AP®

Solving Linear & Rotational Force/Torque Problems

Unit: Rotational Statics & Dynamics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-1, HS-PS2-10(MA)

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 5.3.A, 5.3.A.1, 5.3.A.2, 5.3.B.1, 5.3.B.1.i, 5.3.B.1.ii, 5.3.B.2

Mastery Objective(s): (Students will be able to…)

- Set up and solve problems involving combinations of linear and rotational dynamics.

Success Criteria:

- Variables are correctly identified and substituted correctly into equations.
- Equations are combined correctly algebraically.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Identify which parts of a problem are linear and which parts are rotational.

Tier 2 Vocabulary: force, rotation, balance, torque

Notes:

Newton's second law—that forces produce acceleration—applies in both linear and rotational contexts. In fact, you can think of the equations as exactly the same, except that one set uses Cartesian coördinates, and the other uses polar or spherical coördinates.

You can substitute rotational variables for linear variables in all of Newton's equations (motion and forces), and the equations are still valid.

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The following is a summary of the variables used for dynamics problems:

Linear			**Angular**		
Var.	**Unit**	**Description**	**Var.**	**Unit**	**Description**
$\vec{x}$	m	position	$\vec{\theta}$	— (rad)	angle; angular position
$\vec{d}, \Delta\vec{x}$	m	displacement	$\Delta\vec{\theta}$	— (rad)	angular displacement
$\vec{v}$	$\frac{m}{s}$	velocity	$\vec{\omega}$	$\frac{1}{s}\left(\frac{rad}{s}\right)$	angular velocity
$\vec{a}$	$\frac{m}{s^2}$	acceleration	$\vec{\alpha}$	$\frac{1}{s^2}\left(\frac{rad}{s^2}\right)$	angular acceleration
t	s	time	t	s	time
m	kg	mass	I	$kg \cdot m^2$	moment of inertia
$\vec{F}$	N	force	$\vec{\tau}$	N·m	torque

Notice that each of the linear variables has an angular counterpart.

Keep in mind that "radian" is not a unit. A radian is a ratio that describes an angle as the ratio of the arc length to the radius. This ratio is dimensionless (has no unit), because the units cancel. This means that an angle described in radians has no unit, and therefore never needs to be converted from one unit to another. However, we often write "rad" after an angle measured in radians to remind ourselves that the quantity describes an angle.

We have learned the following equations for solving motion problems:

Linear Equation	**Angular Equation**	**Relation**	**Comments**
$\vec{F} = m\vec{a}$	$\vec{\tau} = I\vec{\alpha}$	$\vec{\tau} = \vec{r} \times \vec{F} = rF_{\perp}$	Quantity that produces acceleration
$\vec{F}_c = m\vec{a}_c = \frac{m\vec{v}^2}{r}$	$\vec{F}_c = m\vec{a}_c = mr\vec{\omega}^2$		Centripetal force (which causes centripetal acceleration)

Note that vector quantities (shown in bold) can be positive or negative, depending on direction.

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Problems Involving Linear and Rotational Dynamics

The main points of the linear Dynamics (Forces) & Gravitation chapter were:

a. A net force produces acceleration. $\vec{F}_{net} = m\vec{a}$

b. If there is no acceleration, then there is no net force, which means all forces must cancel in all directions. No acceleration may mean a static situation (nothing is moving) or constant velocity.

c. Forces are vectors. Perpendicular vectors do not affect each other, which means perpendicular forces do not affect each other.

The analogous points hold true for torques:

1. A net torque produces angular acceleration. $\vec{\tau}_{net} = I\vec{\alpha}$

2. If there is no angular acceleration, then there is no net torque, which means all torques must cancel. No angular acceleration may mean a static situation (nothing is rotating) or it may mean that there is rotation with constant angular velocity.

