Diffraction

Unit: Light & Optics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 14.8.A, 14.8.A.1, 14.8.A.1.i, 14.8.A.1.ii, 14.8.A.1.iii, 14.8.A.1.iv, 14.8.A.1.v, 14.8.A.1.vi, 14.8.A.2, 14.8.A.3, 14.8.A.4, 14.8.A.5

Mastery Objective(s): (Students will be able to…)

- Explain how light "spreads" beyond an opening or around an obstacle.
- Perform calculations relating to the location of bright and dim regions when light passes through a diffraction grating.

Success Criteria:

- Explanations account for observed behavior.
- Calculations are correct with correct algebra.

Language Objectives:

- Explain why looking through a diffraction grating produces a "rainbow".

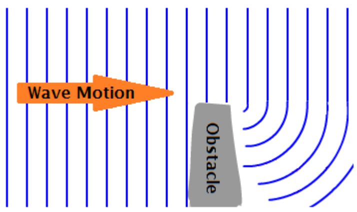
Tier 2 Vocabulary: light, diffraction, slit

Labs, Activities & Demonstrations:

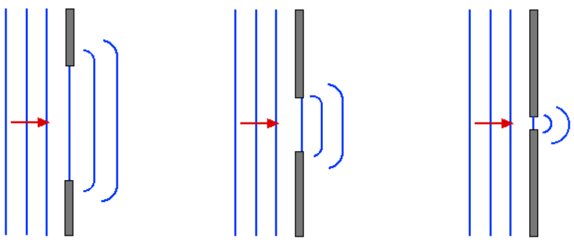
- thickness of human hair
- double slit experiment with laser & diffraction grating

Notes:

diffraction: the slight bending of a wave as it passes around the edge of an object or through a slit:



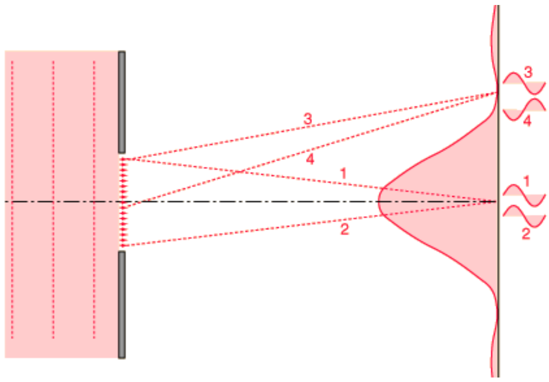
When light passes straight through a wide opening, the rays continue in a straight line. However, if we make the slit so narrow that the width is approximately equal to the wavelength, then the slit effectively becomes a point, and diffraction occurs in all directions from it.



If we shine light through a slit whose thickness is approximately the same order of magnitude as the wavelength, the light can only hit the wall in specific locations.

In this diagram, light travels the same distance for paths 1 and 2—the same number of wavelengths. Light waves hitting this point will add constructively, which makes the light brighter.

However, for paths 3 and 4, path 4 is ½ wavelength longer than path 3. Light taking path 4 is ½ wavelength out of phase with light from path 3. The waves add destructively (cancel), and there is no light:



Farther up or down on the right side will be alternating locations where the difference in path length (ΔD) results in waves that are different by an exact multiple of the wavelength (in phase = constructive interference = bright spot), *vs.* by a multiple of the wavelength plus ½ (out-of-phase = destructive interference = dark spot).

The difference in path length is given by the equation:

$$\Delta D = a \sin\theta$$

where:

ΔD = the difference in path length between light coming from the center of the slit and its edge

a = the width of the opening

θ = the angle between a line normal to the opening and the direction of propagation of the actual wave

For small angles ($\theta < 10°$), the "small-angle approximation" can be used:

$$a\left(\frac{y_{min}}{L}\right) \approx m\lambda$$

where:

a = the width of the slit

y_{min} = the distance from the central (bright) fringe to the m^{th} order of minimum brightness on the screen.

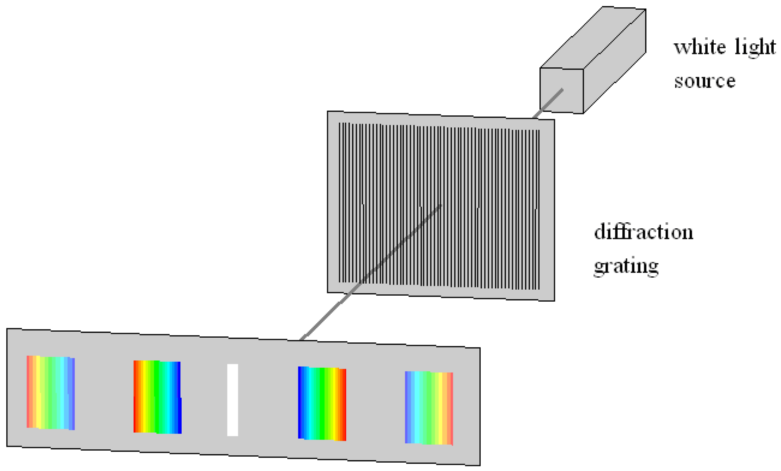
L = the distance between the diffraction grating and the screen.

m = the number of waves that equals the difference in the lengths of the two paths (integer)

