Conservation of Energy

Unit: Energy, Work & Power

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-1

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 3.4.A, 3.4.A.1, 3.4.A.2, 3.4.B, 3.4.B.1, 3.4.B.2, 3.4.B.3, 3.4.B.4, 3.4.C, 3.4.C.1, 3.4.C.2, 3.4.C.3

Mastery Objective(s): (Students will be able to…)

- Solve problems that involve the conversion of energy from one form to another.

Success Criteria:

- Correct equations are chosen for the situation.
- Variables are correctly identified and substituted correctly into equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Describe the type(s) of energy that an object has in different situations.

Tier 2 Vocabulary: work, energy, potential

Labs, Activities & Demonstrations:

- Golf ball loop-the-loop.
- Marble raceways.
- Bowling ball pendulum.

Notes:

In a *closed system* (meaning a system in which there is no exchange of matter or energy between the system and the surroundings), the total energy is constant. Energy can be converted from one form to another. When this happens, the increase in any one form of energy is the result of a corresponding decrease in another form of energy.

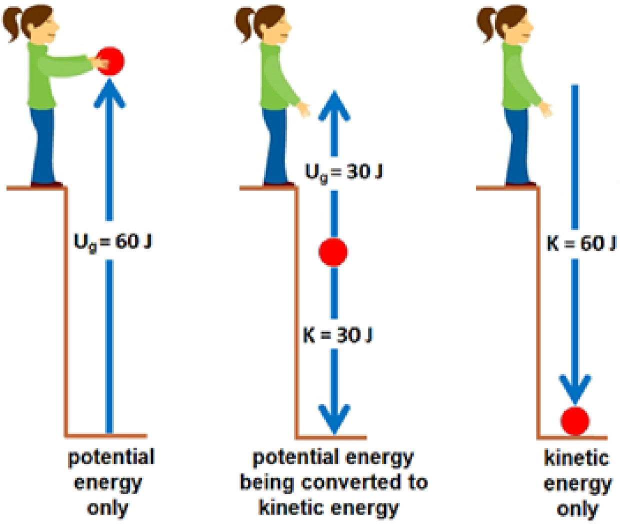
<u>mechanical energy</u>: kinetic energy plus gravitational potential energy.

In a system that has potential energy and kinetic energy, the total mechanical energy is given by:

$$TME = U + K$$

If there is no work done on a system and there are no nonconservative interactions, then the total mechanical energy of the system is constant.

In the following diagram, suppose that a student drops a ball with a mass of 2 kg from a height of 3 m.



Before the student lets go of the ball, it has 60 J of potential energy. As the ball falls to the ground, potential energy is gradually converted to kinetic energy. The potential energy continuously decreases, and the kinetic energy continuously increases, but the total energy is always 60 J. After the ball hits the ground, 60 J of work was done by gravity, and the 60 J of kinetic energy is converted to other forms. For example, if the ball bounces back up, some of the kinetic energy is converted back to potential energy. If the ball does not reach its original height, that means the rest of the energy was converted into other forms, such as thermal energy (the temperatures of the ball and the ground increase infinitesimally), sound, *etc.*

Work-Energy Theorem

We have already seen that work is the action of a force applied over a distance. A broader and more useful definition is that work is the change in the energy of an object or system. If we think of a system as having imaginary boundaries, then work is the flow of energy across those boundaries, either into or out of the system.

For a system that has only mechanical energy, work changes the amount of potential and/or kinetic energy in the system.

$$W = \Delta K + \Delta U$$

As mentioned earlier, although work is a scalar quantity, we ***use generally use a positive number for work coming into the system*** ("work is done on the system"), and ***a negative number for work going out of the system*** ("work is done by the system on the surroundings").

The units for work are sometimes shown as newton-meters (N·m). Because work is equivalent to energy, the units for work and energy—newton-meters and joules—are equivalent.

$$1\,\text{J} \equiv 1\,\text{N}\cdot\text{m} \equiv 1\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

Work-energy theorem problems will give you information related to the gravitational potential and/or kinetic energy of an object (such as its mass and a change in velocity) and ask you how much work was done.

