Assigning & Substituting Variables

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 1 Learning Objectives: N/A

Knowledge/Understanding Goals:

- be able to declare (assign) variables from a word problem
- be able to substitute values for variables in an equation

Language Objectives:

- understand and correctly use the terms "variable" and "subscript."
- accurately describe and apply the concepts described in this section, using appropriate academic language

Notes:

Math is a language. Like other languages, it has nouns (numbers), pronouns (variables), verbs (operations), and sentences (equations), all of which must follow certain rules of syntax and grammar.

This means that turning a word problem into an equation is translation from English to math.

Mathematical Operations

You have probably been taught translations for most of the common math operations:

word	meaning	word	meaning	word	meaning
and, more than (but not "is more than")	+	percent ("per" + "cent")	÷ 100	is at least	≥
less than (but not "is less than")	−	change in *x*, difference in *x*	Δx	is more than	>
of	×	is	=	is at most	≤
per, out of	÷			is less than	<

Use this space for summary and/or additional notes.

Identifying Variables

In science, almost every measurement must have a unit. These units are your key to what kind of quantity the numbers describe. Some common quantities in physics and their units are:

quantity	**S.I. unit**	**variable**
mass	kg	m
distance, length	m	d, ℓ
area	m^2	A
acceleration	m/s^2	a
volume	m^3	V
velocity (speed)	m/s	v
pressure	Pa	P*
momentum	N·s	p*
density	kg/m^3	ρ*
moles	mol	n
time	s	t
temperature	K	T
heat	J	Q
electric charge	C	q

*Note the subtle differences between uppercase "P", lowercase "p", and the Greek letter ρ ("rho").

Any time you see a number in a word problem that has a unit you recognize (such as one listed in this table), notice which quantity the unit is measuring and label the quantity with the appropriate variable.

Be especially careful with uppercase and lowercase letters. In physics, the same uppercase and lowercase letter may be used for completely different quantities.

Use this space for summary and/or additional notes.

Variable Substitution

Variable substitution simply means taking the numbers you have from the problem and substituting those numbers for the corresponding variable in an equation. A simple version of this is a density problem:

If you have the formula:

$\rho = \frac{m}{V}$ and you're given: $m = 12.3$ g and $V = 2.8$ cm^3

simply substitute 12.3 g for *m*, and 2.8 cm^3 for *V*, giving:

$$\rho = \frac{12.3\text{ g}}{2.8\text{ cm}^3} = 4.4\ \tfrac{\text{g}}{\text{cm}^3}$$

Because variables and units both use letters, it is often easier to leave the units out when you substitute numbers for variables and then add them back in at the end:

$$\rho = \frac{12.3}{2.8} = 4.4\ \tfrac{\text{g}}{\text{cm}^3}$$

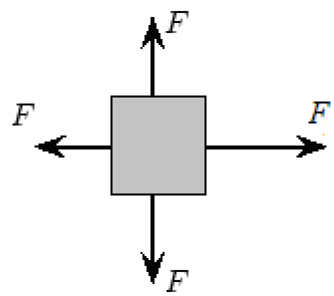
Use this space for summary and/or additional notes.

Subscripts

In physics, one problem can often have several instances of the same quantity. For example, consider a box with four forces on it:

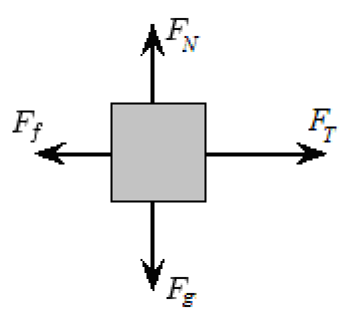
1. The force of gravity, pulling downward.
2. The "normal" force of the table resisting gravity and holding the box up.
3. The tension force in the rope, pulling the box to the right.
4. The force of friction, resisting the motion of the box and pulling to the left.

The variable for force is "F", so the diagram would look like this:



In order to distinguish between the forces and make the diagram easier to understand, we add subscripts to the variables:

1. F_g is the force of gravity.
2. F_N is the normal force.
3. F_T is the tension in the rope.
4. F_f is friction.



Use this space for summary and/or additional notes.

