol{v}} =$ velocity of source or detector $\left(\frac{\text{m}}{\text{s}}\right)$
- $f =$ frequency (Hz)
- $\lambda =$ wavelength (m)
- $A =$ amplitude (m)
- $x =$ position (m)
- $T =$ period (of time) (s)
- $F_T =$ tension (force) on string (N)
- $\mu =$ elastic modulus of string $\left(\frac{\text{kg}}{\text{m}}\right)$
- $\theta =$ angle (°, rad)
- $\phi =$ phase offset (°, rad)
- $\theta_i =$ angle of incidence (°, rad)
- $\theta_r =$ angle of reflection (°, rad)
- $\theta_c =$ critical angle (°, rad)
- $n =$ index of refraction* (*dimensionless*)
- $c =$ speed of light in a vacuum $= 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$
- $f = s_f = d_f =$ distance to focus of mirror/lens (m)
- $r_c =$ radius of curvature of spherical mirror (m)
- $s_i = d_i =$ distance from mirror/lens to image (m)
- $s_o = d_o =$ distance from mirror/lens to object (m)
- $h_i =$ height of image (m)
- $h_o =$ height of object (m)
- $M =$ magnification (*dimensionless*)
- $d =$ separation (m)
- $L =$ distance from the opening (m)
- $m =$ an integer

*characteristic property of a substance (to be looked up)

Table R. Absolute Indices of Refraction

Measured using $f = 5.89 \times 10^{14}$ Hz (yellow light) at 20 °C unless otherwise specified

Substance	Index of Refraction	Substance	Index of Refraction
air (0 °C and 1 atm)	1.000293	silica (quartz), fused	1.459
ice (0 °C)	1.309	Plexiglas	1.488
water	1.3330	Lucite	1.495
ethyl alcohol	1.36	glass, borosilicate (Pyrex)	1.474
human eye, cornea	1.38	glass, crown	1.50–1.54
human eye, lens	1.41	glass, flint	1.569–1.805
safflower oil	1.466	sodium chloride, solid	1.516
corn oil	1.47	PET (#1 plastic)	1.575
glycerol	1.473	zircon	1.777–1.987
honey	1.484–1.504	cubic zirconia	2.173–2.21
silicone oil	1.52	diamond	2.417
carbon disulfide	1.628	silicon	3.96

