Tension

Unit: Forces in One Dimension

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 2.3.A, 2.3.A.1, 2.3.A.2, 2.3.A.3, 2.3.A.3.i, 2.3.A.3.ii, 2.3.A.3.iii, 2.3.A.3.iv,

Mastery Objective(s): (Students will be able to…)

- Set up and solve problems involving pulleys and ropes under tension.

Success Criteria:

- Expressions involving tension and acceleration are correct including the sign (direction).
- Equations for all parts of the system are combined correctly algebraically.
- Algebra is correct and rounding to an appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how the sign of all of the forces in a pulley system relate to the direction that the system will move.

Tier 2 Vocabulary: pulley, tension

Labs, Activities & Demonstrations:

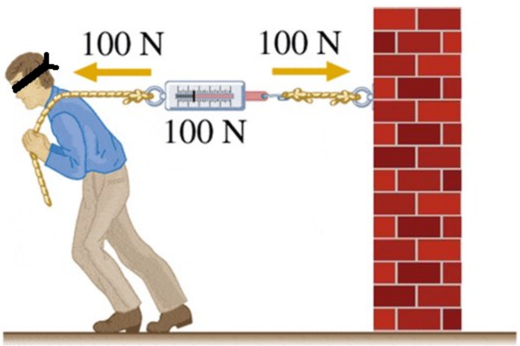
- Atwood machine

Notes:

tension ($\vec{F}_T$, $\vec{T}$): the pulling force on a rope, string, chain, cable, *etc.*

Tension is its own reaction force; tension always travels through the rope in both directions at once, and unless there are additional forces between one end of the rope and the other, the tension at every point along the rope is the same. The direction of tension is always along the rope.

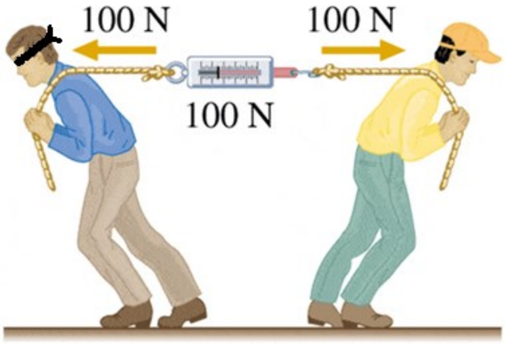
For example, in the following picture a blindfolded person pulls on a rope with a force of 100 N. The rope transmits the force to the scale, which transmits the force to the other rope and then to the wall. This causes a reaction force (also tension) of 100 N in the opposite direction.



The scale attached to the rope measures 100 N, because that is the amount of force (tension) that is stretching the spring in the scale.

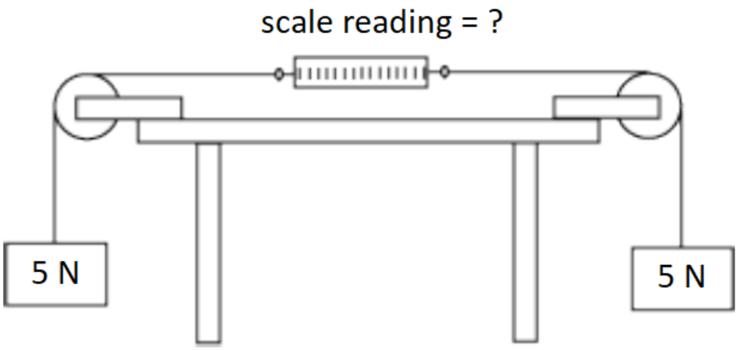
Use this space for summary and/or additional notes:

If we replace the brick wall with a person who is pulling with a force of 100 N, the blindfolded person has no idea whether the 100 N of resistance is coming from a brick wall or another person. Thus, the forces acting on the blindfolded person (and the scale) are the same.



Of course, the scale doesn't "know" either, so it still reads 100 N.