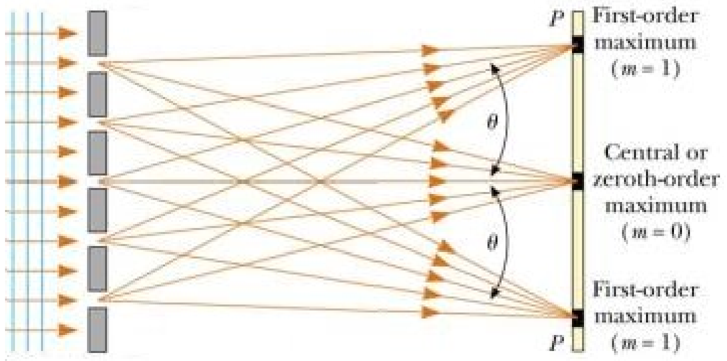
λ = the wavelength of the light

Diffraction Gratings

<u>diffraction grating</u>: a screen with a series of evenly-spaced slits that scatters light in a repeating, predictable pattern.



When light shines through a diffraction grating, the following happens:



The arrows represent pathways that differ by an integer number of wavelengths of light. The patterns of light surrounding the center are therefore the points where the waves of light add constructively.

Notice that blue and violet light (with the shortest wavelengths) is diffracted least, and appears closest to the center; red light (with the longest wavelengths) is diffracted most and appears farthest away.

This is why shorter waves "bend" more around the edges of a slit, because they need to turn less far to fit through the slits than longer waves do.

In a diffraction grating that has multiple slits that are a distance *d* apart, the difference in path length will be the same $\Delta D = d\sin\theta$ as we saw above. Therefore, there will be maximum constructive interference when ΔD is an exact (integer) multiple of the wavelength. This gives the equation:

$$d\sin\theta_m = m\lambda$$

where:

- d = the distance between the slits
- θ_m = the angle of emergence (or angle of deviation) in order for light from one slit to add constructively to light from a neighboring slit that is *m* wavelengths away.
- m = the number of waves that equals the difference in the lengths of the two paths (integer)
- λ = the wavelength of the light

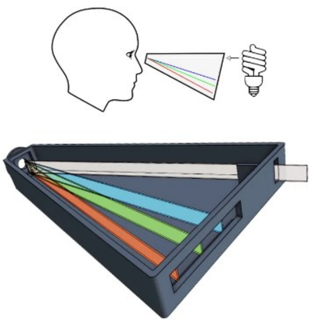
For small angles ($\theta < 10°$), the "small-angle approximation" is similar to what we saw above:

$$d\left(\frac{y_{max}}{L}\right) \approx m\lambda$$

where:

- d = the distance between the slits
- y_{max} = the distance from the central (bright) fringe to the mth order of maximum brightness on the screen.
- L = the distance between the diffraction grating and the screen.
- m = the number of waves that equals the difference in the lengths of the two paths (integer)
- λ = the wavelength of the light

Diffraction gratings are used in spectroscopes, which you have probably used in chemistry and/or physics classes to see different wavelengths of light produced by a light source.



Sample Problem:

Q: Consider three laser pointers: a red laser with a wavelength of 650 nm, a green laser with a wavelength of 532 nm, and a blue laser with a wavelength of 405 nm. If each of these is shone through a diffraction grating with 5 000 lines per cm, what will be the angle of emergence for each color?

A: For our diffraction grating, 5 000 lines per cm equals 500 000 lines per meter.

$$d = \frac{1}{500\,000} = 2\times10^{-6}\text{ m}$$

For the red laser, 650 nm equals $\lambda = 650\text{ nm} = 6.50\times10^{-7}\text{m}$

The equation is:

$$d\sin\theta_m = m\lambda$$

For the red laser at m = 1, this becomes:

$$(2\times10^{-6})\sin\theta = (1)(6.50\times10^{-7})$$
$$\sin\theta = \frac{6.50\times10^{-7}}{2\times10^{-6}} = 0.325$$
$$\theta = \sin^{-1}(0.325) = 19.0°$$

For the green laser $(\lambda = 532\text{ nm} = 5.32\times10^{-7}\text{m})$ and the blue laser also at m = 1 $(\lambda = 405\text{ nm} = 4.05\times10^{-7}\text{m})$:

$$\sin\theta = \frac{5.32\times10^{-7}}{2\times10^{-6}} = 0.266$$
$$\theta = \sin^{-1}(0.266) = 15.4°$$

and

$$\sin\theta = \frac{4.05\times10^{-7}}{2\times10^{-6}} = 0.203$$
$$\theta = \sin^{-1}(0.203) = 11.7°$$

Homework Problems

In a Young's double slit experiment using yellow light of wavelength 550 nm, the fringe separation (separation between bright spots) is 0.275 mm.

1. **(M)** Find the slit separation if the fringes are 2.0 m from the slit.

   Answer: 0.004 m (= 4 mm)

2. **(M)** The yellow lamp is replaced with a purple one whose light is made of two colors, red light with a wavelength of 700 nm, and violet light with a wavelength of 400 nm.

   a. Find the distance between the violet fringes.

      Answer: 0.000 2 m (= 0.2 mm)

   b. Find the distance between the red fringes.

      Answer: 0.000 35 m (= 0.35 mm)