Figure S. The Electromagnetic Spectrum

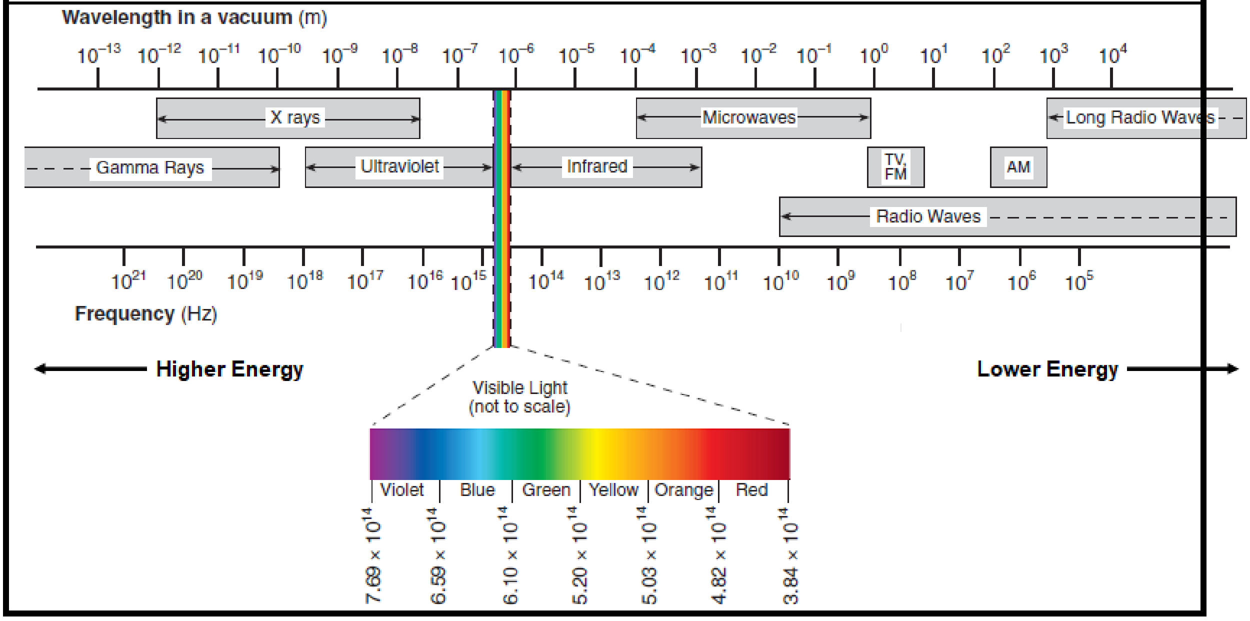


Table T. Planetary Data

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Distance from Sun (m)	5.79×10^{10}	1.08×10^{11}	1.50×10^{11}	2.28×10^{11}	7.79×10^{11}	1.43×10^{12}	2.87×10^{12}	4.52×10^{12}	5.91×10^{12}
Radius (m)	2.44×10^{6}	6.05×10^{6}	6.38×10^{6}	3.40×10^{6}	7.15×10^{7}	6.03×10^{7}	2.56×10^{7}	2.48×10^{7}	1.19×10^{6}
Mass (kg)	3.30×10^{23}	4.87×10^{24}	5.97×10^{24}	6.42×10^{23}	1.90×10^{27}	5.68×10^{26}	8.68×10^{25}	1.02×10^{26}	1.30×10^{22}
Density $\left(\frac{kg}{m^3}\right)$	5429	5243	5514	3934	1326	687	1270	1638	1850
Orbit (years)	0.24	0.61	1.00	1.88	11.8	29	84	164	248
Rotation Period (hours)	1408	−5833	23.9	24.6	9.9	10.7	−17.2	16.1	−153.3
Tilt of axis	0.034°	177.4°	23.4°	25.2°	3.1°	26.7°	97.8°	28.3°	122.5°
# of observed satellites	0	0	1	2	92	83	27	14	5
Mean temp. (°C)	167	464	15	−65	−110	−140	−195	−200	−225
Global magnetic field	Yes	No	Yes	No	Yes	Yes	Yes	Yes	Yes

Data from NASA Planetary Fact Sheet, https://nssdc.gsfc.nasa.gov/planetary/factsheet/ last updated 11 February 2023.

Table U. Sun & Moon Data

Radius of the sun (m)	6.96×10^{8}
Mass of the sun (kg)	1.99×10^{30}
Radius of the moon (m)	1.74×10^{6}
Mass of the moon (kg)	7.35×10^{22}
Distance of moon from Earth (m)	3.84×10^{8}

Table V. Fluids Formulas and Equations

Fluids	$\rho = \frac{m}{V}$ $P = \frac{F}{A}$ $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ $P_{\text{hydrostatic}} = P_H = \rho g h$ $F_B = \rho V_d g$ $P_{\text{dynamic}} = P_D = \frac{1}{2}\rho v^2$ $A_1 v_1 = A_2 v_2$ $P_{total} = P_{ext.} + P_H + P_D$ $P_1 + P_{H,1} + P_{D,1} = P_2 + P_{H,2} + P_{D,2}$ $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$	*var.* = name of quantity (unit) Δ = change in something. (*E.g.*, Δx = change in *x*) ρ = density $\left(\frac{\text{kg}}{\text{m}^3}\right)$ m = mass (kg) V = volume (m^3) P = presure (Pa) g = gravitational field $= 9.8\,\frac{\text{N}}{\text{kg}} \approx 10\,\frac{\text{N}}{\text{kg}}$ h = height or depth (m) A = area (m^2) v = velocity (of fluid) $\left(\frac{\text{m}}{\text{s}}\right)$ F = force (N) *characteristic property of a substance (to be looked up)

Table W. Properties of Water and Air

Temp. (°C)	Water: Density $\left(\frac{\text{kg}}{\text{m}^3}\right)$	Water: Speed of Sound $\left(\frac{\text{m}}{\text{s}}\right)$	Water: Vapor Pressure (Pa)	Air: Density $\left(\frac{\text{kg}}{\text{m}^3}\right)$	Air: Speed of Sound $\left(\frac{\text{m}}{\text{s}}\right)$
0	999.78	1 403	611.73	1.288	331.30
5	999.94	1 427	872.60	1.265	334.32
10	999.69	1 447	1 228.1	1.243	337.31
20	998.19	1 481	2 338.8	1.200	343.22
25	997.02	1 496	3 169.1	1.180	346.13
30	995.61	1 507	4 245.5	1.161	349.02
40	992.17	1 526	7 381.4	1.124	354.73
50	990.17	1 541	9 589.8	1.089	360.35
60	983.16	1 552	19 932	1.056	365.88
70	980.53	1 555	25 022	1.025	371.33
80	971.79	1 555	47 373	0.996	376.71
90	965.33	1 550	70 117	0.969	382.00
100	954.75	1 543	101 325	0.943	387.23

Table X. Atomic & Particle Physics (Modern Physics)

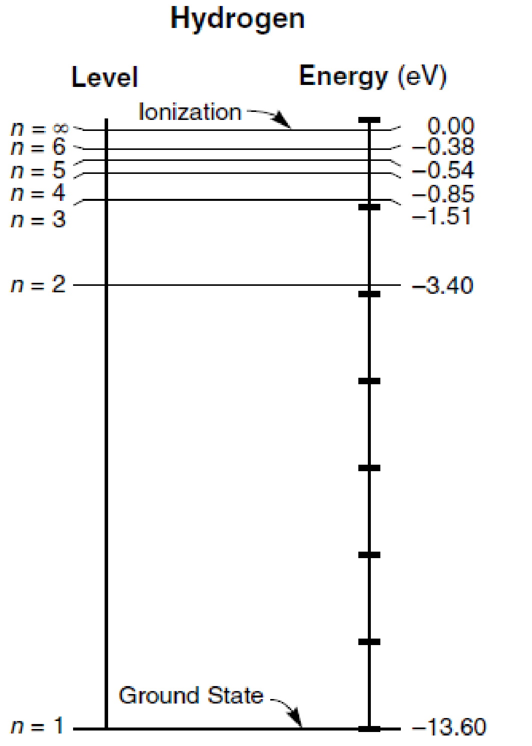
Energy	$E_{photon} = hf = \frac{hc}{\lambda} = pc = \hbar\omega$ $E_{k,\max} = hf - \phi$ $\lambda = \frac{h}{p}$ $E_{photon} = E_i - E_f$ $E^2 = (pc)^2 + (mc^2)^2$ $\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$
Special Relativity	$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ $\gamma = \frac{L_o}{L} = \frac{\Delta t'}{\Delta t} = \frac{m_{rel}}{m_o}$

var. = name of quantity (unit)

