Capacitance

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 10.6.A, 10.6.A.1, 10.6.A.2, 10.6.A.2.i, 10.6.A.2.ii, 10.6.A.3, 10.6.A.3.i, 10.6.A.3.ii, 10.6.A.4, 10.6.A.5, 10.6.A.6

Mastery Objective(s): (Students will be able to…)

- Solve problems involving relationships between capacitance, charge and voltage.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Describe what a capacitor does.

Tier 2 Vocabulary: charge, capacitance

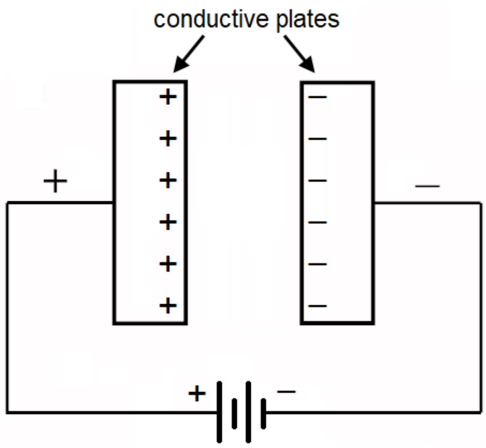
Labs, Activities & Demonstrations:

- build a capacitor

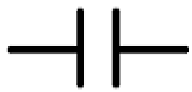
Notes:

capacitor: an electrical component that stores electrical charge but does not allow current to flow through.

The simplest capacitors are made of metal plates that are separated by some fixed distance.



The electrical symbol for a capacitor is a representation of the parallel plates.



When an electric potential difference (voltage) is applied to a circuit that contains a capacitor, one side of the capacitor will acquire a positive charge, and the other side will acquire an equal negative charge. This process is called *charging the capacitor*.

When a charged capacitor is placed in a circuit (perhaps it was charged previously, and then the voltage source was switched off), charge flows out of the capacitor into the circuit. This process is called *discharging the capacitor*.

No current actually flows through the capacitor, but as it charges, the positive charges that accumulate on one side of the capacitor repel positive charges from the other side into the rest of the circuit. (The same is true from the negative charges on the other side.) This means that ***an uncharged capacitor acts like a wire*** when it first begins to charge.

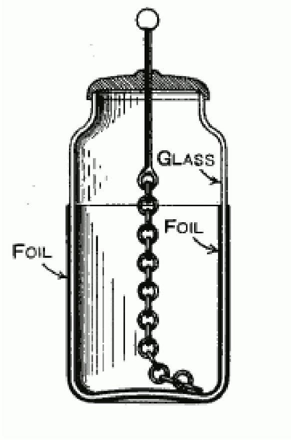
As charge builds up in the capacitor, its electric potential increases on the positive side, and decreases on the negative side. This means that the potential difference between the battery (or other voltage source) and the capacitor decreases, due to the charges repelling the additional charges that the battery is trying to add.

Eventually, the capacitor becomes fully charged, and the electric potential difference in the circuit is unable to add any more charge. When this happens, no more charges flow into or out of the capacitor. This means that ***a fully-charged capacitor*** *in a circuit that has a power supply (e.g., a battery)* ***acts like an open switch or a broken wire***.

If you disconnect the battery and reconnect the fully-charged capacitor to a circuit that allows the capacitor to discharge, charges will flow out of the capacitor and through the circuit. This means that ***a fully charged capacitor in a circuit without a separate power supply acts like a battery*** when it first begins to discharge.

Toys from joke shops that shock people use simple battery-and-capacitor circuits. The battery charges the capacitor gradually over time until a significant amount of charge has built up. When the person grabs the object, the person completes a circuit that discharges the capacitor, resulting in a sudden, unpleasant electric shock.

The first capacitors were made independently in 1745 by the German cleric Ewald Georg von Kleist, and by the Dutch scientist Pieter van Musschenbroek. Both von Kleist and van Musschenbroek lined a glass jar with metal foil on the outside and filled the jar with water. (Recall that water with ions dissolved in it conducts electricity.) Both scientists charged the devices with electricity, and both received a severe shock when they accidentally discharged the jars through themselves.



This type of capacitor is named after is called a Leyden jar, after the city of Leiden (Leyden) where van Musschenbroek lived.

Modern Leyden jars are lined on the inside and outside with conductive metal foil. As a potential difference is applied between the inside and outside of the jar, charge builds up between them. The glass, which acts as an insulator (a substance that does not conduct electricity), keeps the two pieces of foil separated and does not allow the charge to flow through.

Because the thickness of the jar is more or less constant, the Leyden jar behaves like a parallel plate capacitor.

Shortly after the invention of the Leyden jar, Daniel Gralath discovered that he could connect several jars in parallel to increase the total possible stored charge. The American scientist and statesman Benjamin Franklin compared this idea with a "battery" of cannon. (The original meaning of the term "battery" was a collection of cannons for the purpose of battering the enemy.) The term is now used to describe a similar arrangement of electrochemical cells.

Franklin's most famous experiment was to capture the charge from a lightning strike in Leyden jars, proving that lightning is an electric discharge.

capacitance: a measure of the ability of a capacitor to store charge. Capacitance is measured in farads (F), named after the English physicist Michael Faraday.