3. Torques are vectors. Perpendicular torques do not affect each other.

4. Torques and linear forces act independently.

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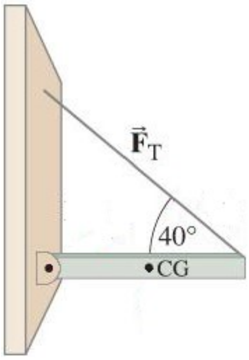
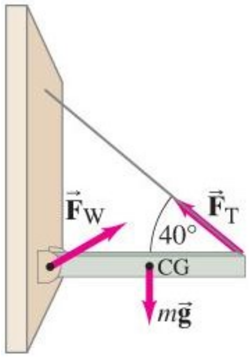
One of the most common types of problem involves a stationary object that has both linear forces and torques, both of which are in balance.

In the diagrams at the right, a beam with a center of gravity (center of mass) in the middle (labeled "CG") is attached to a wall with a hinge. The end of the beam is held up with a rope at an angle of 40° above the horizontal.



The rope applies a torque to the beam at the end at an angle of rotation with a radius equal to the length of the beam. Gravity applies a force straight down on the beam.



1. Because the beam is not rotating, we know that $\vec{\tau}_{net}$ must be zero, which means the wall must apply a torque that counteracts the torque applied by the rope. (Note that the axis of rotation for the torque from the wall is the opposite end of the beam.)

2. Because the beam is not moving (translationally), we know that $\vec{F}_{net}$ must be zero in both the vertical and horizontal directions. This means that the wall must apply a force $\vec{F}_W$ to balance the vertical and horizontal components of $\vec{F}_T$ and $m\vec{g}$. Therefore, the vertical component of $\vec{F}_W$ plus the vertical component of $\vec{F}_T$ must add up to $m\vec{g}$, and the horizontal components of $\vec{F}_T$ and $\vec{F}_W$ must cancel.

AP questions often combine pulleys with torque. (See the section on Tension starting on page 301.) These questions usually require you to combine the following concepts/equations:

1. A torque is the action of a force acting perpendicular to the radius at some distance from the axis of rotation: $\tau = rF_{\perp}$

2. Net torque produces angular acceleration according to the formula: $\tau_{net} = I\alpha$

3. The relationships between tangential and angular velocity and acceleration are: $v_T = r\omega$ and $a_T = r\alpha$

AP free-response problems are always scaffolded, meaning that each part leads to the next.

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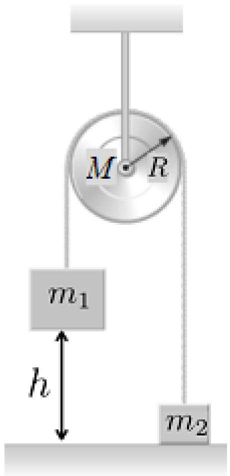
Sample AP-Style Problem



Q: Two masses, $m_1 = 23.0\,\text{kg}$ and $m_2 = 14.0\,\text{kg}$ are suspended by a rope that goes over a pulley that has a radius of R = 0.350 m and a mass of M = 40 kg, as shown in the diagram to the right. (You may assume that the pulley is a solid cylinder.) Initially, mass m_2 is on the ground, and mass m_1 is suspended at a height of h = 0.5 m above the ground.

a. What is the net torque on the pulley?

CCW: *the torque is caused by mass m_1 at a distance of R, which is given by:*

$$\tau_1 = m_1 gR = (23.0)(10)(0.350) = 80.5\,\text{N}\cdot\text{m}$$

(Note that we are using positive numbers for counter-clockwise torques and negative numbers for clockwise torques.)

CW: *the torque is caused by mass m_2 at a distance of R, so:*

$$\tau_2 = m_2 gR = -(14.0)(10)(0.350) = -49.0\,\text{N}\cdot\text{m}$$

Net: *The net torque is just the sum of all of the torques:*

$$\tau_{net} = 80.5 + (-49.0) = +31.5\,\text{N}\cdot\text{m}\,(\text{CCW})$$

b. What is the angular acceleration of the pulley?

Now that we know the net torque, we can use the equation $\tau_{\text{net}} = I\alpha$ to calculate α (but we have to calculate I first).

$$I = \tfrac{1}{2}MR^2 = (\tfrac{1}{2})(40)(0.35)^2 = 2.45\,\text{N}\cdot\text{m}^2$$

$$\tau_{\text{net}} = I\alpha$$
$$31.5 = 2.45\alpha$$
$$\alpha = 12.9\,\tfrac{\text{rad}}{\text{s}^2}$$

c. What is the linear acceleration of the blocks?