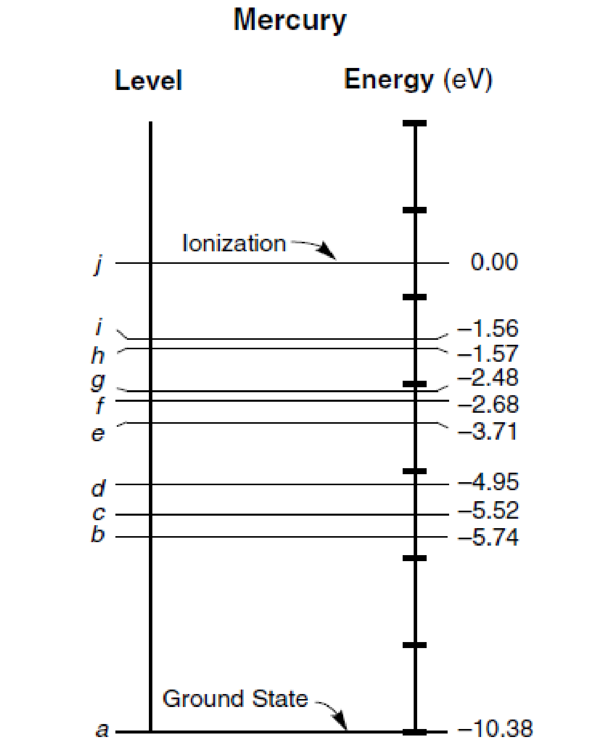
Δ = change in something. (*E.g.,* Δx = change in x)
E = energy (J)
h = Planck's constant $= 6.63\times 10^{-34}\ \mathrm{J\cdot s}$
$\hbar$ = reduced Planck's constant $= \frac{h}{2\pi} = 1.05\times 10^{-34}\ \mathrm{J\cdot s}$
f = frequency (Hz)
v = velocity $\left(\frac{\mathrm{m}}{\mathrm{s}}\right)$
c = speed of light $= 3.00\times 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$
λ = wavelength (m)
p = momentum (N·s)
m = mass (kg)
K = kinetic energy (J)
ϕ = work function* (J)
R_H = Rydberg constant $= 1.10\times 10^{7}\ \mathrm{m}^{-1}$
γ = Lorentz factor (dimensionless)
L = length in moving reference frame (m)
L_o = length in stationary reference frame (m)
$\Delta t'$ = time in stationary reference frame (s)
Δt = time in moving reference frame (s)
m_o = mass in stationary reference frame (kg)
m_{rel} = apparent mass in moving reference frame (kg)

*characteristic property of a substance (to be looked up)

