Vector Multiplication

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 1 Learning Objectives: N/A

Skills:

- dot product & cross product of two vectors

Language Objectives:

- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

With scalar (ordinary) numbers, there is only one way to multiply them, which you learned in elementary school. Vectors, however, can be multiplied in three different ways.

dot product: multiplication of two vectors that results in a scalar.

cross product: multiplication of two vectors that results in a new vector.

tensor product: multiplication of two vectors that results in a tensor. (A tensor is an array of vectors that describes the effect of each vector on each other vector within the array. We will not use tensors in a high school physics course.)

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Multiplying a Vector by a Scalar

Multiplying a vector by a scalar is like multiplying a variable by a number. The magnitude changes, but the direction does not. For example, in physics, displacement equals velocity times time:

$$\vec{d} = \vec{v}t$$

Velocity is a vector; time is a scalar. The magnitude is the velocity times the time, and the direction of the displacement is the same as the direction of the velocity.

The Dot (Scalar) Product of Two Vectors

The scalar product of two vectors is called the "dot product". Dot product multiplication of vectors is represented with a dot:

$$\vec{A} \bullet \vec{B}$$ [*]

The dot product of $\vec{A}$ and $\vec{B}$ is:

$$\vec{A} \bullet \vec{B} = AB\cos\theta$$

where A is the magnitude of $\vec{A}$, B is the magnitude of $\vec{B}$, and θ is the angle between the two vectors $\vec{A}$ and $\vec{B}$.

For example, in physics, *work* (a scalar quantity) is the dot product of the vectors *force* and *displacement* (distance):

$$W = \vec{F} \bullet \vec{d} = Fd\cos\theta$$

[*] pronounced "A dot B"

Use this space for summary and/or additional notes.

The Cross (Vector) Product of Two Vectors

The vector product of two vectors is called the cross product. Cross product multiplication of vectors is represented with a multiplication sign:

$$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}^{*}$$

The cross product of vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ that have an angle of θ between them is given by the formula:

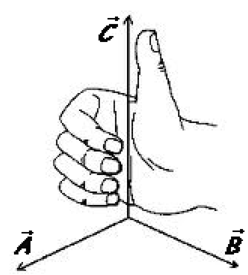
$$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = AB\sin\theta\,\hat{\boldsymbol{n}}$$

where the magnitude is $AB \sin \theta$, and the vector $\hat{\boldsymbol{n}}$ is the direction. ($AB \sin\theta$ is a scalar. The unit vector $\hat{\boldsymbol{n}}$ is what gives the vector its direction.)

The direction of the cross product is a little difficult to make sense out of. You can figure it out using the "right hand rule":

Position your right hand so that your fingers curl from the first vector to the second. Your thumb points in the direction of the resultant vector ($\hat{\boldsymbol{n}}$).

Note that this means that the resultant vectors for $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$ and $\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}}$ will point in *opposite* directions, *i.e.,* the cross product of two vectors is _not_ commutative!



$$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = \vec{\boldsymbol{C}}$$
$$\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}} = -\vec{\boldsymbol{C}}$$

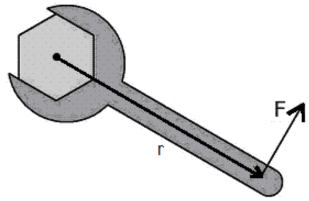
A vector coming out of the page is denoted by a series of ⊙ ⊙ ⊙ ⊙ ⊙ symbols, and a vector going into the page is denoted by a series of ⊗ ⊗ ⊗ ⊗ ⊗ symbols. The symbols represent an arrow inside a tube. The dot represents the tip of the arrow coming toward you, and the "X" represents the fletches (feathers) on the tail of the arrow going away from you.)

* pronounced "A cross B"

Use this space for summary and/or additional notes.

Vector Multiplication

In physics, torque is a vector quantity that is derived by a cross product.



The torque produced by a force $\vec{F}$ acting at a radius $\vec{r}$ is given by the equation:

$$\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\theta\, \hat{n}$$

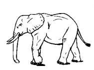
Because the direction of the force is usually perpendicular to the displacement, it is usually true that $\sin\theta = \sin 90° = 1$. This means the magnitude $rF \sin\theta = rF(1) = rF$. Using the right-hand rule, we determine that the *direction* of the resultant torque vector ($\hat{n}$) is coming out of the page.

Thus, if you are tightening or loosening a nut or bolt that has right-handed (standard) thread, the torque vector will be in the direction that the nut or bolt moves.

Vector Jokes

Now that you understand vectors, here are some bad vector jokes:

Q: What do you get when you cross an elephant with a bunch of grapes?

A:   $\sin\theta\, \hat{n}$

Q: What do you get when you cross an elephant with a mountain climber?

A: You can't do that! A mountain climber is a scalar ("scaler," meaning someone who scales a mountain).

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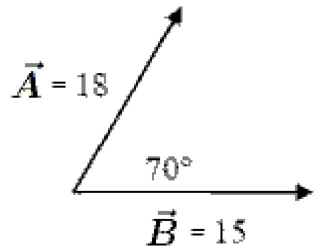
Homework Problems

For the following vectors $\vec{\boldsymbol{A}}$ & $\vec{\boldsymbol{B}}$:



1. Determine $\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}}$

2. Determine $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$ (both magnitude and direction)

Use this space for summary and/or additional notes.