Torque

Unit: Rotational Statics & Dynamics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 5.3.A, 5.3.A.1, 5.3.A.2, 5.3.B.1, 5.3.B.1.i, 5.3.B.1.ii, 5.3.B.2

Mastery Objective(s): (Students will be able to…)

- Calculate the torque on an object.
- Calculate the location of the fulcrum of a system using balanced torques.
- Calculate the amount and distance from the fulcrum of the mass needed to balance a system.

Success Criteria:

- Variables are correctly identified and substituted correctly into equations.
- Equations for torques on different masses are combined correctly algebraically.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why a longer lever arm is more effective.

Tier 2 Vocabulary: balance, torque

Labs, Activities & Demonstrations:

- Balance an object on two fingers and slide both toward the center.
- Clever wine bottle stand.

Notes:

<u>torque</u> ($\vec{\boldsymbol{\tau}}$): a vector quantity that measures the effectiveness of a force in causing rotation. Take care to distinguish the Greek letter "τ" from the Roman letter "t". Torque is measured in units of newton-meters:

$$1\,\text{N}\cdot\text{m} = 1\,\tfrac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

Note that work and energy (which we will study later) are also measured in newton-meters. However, work and energy are different quantities from torque, and are not interchangeable. (Among other differences, work and energy are scalar quantities, and torque is a vector quantity.)

<u>axis of rotation</u>: the point around which an object rotates.

<u>fulcrum</u>: the point around which a lever pivots. Also called the pivot.

<u>lever arm</u>: the distance from the axis of rotation that a force is applied, causing a torque.

Use this space for summary and/or additional notes:

Just as force is the quantity that causes linear acceleration, torque is the quantity that causes a change in the speed of rotation (rotational acceleration).

Because inertia is a property of mass, Newton's second law is the relationship between force and inertia. Newton's second law in rotational systems looks similar to Newton's second law in linear systems:

$$\vec{\boldsymbol{a}} = \frac{\sum \vec{\boldsymbol{F}}}{m} = \frac{\vec{\boldsymbol{F}}_{net}}{m} \qquad \vec{\boldsymbol{\alpha}} = \frac{\Sigma \vec{\boldsymbol{\tau}}}{I} = \frac{\vec{\boldsymbol{\tau}}_{net}}{I}$$

$$\vec{\boldsymbol{F}}_{net} = m\vec{\boldsymbol{a}} \qquad \vec{\boldsymbol{\tau}}_{net} = I\vec{\boldsymbol{\alpha}}\ ^{*}$$

linear rotational

As you should remember, a net force of zero, that means all forces cancel in all directions and there is no acceleration. If there is no acceleration $(\vec{\mathbf{a}} = 0)$, the velocity remains constant (which may or may not equal zero).

Similarly, if the net torque is zero, then the torques cancel in all directions and there is no angular acceleration. If there is no angular acceleration $(\vec{\boldsymbol{\alpha}} = 0)$, then the angular velocity remains constant (which may or may not equal zero).

rotational equilibrium: when all of the torques on an object cancel each other's effects (resulting in a net force of zero) and the object either does not rotate or rotates with a constant angular velocity.

Torque is also the cross product of distance from the center of rotation ("lever arm") × force:

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} \quad \text{which gives:} \quad \|\vec{\boldsymbol{\tau}}\| = \tau = rF\sin\theta = rF_{\perp}$$

where θ is the angle between the lever arm and the applied force.

We use the variable *r* for the lever arm (which is a distance) because torque causes rotation, and *r* is the distance from the center of the circle (radius) at which the force is applied.