Figure Y. Quantum Energy Levels



Energy Levels for the Hydrogen Atom



A Few Energy Levels for the Mercury Atom

Figure Z. Particle Sizes

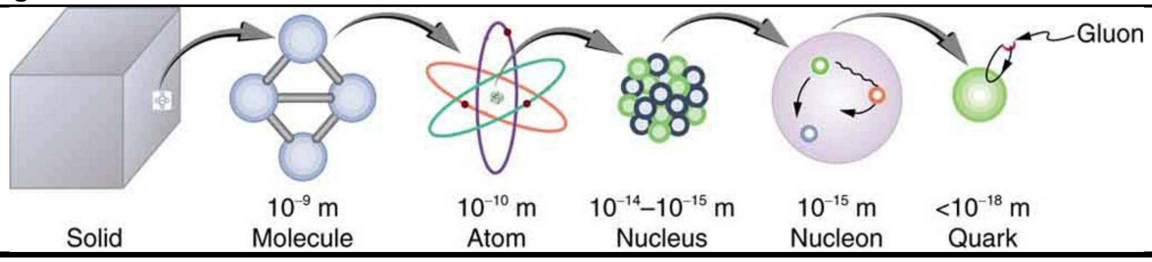


Figure AA. Classification of Matter

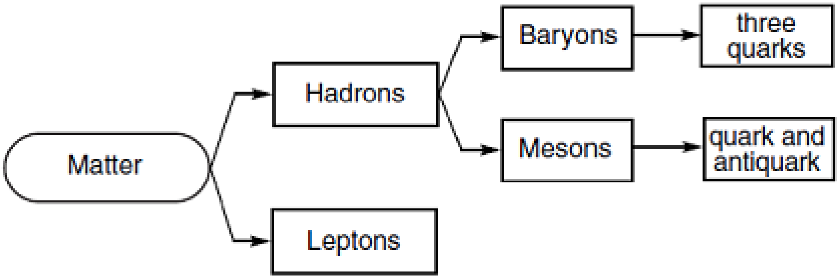


Table BB. The Standard Model of Elementary Particles

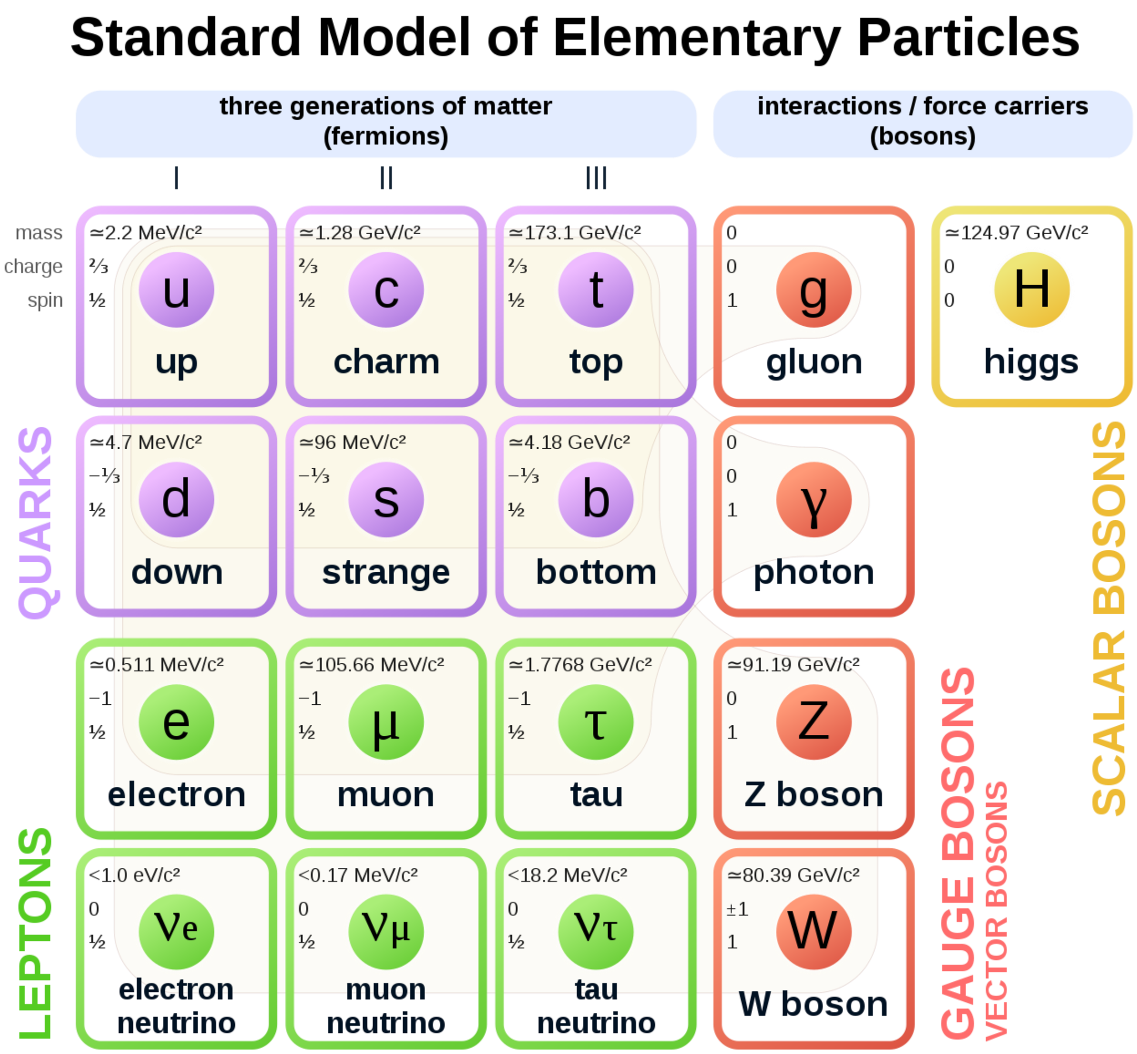


Figure CC. Periodic Table of the Elements

Period	1 I A	2 II A	3 III B	4 IV B	5 V B	6 VI B	7 VII B	8 VIII B	9 VIII B	10 VIII B	11 I B	12 II B	13 III A	14 IV A	15 V A	16 VI A	17 VII A	18 VIII A
1	1 H hydrogen 1.008																	2 He helium 4.003
2	3 Li lithium 6.968	4 Be beryllium 9.012											5 B boron 10.81	6 C carbon 12.01	7 N nitrogen 14.01	8 O oxygen 16.00	9 F fluorine 19.00	10 Ne neon 20.18
3	11 Na sodium 22.99	12 Mg magnesium 24.31											13 Al aluminum 26.98	14 Si silicon 28.09	15 P phosphorus 30.97	16 S sulfur 32.07	17 Cl chlorine 35.45	18 Ar argon 39.95
4	19 K potassium 39.10	20 Ca calcium 40.08	21 Sc scandium 44.96	22 Ti titanium 47.87	23 V vanadium 50.94	24 Cr chromium 52.00	25 Mn manganese 54.94	26 Fe iron 55.85	27 Co cobalt 58.93	28 Ni nickel 58.69	29 Cu copper 63.55	30 Zn zinc 65.38	31 Ga gallium 69.72	32 Ge germanium 72.63	33 As arsenic 74.92	34 Se selenium 78.97	35 Br bromine 79.90	36 Kr krypton 83.80
5	37 Rb rubidium 85.47	38 Sr strontium 87.62	39 Y yttrium 88.91	40 Zr zirconium 91.22	41 Nb niobium 92.91	42 Mo molybdenum 95.95	43 Tc technetium 98	44 Ru ruthenium 101.1	45 Rh rhodium 102.9	46 Pd palladium 106.4	47 Ag silver 107.9	48 Cd cadmium 112.4	49 In indium 114.8	50 Sn tin 118.7	51 Sb antimony 121.8	52 Te tellurium 127.6	53 I iodine 126.9	54 Xe xenon 131.3
6	55 Cs cesium 132.9	56 Ba barium 137.3	71 Lu lutetium 175.0	72 Hf hafnium 178.5	73 Ta tantalum 180.9	74 W tungsten 183.8	75 Re rhenium 186.2	76 Os osmium 190.2	77 Ir iridium 192.2	78 Pt platinum 195.1	79 Au gold 197.0	80 Hg mercury 200.6	81 Tl thallium 204.4	82 Pb lead 207.2	83 Bi bismuth 209.0	84 Po polonium 209	85 At astatine 210	86 Rn radon 222
7	87 Fr francium 223	88 Ra radium 226	103 Lr lawrencium 262	104 Rf rutherfordium 267	105 Db dubnium 268	106 Sg seaborgium 271	107 Bh bohrium 272	108 Hs hassium 270	109 Mt meitnerium 276	110 Ds darmstadtium 281	111 Rg roentgentium 280	112 Cn copernicum 285	113 Nh nihonium 284	114 Fl flerovium 289	115 Mc moscovium 288	116 Lv livermorium 293	117 Ts tennessine 292	118 Og oganesson 294