The linear acceleration of the blocks is the same as the acceleration of the rope, which is the same as the tangential acceleration of the pulley:

$$a_T = r\alpha = (0.35)(12.9) = 4.5\,\tfrac{\text{m}}{\text{s}^2}$$

d. How much time does it take for mass m_1 to hit the floor?

We never truly get away from kinematics problems!

$$d = v_o t + \tfrac{1}{2}at^2 \qquad t^2 = 0.222$$
$$0.5 = (\tfrac{1}{2})(4.5)t^2 \qquad t = \sqrt{0.222} = 0.47\,\text{s}$$

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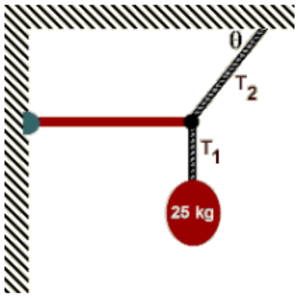
Homework Problems

1. **(M – AP®; A – honors & CP1)** A 25 kg bag is suspended from the end of a uniform 100 N beam of length *L*, which is attached to the wall by an ideal (freely-swinging, frictionless) hinge, as shown in the figure to the right. The angle of rope hanging from the ceiling is $\theta = 30°$.



What is the tension, T_2, in the rope that hangs from the ceiling?

Answer: 600 N

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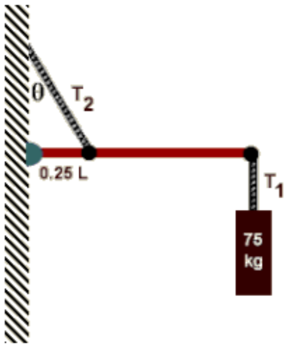
2. **(M – AP®; A – honors & CP1)** A 75 kg block is suspended from the end of a uniform 100 N beam of length *L*, which is attached to the wall by an ideal hinge. A support rope is attached ¼ of the way to the end of the beam at an angle from the wall of $\theta = 30°$.



What is the tension in the support rope (T_2)?

Answer: 3 695 N

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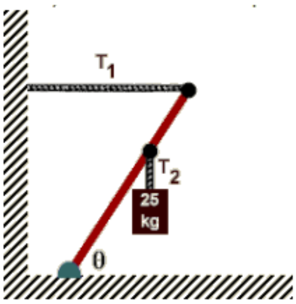
3. **(M – AP®; A – honors & CP1)** A 25 kg box is suspended $^2/_3$ of the way up a uniform 100 N beam of length L, which is attached to the floor by an ideal hinge, as shown in the picture to the right. The angle of the beam above the horizontal is θ = 37°.



What is the tension, T_1, in the horizontal support rope?

Answer: 288 N

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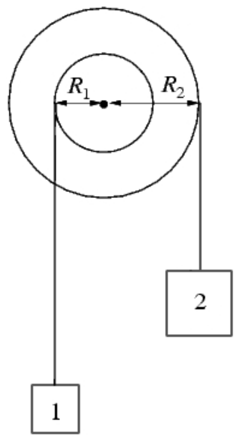
4. **(M – AP®; A – honors & CP1)** Two blocks are suspended from a double pulley as shown in the picture to the right. Block #1 has a mass of 2 kg and is attached to a pulley with radius $R_1 = 0.25$ m. Block #2 has a mass of 3.5 kg and is attached to a pulley with radius $R_2 = 0.40$ m. The pulley has a moment of inertia of 1.5 kg·m^2.



When the weights are released and are allowed to fall,

a. **(M – AP®; A – honors & CP1)** What will be the net torque on the system?

Answer: 9 N·m CW (↻)

b. **(M – AP®; A – honors & CP1)** What will be the angular acceleration of the pulley?

Answer: $6\ \frac{\text{rad}}{\text{s}^2}$

c. **(M – AP®; A – honors & CP1)** What will be the linear accelerations of blocks #1 and #2?

Answer: block #1: $1.5\ \frac{\text{m}}{\text{s}^2}$; block #2: $2.4\ \frac{\text{m}}{\text{s}^2}$

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