Potential Energy

Unit: Energy, Work & Power

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-1

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 3.1.A, 3.1.A.1, 3.1.A.2, 3.1.A.3, 3.3.A, 3.3.A.1, 3.3.A.2, 3.3.A.3, 3.3.A.4, 3.3.A.4.i, 3.3.A.4.ii, 3.3.A.4.iii, 3.3.A.5

Mastery Objective(s): (Students will be able to…)

- Calculate the gravitational potential energy of an object.

Success Criteria:

- Correct equation(s) are chosen for the situation.
- Variables are correctly identified and substituted correctly into equation(s).
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain when & why an object has potential energy.

Tier 2 Vocabulary: work, energy

Summary of Concepts & Equations:

potential energy: energy at a distance caused by an attractive force between two or more objects.

The potential energy between two objects is zero at an infinite distance, and negative at any finite distance.

When we talk about "gravitational potential energy", we mean the difference in potential energy that an object would have in one location *vs.* another.

The equation for gravitational potential energy is $\Delta U_g = mg\Delta h$, but it is almost always written $U_g = mgh$.

Labs, Activities & Demonstrations:

- Popper lab.

potential energy (U): the ability of objects to move toward each other due to attractive forces between them.

Potential energy is sometimes described as "stored" energy, and is related to an object's position relative to another object, and the strength and nature of the forces between them.

Conservation of Energy (which will be discussed in detail starting on page 464), tells us that energy may be converted between one form and another. Thus, potential energy is useful because it is available to be turned into some other form, such as kinetic energy, internal (thermal) energy, *etc.*

Gravitational Potential Energy (GPE)

There is gravitational potential energy (GPE) between any two objects that have mass, because there is a gravitational force that attracts them to each other.

As we saw in the section on *Universal Gravitation*, starting on page 426, when two objects with mass (m_1 and m_2) objects are separated, the gravitational force between them is given by:

$$F_g = -\frac{Gm_1m_2}{r^2}$$

The potential energy between these objects is due to the action of the gravitational force at every possible distance between the two objects as they move apart. Using calculus, we can add up the actions of these forces by taking the integral over the distance:

$$U_g = \int_r^\infty -\frac{Gm_1m_2}{r^2}\,dr = -\frac{Gm_1m_2}{r^2}\cdot r = -\frac{Gm_1m_2}{r}$$

Don't worry about the derivation if you haven't learned calculus. It simply means:

$$U_g = -\frac{Gm_1m_2}{r} = F_g r$$

With energy, positive and negative denote the direction that energy is transferred. In most situations we are not concerned with the direction of energy transfer, which means we usually omit the negative sign.

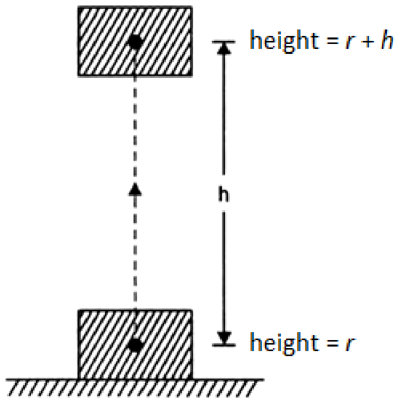
Gravitational Potential Energy Near the Surface of the Earth

Recall from *Universal Gravitation*, starting on page 426, that the strength of the gravitational field near Earth's surface is denoted by the constant *g*, and that:

$$g = \frac{Gm_1}{r^2}$$

where *G* is the universal gravitational constant, m_1 is the mass of the Earth, and *r* is the distance from the Earth's surface to its center of mass. (If we assume that the Earth has a uniform density, this is at the Earth's geometric center.)

If the potential energy of an object on the ground (the surface of the Earth) is a distance *r* from the center of the Earth, then an object that is a height *h* above the ground would be the distance $r+h$ above the center of the Earth. The potential energy of the object would therefore be $U_g = F_g(r+h)$.



The difference in potential energy would therefore be:

$$\Delta U_g = F_g(r+h) - F_g r = F_g h$$

Because $F_g = mg$, this means:

$$\Delta U_g = mgh$$

Dealing with Vocabulary in an Imperfect World

If this were a perfect world, the phrase "potential energy" would *always* be negative, and it would always refer to the amount of energy that it would take to separate the two objects to an infinite distance. The phrase "difference in potential energy" would *always* mean the amount of energy that it would take to increase the separation between the objects by a specific distance.

However, in our imperfect world, words mean whatever the person saying (or writing them) intends for them to mean. In our imperfect world, the phrase "potential energy" is used interchangeably with "difference in potential energy". If we lift an object that has a mass of 2 kg to a height of 3 m above the ground, we would say that there is $U_g = mgh = (2)(10)(3) = 30\text{ J}$ of potential energy between the object and the Earth.