lanthanides (rare earth metals)	57 La lanthanum 138.9	58 Ce cerium 140.1	59 Pr praseodymium 140.9	60 Nd neodymium 144.2	61 Pm promethium 145	62 Sm samarium 150.4	63 Eu europium 152.0	64 Gd gadolinium 157.3	65 Tb terbium 158.9	66 Dy dysprosium 162.5	67 Ho holmium 164.9	68 Er erbium 167.3	69 Tm thulium 168.9	70 Yb ytterbium 173.1
actinides	89 Ac actinium 227	90 Th thorium 232.0	91 Pa protactinium 231.0	92 U uranium 238.0	93 Np neptunium 237	94 Pu plutonium 244	95 Am americium 243	96 Cm curium 247	97 Bk berkelium 247	98 Cf californium 251	99 Es einsteinium 252	100 Fm fermium 257	101 Md mendelevium 258	102 No nobelium 259

Table DD. Symbols Used in Nuclear Physics

Name	Notation	Symbol
alpha particle	${}^{4}_{2}\mathrm{He}$ or ${}^{4}_{2}\alpha$	□
beta particle (electron)	${}^{0}_{-1}e$ or ${}^{0}_{-1}\beta$	□ –
gamma radiation	${}^{0}_{0}\gamma$	□
neutron	${}^{1}_{0}n$	*n*
proton	${}^{1}_{1}\mathrm{H}$ or ${}^{1}_{1}p$	*p*
positron	${}^{0}_{+1}e$ or ${}^{0}_{+1}\beta$	□ +

Table FF. Constants Used in Nuclear Physics

Constant	Value
mass of an electron (m_e)	0.00055 amu
mass of a proton (m_p)	1.00728 amu
mass of a neutron (m_n)	1.00867 amu
Bequerel (Bq)	1 disintegration/second
Curie (Ci)	3.7 x 10^{10} Bq