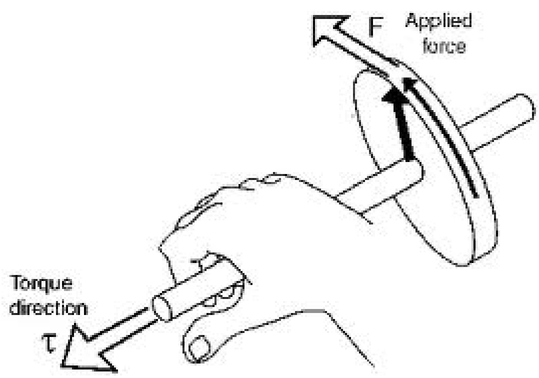
$F\sin\theta$ is sometimes written as $F_{\perp}$ (the component of the force that is perpendicular to the radius) and sometimes $F_{||}$ (the component of the force that is parallel to the direction of motion). These notes will use $F_{\perp}$, because in many cases the force is applied to a lever, and the component of the force that causes the torque is perpendicular to the lever itself, so it is easy to think of it as "the amount of force that is perpendicular to the lever". This gives the equation:

$$\tau = rF_{\perp}$$

* In this equation, $\vec{\alpha}$ is angular acceleration, which is studied in AP® Physics 1, but is beyond the scope of the CP1 and honors physics course. Qualitatively, angular acceleration is a change in how fast something is rotating.

Use this space for summary and/or additional notes:

Of course, because torque is the cross product of two vectors, it is a vector whose direction is perpendicular to both the lever arm and the force.



This is an application of the "right hand rule." If your fingers of your right hand curl from the first vector ($\vec{r}$) to the second ($\vec{F}$), then your thumb points in the direction of the resultant vector ($\vec{\tau}$). Note that the direction of the torque vector is parallel to the axis of rotation.

Note, however, that you can't "feel" torque; you can only "feel" force. Most people think of the "direction" of a torque as the direction of the rotation that the torque would produce (clockwise or counterclockwise). In fact, the College Board usually uses this convention.

Mathematically, the direction of the torque vector is needed only to give torques a positive or negative sign, so torques in the same direction add and torques in opposite directions subtract. In practice, most people find it easier to define the positive direction for rotation (clockwise ↻ or counterclockwise ↺) and use those for positive or negative torques in the problem, regardless of the direction of the torque vector.

Note that diagrams showing forces in rotating systems are *force diagrams*, but are not properly called "free-body diagrams", because a rotating system is constrained to rotate around its axis, and is not technically a "free body". However, for the purposes of this course, force diagrams and free-body diagrams work the same way and may be considered equivalent.

Sample Problem:

Q: If a perpendicular force of 20 N is applied to a wrench with a 25 cm handle, what is the torque applied to the bolt?

A: $\tau = r F_{\perp}$

$\tau = (0.25\,\text{m})(20\,\text{N})$

$\tau = 4\,\text{N}\cdot\text{m}$

Use this space for summary and/or additional notes:

Seesaw Problems

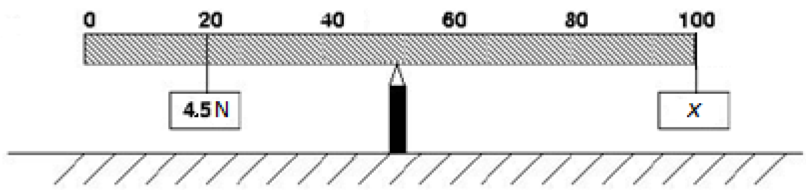
A seesaw problem is one in which objects on opposite sides of a lever (such as a seesaw) balance one another.

To solve seesaw problems, if the seesaw is not moving, then the torques must balance and the net torque must be zero.

The total torque on each side is the sum of the separate torques caused by the separate masses. Each of these masses can be considered as a point mass (infinitely small object) placed at the object's _center of mass_.

Sample Problems:

Q: A 100 cm meter stick is balanced at its center (the 50-cm mark) with two objects hanging from it, as shown below:



One of the objects weighs 4.5 N and is hung from the 20-cm mark (30 cm = 0.3 m from the fulcrum). A second object is hung at the opposite end (50 cm = 0.5 m from the fulcrum). What is the weight of the second object?

