Newton's Equations of Motion

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.1, 1.2

AP Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3

Skills:

- use Isaac Newton's motion equations to calculate position, velocity and acceleration for problems that involve movement in one direction

Language Objectives:

- Understand, solve and explain problems involving motion and acceleration.

Notes:

Most motion problems can be calculated from Isaac Newton's equations of motion. The following is a summary of the equations presented in the previous sections:

Equation	Description
$\vec{d} = \Delta\vec{x} = \vec{x} - \vec{x}_o$	Definition of displacement.
$\bar{\vec{v}} = \frac{\Delta\vec{x}}{t} = \frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$	Average velocity is the distance per unit of time, which also equals the calculated value of average velocity.
$\vec{a} = \frac{\Delta\vec{v}}{t} = \frac{\vec{v} - \vec{v}_o}{t}$	Acceleration is a change in velocity divided by time.
$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2}\vec{a}t^2$	Position at the end equals position at the beginning (x_o) plus the displacement due to velocity ($v_o t$), plus the displacement due to acceleration ($\frac{1}{2}at^2$).
$\vec{v}^2 = \vec{v}_o^2 + 2\vec{a}\Delta\vec{x}$	Velocity at the end can be calculated from velocity at the beginning, acceleration, and displacement.

Use this space for summary and/or additional notes.

Selecting the Right Equation

When you are faced with a problem, choose an equation based on the following criteria:

- The equation *must* contain the variable you are looking for.
- All other quantities in the equation must be either given in the problem or assumed from the description of the problem.
  - If an object starts at rest (not moving), that means $\vec{v}_o = 0$.
  - If an object comes to a stop, that means $\vec{v} = 0$. (Remember that $\vec{v}$ is the velocity at the end.)
  - If gravity is involved (*e.g.,* the object is falling), that means $\vec{a} = \vec{g} \approx 10\,\frac{\text{m}}{\text{s}^2}$. (Applies to linear acceleration problems only.)

This means you can choose the appropriate equation by making a list of what you are looking for and what you know. The equation in which you know everything except what you are looking for is the one to use.

Representing Vectors with Positive and Negative Numbers

Remember that position ($\vec{x}$), velocity ($\vec{v}$), and acceleration ($\vec{a}$) are all vectors, which means they can be positive or negative, depending on the direction.

- If an object is located is on the positive side of the origin (position zero), then its position, $\vec{x}$, is positive. If the object is located on the negative side of the origin, its position is negative.
- If an object is moving in the positive direction, then its velocity, $\vec{v}$, is positive. If the object is moving in the negative direction, then its velocity is negative.
- If an object's velocity is either increasing in the positive direction *or decreasing in the negative direction*, then its acceleration, $\vec{a}$, is positive. If the object's velocity is either decreasing in the positive direction *or increasing in the negative direction*, then its acceleration is negative.
- An object can have positive velocity and negative acceleration at the same time (or *vice versa*).
- An object can have a velocity of zero for an instant but can still be accelerating.

Use this space for summary and/or additional notes.

Homework Problems: Motion Equations Set #1

1. A racecar, traveling at constant speed, makes one lap around a circular track of radius 100 m. When the car has traveled halfway around the track, what is the magnitude of its displacement from the starting point? (*Hint: it may be helpful to draw a sketch.*)

2. An elevator is moving upward with a speed of $11\,\frac{\text{m}}{\text{s}}$. Three seconds later, the elevator is still moving upward, but its speed has been reduced to $5.0\,\frac{\text{m}}{\text{s}}$. What is the average acceleration of the elevator during the 3.0 s interval?

   Answer: $-2\,\frac{\text{m}}{\text{s}^2}$

3. A car, starting from rest, accelerates in a straight-line path at a constant rate of $2.5\,\frac{\text{m}}{\text{s}^2}$. How far will the car travel in 12 seconds?

   Answer: 180 m

Use this space for summary and/or additional notes.

4. 4. A body initially at rest is accelerated at a constant rate for 5.0 seconds in the positive *x* direction. If the final speed of the body is $20.0\,\frac{m}{s}$, what was the body's acceleration?

   Answer: $4\,\frac{m}{s^2}$

5. An object starts from rest and accelerates uniformly in a straight line in the positive *x* direction. After 10. seconds its speed is $70.\,\frac{m}{s}$.

   a. Determine the acceleration of the object.

      Answer: $7\,\frac{m}{s^2}$

   b. How far does the object travel during those first 10 seconds?