A popular demonstration in physics classrooms is to set up the equivalent situation, using a scale with hanging weights on both sides:



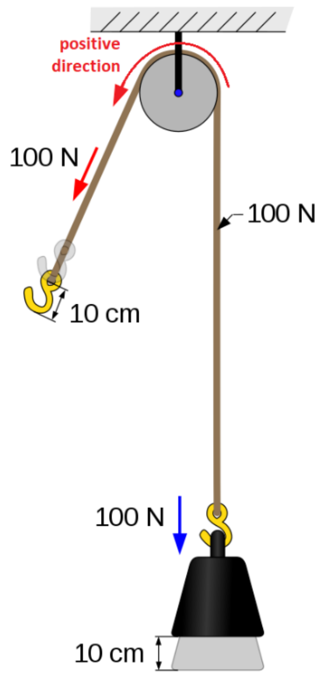
As you have undoubtedly realized, each rope pulls against the scale with a force of 5 N. The spring inside the scale pulls back with the same 5 N of force (in *both* directions), so the scale must read 5 N.

Use this space for summary and/or additional notes:

Pulleys

<u>pulley</u>: a wheel used to change the direction of tension on a rope

The tension remains the same in all parts of the rope.



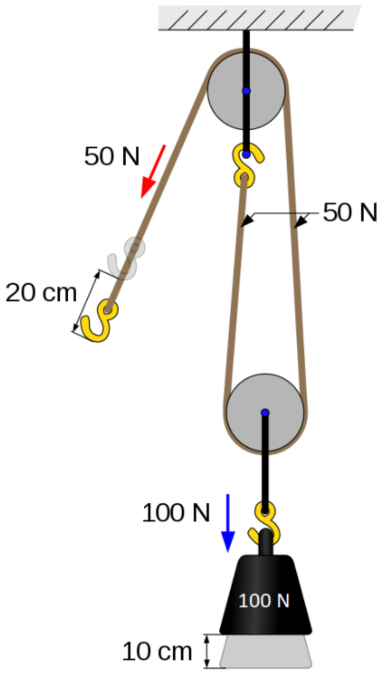
In the example at the right (with one pulley), it takes 100 N of force to lift a 100 N weight. The pulley changes the direction of the force, but the amount of force does not change. If the rope is pulled 10 cm, the weight is lifted by the same 10 cm.

Up to this point, we have chosen a single direction (left/right or up/down) to be the positive direction. With pulleys, we usually define the positive and negative directions to follow the rope. In this example, we would most likely choose the positive direction to be the direction that the rope is pulled. Instead of saying that positive is upward for the weight and downward for the hook, we would usually say that the positive direction is counter-clockwise (↺), because that is the direction that the pulley will turn.

CP1 & honors (not AP®)

Mechanical Advantage

If we place a second pulley just above the weight that we want to lift, two things will happen when we pull on the rope:



1. As we pull on the rope, there is less rope between the two pulleys. This means the lower pulley will move upward.

2. The rope going around the lower pulley will be lifting the 100 N weight from both sides. This means each side will support half of the weight (50 N). Therefore, the tension in every part of the rope is 50 N, which means it takes half as much force to lift the weight.

3. The length of rope that is pulled is divided between the two sections that go around the lower pulley. This means that pulling the rope 20 cm will raise the weight half as much (10 cm).

Use this space for summary and/or additional notes:

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Notice that when the force is cut in half, the length of rope is doubled. The double pulley is effectively trading force for distance. Later, in the *Introduction: Energy, Work* & Power unit starting on page 407, we will see that force times distance is work (change in energy). This means using half as much force but pulling the rope twice as much distance takes the same amount of energy to lift the weight.