Figure GG. Neutron/Proton Stability Band

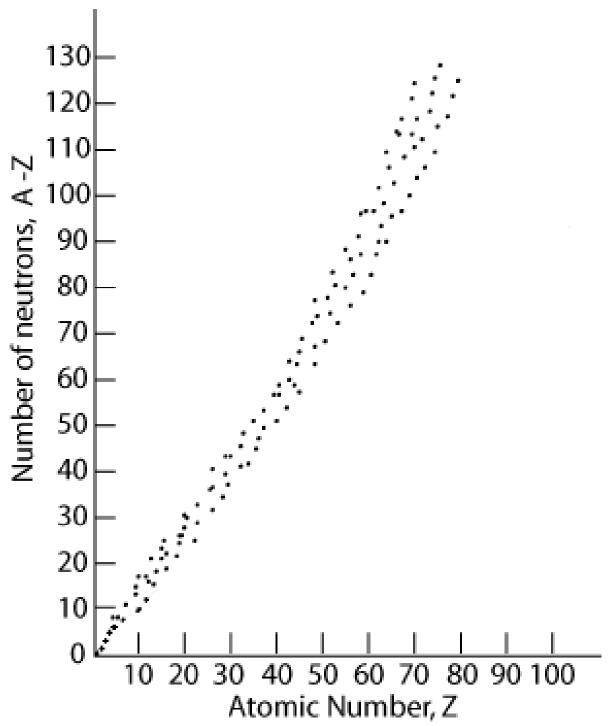


Table EE. Selected Radioisotopes

Nuclide	Half-Life	Decay Mode
${}^{3}\mathrm{H}$	12.26 y	□ –
${}^{14}\mathrm{C}$	5730 y	□ –
${}^{16}\mathrm{N}$	7.2 s	□ –
${}^{19}\mathrm{Ne}$	17.2 s	□ +
${}^{24}\mathrm{Na}$	15 h	□ –
${}^{27}\mathrm{Mg}$	9.5 min	□ –
${}^{32}\mathrm{P}$	14.3 d	□ –
${}^{36}\mathrm{Cl}$	3.01 × 10^{5} y	□ –
${}^{37}\mathrm{K}$	1.23 s	□ +
${}^{40}\mathrm{K}$	1.26 × 10^{9} y	□ +
${}^{42}\mathrm{K}$	12.4 h	□ –
${}^{37}\mathrm{Ca}$	0.175 s	□ –
${}^{51}\mathrm{Cr}$	27.7 d	□
${}^{53}\mathrm{Fe}$	8.51 min	□ –
${}^{59}\mathrm{Fe}$	46.3 d	□ –
${}^{60}\mathrm{Co}$	5.26 y	□ –
${}^{85}\mathrm{Kr}$	10.76 y	□ –
${}^{87}\mathrm{Rb}$	4.8 × 10^{10} y	□ –
${}^{90}\mathrm{Sr}$	28.1 y	□ –
${}^{99}\mathrm{Tc}$	2.13 x 10^{5} y	□ –
${}^{131}\mathrm{I}$	8.07 d	□ –
${}^{137}\mathrm{Cs}$	30.23 y	□ –
${}^{153}\mathrm{Sm}$	1.93 d	□ –
${}^{198}\mathrm{Au}$	2.69 d	□ –
${}^{222}\mathrm{Rn}$	3.82 d	□
${}^{220}\mathrm{Fr}$	27.5 s	□
${}^{226}\mathrm{Ra}$	1600 y	□
${}^{232}\mathrm{Th}$	1.4 x 10^{10} y	□
${}^{233}\mathrm{U}$	1.62 x 10^{5} y	□
${}^{235}\mathrm{U}$	7.1 x 10^{8} y	□
${}^{238}\mathrm{U}$	4.51 x 10^{9} y	□
${}^{239}\mathrm{Pu}$	2.44 x 10^{4} y	□
${}^{241}\mathrm{Am}$	432 y	□

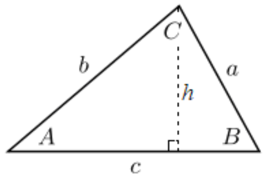
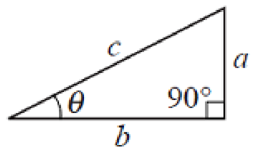
<table>
<tr><th colspan="3">Table HH. Mathematics Formulas</th></tr>
<tr><td>Scientific Notation</td><td colspan="2">$3\times10^{4}=3\times10\,000=30\,000 \qquad 2\times10^{-3}=2\times0.001=0.002$
$(3\times10^{4})(2\times10^{-3})=(3\cdot2)(10^{4}\cdot10^{-3})=6\times10^{4+(-3)}=6\times10^{1}=60$</td></tr>
<tr><td>Rounding
(to underlined place)</td><td colspan="2">$15\,\underline{3}54\rightarrow15\,\underline{4}00 \qquad 27\,\underline{2}49.99\rightarrow27\,\underline{2}00$
$0.037\,\underline{5}00\rightarrow0.037\underline{5}$</td></tr>
<tr><td>Algebra with Fractions</td><td colspan="2">$\frac{a}{b}+\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{d}+\frac{c}{d}\cdot\frac{b}{b}=\frac{ad+cb}{bd}$
$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd} \qquad \frac{a}{b/c}=a\cdot\frac{c}{b}$
$\frac{a}{x}=b\rightarrow\cancel{x}\cdot\frac{a}{\cancel{x}}=b\cdot x\rightarrow a=bx\rightarrow\frac{a}{b}=\frac{bx}{\cancel{b}}\rightarrow\frac{a}{b}=x$</td></tr>
<tr><td>Quadratic Equation</td><td colspan="2">$ax^{2}+bx+c=0 \quad\rightarrow\quad x=\frac{-b\pm\sqrt{b^{2}-4ac}}{2a}$</td></tr>
<tr><td>All Triangles</td><td>$A=\tfrac{1}{2}bh$
$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
$c^{2}=a^{2}+b^{2}-2ab\cos C$</td><td></td></tr>
<tr><td>Right Triangles</td><td>$c^{2}=a^{2}+b^{2}$
$\sin\theta=\frac{a}{c}=\frac{\text{opposite}}{\text{hypotenuse}}$
$\cos\theta=\frac{b}{c}=\frac{\text{adjacent}}{\text{hypotenuse}}$
$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{a}{b}=\frac{\text{opposite}}{\text{adjacent}}$
$b=c\cos\theta$
$a=c\sin\theta$</td><td></td></tr>
<tr><td>Rectangles, Parallelograms and Trapezoids</td><td>$A=\bar{b}h$</td><td rowspan="5">$a,b,c=$ length of a side of a triangle
$\theta=$ angle
$A=$ area
$C=$ circumference
$S=$ surface area
$V=$ volume
$b=$ base
$\bar{b}=$ average base $=\frac{b_1+b_2}{2}$
$h=$ height
$L=$ length
$w=$ width
$r=$ radius</td></tr>
<tr><td>Rectangular Solids</td><td>$V=Lwh$</td></tr>
<tr><td>Circles</td><td>$C=2\pi r$
$A=\pi r^{2}$</td></tr>
<tr><td>Cylinders</td><td>$S=2\pi rL+2\pi r^{2}=2\pi r(L+r)$
$V=\pi r^{2}L$</td></tr>
<tr><td>Spheres</td><td>$S=4\pi r^{2}$
$V=\frac{4}{3}\pi r^{3}$</td></tr>
</table>