A simple rule of thumb (meaning that it's helpful, though not always strictly true) is:

- Potential energy is energy in the *future* (energy that is available for use).
- Kinetic energy is energy in the *present* (the energy of an object that is currently in motion).
- Work is the result of energy in the *past* (energy that has already been added to or taken from an object).

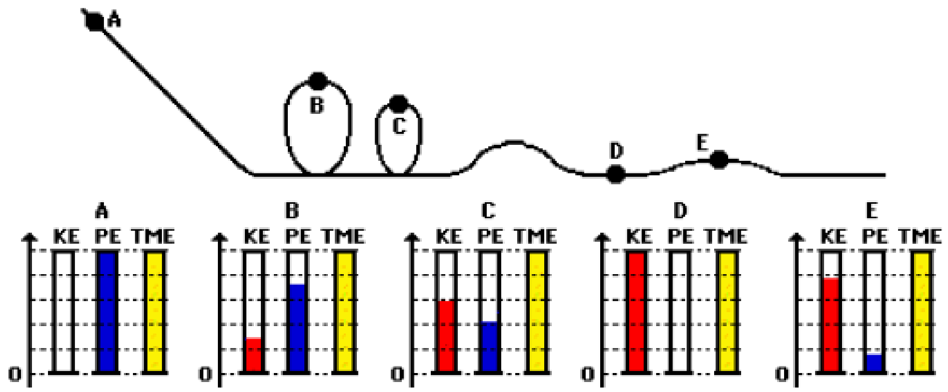
Conservation of Energy

In physics, if a quantity is "conserved", that means when some change happens to a system, there is the same amount of that quantity after the change as there was before.

Energy Bar Charts

A useful way to represent conservation of energy is through bar graphs that represent kinetic energy (*K* or "KE"), gravitational potential energy (U_g or "PE"), and total mechanical energy (TME). (We use the term "chart" rather than "graph" because the scale is usually arbitrary, and the chart is not meant to be used quantitatively.)

The following is an energy bar chart for a roller coaster, starting from point A and traveling through points B, C, D, and E.



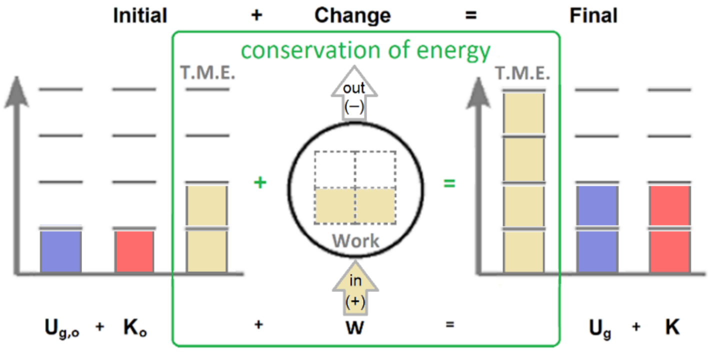
Notice, in this example, that:

3. The total mechanical energy always remains the same. (This is the case in conservation of energy problems if there is no work added to or removed from the system.)
4. KE is zero at point **A** because the roller coaster is not moving. All of the energy is PE, so PE = TME.
5. PE is zero at point **D** because the roller coaster is at its lowest point. All of the energy is KE, so KE = TME.
6. At all points (including points **A** and **D**), KE + PE = TME

It can be helpful to sketch energy bar charts representing the different points in complicated conservation of energy problems. If energy is being added to or removed from the system, add an Energy Flow diagram to show energy that is being added to or removed from the system.

Typically, energy bar charts represent the initial ("before") and final ("after") mechanical energy as a bar graph, and we represent the system in the center as a circle with work available to go in or out ("change").