A: In order for the ruler to balance, the torque on the left side (which is trying to rotate the ruler counter-clockwise) must be equal to the torque on the right side (which is trying to rotate the ruler clockwise). The torques from the two halves of the ruler are the same (because the ruler is balanced in the middle), so this means the torques applied by the objects also must be equal.

The torque applied by the object on the left is:

$$\tau = rF = (0.30)(4.5) = 1.35\,\text{N}\cdot\text{m}$$

The torque applied by the object on the right must also be 1.35 N·m, so we can calculate the force:

$$\tau = rF$$
$$1.35 = 0.50F$$
$$F = \frac{1.35}{0.50} = 2.7\,\text{N}$$

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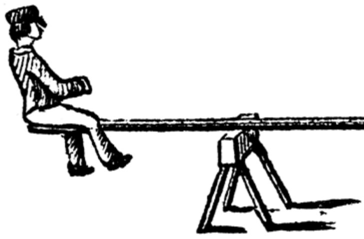
Q: In the following diagram, the mass of the person on the left is 90. kg and the mass of the person on the right is 50. kg. The board is 6.0 m long and has a mass of 20. kg.



Where should the board be positioned in order to balance the seesaw?

A: When the seesaw is balanced, the torques on the left have to equal the torques on the right.

This problem is more challenging because the board has mass and is not balanced at its center. This means the two sides of the board apply different (unequal) torques, so we have to take into account the torque applied by each fraction of the board as well as the torque by each person.

Let's say that the person on the left is sitting at a distance of x meters from the fulcrum. The board is 6 m long, which means the person on the right must be $(6 - x)$ meters from the fulcrum.

Our strategy is:

1. Calculate and add up the counter-clockwise (CCW = ↺) torques. These are the torques that would turn the seesaw in a counter-clockwise direction, which are on the left side. They are caused by the force of gravity acting on the person, at distance x, and the left side of the board, centered at distance $\frac{x}{2}$.
2. Calculate and add up the clockwise (CW = ↻) torques. These are caused by the force of gravity acting on the person, at distance $(6 - x)$, and the left side of the board, centered at distance $\frac{6-x}{2}$.
3. Set the two torques equal to each other and solve for x.

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Left Side (CCW = ↺)

Person

The person has a mass of 90 kg and is sitting at a distance *x* from the fulcrum:

$$\tau_{LP} = rF$$
$$\tau_{LP} = x(mg) = x(90\,\text{kg})(10\,\tfrac{\text{m}}{\text{s}^2})$$
$$\tau_{LP} = 900x$$

Board

The center of mass of the left part of the board is at a distance of $\frac{x}{2}$.

The weight (F_g) of the board to the left of the fulcrum is $\left(\frac{x}{6}\right)(20)(10)$

$$\tau_{LB} = rF$$
$$\tau_{LB} = r(mg) = \left(\frac{x}{2}\right)\left(\frac{x}{6.0}\right)(20)(10)$$
$$\tau_{LB} = 16.\overline{6}\,x^2$$

Total

$$\tau_{ccw} = \tau_{LB} + \tau_{LP}$$
$$\tau_{ccw} = 16.\overline{6}x^2 + 900x$$

Right Side (CW = ↻)

Person

The person on the right has a mass of 50 kg and is sitting at a distance of 6 − x from the fulcrum:

$$\tau_{RP} = rF$$
$$\tau_{RP} = r(mg) = (6-x)(50\,\text{kg})(10\,\tfrac{\text{m}}{\text{s}^2})$$
$$\tau_{RP} = 500\,(6-x)$$
$$\tau_{RP} = 3000 - 500x$$

Board

The center of mass of the right part of the board is at a distance of $\frac{6-x}{2}$.

