Binding Energies & Mass Defect

Unit: Atomic and Nuclear Physics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS1-8

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 15.3.A.5, 15.7.A.4

Mastery Objective(s): (Students will be able to…)

- Calculate the nuclear binding energy of an atom.
- Calculate the energy given off by a radioactive decay based on the binding energies before and after.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain where the energy behind the strong force (which holds the nucleus together) comes from.

Tier 2 Vocabulary: defect

Notes:

binding energy: the amount of energy required to separate objects that are bound together by a force.

There are several types of binding energies:

gravitational binding energy: the energy required to separate objects that are held together by a gravitational force to an infinite distance. This is the inverse of the gravitational potential energy between two objects.

bond dissociation energy (sometimes called bond energy): the energy required to separate a molecule into its constituent atoms, *i.e.,* the energy required to break all of the chemical bonds in a molecule.

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ionization energy (electron binding energy): the energy required to remove an electron from its atomic orbital or from a solid. Because atoms are electrically neutral, removing an electron turns the neutral atom into an ion, hence the name.

atomic binding energy: the energy required to remove all of the electrons from an atom. This is the sum of all of the ionization energies (first, second, third, *etc.*) of the atom.

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nuclear binding energy: the energy that holds the nucleus of an atom together through the strong nuclear force

The nuclear binding energy comes from the small amount of mass (the mass defect) that was released as energy when the nucleus was formed, given by the equation:

$$E = mc^2$$

where E is the nuclear binding energy, m is the mass defect, and c is the speed of light ($3\times10^{8}\ \frac{\text{m}}{\text{s}}$), which means c^2 is $9\times10^{16}\ \frac{\text{m}^2}{\text{s}^2}$ (a very large number)!

You can figure out how much energy is produced by spontaneous radioactive decay by calculating the difference in the sum of the nuclear binding energies of the atoms before and after the decay.

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quantum chromodynamics (QCD) binding energy: the energy binding quarks together into hadrons. It is the energy of the field of the strong nuclear force, which is mediated by gluons. Most of the mass of hadrons (which include protons and neutrons, and therefore most of the matter in the universe) is actually QCD binding energy.

mass defect: the difference between the actual mass of an atom, and the sum of the masses of the protons, neutrons, and electrons that it contains. The mass defect is the amount of "missing" mass that was turned into binding energy.

- A proton has a mass of 1.6726×10^{-27} kg = 1.0073 amu
- A neutron has a mass of 1.6749×10^{-27} kg = 1.0087 amu
- An electron has a mass of 9.1094×10^{-31} kg = 0.0005486 amu

To calculate the mass defect, total up the masses of each of the protons, neutrons, and electrons in an atom. The actual (observed) atomic mass of the atom is always *less* than this number. The "missing mass" is called the mass defect.

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Sample problem:

Q: Calculate the mass defect of 1 mole of uranium-238.

A: $^{238}_{92}U$ has 92 protons, 146 neutrons, and 92 electrons. This means the total mass of one atom of $^{238}_{92}U$ should theoretically be:

92 protons × 1.0073 amu = 92.6704 amu

146 neutrons × 1.0087 amu = 147.2661 amu

92 electrons × 0.000 5486 amu = 0.0505 amu

92.6704 + 147.2661 + 0.0505 = 239.9870 amu

The actual observed mass of one atom of $^{238}_{92}U$ is 238.0003 amu.

The mass defect of one atom of $^{238}_{92}U$ is therefore
239.9870 − 238.0003 = 1.9867 amu.

One mole of $^{238}_{92}U$ would have a mass of 238.0003 g, and therefore a total mass defect of 1.9867 g, or 0.001 9867 kg.

Because $E = mc^2$, that means the binding energy of one mole of $^{238}_{92}U$ is:

$$0.001\,9867\,\text{kg} \times (3.00 \times 10^8)^2 = 1.79 \times 10^{14}\ \text{J}$$

In case you don't realize just how large that number is, the binding energy of just 238 g (1 mole) of $^{238}_{92}U$ would be enough energy to heat every house on Earth for an entire winter!