When writing variables with subscripts, be especially careful that the subscript looks like a subscript—it needs to be smaller than the other letters and lowered slightly. For example, when we write F_g, the variable is F (force) and the subscript $_g$ attached to it tells which kind of force it is (gravity). This might occur in the following equation:

$F_g = mg$ ← right ☺

It is important that the subscript $_g$ on the left does not get confused with the variable g on the right. Otherwise, the following error might occur:

$$Fg = mg$$

$F\not{g} = m\not{g}$ ← wrong! ☹

$$F = m$$

Another common use of subscripts is the subscript "0" to mean "initial". For example, if an object is moving slowly at the beginning of a problem and then it speeds up, we need subscripts to distinguish between the initial velocity and the final velocity. Physicists do this by calling the initial velocity "v_o" where the subscript "0" means "at time zero", *i.e.,* at the beginning of the problem, when the "time" on the "problem clock" would be zero. The final velocity is simply "v" without the zero.

Use this space for summary and/or additional notes.

The Problem-Solving Process

1. Identify the quantities in the problem, based on the units and any other information in the problem.
2. Assign the appropriate variables to those quantities.
3. Find an equation that relates all of the variables.
4. Substitute the values of the variables into the equation.
    a. If you have only one variable left, it should be the one you're looking for.
    b. If you have more than one variable left, find another equation that uses one of the variables you have left, plus other quantities that you know.
5. Solve the equation(s), using basic algebra.
6. Apply the appropriate unit(s) to the result.

Use this space for summary and/or additional notes.

Sample Problem

A force of 30 N acts on an object with a mass of 1.5 kg. What is the acceleration of the object?

We have units of N and kg, and we're looking for acceleration. We need to look these up in our reference tables.

From Table D of our Reference Tables ("Quantities, Variables and Units") on page 601, we find:

Unit Symbol	Unit Name	Quantity	Variable
N	newton	force	$\vec{\boldsymbol{F}}$
kg	kilogram	mass	m
		acceleration	$\vec{\boldsymbol{a}}$

Now we know that we need an equation that relates the variables $\vec{\boldsymbol{F}}$, m, and a. ($\vec{\boldsymbol{F}}$ and $\vec{\boldsymbol{a}}$ *are* in boldface with an arrow above them because they are vectors. We'll discuss vectors a little later in the course.)

Now that we have the variables, we find a formula that relates them. From the second formula box in Table E ("Mechanics Formulas and Equations") on page 602 of our Reference Tables, we find that:

$$\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$$

So we substitute:

$$30 = 1.5a$$
$$20 = a$$

Again from Table D, we find that acceleration has units of meters per second squared, so our final answer is $\boxed{20\,\frac{\text{m}}{\text{s}^2}}$.

Use this space for summary and/or additional notes.

Homework Problems

To solve these problems, refer to your AP Physics 1 Equation Tables starting on page 597 and your Physics Reference Tables starting on page 599. To make the equations easier to find, the table and section of the table in your Physics Reference Tables where the equation can be found is given in parentheses.

1. What is the velocity of a car that travels 90. m in 4.5 s?
   (*mechanics/kinematics*)

   Answer: $20.\,\frac{\text{m}}{\text{s}}$

2. If a force of 100. N acts on a mass of 5.0 kg, what is its acceleration?
   (*mechanics/forces*)

   Answer: $20.\,\frac{\text{m}}{\text{s}^2}$

3. If the momentum of a block is 18 $\text{N}\cdot\text{s}$ and its velocity is $3\,\frac{\text{m}}{\text{s}}$, what is the mass of the block?
   (*mechanics/momentum*)

   Answer: 6 kg

4. If the momentum of a block is *p* and its velocity is *v*, derive an expression for the mass, *m*, of the block. (You may use your work from question #3 above to guide your algebra.)
   (*mechanics/momentum*)

   Answer: $m = \frac{p}{v}$

Use this space for summary and/or additional notes.