Table II. Values of Trigonometric Functions

degree	radian	sine	cosine	tangent	degree	radian	sine	cosine	tangent
0°	0.000	0.000	1.000	0.000					
1°	0.017	0.017	1.000	0.017	46°	0.803	0.719	0.695	1.036
2°	0.035	0.035	0.999	0.035	47°	0.820	0.731	0.682	1.072
3°	0.052	0.052	0.999	0.052	48°	0.838	0.743	0.669	1.111
4°	0.070	0.070	0.998	0.070	49°	0.855	0.755	0.656	1.150
5°	0.087	0.087	0.996	0.087	50°	0.873	0.766	0.643	1.192
6°	0.105	0.105	0.995	0.105	51°	0.890	0.777	0.629	1.235
7°	0.122	0.122	0.993	0.123	52°	0.908	0.788	0.616	1.280
8°	0.140	0.139	0.990	0.141	53°	0.925	0.799	0.602	1.327
9°	0.157	0.156	0.988	0.158	54°	0.942	0.809	0.588	1.376
10°	0.175	0.174	0.985	0.176	55°	0.960	0.819	0.574	1.428
11°	0.192	0.191	0.982	0.194	56°	0.977	0.829	0.559	1.483
12°	0.209	0.208	0.978	0.213	57°	0.995	0.839	0.545	1.540
13°	0.227	0.225	0.974	0.231	58°	1.012	0.848	0.530	1.600
14°	0.244	0.242	0.970	0.249	59°	1.030	0.857	0.515	1.664
15°	0.262	0.259	0.966	0.268	60°	1.047	0.866	0.500	1.732
16°	0.279	0.276	0.961	0.287	61°	1.065	0.875	0.485	1.804
17°	0.297	0.292	0.956	0.306	62°	1.082	0.883	0.469	1.881
18°	0.314	0.309	0.951	0.325	63°	1.100	0.891	0.454	1.963
19°	0.332	0.326	0.946	0.344	64°	1.117	0.899	0.438	2.050
20°	0.349	0.342	0.940	0.364	65°	1.134	0.906	0.423	2.145
21°	0.367	0.358	0.934	0.384	66°	1.152	0.914	0.407	2.246
22°	0.384	0.375	0.927	0.404	67°	1.169	0.921	0.391	2.356
23°	0.401	0.391	0.921	0.424	68°	1.187	0.927	0.375	2.475
24°	0.419	0.407	0.914	0.445	69°	1.204	0.934	0.358	2.605
25°	0.436	0.423	0.906	0.466	70°	1.222	0.940	0.342	2.747
26°	0.454	0.438	0.899	0.488	71°	1.239	0.946	0.326	2.904
27°	0.471	0.454	0.891	0.510	72°	1.257	0.951	0.309	3.078
28°	0.489	0.469	0.883	0.532	73°	1.274	0.956	0.292	3.271
29°	0.506	0.485	0.875	0.554	74°	1.292	0.961	0.276	3.487
30°	0.524	0.500	0.866	0.577	75°	1.309	0.966	0.259	3.732
31°	0.541	0.515	0.857	0.601	76°	1.326	0.970	0.242	4.011
32°	0.559	0.530	0.848	0.625	77°	1.344	0.974	0.225	4.331
33°	0.576	0.545	0.839	0.649	78°	1.361	0.978	0.208	4.705
34°	0.593	0.559	0.829	0.675	79°	1.379	0.982	0.191	5.145
35°	0.611	0.574	0.819	0.700	80°	1.396	0.985	0.174	5.671
36°	0.628	0.588	0.809	0.727	81°	1.414	0.988	0.156	6.314
37°	0.646	0.602	0.799	0.754	82°	1.431	0.990	0.139	7.115
38°	0.663	0.616	0.788	0.781	83°	1.449	0.993	0.122	8.144
39°	0.681	0.629	0.777	0.810	84°	1.466	0.995	0.105	9.514
40°	0.698	0.643	0.766	0.839	85°	1.484	0.996	0.087	11.430
41°	0.716	0.656	0.755	0.869	86°	1.501	0.998	0.070	14.301
42°	0.733	0.669	0.743	0.900	87°	1.518	0.999	0.052	19.081
43°	0.750	0.682	0.731	0.933	88°	1.536	0.999	0.035	28.636
44°	0.768	0.695	0.719	0.966	89°	1.553	1.000	0.017	57.290
45°	0.785	0.707	0.707	1.000	90°	1.571	1.000	0.000	∞