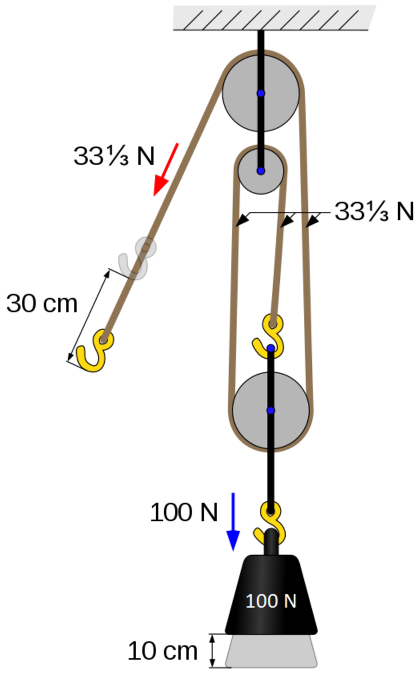
As you would expect, as we add more pulleys, the force needed is reduced and the distance increases. This reduction in force is called mechanical advantage.

mechanical advantage: the ratio of the force applied by a machine divided by the force needed to operate it.

The mechanical advantage of a pulley system is equal to the number of ropes supporting the hanging weight. It is therefore also equal to the number of pulleys.

The mechanical advantage of the above system is 2:1 (or just 2).

If we add a third pulley, we can see that there are now three sections of the rope that are lifting the 100 N weight. This means that each section is holding up ⅓ of the weight. This means that the tension in the rope is ⅓ of 100 N, or 33⅓ N, but we now need to pull three times as much rope to lift the weight the same distance.



A two-pulley system has a mechanical advantage of 2, because it applies twice as much force to the weight as you need to apply to the rope. Similarly, a 3-pulley system has a mechanical advantage of 3, and so on.

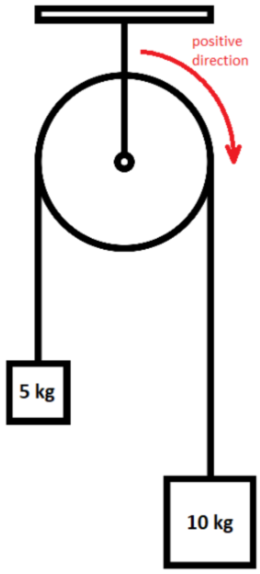
The mechanical advantage of any pulley system equals the number of ropes participating in the lifting.

block and tackle: a system of two or more pulleys (which therefore has a mechanical advantage of 2 or more) that is used for lifting heavy loads.

Use this space for summary and/or additional notes:

Atwood's Machine

Atwood's machine is named for the English mathematician George Atwood. The machine is a device with a single pulley in which one weight, which is pulled down by gravity, is used to lift a second weight. Atwood invented the machine in 1784 to verify Isaac Newton's equations of motion with constant acceleration.



To illustrate how Atwood's experiment works, consider the system to the right. To simplify the problem, we will assume that the rope and the pulley have negligible mass, and the pulley operates with negligible friction. Let us choose the positive direction as the direction that turns the pulley clockwise (↻). (We could have chosen either direction to be positive, but it makes intuitive sense to choose the direction that the system will move when we release the weights.)

The force on the mass on the right is its weight, which is $m\vec{g} = (10)(+10) = 100\,\text{N}$. (We use a positive value for $\vec{g}$ because gravity is attempting to pull this weight in the positive direction.)

The force on the mass on the left is $m\vec{g} = (5)(-10) = -50\,\text{N}$. (We use a negative value for $\vec{g}$ because gravity is attempting to pull this weight in the positive direction.)

The net force on the system is therefore $\vec{F}_{net} = \sum \vec{F} = 100 + (-50) = 50\,\text{N}$.

The masses are connected by a rope, which means both masses will accelerate together. The total mass is 15 kg.