      Answer: 350 m

6. A racecar has a speed of $80.\,\frac{m}{s}$ when the driver releases a drag parachute. If the parachute causes a deceleration of $-4\,\frac{m}{s^2}$, how far will the car travel before it stops?

   Answer: 800 m

Use this space for summary and/or additional notes.

7. A racecar has a speed of v_o when the driver releases a drag parachute. If the parachute causes a deceleration of a, derive an expression for how far the car will travel before it stops. (You may use your work from problem #6 above to guide your algebra.)

   Answer: $d = \frac{v_o^2}{2a}$

8. A brick is dropped from rest from a height of 5.0 m. How long does it take for the brick to reach the ground?

   Answer: 1 s

9. A ball is dropped from rest from a tower and strikes the ground 125 m below. Approximately how many seconds does it take for the ball to strike the ground after being dropped? (Neglect air resistance.)

   Answer: 5.0 s

Use this space for summary and/or additional notes.

10. A ball is shot straight up from the surface of the earth with an initial speed of $30.\frac{\text{m}}{\text{s}}$. Neglect any effects due to air resistance.

    a. What is the maximum height that the ball will reach?

    Answer: 45 m

    b. How much time elapses between the throwing of the ball and its return to the original launch point?

    Answer: 6.0 s

11. Water drips from rest from a leaf that is 20 meters above the ground. Neglecting air resistance, what is the speed of each water drop when it hits the ground?

    Answer: $20.0\,\frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes.

12. What is the maximum height that will be reached by a stone thrown straight up with an initial speed of $35\,\frac{m}{s}$?

Answer: 61.25 m

Homework Problems: Motion Equations Set #2

These problems are more challenging than Set #1.

1. A car starts from rest at 50 m to the west of a road sign. It travels to the east reaching $20\,\frac{m}{s}$ after 15 s. Determine the position of the car relative to the road sign.

   Answer: 100 m east

2. A car starts from rest at 50 m west of a road sign. It has a velocity of $20\,\frac{m}{s}$ east when it is 50 m east of the road sign. Determine the acceleration of the car.

   Answer: $2\,\frac{m}{s^2}$

Use this space for summary and/or additional notes.

3. During a 10 s period, a car has an average velocity of $25\,\frac{\text{m}}{\text{s}}$ and an acceleration of $2\,\frac{\text{m}}{\text{s}^2}$. Determine the initial and final velocities of the car.

Answer: $v_o = 15\,\frac{\text{m}}{\text{s}}$; $v = 35\,\frac{\text{m}}{\text{s}}$

4. A racing car increases its speed from an unknown initial velocity to $30\,\frac{\text{m}}{\text{s}}$ over a distance of 80 m in 4 s. Calculate the initial velocity of the car and the acceleration.

Answer: $v_o = 10\,\frac{\text{m}}{\text{s}}$; $a = 5\,\frac{\text{m}}{\text{s}^2}$

5. A tennis ball is shot vertically upwards from the ground. It takes 3.2 s for it to return to the ground. Find the total distance the ball traveled.

Answer: 25.6 m

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6. A kangaroo jumps vertically to a height of 2.7 m. How long will it be in the air before returning to the earth?

   Answer: 1.5 s

7. A falling stone takes 0.30 s to travel past a window that is 2.2 m tall. From what height above the window did the stone fall?

   Answer: 1.70 m

Use this space for summary and/or additional notes.

8. A helicopter is ascending vertically with a speed of $5.50\,\frac{\text{m}}{\text{s}}$. At a height of 100 m above the Earth, a package is dropped from the helicopter. How much time does it take for the package to reach the ground?

   Answer: 5.06 s

9. A helicopter is ascending vertically with a speed of v_o. At a height h above the Earth, a package is dropped from the helicopter. Derive an expression for the time, t, that it takes for the package to reach the ground. (You may use your work from problem #7 above to guide your algebra.)

   Answer: $t = \dfrac{-v_o \pm \sqrt{v_o^2 + 2gh}}{-g}$, disregarding the negative answer

Use this space for summary and/or additional notes.

10. A stone is thrown vertically upward with a speed of $12.0\,\frac{m}{s}$ from the edge of a cliff that is 75.0 m high.

    a. How much later does it reach the bottom of the cliff?

    Answer: 5.25 s

    b. What is its speed just before it hits the ground?

    Answer: $-40.5\,\frac{m}{s}$

    c. What is the total distance the stone travels?

    Answer: 89.0 m

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