Table JJ. Some Exact and Approximate Conversions

Length	1 cm	≈	width of a small paper clip		
	1 inch (in.)	≡	2.54 cm		
	length of a US dollar bill	=	6.14 in.	=	15.6 cm
	12 in.	≡	1 foot (ft.)	≈	30 cm
	3 ft.	≡	1 yard (yd.)	≈	1 m
	1 m	≡	0.3048 ft.	=	39.37 in.
	1 km	≈	0.6 mi.		
	5,280 ft.	≡	1 mile (mi.)	≈	1.6 km
Mass / Weight	1 small paper clip	≈	0.5 g		
	US 1¢ coin (1983–present)	=	2.5 g		
	US 5¢ coin	=	5 g		
	1 oz.	≈	30 g		
	one medium-sized apple	≈	1 N	≈	3.6 oz.
	1 pound (lb.)	≡	16 oz.	≈	454 g
	1 pound (lb.)	≈	4.45 N		
	1 ton	≡	2000 lb.	≈	0.9 tonne
	1 tonne	≡	1000 kg	≈	1.1 ton
Volume	1 pinch	≈	$^{1}/_{16}$ teaspoon (tsp.)		
	1 dash	≈	$^{1}/_{8}$ teaspoon (tsp.)		
	1 mL	≈	10 drops		
	1 tsp.	≈	5 mL	≈	60 drops
	3 tsp.	≡	1 tablespoon (Tbsp.)	≈	15 mL
	2 Tbsp.	≡	1 fluid ounce (fl. oz.)	≈	30 mL
	8 fl. oz.	≡	1 cup (C)	≈	250 mL
	16 fl. oz.	≡	1 U.S. pint (pt.)	≈	500 mL
	20 fl. oz.	≡	1 Imperial pint (UK)	≈	600 mL
	2 pt. (U.S.)	≡	1 U.S. quart (qt.)	≈	1 L
	4 qt. (U.S.)	≡	1 U.S. gallon (gal.)	≈	3.8 L
	4 qt. (UK) ≡ 5 qt. (U.S.)	≡	1 Imperial gal. (UK)	≈	4.7 L
Speed / Velocity	1 $^{\text{m}}/_{\text{s}}$	=	3.6 $^{\text{km}}/_{\text{h}}$	≈	2.24 $^{\text{mi.}}/_{\text{h}}$
	60 $^{\text{mi.}}/_{\text{h}}$	≈	100 $^{\text{km}}/_{\text{h}}$	≈	27 $^{\text{m}}/_{\text{s}}$
Energy	1 cal	≈	4.18 J		
	1 Calorie (food)	≡	1 kcal	≈	4.18 kJ
	1 BTU	≈	1.06 kJ		
Power	1 hp	≈	746 W		
	1 kW	≈	1.34 hp		
Temper-ature	0 K	≡	−273.15 °C	=	absolute zero
	0 °R	≡	−459.67 °F	=	absolute zero
	0 °F	≈	−18 °C ≡ 459.67 °R		
	32 °F	=	0 °C ≡ 273.15 K	=	water freezes
	70 °F	≈	21 °C	≈	room temperature
	212 °F	=	100 °C	=	water boils
Speed of light	300 000 000 $^{\text{m}}/_{\text{s}}$	≈	186 000 $^{\text{mi.}}/_{\text{s}}$	≈	1 $^{\text{ft.}}/_{\text{ns}}$

Table KK. Greek Alphabet

Α	α	alpha
Β	β	beta
Γ	γ	gamma
Δ	δ	delta
Ε	ε	epsilon
Ζ	ζ	zeta
Η	η	eta
Θ	θ	theta
Ι	ι	iota
Κ	κ	kappa
Λ	λ	lambda
Μ	μ	mu
Ν	ν	nu
Ξ	ξ	xi
Ο	ο	omicron
Π	π	pi
Ρ	ρ	rho
Σ	σ	sigma
Τ	τ	tau
Υ	υ	upsilon
Φ	φ	phi
Χ	χ	chi
Ψ	ψ	psi
Ω	ω	omega

Table LL. Decimal Equivalents

$\frac{1}{2}=0.5$	$\frac{1}{5}=0.2$
$\frac{1}{3}=0.33\overline{3}$	$\frac{2}{5}=0.4$
$\frac{2}{3}=0.66\overline{6}$	$\frac{3}{5}=0.6$
$\frac{1}{4}=0.25$	$\frac{4}{5}=0.8$
$\frac{3}{4}=0.75$	$\frac{1}{8}=0.125$
$\frac{1}{6}=0.166\overline{6}$	$\frac{3}{8}=0.375$
$\frac{5}{6}=0.833\overline{3}$	$\frac{5}{8}=0.625$
$\frac{1}{7}=0.\overline{142857}$	$\frac{7}{8}=0.875$
$\frac{2}{7}=0.\overline{285714}$	$\frac{1}{9}=0.11\overline{1}$
$\frac{3}{7}=0.\overline{428571}$	$\frac{2}{9}=0.22\overline{2}$
$\frac{4}{7}=0.\overline{571428}$	$\frac{4}{9}=0.44\overline{4}$
$\frac{5}{7}=0.\overline{714285}$	$\frac{5}{9}=0.55\overline{5}$
$\frac{6}{7}=0.\overline{857142}$	$\frac{7}{9}=0.77\overline{7}$
$\frac{1}{11}=0.09\overline{09}$	$\frac{8}{9}=0.88\overline{8}$
$\frac{2}{11}=0.18\overline{18}$	$\frac{1}{16}=0.0625$
$\frac{3}{11}=0.27\overline{27}$	$\frac{3}{16}=0.1875$
$\frac{4}{11}=0.36\overline{36}$	$\frac{5}{16}=0.3125$
$\frac{5}{11}=0.45\overline{45}$	$\frac{7}{16}=0.4375$
$\frac{6}{11}=0.54\overline{54}$	$\frac{9}{16}=0.5625$
$\frac{7}{11}=0.63\overline{63}$	$\frac{11}{16}=0.6875$
$\frac{8}{11}=0.72\overline{72}$	$\frac{13}{16}=0.8125$
$\frac{9}{11}=0.81\overline{81}$	$\frac{15}{16}=0.9375$
$\frac{10}{11}=0.90\overline{90}$	

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