Newton's Second Law says:

$$\vec{F}_{net} = \sum \vec{F} = m\vec{a}$$
$$+50 = 15\vec{a}$$
$$\vec{a} = \frac{50}{15} = +3.\bar{3}\,\tfrac{\text{m}}{\text{s}^2}$$

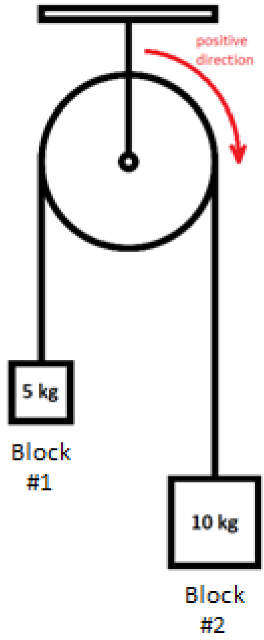
I.e., the system will accelerate at $3.3\,\frac{\text{m}}{\text{s}^2}$ in the positive direction (clockwise).

Atwood performed experiments with different masses and observed behavior that was consistent with both Newton's second law, and with Newton's equations of motion.

Notice that the solution to finding acceleration in a problem involving Atwood's machine is to simply find the net force, add up the total mass, and use $\vec{F}_{net} = m\vec{a}$.

Use this space for summary and/or additional notes:

An important feature of Newton's second law is that it can be applied to an <u>entire system</u>, or to <u>any component</u> of the system.



For the Atwood's machine pictured, we found that:

Entire system:

$$\vec{F}_{net} = m\vec{a}$$
$$+50\,\text{N} = (5\,\text{kg} + 10\,\text{kg})(3.\overline{3}\,\tfrac{\text{m}}{\text{s}^2})$$

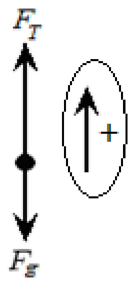
We can apply Newton's second law to each block separately:

$$\vec{F}_{1,net} = m_1\vec{a}$$
$$\vec{F}_{2,net} = m_2\vec{a}$$

Because the blocks are connected via the same rope, the acceleration is the same for both blocks.

This means that we can apply Newton's second law to either of the blocks to determine the tension in the rope:

Block #1:

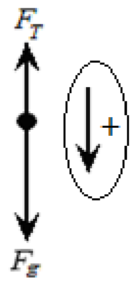


$$\vec{F}_{1,net} = (\vec{F}_T - \vec{F}_{g,1}) \qquad \vec{F}_{1,net} = m_1\vec{a}$$
$$(\vec{F}_T - m_1\vec{g}) = m_1\vec{a}$$
$$[\vec{F}_T - (5)(10)] = (5)(3.\overline{3})$$
$$\vec{F}_T - (-50) = 16.\overline{6}$$
$$\vec{F}_T = 66.\overline{6}\,\text{N}$$

Block #2: (same calculation; yields the same result)

We can do the same calculation for Block #2, with the same result for $\vec{F}_T$.

(Remember that we chose the positive direction to be the direction that the system moves. This means the positive direction is up for block #1, but down for block #2.)



$$\vec{F}_{2,net} = (\vec{F}_{g,2} - \vec{F}_T) \qquad \vec{F}_{2,net} = m_2\vec{a}$$
$$(m_2\vec{g} - \vec{F}_T) = m_2\vec{a}$$
$$[(10)(10) - \vec{F}_T] = (10)(3.\overline{3})$$
$$100 - \vec{F}_T = 33.\overline{3}$$
$$\vec{F}_T = 66.\overline{6}\,\text{N}$$

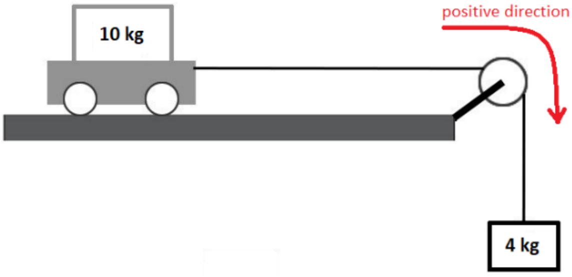
Q.E.D.

Notice that the tension $(66.\overline{6}\,\text{N})$ must be greater than the weight of the smaller block (50 N), and less than the weight of the larger block (100 N). (This should be obvious from the free-body diagrams.)

