Half-Life

Unit: Atomic and Nuclear Physics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP Physics 2 Learning Objectives/Essential Knowledge (2024): 15.7.B, 15.7.B.1, 15.7.B.1.i, 15.7.B.1.ii, 15.7.B.1.iii, 15.7.B.2, 15.7.B.3

Mastery Objective(s): (Students will be able to…)

- Calculate the amount of material remaining after an amount of time.
- Calculate the elapsed time based on the amount of material remaining.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why the *mass* of material that decays keeps decreasing.

Tier 2 Vocabulary: life, decay

Labs, Activities & Demonstrations:

- half-life of dice or M & M candies

Notes:

The atoms of radioactive elements are unstable, and they spontaneously decay (change) into atoms of other elements.

For any given atom, there is a certain probability, *P*, that it will undergo radioactive decay in a given amount of time. The half-life, τ, is how much time it would take to have a 50% probability of the atom decaying. If you start with *n* atoms, after one half-life, half of them (0.5*n*) will have decayed.

If we started with 32 g of ^{53}Fe, which has a half-life (τ) of 8.5 minutes, we would observe the following:

# minutes	0	8.5	17	25.5	34
# half-lives	0	1	2	3	4
amount left	32 g	16 g	8 g	4 g	2 g

Amount of Material Remaining

Most half-life problems in a first-year high school physics course involve a whole number of half-lives and can be solved by making a table like the one above. However, on the AP® exam you can expect problems that do not involve a whole number of half-lives, and you need to use the exponential decay equation.

Because *n* is decreasing, the number of atoms (and consequently also the mass) remaining after any specific period of time follows the exponential decay function:

$$N = N_o\left(\tfrac{1}{2}\right)^n$$

where N is the amount you have now, N_o is the amount you started with, and n is the number of half-lives that have elapsed.

Because the number of half-lives equals the total time elapsed (t) divided by the half-life $t_{1/2}$, we can replace $n = \frac{t}{t_{1/2}}$ and rewrite the equation as:

$$N = N_o\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}} \text{ or } \frac{N}{N_o} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

If you want to find either N or N_o, you can plug the values for t and $t_{1/2}$ into the above equation.

Sample Problem:

Q: If you start with 228 g of ^{90}Sr, how much would remain after 112.4 years?

A: N_0 = 228 g

N = N

$t_{1/2}$ = 28.1 years (from the "Selected Radioisotopes" table in your reference tables)

t = 112.4 years

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$N = (228)\left(\frac{1}{2}\right)^{112.4/28.1} = (228)\left(\frac{1}{2}\right)^4 = (228)\left(\frac{1}{16}\right) = 14.25\text{ g}$$

Or, if the decay happens to occur over an integer number of half-lives (as in this example), you can use a chart:

# years	0	28.1	56.2	84.3	112.4
# half-lives	0	1	2	3	4
amount left	228 g	114 g	57 g	28.5 g	14.25 g

Finding the Time that has Passed

Integer Number of Half-Lives

If the amount you started with divided by the amount left is an exact power of two, you have an integer number of half-lives and you can just make a table.

Sample problem:

Q: If you started with 64 g of ^{131}I, how long would it take until there was only 4 g remaining? The half-life ($t_{1/2}$) of ^{131}I is 8.07 days.

A: $\frac{64}{4} = 16$ which is a power of 2, so we can simply make a table:

# half-lives	0	1	2	3	4
amount remaining	64 g	32 g	16 g	8 g	4 g

From the table, after 4 half-lives, we have 4 g remaining.

The half-life ($t_{1/2}$) of ^{131}I is 8.07 days.

$$8.07 \times 4 = 32.3 \text{ days}$$

Non-Integer Number of Half-Lives

If you need to find the elapsed time and it is not an exact half-life, you need to use logarithms.

In mathematics, *the only reason you ever need to use logarithms is when you need to solve for a variable that's in an exponent*. For example, suppose we have the expression of the form $a^b = c$.

If b is a constant, we can solve for either a or c, as in the expressions:

$$a^3 = 21 \quad (\sqrt[3]{a^3} = \sqrt[3]{21} = 2.76)$$
$$6^2 = c \quad (6^2 = 36)$$

However, we can't do this if a and c are constants and we need to solve for b, as in the expression:

$$3^b = 17$$

To solve for b, we need to get b out of the exponent. We do this by taking the logarithm of both sides:

$$b\log(3) = \log(17)$$
$$b = \frac{\log(17)}{\log(3)} = \frac{1.23}{0.477} = 2.58$$

It doesn't matter which base you use. Using ln instead of log gives the same result:

$$b\ln(3) = \ln(17)$$
$$b = \frac{ln(17)}{ln(3)} = \frac{2.83}{1.10} = 2.58$$

We can apply this same logic to the half-life equation:

$$\frac{N}{N_o} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$
$$\ln N - \ln N_o = \frac{t}{t_{1/2}} \ln\left(\frac{1}{2}\right)$$

The College Board prefers to define a decay constant $\lambda = \frac{\ln 2}{t_{1/2}}$, which gives

$$N = N_o e^{-\lambda t} \quad \text{and} \quad \ln\frac{N}{N_o} = -\lambda t$$

Sample problem:

Q: If you started with 64 g of ^{131}I, how long would it take until there was only 5.75 g remaining? The half-life (τ) of ^{131}I is 8.07 days.

A: We have 5.75 g remaining. However, $\frac{64}{5.75}=11.13$, which is not a power of two.

This means we don't have an integer number of half-lives, so we need to use logarithms:

$$\frac{N}{N_o}=\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$\ln N-\ln N_o=\frac{t}{t_{1/2}}\ln\left(\frac{1}{2}\right)$$

$$\ln 5.75-\ln 64=\frac{t}{8.07}\ln\left(\frac{1}{2}\right)$$

$$1.7492-4.1589=\frac{t}{8.07}(-0.6931)$$

$$-2.4097=-0.0859\,t$$

$$28.1\text{ days}=t$$

Homework Problems

For these problems, you will need to use half-life information from *Table EE. Selected Radioisotopes* on page 513 of your physics reference tables.

1. **(M)** If a lab had 128 g of ^{3}H waste 49 years ago, how much of it would be left today? (*Note: you may round off to a whole number of half-lives.*)

Answer: 8 g

2. **(S)** Suppose you set aside a 20. g sample of ^{42}K at 5:00pm on a Friday for an experiment, but you are not able to perform the experiment until 9:00am on Monday (64 hours later). How much of the ^{42}K will be left?

   Answer: 0.56 g

3. **(M)** If a school wants to dispose of small amounts of radioactive waste, they can store the materials for ten half-lives and then dispose of the materials as regular trash.

   a. If we had a sample of ^{32}P, how long would we need to store it before disposing of it?

      Answer: 143 days

   b. If we had started with 64 g of ^{32}P, how much ^{32}P would be left after ten half-lives? Approximately what fraction of the original amount would be left?

      Answer: 0.063 g; approximately $\frac{1}{1000}$ of the original amount.

4. **(M)** If the carbon in a sample of human bone contained 30. % of the expected amount of ^{14}C, approximately how old is the sample?

   Answer: 9 950 years