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Ramp Problems

Unit: Forces in Multiple Dimensions

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP Physics 1 Learning Objectives/Essential Knowledge (2024): 1.C.1.1, 2.B.1.1, 3.A.2.1, 3.B.1.1, 3.B.1.2, 3.B.1.3, 3.B.2.1, 4.A.2.3, 4.A.3.1, 4.A.3.2

Mastery Objective(s): (Students will be able to…)

- Calculate forces on an object on a ramp.

Success Criteria:

- Forces are split or combined correctly using the Pythagorean Theorem and trigonometry.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how the forces on an object on a ramp depend on the angle of inclination of the ramp.

Tier 2 Vocabulary: force, ramp, inclined, normal

Labs, Activities & Demonstrations:

- Objects sliding down a ramp at different angles.
- Set up ramp with cart & pulley and measure forces at different angles.

Notes:

The direction of the normal force does not always directly oppose gravity. For example, if a block is resting on a (frictionless) ramp, the weight of the block is $\vec{F}_g$, in the direction of gravity. However, the normal force is perpendicular to the ramp, not to gravity.

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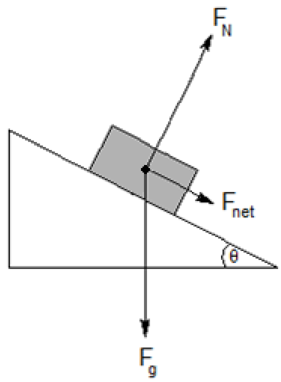
If we were to add the vectors representing the two forces, we would see that the resultant—the net force—acts down the ramp:



Intuitively, we know that if the ramp is horizontal $(\theta = 0)$, the net force is zero and $\vec{F}_N = \vec{F}_g$, because they are equal and opposite.

We also know intuitively that if the ramp is vertical $(\theta = 90°)$, the net force is $\vec{F}_g$ and $\vec{F}_N = 0$.

If the angle is between 0 and 90°, the net force must be between 0 and $\vec{F}_g$, and the proportion must be related to the angle (trigonometry!). Note that $\sin(0°) = 0$ and $\sin(90°) = 1$. Intuitively, it makes sense that the steeper the angle, the greater the net force, and therefore multiplying $\vec{F}_g$ by the sine of the angle should give the net force down the ramp for any angle between 0 and 90°.

Similarly, If the angle is between 0 and 90°, the normal force must be between $\vec{F}_g$ (at 0) and 0 (at 90°). Again, the proportion must be related to the angle (trigonometry!). Note that $\cos(0°) = 1$ and $\cos(90°) = 0$. Intuitively, it makes sense that the shallower the angle, the greater the normal force, and therefore multiplying $\vec{F}_g$ by the cosine of the angle should give the normal force for any angle 0 and 90°.

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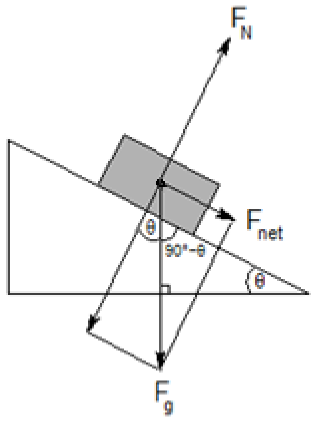
Let's look at a geometric explanation:

From geometry, we can determine that the angle of the ramp, θ, is the same as the angle between gravity and the normal force.

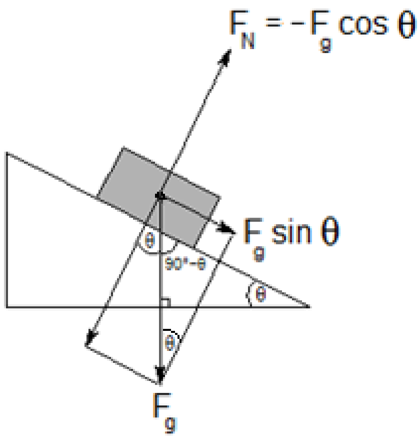


From trigonometry, we can calculate that the component of gravity parallel to the ramp (which equals the net force down the ramp) is the side *opposite* angle θ. This means:

$$F_{net} = F_g \sin\theta$$

The component of gravity perpendicular to the ramp is $F_g \cos\theta$, which means the normal force is:

$$F_N = -F_g \cos\theta$$



(The negative sign is because the normal force is in the opposite direction from $F_g \cos\theta$.)

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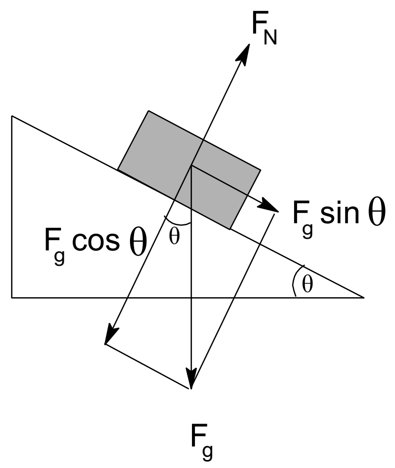
Sample Problems:

Q: A block with a mass of 2.5 kg sits on a frictionless ramp with an angle of inclination of 35°. How fast does the block accelerate down the ramp?