Use this space for summary and/or additional notes:

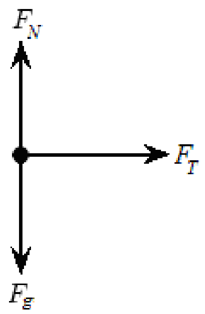
A variation of Atwood's machine is to have one of the masses on a horizontal table (possibly on a cart to reduce friction). This means that the net force is only the action of gravity on the hanging mass.

Consider the problem illustrated to the right. To simplify the problem, we will assume that the pulley has negligible mass, and that both the pulley and the cart are frictionless. The 10 kg for the mass on the left includes the mass of the cart.



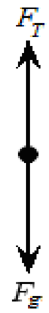
The forces on the two masses are:

FBD for the cart:



$\vec{F}_{1,net} = \vec{F}_T$

FBD for the hanging mass:



$\vec{F}_{2,net} = \vec{F}_g - \vec{F}_T$

Gravity and the normal force cancel for the cart. The tensions cancel because they are equal (it's the same rope) and are in opposite directions. This means that the only uncancelled force is the force of gravity on the 4 kg mass. This uncancelled force is the net force, which is $\vec{F}_{net} = mg = (4)(10) = 40\,\text{N}$.

The total mass is 10 + 4 = 14 kg.

Now that we have the net force and the total mass, we can find the acceleration using Newton's Second Law:

$$\vec{F}_{net} = m\vec{a}$$
$$40 = 14\vec{a}$$
$$\vec{a} = \frac{40}{14} = 2.86\,\tfrac{\text{m}}{\text{s}^2}$$

Use this space for summary and/or additional notes:

To find the tension, we can apply Newton's second law to the cart:

$$F_{net,cart} = F_T$$
$$m_{cart}a = F_T$$
$$(10)(2.86) = 28.6\,\text{N}$$

Again, we can get the same result by applying Newton's second law to the hanging mass:

$$F_{net,hang} = F_g - F_T$$
$$m_{hang}a = F_g - F_T$$
$$(4)(2.86) = (4)(10) - F_T$$
$$11.4 = 40 - F_T$$
$$F_T = 40 - 11.4 = 28.6\,\text{N}$$

Notice that the tension (28.6 N) must be less than the weight of the hanging block (40 N). (Again, this should be obvious from the free-body diagram for the hanging block.)

Alternative Approach

In most physics textbooks, the solution to Atwood's machine problems is presented as a system of equations. The strategy is:

- Draw a free-body diagram for each block.
- Apply Newton's 2nd Law to each block separately, giving $F_{net} = m_1 a$ for block 1 and $F_{net} = F_g - F_T = m_2 a$, which becomes $F_{net} = m_2 g - F_T = m_2 a$ for block 2.
- Set the two F_{net} equations equal to each other, eliminate one of F_T or a, and solve for the other.

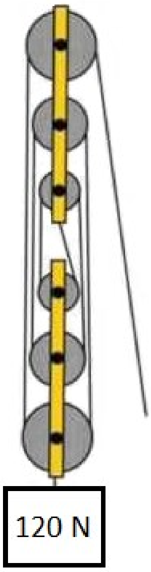
This is really just a different presentation of the same approach, but most students find it less intuitive.

Use this space for summary and/or additional notes:

Homework Problems

CP1 & honors (not AP®)

1. **(M)** For the pulley system shown at the right:



   a. **(M)** What is the mechanical advantage of the system?

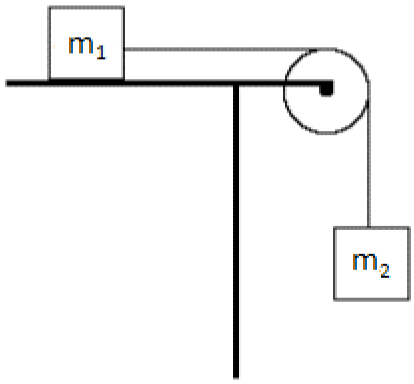
   b. **(M)** How much force needs to be applied to the rope in order to lift the hanging weight?

   c. **(M)** If the 120 N weight is to be lifted 0.5 m, how far will the rope need to be pulled?

