Springs

Unit: Forces in One Dimension

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 2.8.A, 2.8.A.1, 2.8.A.2, 2.8.A.3

Mastery Objective(s): (Students will be able to…)

- Set up and solve problems involving springs.

Success Criteria:

- Expressions involving springs are correct including the sign (direction).
- Algebra is correct and rounding to an appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the direction of the force applied by a spring.

Tier 2 Vocabulary: spring

Notes:

spring: a device made of an elastic, but rigid material (usually metal) bent into a form (often a coil) that can return to its natural shape after being extended or compressed.

equilibrium position: the position of an object attached to a spring when there is no force on it.

closed coil spring (tension spring): a spring whose coils are touching when the spring is in its equilibrium position. A closed coil spring can be extended but cannot be compressed.

open coil spring (compression spring): a spring whose coils are not touching when the spring is in its equilibrium position. An open coil spring can be either extended or compressed. Unless otherwise specified, assume that all springs are open coil springs.

spring force (F_s): the force exerted by a spring as it attempts to return to its natural shape.

The spring force is a reaction force that is caused by the force that displaces the spring from its equilibrium position.

Use this space for summary and/or additional notes:

spring constant (k): the amount of force needed to extend or compress a spring a specific distance (measured in $\frac{N}{m}$).

The larger the spring constant, the more force is needed to extend or compress the spring. For example, a Slinky has a spring constant of about $0.5\frac{N}{m}$, while a heavy garage door spring might have a spring constant of $700\frac{N}{m}$ or more.

Note that ***the spring constant is specific to an individual spring***, not just the material that it is made of.

In English units, the spring constant is often called the "spring rate", expressed in $\frac{lbs.}{in.}$. $1\frac{lb.}{in.} \approx 175\frac{N}{m}$.

ideal spring: a spring that has negligible mass and that exerts a force proportional to its change in length.

For an ideal spring, the spring force is given by Hooke's law, named for the 17th-century British physicist Robert Hooke:

$$\vec{F}_s = -k\Delta\vec{x}$$

where:

- $\vec{F}_s$ = spring force (N)
- k = spring constant $(\frac{N}{m})$
- $\Delta\vec{x}$ = displacement of the spring (either extended or compressed) (m)

The negative sign in the equation is because the force is always in the ***opposite direction*** from the displacement, *i.e.,* the force is always back toward the equilibrium position of the object-spring system.

Sample Problem:

Q: A weight of 7 N is hung from a spring, causing the spring to stretch 0.25 m. What is the spring constant for this spring?

A: $\vec{F}_s = -k\Delta\vec{x}$

$$k = \frac{F_s}{\Delta x} = \frac{7}{0.25} = 28 \text{ N}$$

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Slinky™ Physics

A Slinky™ is a toy that is simply a spring with a very low spring constant (about $0.5\frac{N}{m}$). It was invented by American naval engineer Richard T. James in 1943.

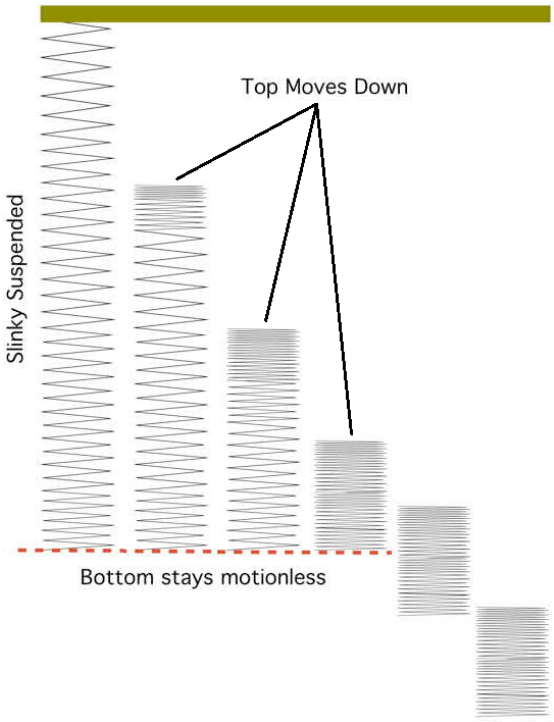
Slinky "Walking" Down Stairs

In order to make a Slinky "walk" down stairs, you need to stretch it and then place one end on a surface that is lower than the surface that the other end is resting on. The spring force will pull both ends of the slinky toward the equilibrium position. As long as the equilibrium position is below the upper surface, gravity will continue to stretch the lower portion, which means the spring force will continue to pull the upper portion, causing the entire Slinky to cascade to the lower level.

Because the Slinky has some velocity in the horizontal direction (which you gave it when you pulled it toward the lower stair), the top of the Slinky continues moving horizontally until it is over the next stair, at which point that end starts falling and the process repeats itself.

Dropping a Slinky

When a Slinky is held vertically, it stretches due to the gravitational force. When the Slinky is in equilibrium, the spring force that is holding up each point on the Slinky is equal to the weight of the Slinky below it, which is pulling down on that same point with the same amount of force.



As soon as the top of the Slinky is released, the spring force pulls the top downward. Because the spring force causes the center of mass to accelerate downward at the same rate as acceleration due to gravity, the bottom of the Slinky remains in position until the entire Slinky has collapsed.

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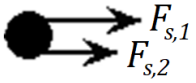
Springs in Parallel

If two or more springs are each independently pulling on the same object, we say that the springs are pulling in *parallel*. (This makes sense, because if the springs are all pulling in the same direction, they are parallel to each other.)