The weight (F_g) of the board to the right of the fulcrum is $\left(\frac{6-x}{6}\right)(20)(10)$

$$\tau_{RB} = rF$$
$$\tau_{RB} = r(mg) = \left(\frac{6-x}{2}\right)\left(\frac{6-x}{6}\right)(20)(10)$$
$$\tau_{RB} = 16.\overline{6}\,(36 - 12x + x^2)$$
$$\tau_{RB} = 600 - 200x + 16.\overline{6}x^2$$

Total

$$\tau_{cw} = \tau_{RB} + \tau_{RP}$$
$$\tau_{cw} = 16.\overline{6}x^2 - 200x + 600 + 3000 - 500x$$
$$\tau_{cw} = 16.\overline{6}x^2 - 700x + 3600$$

Because the seesaw is not rotating, the net torque must be zero. So, we need to define the positive and negative directions. A common convention is to define counter-clockwise as the positive direction. (Most math classes already do this—a positive angle means counter-clockwise starting from zero at the *x*-axis.)

This gives:

$$\tau_{ccw} = 16.\overline{6}x^2 + 900x \qquad \tau_{cw} = -(16.\overline{6}x^2 - 700x + 3600) = -16.\overline{6}x^2 + 700x - 3600$$

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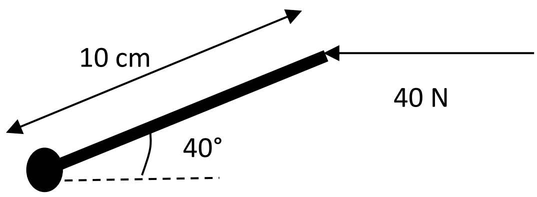
Because the seesaw is not rotating we set $\tau_{CCW} + \tau_{CW} = 0$ and solve:

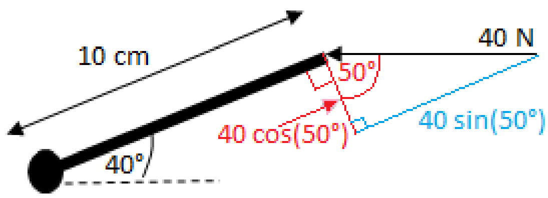
$$0 = \tau_{net} = \sum \tau = \tau_{ccw} + \tau_{cw} = 16.\bar{6}x^2 + 900x - 16.\bar{6}x^2 + 700x - 3600$$
$$0 = 900x + 700x - 3600$$
$$0 = 1600x - 3600$$
$$1600x = 3600$$
$$x = \frac{3600}{1600} = 2.25\,\text{m}$$

The board should therefore be placed with the fulcrum 2.25 m away from the person on the left.

Q: Calculate the torque on the following 10 cm lever. The lever is angled 40° up from horizontal, and a force of 40 N force is applied parallel to the ground.



A: This is an exercise in geometry. We need the component of the 40 N force that is perpendicular to the lever $(F_{\perp})$. To find this, we draw a right triangle in which the hypotenuse is the applied force. (R*emember that the hypotenuse is the longest side, and the total force must be greater than or equal to any of its components.*)



Now, we simply use our calculators (or trigonometry tables):

$$40\cos(50^\circ) = (40)(0.643) = 25.7\,\text{N}$$

CP1 & honors (not AP®)

Extension

Just as yank is the rate of change of force with respect to time, the rate of change of torque with respect to time is called rotatum: $\vec{\boldsymbol{P}} = \frac{\Delta\vec{\boldsymbol{\tau}}}{\Delta t} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{Y}}$. Rotatum is also sometimes called the "moment of a yank," because it is the rotational analogue to yank.

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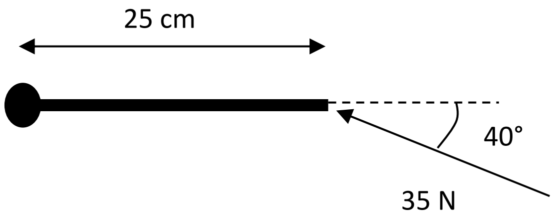
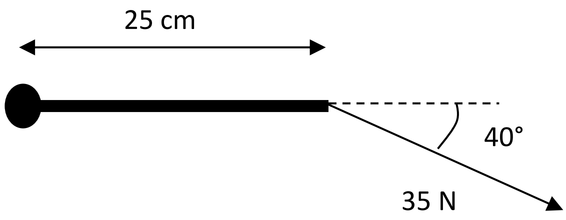
Homework Problems

For each of the following diagrams, find the torque about the axis indicated by the black dot. Assume that the lever itself has negligible mass.