Use this space for summary and/or additional notes:

honors & AP®

2. **(M – AP® & honors; A – CP1)** A block with a mass of m_1 sitting on a frictionless horizontal table is connected to a hanging block of mass m_2 by a string that passes over a pulley, as shown in the figure below.



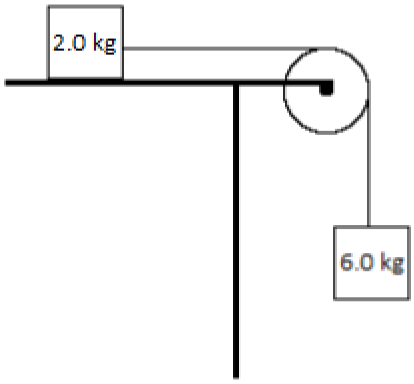
Assuming that friction, the mass of the string, and the mass of the pulley are negligible, derive expressions for the rate at which the blocks accelerate and the tension in the rope.

(*If you are not sure how to solve this problem, do #3 below and use the steps to guide your algebra.*)

Answer: $a = \dfrac{m_2 g}{m_1 + m_2}$; $F_T = \dfrac{m_1 m_2 g}{m_1 + m_2}$

Use this space for summary and/or additional notes:

3. **(S – AP® & honors; M – CP1)** A block with a mass of 2.0 kg sitting on a frictionless horizontal table is connected to a hanging block of mass 6.0 kg by a string that passes over a pulley, as shown in the figure below.

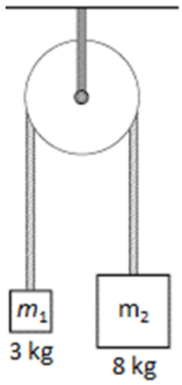


Assuming that friction, the mass of the string, and the mass of the pulley are negligible, at what rate do the blocks accelerate? What is the tension in the rope?

(<u>You must start with the equations in your Physics Reference Tables *and show all of the steps of GUESS.*</u> *You may not use the answer to question #2 above as a starting point unless you have already solved that problem.*)

Answer: $a = 7.5\,\frac{\text{m}}{\text{s}^2}$; $F_T = 15\,\text{N}$

4. **(M)** Two masses, m_1 = 3 kg and m_2 = 8 kg, are connected by an ideal (massless) rope over an ideal pulley (massless and frictionless).



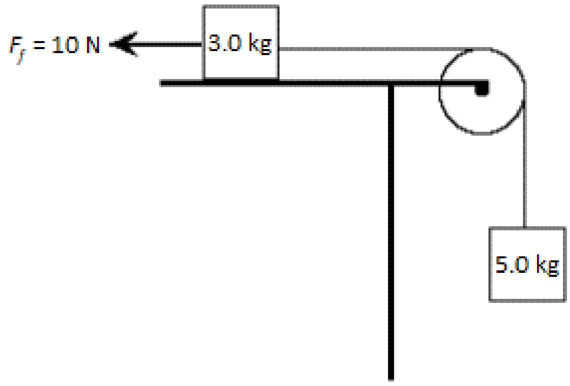
What is the acceleration of the system? What is the tension in the rope?

Answer: $a = 4.5\,\frac{\text{m}}{\text{s}^2}$; $F_T = 43.6\,\text{N}$

Use this space for summary and/or additional notes:

honors & AP®

5. **(S)** A block with a mass of 3.0 kg sitting on a horizontal table is connected to a hanging block of mass 5.0 kg by a string that passes over a pulley, as shown in the figure below. The force of friction between the upper block and the table is 10 N.



At what rate do the blocks accelerate? What is the tension in the rope?

Answer: $a = 5\,\frac{\text{m}}{\text{s}^2}$; $F_T = 25\,\text{N}$

Use this space for summary and/or additional notes: