Parallel Circuits (Resistance Only)

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.5.A, 11.5.A.1, 11.5.A.1.ii, 11.5.A.2, 11.5.A.2.ii, 11.5.A.2.iii

Mastery Objective(s): (Students will be able to…)

- Calculate voltage, current, resistance and power in parallel circuits.

Success Criteria:

- Correct relationships are applied for voltage, current, resistance and power in parallel circuits.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the relationships for voltages, current, resistance and power in parallel circuits.

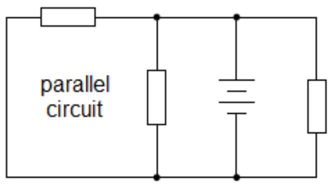
Tier 2 Vocabulary: parallel

Labs, Activities & Demonstrations:

- Circuit with light bulbs wired in parallel.

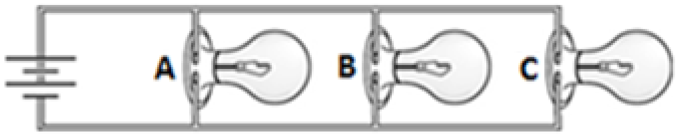
Notes:

<u>parallel</u>: Components in <u>parallel</u> lie in separate paths.



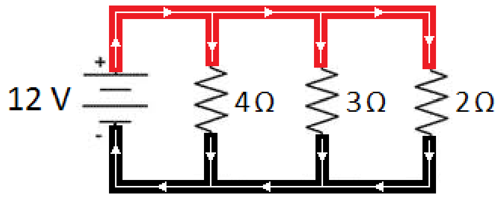
In a parallel circuit, the current divides at each junction, with some of the current flowing through each path. If the current is interrupted in one path, current can still flow through the other paths.

For example, in the parallel circuit below, if any of light bulbs A, B, or C is removed, current still flows through the remaining bulbs.



Because the voltage across each branch is equal to the total voltage, all of the bulbs will light up with full brightness, regardless of how many bulbs are in the circuit. (However, each separate light bulb draws the same amount of current as if it were the only thing in the circuit, so ***the total current in the circuit increases with each new branch***. This is why you trip a circuit breaker or blow a fuse if you have too many high-power components plugged into the same circuit.)

The following circuit shows a battery and three resistors in parallel:



Current

The current divides at each junction (as indicated by the arrows). This means the current through each path must add up to the total current:

$$I_{total} = I_1 + I_2 + I_3 + \ldots$$

Voltage

In a parallel circuit, the potential difference (voltage) across the battery is always the same (12 V in the above example). Therefore, the potential difference between *any point* on the top wire and *any point* on the bottom wire must be the same. This means the voltage is the same across each path:

$$V_{total} = V_1 = V_2 = V_3 = \ldots$$

Resistance

If there are multiple resistors, the effective resistance of each path becomes less as there are more paths for the current to flow through. The total resistance is given by the formula:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Some students find it confusing that the combined resistance of a group of resistors in series is always less than any single resistor by itself.

Power

Just as with series circuits, in a parallel circuit, any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \ldots$$

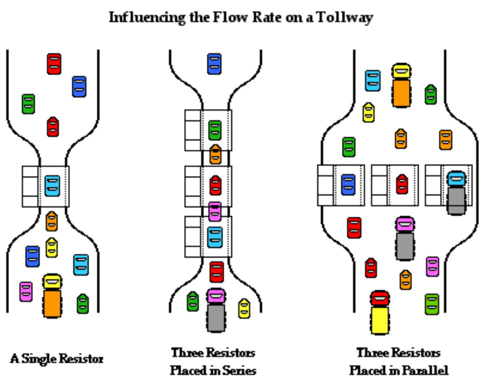
Electric current is analogous to water in a pipe:

- The current corresponds to the flow rate.
- The voltage corresponds to the pressure between one side and the other.
- The resistance would correspond to how small the pipe is (i.e., how hard it is to push water through the pipes). A smaller pipe has more resistance; a larger pipe will let water flow through more easily than a smaller pipe.

The voltage (pressure) drop is the same between one side and the other because less water flows through the smaller pipes and more water flows through the larger ones until the pressure is completely balanced. The same is true for electrons in a parallel circuit.

The water will flow through the *set* of pipes more easily than it would through any one pipe by itself. The same is true for resistors. As you add more resistors, you add more pathways for the current, which means less total resistance.

Another common analogy is to compare resistors with toll booths on a highway.



One toll booth slows cars down while the drivers pay the toll.

Multiple toll booths in a row would slow traffic down more. This is analogous to resistors in series.

Multiple toll booths next to each other (in parallel) make traffic flow faster because there are more paths for the cars to follow. Each toll booth further reduces the resistance to the flow of traffic.

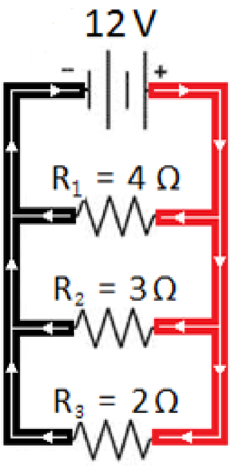
Calculations

Just as with series circuits, you can calculate the voltage, current, resistance, and power of each component and the entire circuit using the equations:

$$\Delta V = IR \qquad P = I\Delta V = I^2R = \frac{(\Delta V)^2}{R}$$

Sample Problem

Suppose we are given the following parallel circuit:



and we are asked to fill in the following table:

	Var.	R_1	R_2	R_3	Total
Voltage (V)	ΔV				12
Current (A)	I				
Resistance (Ω)	R	4	3	2	
Power (W)	P				

Because this is a parallel circuit, the total voltage equals the voltage across all three branches, so we can fill in 12 V for each resistor.

The next thing we can do is use $\Delta V = IR$ to find the current through each resistor:

	Var.	R_1	R_2	R_3	Total
Voltage (V)	ΔV	***12***	***12***	***12***	12
Current (A)	I	***3***	***4***	***6***	***13***
Resistance (Ω)	R	4	3	2	
Power (W)	P				

In a parallel circuit, the current adds, so the total current is 3 + 4 + 6 = 13 A.

Now, we have two ways of finding the total resistance. We can use $\Delta V = IR$ with the total voltage and current:

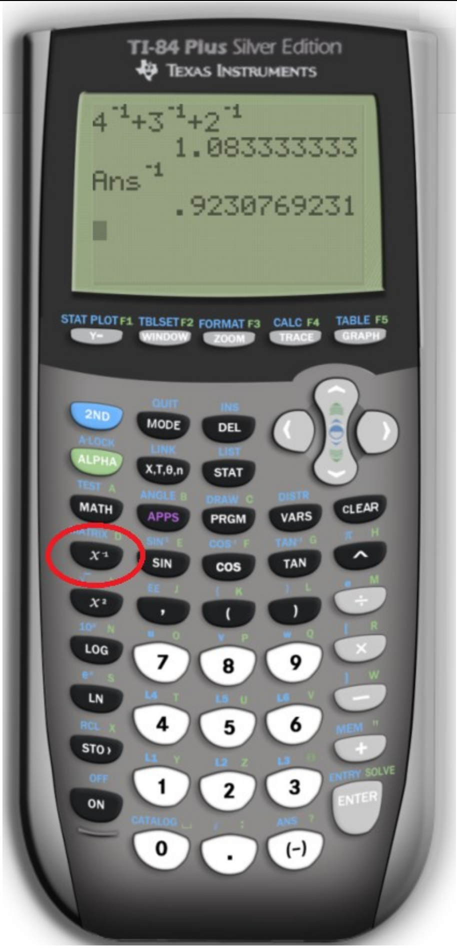
$$\Delta V = IR$$
$$12 = 13R$$
$$R = \frac{12}{13} = 0.923\,\Omega$$

or we can use the formula for resistances in parallel:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{total}} = \frac{1}{4} + \frac{1}{3} + \frac{1}{2} = \frac{3}{12} + \frac{4}{12} + \frac{6}{12} = \frac{13}{12}$$

$$R_{total} = \frac{12}{13} = 0.923\,\Omega$$



Instead of finding a common denominator, it is usually easiest to use the $1/x$ or x^{-1} function on a calculator. On a TI-84 calculator, the calculation would look like the picture to the right. (The x^{-1} button is circled in the picture.)

Now we have:

	Var.	R_1	R_2	R_3	Total
Voltage (V)	ΔV	12 V	12 V	12 V	12 V
Current (A)	I	3 A	4 A	6 A	13 A
Resistance (Ω)	R	4 Ω	3 Ω	2 Ω	***0.923 Ω***
Power (W)	P				

As we did with series circuits, we can calculate the power, using $P = I\,\Delta V$:

	Var.	R_1	R_2	R_3	Total
Voltage (V)	ΔV	12	12	12	12
Current (A)	I	3	4	6	13
Resistance (Ω)	R	4	3	2	0.923
Power (W)	P	***36***	***48***	***72***	***156***

Batteries in Parallel

One question that has not been answered yet is what happens when batteries are connected in parallel.

If the batteries have the same voltage, the potential difference (voltage) remains the same, but the total current is the combined current from the two batteries.

However, if the batteries have different voltages there is a problem, because each battery attempts to maintain a constant potential difference (voltage) between its terminals. This results in the higher voltage battery overcharging the lower voltage battery.

Remember that physically, batteries are electrochemical cells—small solid-state chemical reactors with redox reactions taking place in each cell. If one battery overcharges the other, material is deposited on the cathode (positive terminal) until the cathode becomes physically too large for its compartment, at which point the battery bursts, and the chemicals leak out.

Light Bulbs

Electric light bulbs, which were commercialized by Thomas Edison in 1880, use electrical energy to produce light. For about 100 years, most light bulbs were incandescent bulbs, which pass electricity through a tungsten filament until it glows white-hot.

Newer light bulbs, such as fluorescent bulbs or light-emitting diode (LED) bulbs, produce similar amounts of light but much less heat, making them much more energy efficient.

The S.I. unit for light intensity is the lumen (lm).

Intensity (lm)	560	800	1100	1600
Incandescent Bulb Power (W)*	40	60	75	100
Incandescent Bulb Resistance (Ω)†	360	240	192	144
LED Bulb Power (W)*	2–3	7–10	10–15	15–20
LED Bulb Resistance (Ω)	5760	1700	1150	820

* Assuming a 120 V household circuit.

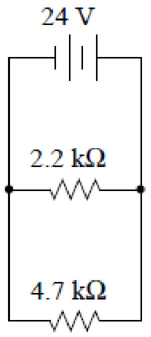
† Note that incandescent light bulbs are not ohmic resistors, meaning their resistance changes as the current changes because the heat generated affects the resistivity of the tungsten filament.

The amount of light that a light bulb produces is proportional to the amount of power it consumes. For over 100 years, incandescent bulbs were sold according to their power rating (in watts), and people developed an understanding of how much light a typical incandescent bulb would produce in a 120 V household circuit. Note that the power consumed by a light bulb is a function of both the current and the voltage: $P = I\Delta V$.

- In a parallel circuit (such as you would find in your house), the voltage is constant. The resistance of a component determines how much current it draws. A component with lower resistance (*e.g.,* a light bulb with a higher "wattage" rating) draws more current and therefore uses more power. This means a bulb with a higher "wattage" rating will be ***brighter*** in a parallel circuit.
- In a series circuit, the current through each bulb is constant (because there is only one path). The voltage across the entire circuit is fixed, but the voltage across each component splits according to the component's resistance. A component with less resistance (*e.g.,* a light bulb with a higher "wattage" rating) "uses up" less of the total voltage and therefore uses less power. This means a bulb with a higher "wattage" rating will be ***dimmer*** in a series circuit.

Homework Problems

1. **(M)** Fill in the table for the following circuit:

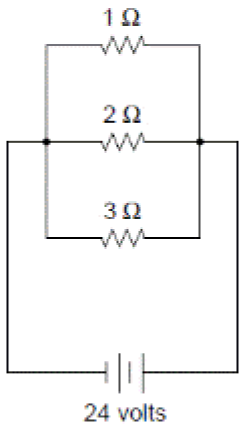


	Var.	R_1	R_2	Total
Voltage (V)	ΔV			24
Current (mA)	I			
Resistance (kΩ)	R	2.2	4.7	
Power (mW)	P			

Note: the above units can be used directly with $\Delta V = IR$ and $P = I\Delta V$ without converting. (Because volts = milliamps × kiloöhms and milliwatts = milliamps × volts.)

(The space below is intentionally left blank for calculations.)

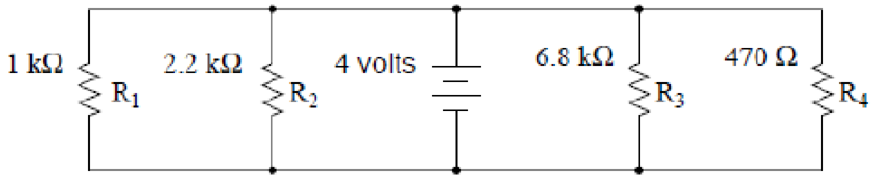
2. **(S)** Fill in the table for the following circuit:



	Var.	R_1	R_2	R_3	Total
Voltage (V)	ΔV				24
Current (A)	I				
Resistance (Ω)	R	1	2	3	
Power (W)	P				

(The space below is intentionally left blank for calculations.)

3. **(M)** Fill in the table for the following circuit:



	Var.	**R_1**	**R_2**	**R_3**	**R_4**	**Total**
Voltage (V)	ΔV					4
Current (mA)	I					
Resistance (kΩ)	R	1	2.2	6.8	0.47	
Power (mW)	P					

Note: the above units can be used directly with $\Delta V = IR$ and $P = I\Delta V$ without converting. (Because volts = milliamps × kiloöhms and milliwatts = milliamps × volts.)

(The space below is intentionally left blank for calculations.)