For example, suppose a car started out moving (which means it started with kinetic energy or KE) and was at the top of a hill (which means it started with gravitational potential energy or GPE). The car ended up on top of a higher hill (which means it ended with more GPE), and was also going faster (which means it also ended with more KE). In order to make the car speed up while it was also going up a hill, the driver had to press the accelerator, causing the engine to do work. The energy bar chart diagram would look like this:



Notice that:

- The initial GPE and initial KE add up to the initial total mechanical energy (T.M.E.).
- The initial T.M.E. plus the work adds up to the final T.M.E.
- The final GPE and final K add up to the final T.M.E.
- The conservation equation is $U_{g,o} + K_o + W = U_g + K$

Charts like this are called "LOL charts" or "LOL diagrams," because the axes on the left and right side resemble the letter "L", and the circle for the system resembles the letter "O".

Once you have the types of energy, replace each type of energy with its equation:

- $W = \vec{F} \bullet \vec{d} = Fd\cos\theta$ $(= Fd$ if force & displacement are in the same direction)
- $U_g = mgh$
- $K = \frac{1}{2}mv^2$

For this problem, the equation would become:

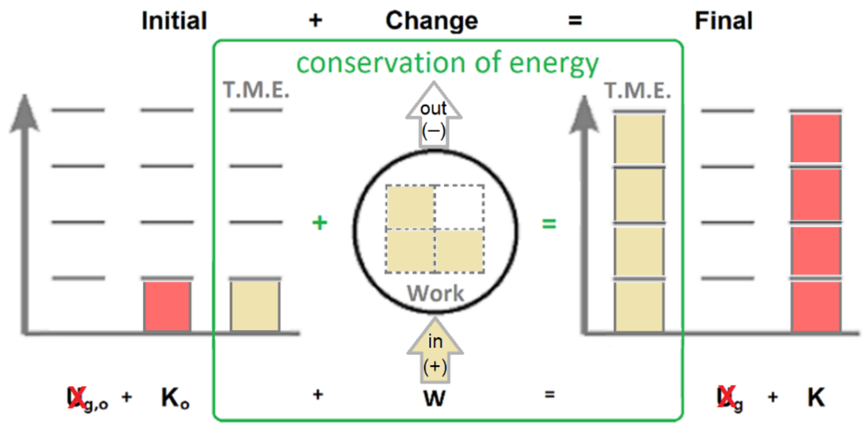
$$U_{g,o} \;+\; K_o \;+ W = \;U_g \;+\; K$$
$$mgh_o + \tfrac{1}{2}mv_o^2 + W = mgh + \tfrac{1}{2}mv^2$$

In most problems, one or more of these quantities will be zero, making the problem easier to solve.

Sample Problems:

Q: An 875 kg car accelerates from $22\,\frac{\text{m}}{\text{s}}$ to $44\,\frac{\text{m}}{\text{s}}$ on level ground.

a. Draw an LOL chart representing the initial and final energies and the flow of energy into or out of the system.



Notice that:

- *The height doesn't change, which means the gravitational potential energy is zero, both before and after, and the only type of energy the car has in this problem is kinetic.*
- *The car is moving, both before and after, so it has kinetic energy. The car is moving faster at the end, so it has more K.E. at the end than at the beginning, and therefore more T.M.E. at the end than at the beginning.*
- *Because the T.M.E. at the end was more than at the beginning, work must have gone into the system.*

b. What were the initial and final kinetic energies of the car? How much work did the engine do to accelerate it?

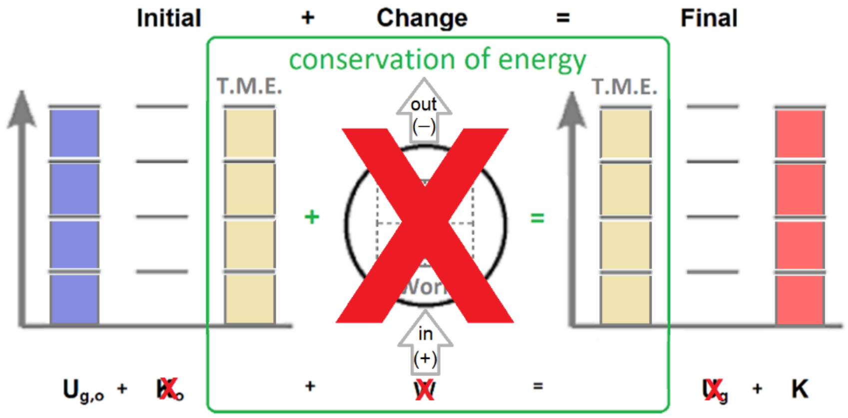
$$K_o + W = K$$
$$\tfrac{1}{2}mv_o^2 + W = \tfrac{1}{2}mv^2$$
$$\tfrac{1}{2}(875)(22)^2 + W = \tfrac{1}{2}(875)(44)^2$$
$$211750 + W = 847000$$
$$W = 847000 - 211750 = 635250\text{ J}$$