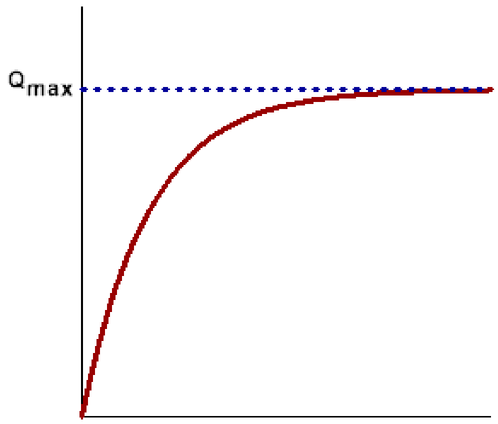
Capacitance is the ratio of the charge stored by a capacitor to the voltage applied:

$$C = \frac{Q}{\Delta V} \quad \text{which is often represented as} \quad Q = C\Delta V$$

Thus, one farad is one coulomb per volt. Note, however, that one farad is a ridiculously large amount of capacitance. The capacitors in most electrical circuits are in the millifarad (mF) to picofarad (pF) range.

Capacitance is the theoretical limit of the charge that a capacitor could store at a given potential difference (voltage) if the charge were allowed to build up over an infinite amount of time.

As described above, as a capacitor is charged, the positive side increasingly repels additional positive charges coming from the voltage source, and the negative side increasingly repels additional negative charges. This means that the capacitor charges rapidly at first, but the amount of charge stored decreases exponentially as the charge builds up.



Note that Q_{max} is sometimes labeled Q_o. Be careful—in this case, the subscript 0 does ***not*** necessarily mean at time = 0.

Energy Stored in a Capacitor

Recall that energy is the ability to do work, and that $W = \Delta U$. Because $W = qV$, if we keep the voltage constant and add charge to the capacitor:

$$W = \Delta U = \Delta V \Delta q$$

Applying calculus[*] gives:

$$dU = \Delta V\,dq \qquad \text{and therefore} \qquad U = \int_0^Q \Delta V\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C}$$

Because $Q = C\Delta V$, we can substitute $C\Delta V$ for Q, giving the equation for the stored (potential) energy in a capacitor:

$$U_C = \frac{Q^2}{2C} = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2$$

[*] Because this is not a calculus-based course, you are not responsible for understanding this derivation. However, you do need to be able to use the resulting equations.

Parallel-Plate Capacitors and Dielectrics

The capacitance of a parallel plate capacitor is given by the following equation:

$$C = \kappa \varepsilon_o \frac{A}{d}$$

where:

$C =$ capacitance

$\kappa = \varepsilon_r =$ relative permittivity (dielectric constant), vacuum $\equiv 1$

$\varepsilon_o =$ electrical permittivity of free space $= 8.85 \times 10^{-12} \, \frac{\text{F}}{\text{m}}$

$A =$ cross-sectional area

$d =$ distance between the plates of the capacitor

When a capacitor is fully charged, the distance between the plates can be so small that a spark could jump from one plate to the other, shorting out and discharging the capacitor. In order to prevent this from happening, the space between the plates is often filled with a chemical (often a solid material or an oil) called a dielectric.

A dielectric is an electrical insulator (charges do not move, which reduces the possibility of the capacitor shorting out), but has a relatively high value of electric permittivity (ability to support an electric field). (See *Electric Permittivity*, starting on page 159.)

Dielectrics in capacitors serve the following purposes:

- Keep the conducting plates from coming into contact, allowing for smaller plate separations and therefore higher capacitances.
- Increase the effective capacitance by spreading out the charge, reducing the electric field strength and allowing the capacitor to hold same charge at a lower voltage.
- Reduce the possibility of the capacitor shorting out by sparking (more formally known as dielectric breakdown) during operation at high voltage.

Note that a higher value of κ[*] and lower value of d both enable the capacitor to have a higher capacitance.

Commonly used solid dielectrics include porcelain, glass or plastic (such as polyethylene). Common liquid dielectrics include mineral oil or castor oil. Common gaseous dielectrics include air, nitrogen and sulfur hexafluoride.

[*] Note that κ is the Greek letter "kappa," not the Roman letter "k".

Electric Field in a Capacitor

From *Electric Fields and Electric Potential*, starting on page 167, $\Delta V = \vec{\boldsymbol{E}} \bullet \vec{\boldsymbol{d}}$.

Because $C = \frac{Q}{\Delta V}$, which means $\Delta V = \frac{Q}{C}$, we can substitute $C = \kappa \varepsilon_o \frac{A}{d}$ to give:

$$\Delta V = \frac{Q}{C} = \frac{Qd}{\kappa \varepsilon_o A} = \vec{\boldsymbol{E}} \bullet \vec{\boldsymbol{d}} = Ed\cos\theta$$

Assuming the electric field and displacement are in the same direction, $\cos\theta = 1$, which means the electric field between the plates of a capacitor is:

$$E_c = \frac{Q}{\kappa \varepsilon_o A}$$