Logarithms

Unit: Laboratory & Mathematics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Knowledge/Understanding:

- What logarithms represent and an intuitive understanding of logarithmic quantities.

Skills:

- Use logarithms to solve for a variable in an exponent.

Language Objectives:

- Understand the use of the terms “exponential” and “logarithm” and understand the vernacular use of “log” (otherwise a Tier 1 word) as an abbreviation for “logarithm”.

Tier 2 Vocabulary: function

Notes:

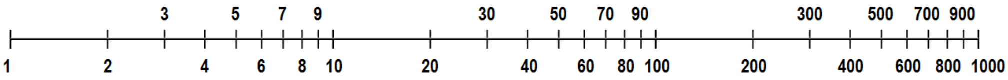
The logarithm may well be the least well-understood function encountered in high school mathematics.

The simplest logarithm to understand is the base-ten logarithm. You can think of the (base-ten) logarithm of a number as the number of zeroes after the number.

x		$\log_{10}(x)$
100 000	10^5	5
10 000	10^4	4
1 000	10^3	3
100	10^2	2
10	10^1	1
1	10^0	0
0.1	10^{-1}	−1
0.01	10^{-2}	−2
0.001	10^{-3}	−3
0.000 1	10^{-4}	−4
0.000 01	10^{-5}	−5

As you can see from the above table, the logarithm of a number turns a set of numbers that vary exponentially (powers of ten) into a set that vary linearly.

You can get a visual sense of the logarithm function from the logarithmic number line below:



Notice that the *distance* from 1 to 10 is the same as the *distance* from 10 to 100 and from 100 to 1000. In fact, the relative distance to every number on this number line is the logarithm of the number.

x	$\log_{10}(x)$	distance from beginning of number line
10^0	0	0
$10^{0.5} \approx 3.16$	0.5	½ cycle
$10^1 = 10$	1	1 cycle
$10^2 = 100$	2	2 cycles
$10^3 = 1000$	3	3 cycles

By inspection, you can see that the same is true for numbers that are not exact powers of ten. The logarithm function compresses correspondingly more as the numbers get larger.

The most useful mathematical property of logarithms is that they move an exponent into the linear part of the equation:

$$\log_{10}(10^3) = 3\log_{10}(10) = (3)(1) = 3$$

In fact, the logarithm function works the same way for any base, not just 10:

$$\log_2(2^7) = 7\log_2(2) = (7)(1) = 7$$

(In this case, the word "base" means the base of the exponent.) The general equation is:

$$\log_x(a^b) = b\log_x(a)$$

This is a powerful tool in solving for the exponent in an equation. This is, in fact, precisely the purpose of using logarithms in most mathematical equations.

Sample problem:

Q: Solve $3^x = 15$ for x.

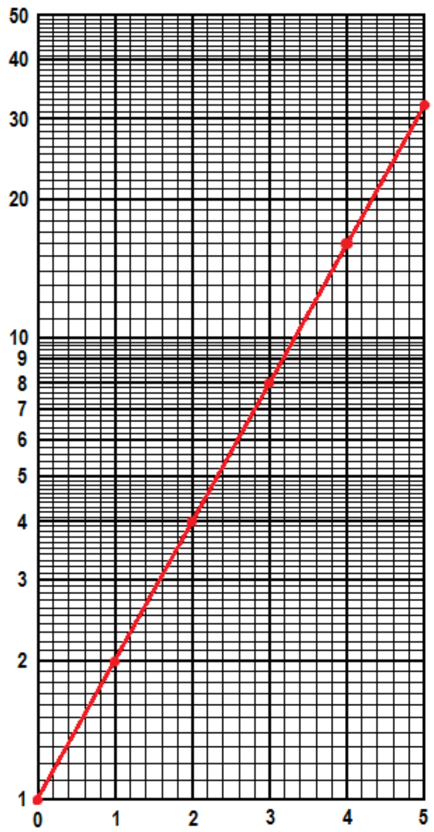
A: Take the logarithm (any base) of both sides. (Note that writing "log" without supplying a base implies that the base is 10.)

$$\log(3^x) = \log(15)$$
$$x \log(3) = \log(15)$$
$$(x)(0.477) = 1.176$$
$$x = \frac{1.176}{0.477} = 2.465$$

This is the correct answer, because $3^{2.465} = 15$

Logarithmic Graphs

A powerful tool that follows from this is using logarithmic graph paper to solve equations. If you plot an exponential function on semilogarithmic ("semi-log") graph paper (meaning graph paper that has a logarithmic scale on one axis but not the other), you get a straight line.



The graph at the right is the function $y = 2^x$. Notice where the following points appear on the graph:

Domain	Range
0	1
1	2
2	4
3	8
4	16
5	32

Notice also that you can use the graph to find intermediate values. For example, at x = 2.6, the graph shows that y = 6.06.

Natural Logarithms

The natural logarithm comes from calculus—it is the solution to the problem:

$$\int \frac{1}{x}\,dx = \log_e(x)$$

where the base of this logarithm, "*e*," is a constant (sometimes called "Euler's number") that is an irrational number equal to approximately 2.71828 18284 59045…

The natural logarithm is denoted "ln", so we would actually write:

$$\int \frac{1}{x}\,dx = \ln(x)$$

The number "*e*" is often called the exponential function. In an algebra-based physics class, the exponential function appears in some equations whose derivations come from calculus, notably some of the equations relating to resistor-capacitor (RC) circuits.

Finally, just as:

$$\log(10^x) = x \text{ and } 10^{\log(x)} = x$$

it is similarly true that:

$$\ln(e^x) = x \text{ and } e^{\ln(x)} = x$$