Springs in Parallel

A free-body diagram for this situation would look like this:



Because each spring pulls separately, the forces add. We can apply Hooke's Law, noting that the total displacement $(\Delta\vec{x})$ is the same:

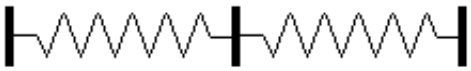
$$\vec{F}_{total} = \vec{F}_{s,1} + \vec{F}_{s,2} = k_1\Delta\vec{x} + k_2\Delta\vec{x} = (k_1 + k_2)\Delta\vec{x}$$

Therefore, the equivalent single spring constant for a combination of springs in parallel is just the sum of the individual spring constants:

$$k_{eq} = k_1 + k_2 + \ldots$$

Springs in Series

If two or more springs are connected in a sequence, one after the other, we say that the springs are in *series*:



Springs in Series

This would effectively create a longer spring, in which each coil would be stretched less (so that the total displacement remains the same). This means that it would take less force to stretch the equivalent spring.

This gives:

$$\vec{F}_{total} = \vec{F}_{s,1} + \vec{F}_{s,2} = k_1\Delta\vec{x}_1 + k_2\Delta\vec{x}_2 \quad \text{and} \quad \Delta\vec{x} = \Delta\vec{x}_1 + \Delta\vec{x}_2$$

The algebra is more complicated, but the equivalent spring constant works out to:

$$k_{eq} = \frac{k_1 \cdot k_2 \cdot k_3 \cdot \ldots}{k_1 + k_2 + k_3 + \ldots}, \text{ which means } \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots$$

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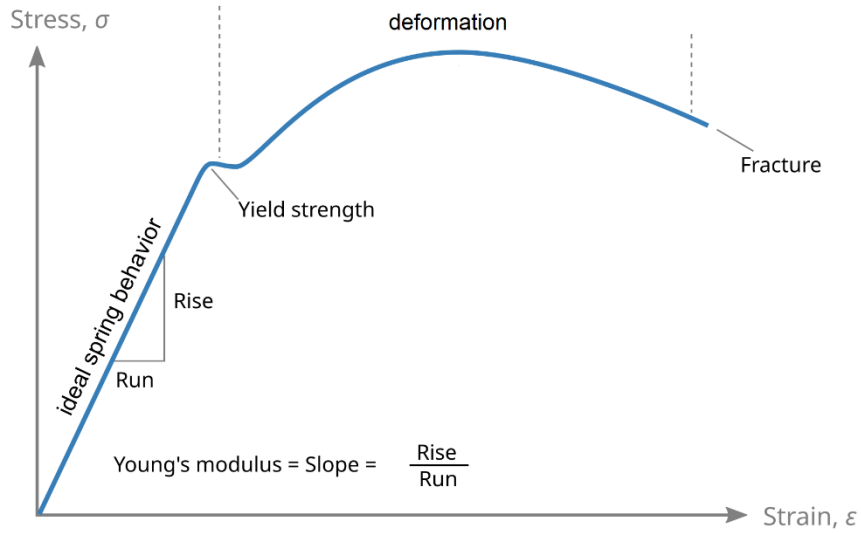
Elastic Limit (Yield Point)

If a spring is stretched beyond its elastic limit (yield point), it deforms (bends irreversibly) and eventually fractures (breaks). The points where these changes happen is shown in a graph of stress *vs.* strain.

stress: the force applied to the spring per unit area of the metal itself.

strain: the amount of proportional deformation (how much the spring expands or compresses)



Young's modulus (Y): the slope of the stress *vs.* strain graph in the linear portion, where the spring behaves as an ideal spring. Named for 19th century British physicist Thomas Young.

The spring constant can be calculated from Young's modulus:

$$k = Y\frac{A}{L}$$

where:

- k = spring constant $\left(\frac{\text{N}}{\text{m}}\right)$
- Y = Young's modulus $\left(\frac{\text{N}}{\text{m}^2} \equiv \text{Pa}\right)$
- A = area (m^2)
- L = length (m)

The linear region of the curve represents the amount of stress and strain under which the spring can be stretched or compressed and will return to its natural shape. If a force greater than the yield strength is applied, the spring will deform (bend) and will no longer return to its natural shape.

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Homework Problems

1. **(M)** One of the springs in in a car's suspension has a spring constant of $42\,000\,\frac{\text{N}}{\text{m}}$. Assume the weight of the car is equally distributed over the four springs, which means each spring is supporting 3 000 N of the car's weight. How far is each spring compressed?

   Answer: 0.07 m (which equals 7 cm).

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2. **(M – honors; A – AP® & CP1)** A 400. N garage door is held up by two springs (in parallel), each of which is stretched 1.05 m when the garage door is closed. A person needs to apply a force of 25 N to lift the garage door.
   a. How much force is applied by the pair of springs together when the springs are fully stretched?

      Answer: 375 N

   b. What is the equivalent spring constant for the two springs in parallel? What is the spring constant for each spring?

      Answer: $k_{eq} = 357\,\frac{\text{N}}{\text{m}}$; for each spring $k = 178.5\,\frac{\text{N}}{\text{m}}$

   c. If a Slinky has a spring constant of $0.5\,\frac{\text{N}}{\text{m}}$, how many Slinkys would it take to provide the same amount of force to the garage door?

      Answer: 714

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