5. What is the potential energy due to gravity of a 95 kg anvil that is about to fall off a 150 m cliff onto Wile E. Coyote's head?
   (*mechanics/energy, work & power*)

   Answer: 139 650 J

6. A 25 Ω resistor is placed in an electrical circuit with a voltage of 110 V. How much current flows through the resistor?
   (*electricity/circuits*)

   Answer: 4.4 A

7. What is the frequency of a wave that is traveling at a velocity of $300.\,\frac{\text{m}}{\text{s}}$ and has a wavelength of 10. m?
   (*waves/waves*)

   Answer: 30. Hz

8. What is the energy of a photon that has a frequency of 6×10^{15} Hz? *Hint: the equation includes a physical constant, which you will need to look up in* Table B *on page 600 of your Reference Tables.*
   (*atomic & particle physics/energy*)

   Answer: 3.96×10^{-18} J

9. A piston with an area of $2.0\,\text{m}^2$ is compressed by a force of 10 000 N. What is the pressure applied by the piston?
   (*fluid mechanics/pressure*)

   Answer: 5 000 Pa

Use this space for summary and/or additional notes.

10. What is the acceleration of a car whose velocity changes from $60.\,\frac{m}{s}$ to $80.\,\frac{m}{s}$ over a period of 5.0 s?
Hint: v_o is the initial velocity and v is the final velocity.
(*mechanics/kinematics*)

Answer: $4.0\,\frac{m}{s^2}$

11. Derive an expression for the acceleration, *a*, of a car whose velocity changes from v_o to *v* in time *t*. You may use your work from problem #10 above to guide your algebra.
(*mechanics/kinematics*)

Answer: $a = \frac{v - v_o}{t}$

12. If the normal force on an object is 100. N and the coëfficient of kinetic friction between the object and the surface it is sliding on is 0.35, what is the force of friction on the object as it slides along the surface?
(*mechanics/forces*)

Answer: 35 N

13. A car has a mass of 1 200 kg and kinetic energy of 240 000 J. What is its velocity?
(*mechanics/energy*)

Answer: $20.\,\frac{m}{s}$

Use this space for summary and/or additional notes.

14. A car has mass *m* and kinetic energy *K*. Derive an expression for its velocity, *v*. You may use your work from problem #13 above to guide your algebra.
(*mechanics/energy*)

Answer: $v = \sqrt{\frac{2K}{m}}$

15. A 1200 W hair dryer is plugged into a electrical circuit with a voltage of 110 V. How much electric current flows through the hair dryer?
(*electricity/circuits*)

Answer: 10.9 A

16. What is the velocity of a photon (wave of light) through a block of clear plastic that has an index of refraction of 1.40?
Hint: the equation includes a physical constant, which you will need to look up in Table B *on page 600 of your Reference Tables.*
(*waves/reflection & refraction*)

Answer: $2.14 \times 10^{8}\ \frac{\text{m}}{\text{s}}$

17. If the distance from a mirror to an object is 0.8 m and the distance from the mirror to the image is 0.6 m, what is the distance from the lens to the focus?
(*waves/mirrors & lenses*)

Answer: 0.343 m

Use this space for summary and/or additional notes.

18. If the distance from a mirror to an object is s_o and the distance from the mirror to the image is s_i, derive an expression for the distance from the lens to the focus, f. You may use your work from problem #17 above to guide your algebra.
(*waves/mirrors & lenses*)

Answer: $f = \dfrac{1}{\frac{1}{s_i} + \frac{1}{s_o}}$

19. What is the momentum of a photon that has a wavelength of 400 nm? *Hint: remember to convert nanometers to meters. Note also that the equation includes a physical constant, which you will need to look up in* Table B *on page 600 of your Reference Tables.*
(*modern physics/energy*)

Answer: $1.65 \times 10^{-27}\ \text{N} \cdot \text{s}$

20. If a pressure of 100 000 Pa is applied to a gas and the volume decreases by $0.05\ \text{m}^3$, how much work was done on the gas?
(*fluid mechanics/work*)

Answer: 5 000 J

Use this space for summary and/or additional notes.