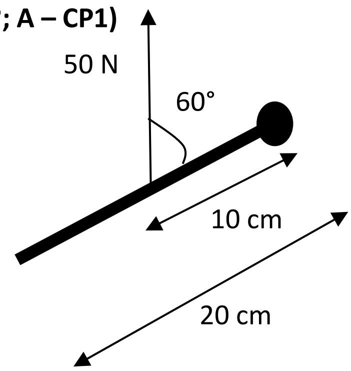
1. **(M – honors & AP®; A – CP1)** Answer: 5.62 N·m CCW (↺)



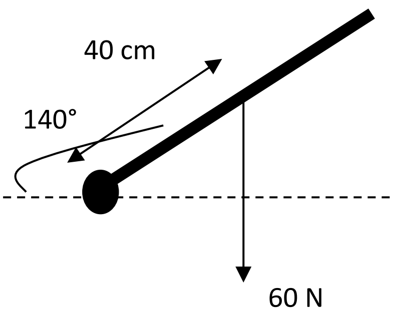
2. **(M – honors & AP®; A – CP1)** Answer: 5.62 N·m CW (↻)



3. **(M – honors & AP®; A – CP1)** Answer: 4.33 N·m CW (↻)

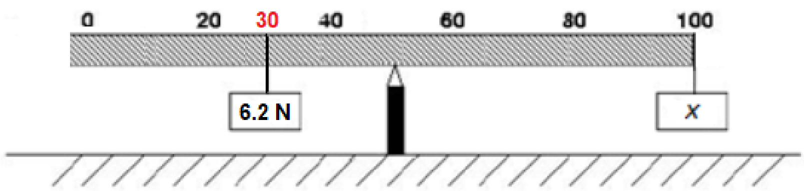


4. **(S – honors & AP®; A – CP1)** Answer: 18.4 N·m CW (↻)



Use this space for summary and/or additional notes:

5. **(M)** In the following diagram, a meter stick is balanced in the center (at the 50 cm mark). A 6.2 N weight is hung from the meter stick at the 30 cm mark. How much weight must be hung at the 100 cm mark in order to balance the meter stick?



Hints:

- *The meter stick has the same amount of mass on both sides of the fulcrum. This means it applies the same amount of torque in both directions and you don't need to include it in your calculations.*
- *The 30 cm mark is 20 cm = 0.2 m from the fulcrum; the 100 cm mark is 50 cm = 0.5 m from the fulcrum.*

Answer: 0.25 kg

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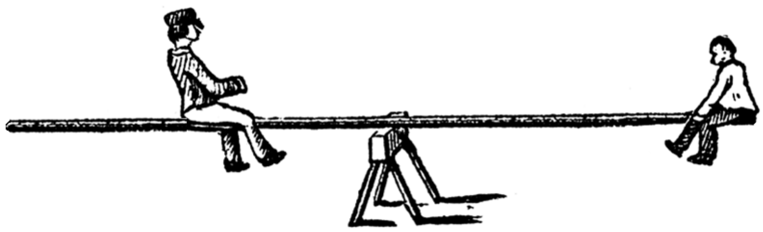
6. **(M – AP®; S – honors; A – CP1)** The seesaw shown in the following diagram balances when no one is sitting on it. The child on the right has a mass of 35 kg and is sitting 2.0 m from the fulcrum. If the adult on the left has a mass of 85 kg, how far should the adult sit from the fulcrum in order for the seesaw to be balanced?



Answer: 0.82 m

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