Worse yet, although it is absolutely true that ***potential energy requires two objects that exert a force on each other***, we often say that the object itself has 30 J of potential energy. (The fact that the Earth must also be involved is implied, but unstated.)

On the AP® Exam, you need to accept that:

- Potential energy requires two objects that exert a force upon each other.
- *Potential energy* sometimes refers to the energy between two objects with finite separation, where zero potential energy can only occur at infinite separation.
- In this course, *potential energy* usually refers to the energy between an object and the Earth due to its height above the Earth. In this case, an object's potential energy is zero when it is resting on the ground, and positive when it is above the ground.
- A single, isolated object cannot have potential energy. However, questions may still ask about the gravitational potential energy that a single object has. In this case, it is assumed that the object must be in a system that includes the Earth.

Potential Energy from a Gravitational Force Field

Because the Earth's contribution to the gravitational pull between two objects is constant at a constant radius, we can model the gravitational pull from the Earth as a force field with a constant strength of approximately $10\,\frac{\text{N}}{\text{kg}}$. (This was previously explained in the section on *299Gravitational Force*, starting on page 299.)

If we model the gravitational force this way, an object has potential energy due to its interaction with the force field. The object's potential energy is equal to the force exerted by the field times the distance over which the field could apply that force.

We would say that the 2 kg object at a height of 3 m from the above example has 60 J of potential energy due to the force field. Again, this the statement claims that the object itself has potential energy; we must remember that It is still true that potential energy requires two objects, because the force field could not exist without the Earth.

Other Types of Potential Energy

Other types of potential energy include:

- <u>electric potential energy</u>: potential energy due to the attraction (or repulsion) between charged objects. As with gravity, electric potential energy may be attributed to the effect of an electric field on a charged object.
- <u>springs</u>: if an object that is attached to a spring that has been stretched or compressed, there is potential energy between the object and the spring. (Again, we often say that the object has potential energy.) When the spring is released and allowed to move, that potential energy becomes the kinetic energy of the object.

- chemical potential: chemicals release energy when they form chemical bonds. This is another example where potential energy is zero when the objects (in this case, the two chemical species) are separated by an infinite distance, and negative when they come together to form bonds. Chemical potential energy is called "free energy". If a chemical compound is said to have negative free energy, it means that amount of energy was released in the process of forming the bonds in that compound.

Gravitational Potential Energy (GPE) for Objects Close to the Earth

We can think of gravitational potential energy (GPE) as the action of the gravitational force in a way that can increase an object's kinetic energy. Because acceleration due to gravity on Earth is approximately $\vec{g} = 10\,\frac{\text{m}}{\text{s}^2}$ near the surface of the Earth, this means that the acceleration caused by the gravitational force is:

$$\vec{F}_g = m\vec{a} = m\vec{g}$$

Because kinetic energy is $K = \frac{1}{2}mv^2$ and $v^2 - v_o^2 = 2ad$, if an object starts from rest $(v_o = 0)$, and is only accelerated by the gravitational force, then:

$$v^2 = 2ad \quad \text{and} \quad \Delta K = \tfrac{1}{2}m(2ad) = mad$$

Remember that $\vec{a} = \vec{g}$. If the distance is vertical, we usually call it height, and we use the variable *h* instead of *d*, which means $\Delta K = mgh$. Therefore, the GPE (U_g) is the amount of kinetic energy that could be added by the object falling from a height:

$$U_g = mgh$$

Deriving Potential Energy from the Gravitational Force as a Force Field

A discussed earlier, we think of the gravitational force as the force caused by a gravitational force field—a region (near a massive object like the Earth) in which a gravitational force acts on all objects that have mass (*m*). If the strength of the gravitational field is $\vec{g}$ (approximately $10\,\frac{\text{N}}{\text{kg}}$ near the surface of the Earth), then the force is $\vec{F}_g = m\vec{g}$. Using the reasoning above, this gives the same equation, $U_g = mgh$.

GPE between Objects that are Far Apart Compared with Their Size

Considering the gravitational force as a force field of constant strength does not work when the objects are very far apart. In that case, we need to consider GPE as the result of a gravitational force between two objects.

As we saw in the section on *Universal Gravitation*, starting on page 426, when two objects with mass (m_1 and m_2) objects are separated, the gravitational force between them is given by:

$$F_g = \frac{Gm_1m_2}{r^2}$$

If *r* is the distance between the objects' centers of mass, then we can apply the same reasoning as above, but using *r* instead of *h* as the distance between the objects. Thus, $U_g = mgh$ becomes $U_g = mgr$, which means:

$$U_g = F_g h = \frac{Gm_1m_2}{r^2} \cdot r = \frac{Gm_1m_2}{r}$$

Potential Energy of a Spring

As mentioned above, potential energy can be stored in a spring that is either stretched or compressed (and is therefore exerting a spring force).

