Power

Unit: Energy, Work & Power

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP Physics 1 Learning Objectives/Essential Knowledge (2024): 3.5.A, 3.5.A.1, 3.5.A.2, 3.5.A.3, 3.5.A.4

Mastery Objective(s): (Students will be able to…)

- Calculate power as a rate of energy consumption.

Success Criteria:

- Variables are correctly identified and substituted correctly into the appropriate equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the difference between total energy and power.

Tier 2 Vocabulary: power

Notes:

power: a measure of the rate at which energy is applied or work is done. The average power is calculated by dividing work (or energy) by time.

$$P_{avg} = \frac{\Delta E}{t} = \frac{W}{t} = \frac{\Delta K + \Delta U}{t}$$

Power is a scalar quantity and is measured in Watts (W).

$$1\,\text{W} = 1\,\tfrac{\text{J}}{\text{s}} = 1\,\tfrac{\text{N}\cdot\text{m}}{\text{s}} = 1\,\tfrac{\text{kg}\cdot\text{m}^2}{\text{s}^3}$$

Note that utility companies measure energy in kilowatt-hours. This is because $P = \frac{W}{t}$, which means energy = $W = Pt$.

Because 1 kW = 1000 W and 1 h = 3600 s, this means
1 kWh = (1000 W)(3600 s) = 3 600 000 J

Because $W = F_{||}d$, this means $P_{avg} = \frac{F_{||}d}{t} = F_{||}\left(\frac{d}{t}\right) = F_{||}v_{avg}$

However, if we use the instantaneous velocity instead of the average velocity, this equation gives us the instantaneous power:

$$P_{inst} = F_{||}v = Fv\cos\theta$$

AP®

Power in Rotational Systems

In a rotational system, the formula for power looks similar to the equation for power in linear systems, with force replaced by torque and linear velocity replaced by angular velocity:

$P = Fv$	$P = \tau\omega$
linear	**rotational**

Solving Power Problems

Many power problems require you to calculate the amount of work done or the change in energy, which you should recall is:

$W = F_{\parallel} d$	if the force is caused by linear displacement
$\Delta K_t = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2$ * $= \frac{1}{2}m(v^2 - v_o^2)$	if the change in energy was caused by a change in velocity
$\Delta U_g = mgh - mgh_o$ $= mg\Delta h$	if the change in energy was caused by a change in height

Solving Rotational Power Problems

AP®

Power is also applicable to rotating systems:

$W = \tau \Delta\theta$	if the work is produced by a torque
$\Delta K_r = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_o^2$ $= \frac{1}{2}I(\omega^2 - \omega_o^2)$	if the change in energy was caused by a change in angular velocity

Once you have the work or energy, you can plug it in for either W, ΔK or ΔU, use the appropriate parts of the formula:

$$P = \frac{W}{t} = \frac{\Delta K + \Delta U}{t} = Fv = \tau\omega$$

and solve for the missing variable.

* K_t is translational kinetic energy. This is the only form of kinetic energy used in CP1 and honors physics. The subscript *t* is used here to distinguish translational kinetic energy from rotational kinetic energy (K_r), because both are used in AP® Physics.

Sample Problems:

Q: What is the power output of an engine that pulls with a force of 500. N over a distance of 100. m in 25 s?

A: $W = Fd = (500)(100) = 50\,000\text{ J}$

$$P = \frac{W}{t} = \frac{50\,000}{25} = 2000\text{ W}$$

Q: A 60. W incandescent light bulb is powered by a generator that is powered by a falling 1.0 kg mass on a rope. Assuming the generator is 100 % efficient (*i.e.,* no energy is lost when the generator converts its motion to electricity), how far must the mass fall in order to power the bulb at full brightness for 1.0 minute?

A:
$$P = \frac{\Delta U_g}{t} = \frac{mg\,\Delta h}{t}$$

$$60 = \frac{(1)(10)\,\Delta h}{60}$$

$$3600 = 10\,\Delta h$$

$$\Delta h = \frac{3600}{10} = 360\text{ m}$$

Note that 360 m is approximately the height of the Empire State Building. This is why changing from incandescent light bulbs to more efficient compact fluorescent or LED bulbs can make a significant difference in energy consumption!

Homework Problems

1. **(S)** A small snowmobile has a 9 000 W (12 hp) engine. It takes a force of 300. N to move a sled load of wood along a pond. How much time will it take to tow the wood across the pond if the distance is measured to be 850 m?

   Answer: 28.3 s

2. **(M)** A winch, which is rated at 720 W, is used to pull an all-terrain vehicle (ATV) out of a mud bog for a distance of 2.3 m. If the average force applied by the winch is 1 500 N, how long will the job take?

   Answer: 4.8 s

3. **(S)** What is your power output if you have a mass of 65 kg and you climb a 5.2 m vertical ladder in 10.4 s?

   Answer: 325 W

4. **(M)** Jack and Jill went up the hill. (The hill was 23m high.) Jack was carrying a 21 kg pail of water.

   a. **(M)** Jack has a mass of 75 kg and he carried the pail up the hill in 45 s. How much power did he apply?

      Answer: 490.7 W

   b. **(M)** Jill has a mass of 55 kg, and she carried the pail up the hill in 35 s. How much power did she apply?

      Answer: 499.4 W

honors & AP®

5. **(M – honors & AP®; A – CP1)** The maximum power output of a particular crane is P. What is the fastest time, t, in which this crane could lift a crate with mass m to a height h?

   (*If you are not sure how to do this problem, do #6 below and use the steps to guide your algebra.*)

   Answer: $t = \frac{mgh}{P}$

6. **(S – honors & AP®; M – CP1)** The maximum power output of a particular crane is 12 kW. What is the fastest time in which this crane could lift a 3 500 kg crate to a height of 6.0 m?

   (*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #5 above as a starting point if you have already solved that problem.*)

   Hint: Remember to convert kilowatts to watts.

   Answer: 17.5 s

AP®

7. **(M – AP®; A – honors & CP1)** A 30 cm diameter solid cylindrical flywheel with a mass of 2 500 kg was accelerated from rest to an angular velocity of 1 800 RPM in 60 s.

   a. How much work was done on the flywheel?

      Answer: $5.0 \times 10^5\ \text{N} \cdot \text{m}$

   b. How much power was exerted?

      Answer: $8.3 \times 10^3\ \text{W}$