Answers: K_i = 211 750 J; K_f = 847 000 J; W = 635 250 J

Q: An 80 kg physics student falls off the roof of a 15 m high school building. How much kinetic energy does he have when he hits the ground? What is his final velocity?

A: There are two approaches to answer this question.

1. Recognize that the student's potential energy at the top of the building is entirely converted to kinetic energy when he hits the ground.



Notice that:

- *No work is done on the student.* * *Total mechanical energy therefore is the same at the beginning and end.*
- *Initially, the student has only gravitational potential energy. At the end, the student has no potential energy and all of his energy has been converted to kinetic.*

$$U_{g,o} = K$$

$$mgh_o = \tfrac{1}{2}mv^2$$

$$(80)(10)(15) = \tfrac{1}{2}(80)v^2$$

$$12000 = 40v^2 \qquad \frac{12000}{40} = 300 = v^2 \qquad v = \sqrt{300} = 17.3\,\tfrac{\text{m}}{\text{s}}$$

Answers: K_f = 12 000 J; v_f = 17.3 $\tfrac{\text{m}}{\text{s}}$

* Actually, we have two options. This solution assumes the Earth-student system, in which no outside energy is added or removed, which means there is no work, and gravitational potential energy is converted to kinetic energy. If we consider the student-only system, then there is no potential energy, and gravity does work on the student to increase their kinetic energy: $W = \vec{F}_g \bullet \vec{d} = mgh$. The two situations are equivalent and give the same answer.

2. You can also use equations of motion to find the student's velocity when he hits the ground, based on the height of the building and acceleration due to gravity. Then use the formula $K=\frac{1}{2}mv^2$.

$$d=\frac{1}{2}at^2$$
$$15=\frac{1}{2}(10)t^2$$
$$t^2=3$$
$$t=\sqrt{3}=1.732$$
$$v=at$$
$$v=(10)(1.732)=17.32\,\tfrac{\text{m}}{\text{s}}$$

$$K=\frac{1}{2}mv^2$$
$$K=\frac{1}{2}(80)(17.32)^2$$
$$K=12\,000\text{ J}$$

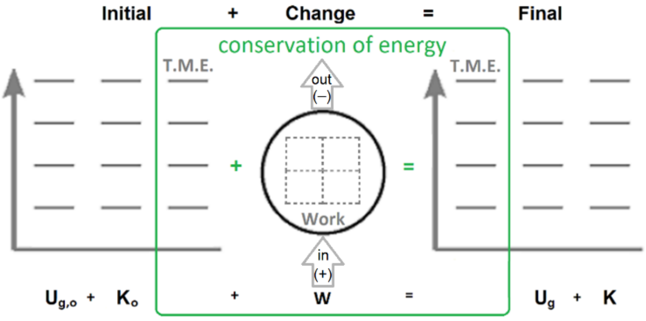
Answers: K_f = 12 000 J; v_f = 17.3 $\frac{\text{m}}{\text{s}}$ as before.

As is the case with this problem, it is often easier to solve motion problems involving free fall using conservation of energy than it is to use the equations of motion.