The elastic potential energy of an ideal spring is given by the equation:

$$U_s = \tfrac{1}{2}k(\Delta x)^2$$

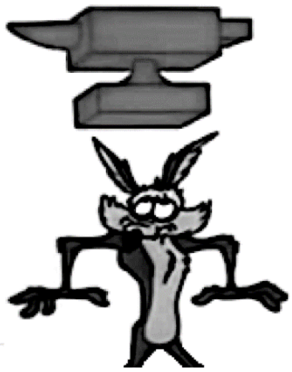
where:

- U_s = potential energy of the spring-object system (J)
- k = spring constant ($\frac{\text{N}}{\text{m}}$)
- Δx = displacement of the object from the spring's equilibrium position (m)

Systems and Potential Energy

Recall that a *system* is a collection of objects for the purpose of describing the interaction of objects within *vs.* outside of that collection. The surroundings is all of the objects outside of the system ("everything else"). (Systems are explained in more detail on page 284.)

Potential energy is an energy relationship between two objects within a system. A single, isolated object cannot have potential energy.



For example, in the coyote-anvil system pictured to the right, both Wile E. Coyote and the anvil have negligible potential energy. (There is a tiny amount of gravitational attraction between them—assuming the anvil has a mass of 200 kg and the coyote has a mass of 20 kg, the gravitational attraction between them would be 3×10^{-7} N.) However, the Earth can attract the entire coyote-anvil system toward itself.

In the coyote-anvil-Earth system, the anvil and the coyote each have GPE with respect to the Earth. As the coyote and anvil both fall toward the Earth, that GPE changes to kinetic energy for both objects, causing both the coyote and the anvil to fall faster and faster…

Remember that potential energy requires:

- Two objects that exert some sort of attractive or repulsive force on each other. (In the case of GPE, this is the gravitational force, which is attractive.)
- A distance between the two objects over which at least one of the objects can move. (In the case of gravitational potential energy, this is the height above the ground.)

This means that a single, isolated object cannot have potential energy.

This also means that regardless of whether we consider gravitational potential energy to be caused by an object and the Earth attracting each other or by the Earth's gravitational field, ***gravitational potential energy can exist only in a system that contains the Earth*** (or other planet/star that has enough mass to exert a significant gravitational force).

Mechanical Energy

Mechanical energy is gravitational potential energy (GPE) plus kinetic energy. Because GPE and kinetic energy are easily interconverted, it is convenient to have a term that represents the combination of the two. There is no single variable for mechanical energy; in this text, we will sometimes use the abbreviation *TME*:

$$TME = U_g + K$$

Chemical Potential Energy

Chemical potential energy is the energy released when two or more chemical species react by forming chemical bonds. This energy comes from the electromagnetic forces that attract the atoms in these bonds.

When the bonds form, the energy that is released often causes an increase in the kinetic energy of the molecules, which we observe as a rise in temperature. When this happens, some of the thermal energy is released into the surroundings as heat. The chemical potential energy that is turned into heat is called the enthalpy of formation and is specific to each chemical compound.

However, chemical potential energy is more complicated than just thermal energy. Chemical potential energy can also be turned into thermal energy that is spread out into a very large number of separate microscopic energy states; thermal energy that is spread in this manner is called entropy. The combination of enthalpy and entropy is called free energy, and the total amount of energy that is released when a compound is formed is called the free energy of formation.

The study of energy in chemical reactions is called chemical thermodynamics, which is beyond the scope of this course, and is studied in detail in AP® Chemistry.

Electric Potential Energy

Electric potential energy is the energy released when electric particles with opposite charges come together, or when electric particles with like charges completely separate (to a distance of infinity).

Electric potential is the electric potential energy per unit of charge, and is what causes electrically charged particles to move through an electric circuit. The energy that moves these charged particles through a circuit ultimately comes from some other source, such as chemical potential energy (*i.e.,* a battery), mechanical energy (*i.e.,* a generator), *etc.* The difference in electric potential between two locations is called the electric potential difference, or more commonly the voltage. Electricity and electric potential are studied in detail in Physics 2.

Homework Problems

1. **(M)** Calculate the gravitational potential energy of a person with a mass of 60. kg at the top of a 10. m flight of stairs.

   Answer: 6 000 J

2. **(M)** Calculate the gravitational potential energy between the Earth and the moon. (You will need to use information from *Table T. Planetary Data* and *Table U. Sun & Moon Data* on page 615.)

   Answer: 7.62×10^{28} J