Center of Mass

Unit: Forces in One Dimension

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 2.1.B, 2.1.B.1, 2.1.B.2, 2.1.B.3

Mastery Objective(s): (Students will be able to…)

- Find the center of mass of an object.

Success Criteria:

- Object balances at its center of mass.

Language Objectives:

- Explain why an object balances at its center of mass.

Tier 2 Vocabulary: center

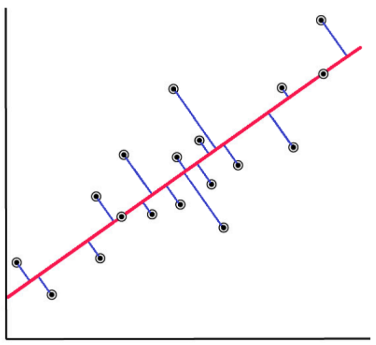
Labs, Activities & Demonstrations:

- Spin an object (*e.g.,* a hammer or drill team rifle) with its center of mass marked.

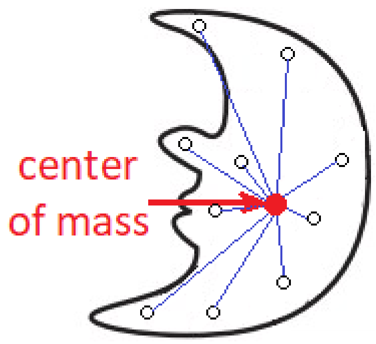
Notes:

center of mass: the point at which all of an object's mass could be placed without changing the results of any forces acting on the object.

You should recall from *Uncertainty & Error Analysis* on page 55 that a best-fit line is the line that minimizes the total accumulated distance from each point to the line. The center of mass is the same concept in three dimensions:



best-fit line



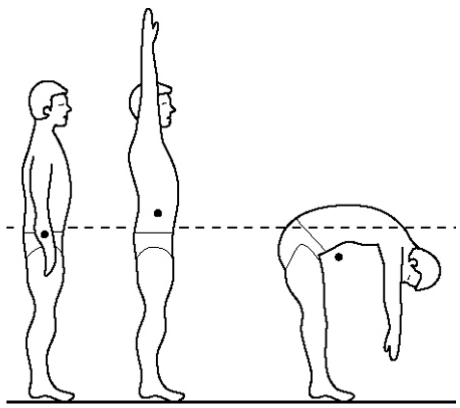
center of mass

Use this space for summary and/or additional notes:

For every physical object, its mass is distributed in some way throughout its volume. In most of the problems that you will see in this course, we can simplify the calculations by pretending that all of the mass of the object is at a single point.

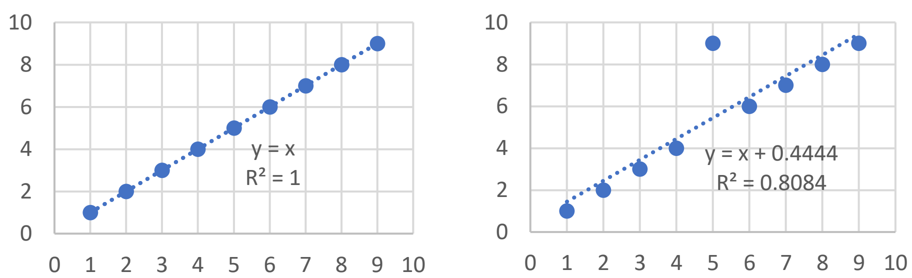
Things you need to understand:

- The center of mass of an object may be outside of the object itself:



- The center of mass of an object is a function of *how far away* each infinitesimal part of the object is from its center of mass. (Of course, it is also a function of the mass of each of those infinitesimal parts.) It is possible for an object to have more mass on one side of its center of mass than the other:

This would be analogous a best-fit line having more points on one side of it than the other. For example, consider these two graphs:



All of the points lie on the best-fit line.

All but one of the points are below the best-fit line.

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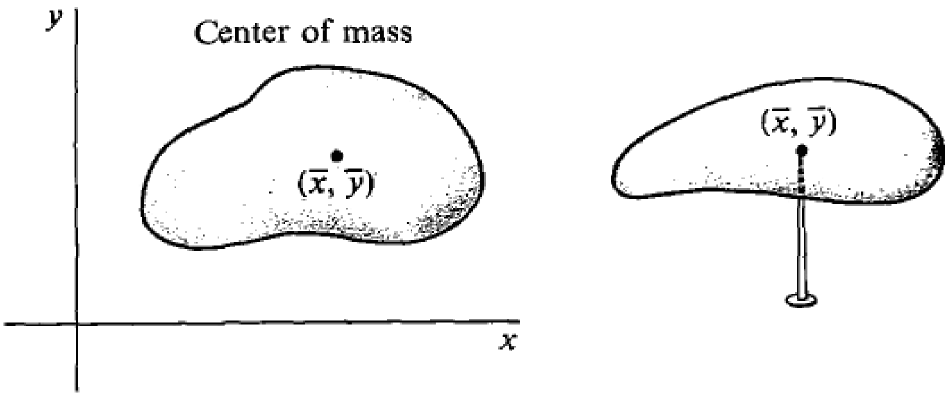
You can find the location of the center of mass of an object using the following formula:

$$\vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$

In this equation, the symbol Σ means "summation." When this symbol appears in a math equation, calculate the equation to the right of the symbol for each set of values, then add them up.