Homework Problems

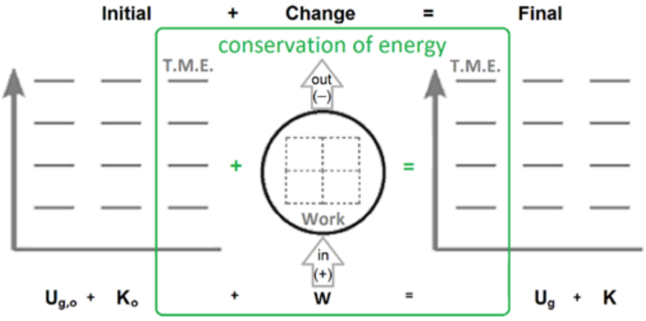
If a problem includes an energy bar chart, you must fill it out *<u>in addition to</u>* calculating the numerical answer.

1. **(S)** A 70. kg pole vaulter converts the kinetic energy of running at ground level into the potential energy needed to clear the crossbar at a height of 4.0 m above the ground. What is the minimum velocity that the pole vaulter must have when taking off from the ground in order to clear the bar?



Answer: $8.9\,\frac{\text{m}}{\text{s}}$

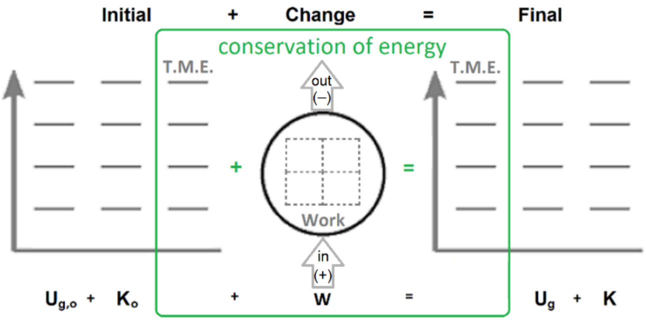
2. **(S)** A 10.0 kg lemur swings on a vine from a point which is 40.0 m above the jungle floor to a point which is 15.0 m above the floor. If the lemur was moving at $2.0\,\frac{\text{m}}{\text{s}}$ initially, what will be its velocity at the 15.0 m point?



Answer: $22.4\,\frac{\text{m}}{\text{s}}$

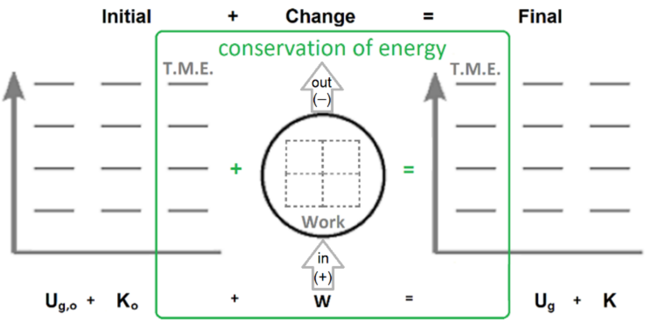
3. **(M)** A 7.25 kg bowling ball hanging from a chain is held against a student's chin, which is 1.45 m above the floor. The student releases the bowling ball, which swings across the room and back, stopping just before it hits the student.

   a. **(M)** What is the maximum velocity of the bowling ball?



   Answer: 5.39 $\frac{m}{s}$

   b. **(M)** What is the velocity of the bowling ball when it is 0.25 m below the person's chin (*i.e.,* 1.2 m above the floor)?

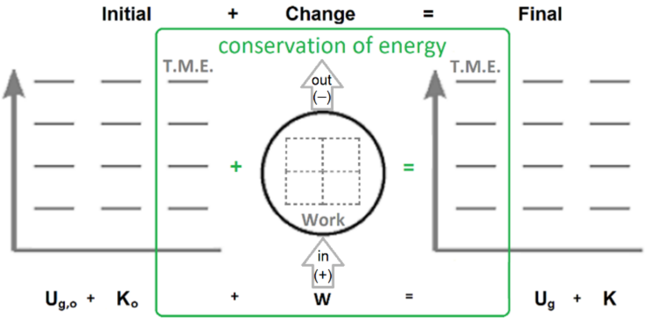


   Answer: 2.24 $\frac{m}{s}$

honors & AP®

4. **(M – honors & AP®; A – CP1)** A roller coaster car with mass m is launched, from ground level with a velocity of v_o. Neglecting friction, how fast will it be moving when it reaches the top of a loop, which is a distance of h above the ground?