A: The weight of the block is $F_g = ma = (2.5)(10) = 25\,\text{N}$, directed straight down. However, the net force must be in the same direction as the acceleration. Therefore, the net force is the component of the force of gravity in the direction that the block can move (down the ramp), which is $F_g \sin\theta$:



$$F_{net} = F_g \sin\theta = 25\sin 35° = (25)(0.574) = 14.3\,\text{N}$$

Now that we know the net force (in the direction of acceleration), we can apply Newton's Second Law:

$$F_{net} = ma$$
$$14.3 = 2.5\,a$$
$$a = 5.7\,\tfrac{\text{m}}{\text{s}^2}$$

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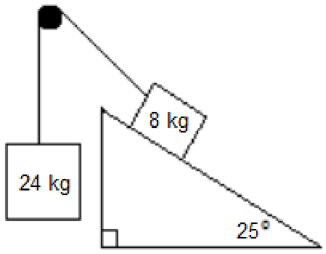
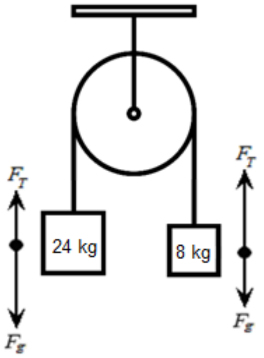
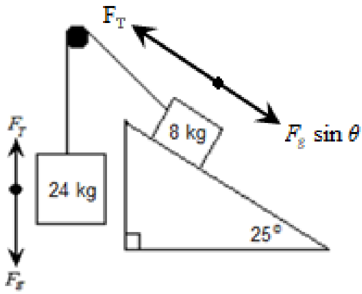
Q: The modified Atwood machine shown in the diagram to the right has a 24 kg mass hanging from a pulley, and an 8 kg mass sitting on a frictionless ramp with an angle of inclination of 25°.

What is the acceleration of the system?



A: This situation is similar to a traditional Atwood machine:





The only difference is that the forces on the 8 kg block are F_T and $F_g \sin\theta$ instead of F_T and F_g.

We solve this just like the traditional Atwood machine:

$$\sum F = ma$$
$$F_{g,24\text{ kg}} - F_{g,8\text{ kg}} \sin\theta = m_{total}a$$
$$(24)(10) - (8)(10)\sin(25°) = (24+8)a$$
$$240 - (80)(0.423) = 32a$$
$$206.2 = 32a$$
$$\boxed{6.44\,\tfrac{\text{m}}{\text{s}^2} = a}$$

If we were then asked to find the tension in the rope, we would continue in the same manner as with any other Atwood machine problem.

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Homework Problem

1. **(M – honors & AP®; A – CP1)** A 10. kg block sits on a frictionless ramp with an angle of inclination of 30°. What is the rate of acceleration of the block?

   Answer: $5.0\,\frac{\text{m}}{\text{s}^2}$

2. **(S – AP®; A – honors & CP1)** A skier is skiing down a slope at a constant and fairly slow velocity (meaning that air resistance is negligible). What is the angle of inclination of the slope?

   Hints:

   - *You will need to look up the coëfficient of kinetic friction for a <u>waxed ski on snow</u>* in *Table E. Approximate Coëfficients of Friction* on page 572 *of your Physics Reference Tables.*
   - *You do not need to know the mass of the skier because it drops out of the equation.*
   - *If the velocity is constant, that means there is no net force, which means the force down the slope (ramp) is equal to the opposing force (friction).*

   Answer: 2.9°

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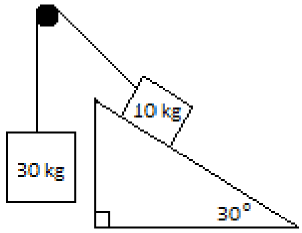
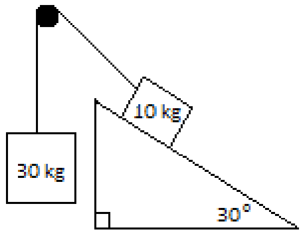
3. **(M – honors & AP®; A – CP1)** A mass of 30. kg is suspended from a massless rope on one side of a massless, frictionless pulley. A mass of 10. kg is connected to the rope on the other side of the pulley and is sitting on a ramp with an angle of inclination of 30°. The system is shown in the diagram to the right.



   a. Assuming the ramp is frictionless, determine the acceleration of the system.

   Answer: $a = 6.25\,\frac{\text{m}}{\text{s}^2}$

   b. **(M – honors & AP®; A – CP1)** Assuming instead that the ramp has a coëfficient of kinetic friction of $\mu_k = 0.3$, determine the acceleration of the system once the blocks start to move.



   Answer: $a = 5.60\,\frac{\text{m}}{\text{s}^2}$

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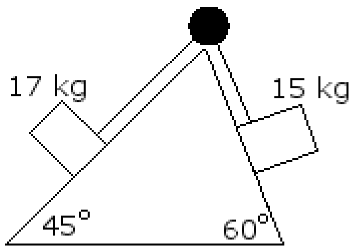
4. **(S – honors & AP®; A – CP1)** Two boxes with masses 17 kg and 15 kg are connected by a light string that passes over a frictionless pulley of negligible mass as shown in the figure below. The surfaces of the planes are frictionless.



   a. **(S – honors & AP®; A – CP1)** When the blocks are released, which direction will the blocks move?

   b. **(S – honors & AP®; A – CP1)** Determine the acceleration of the system.

   Answer: $0.303\,\frac{m}{s^2}$

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