In this case, for each object (designated by a subscript), first multiply the mass (*m*) of each part of the system by its distance from some reference point ($\vec{r}$). Add all of those individual $m_i\vec{r}_i$ pieces and divide by the total mass. The resulting value of $\vec{r}$ is the distance from that reference point.

If each of the individual values of $\vec{r}_i$ has coordinates $(\vec{x}_i, \vec{y}_i)$, then the coordinates of the $\vec{r}$ that we calculate are the coordinates of the center of mass.



On the AP® Physics 1 exam, you will only need to perform this calculation in one dimension, which means the above equation becomes:

$$\vec{x}_{cm} = \frac{\sum_i m_i \vec{x}_i}{\sum_i m_i}$$

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Sample Problem:

Q: Two people sit at the ends of a massless 3.5 m long seesaw. One person has a mass of 59 kg, and the other has a mass of 71 kg. Where is their center of mass?

A: (Yes, there's no such thing as a massless seesaw. This is an idealization to make the problem easy to solve.)

In order to make this problem simple, let us place the 59-kg person at a distance of zero.

$$r_{cm} = \frac{\sum_i m_i r_i}{\sum_i m_i}$$

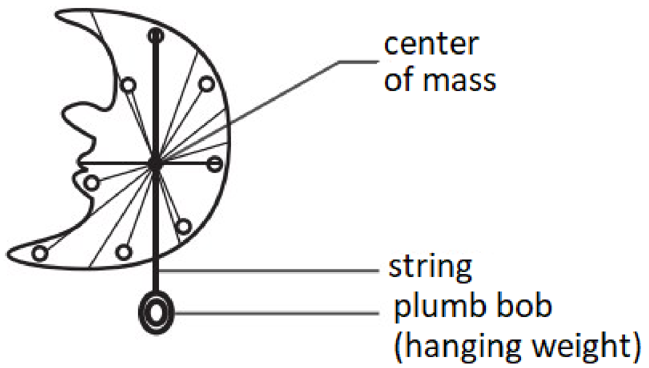
$$r_{cm} = \frac{(59)(0)+(71)(3.5)}{(59+71)}$$

$$r_{cm} = \frac{248.5}{130} = 1.91\,\text{m}$$

Their center of mass is 1.91 m away from the 59-kg person.

An object's center of mass is also the point at which the object will balance on a point. (Actually, because gravity is involved, the object balances because the torques around the center of mass cancel. This is discussed in detail in the *Torque* section, starting on page 373.) For this reason, the center of mass is often called the "center of gravity".

You can find the center of mass of a 2-dimensional object (such as a random shape cut from a piece of paper) by hanging it by a string from each of several different points and drawing a "plumb line" (a line straight downward) from each of those points. The location where those plumb lines intersect is the center of mass.



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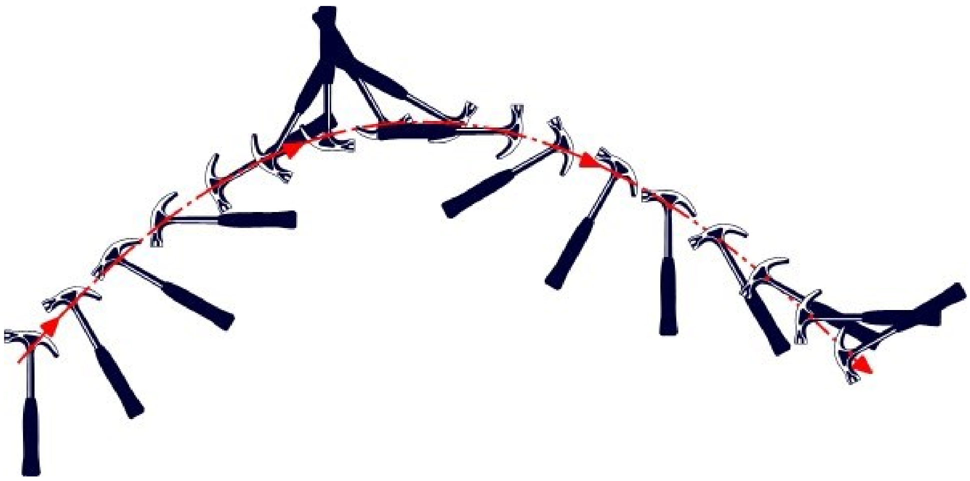
We can apply the concept of center of mass to Newton's Laws and systems:

- If Newton's First Law applies (all of the forces on the system are balanced and there is no net force), then the velocity of the center of mass of the system does not change, regardless of what is happening inside the system.
- If Newton's Second Law applies (there is at least one unbalanced force on the system, which means there is a net force), then the velocity of the center of mass changes, regardless of what is happening inside the system.
- Because of Newton's Third Law, forces that exist entirely within the system do not affect the motion of the center of mass of the system, because the action force and the reaction force both act within the system.

In order to illustrate the concept that "whatever is happening inside the system doesn't affect the motion of the center of mass", consider object that is rotating freely in space. The object will rotate about its center of mass.

If we throw a spinning hammer, its center of mass will move in the same manner as if we had thrown a ball, showing that the motion of the center of mass is not affected by the rotation of the object.



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