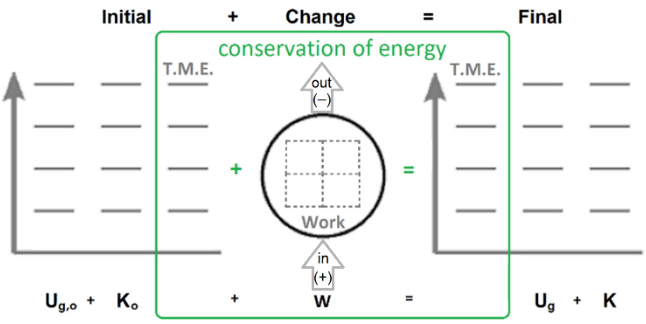
   (*If you are not sure how to solve this problem, do #5 below and use the steps to guide your algebra.*)



   Answer: $\sqrt{v_o^2 - 2gh}$

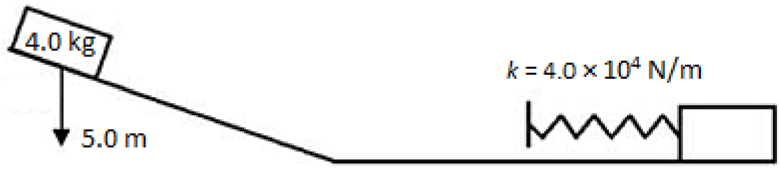
5. **(S – honors & AP®; M – CP1)** A 500. kg roller coaster car is launched, from ground level, at $20.\,\frac{m}{s}$. Neglecting friction, how fast will it be moving when it reaches the top of a loop, which is 15 m above the ground?

   (*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #3 above as a starting point if you have already solved that problem.*)



   Answer: $10\,\frac{m}{s}$

6. **(M – honors & AP®; S – CP1)** A 4.0 kg block is released from a height of 5.0 m on a frictionless ramp. When the block reaches the bottom of the ramp, it slides along a frictionless surface and hits an open spring with a spring constant of $4.0\times10^4\,\frac{\text{N}}{\text{m}}$ as shown in the diagram below:



What is the maximum distance that the spring is compressed after the impact?

Answer: 0.10 m

7. **(M – honors & AP®; S – CP1)** A 1.6 kg block is pressed against an open spring that has a spring constant of $1000\,\frac{\text{N}}{\text{kg}}$. The spring is compressed a distance of 0.02 m, and the block is released from rest onto a frictionless surface. What is the speed of the block as it moves away from the spring? (*Hint: The block separates from the spring when the spring is at its equilibrium point, because the spring starts slowing down at that point.*)

Answer: $0.5\,\frac{\text{m}}{\text{s}}$

8. **(S)** The engine of a 0.200 kg model rocket provides a constant thrust of 10. N for 1.0 s.

   a. **(S)** What is the net force that the engine applies to the rocket?
   (*Hint: This is a forces problem. Draw a free-body diagram.*)

   Answer: 8.0 N

   b. **(S)** What is the velocity of the rocket when the engine shuts off? What is its height at that time?
   (*Hint: Use* $F_{net} = ma$ *to find the acceleration. Then use motion equations to find the velocity and height.*)

   Answer: $v = 40.\,\frac{\text{m}}{\text{s}}$; h = 20. m

   c. **(S)** What is the final height of the rocket?
   (*Hint: calculate the kinetic energy of the rocket when the engine shuts off. This will become additional potential energy when the rocket reaches its highest point. Add this to the work from part b above to get the total energy at the end, which is all potential. Finally, use* $U_g = mgh$ *to find the height.*)

   Answer: 100 m

   d. **(S)** How much work did the engine do on the rocket?

   Answer: 200 N·m