Angular Motion, Speed and Velocity

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.1, 1.2

AP Physics 1 Learning Objectives: 3.A.1.1, 3.A.1.3

Knowledge/Understanding Goals:

- understand terms relating to angular position, speed & velocity

Language Objectives:

- Understand and correctly use the terms “angle” and “angular velocity.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

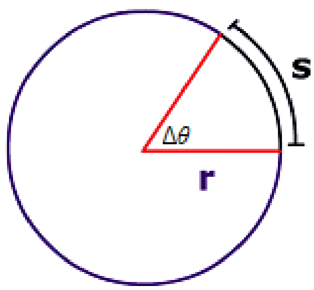
Labs, Activities & Demonstrations:

- Swing an object on a string.

Notes:

If an object is rotating (traveling in a circle), then its position at any given time can be described using polar coördinates by its distance from the center of the circle (*r*) and its angle (θ) relative to some reference angle (which we will call $\theta = 0$).

arc length (*s*): the length of an arc; the distance traveled around part of a circle.



$$s = r\Delta\theta$$

Use this space for summary and/or additional notes.

angular velocity (ω): the rotational velocity of an object as it travels around a circle, *i.e.,* its change in angle per unit of time. (For purposes of comparison, the definition of angular velocity is presented along with its linear counterpart.)

$$\vec{\boldsymbol{v}} = \frac{d}{t} = \frac{\Delta x}{\Delta t} = \frac{x - x_o}{t} \qquad \vec{\boldsymbol{\omega}} = \frac{\Delta \theta}{\Delta t} = \frac{\theta - \theta_o}{t}$$

linear **angular**

In general, physicists use Greek letters for angular variables. The variable for angular velocity is the lower case Greek letter omega (ω). Be careful to distinguish in your writing between the Greek letter "ω" and the Roman letter "*w*".

tangential velocity: the linear velocity of a point on a rigid, rotating body. The term tangential velocity is used because the instantaneous direction of the velocity is tangential to the direction of rotation.

To find the tangential velocity of a point on a rotating (rigid) body, the point travels an arc length of *s* in time *t*. If angle θ is in radians, then $s = r\Delta\theta$. This means:

$$\bar{\vec{\boldsymbol{v}}}_T = \frac{\Delta s}{\Delta t} = \frac{r\Delta\theta}{\Delta t} = r\bar{\vec{\boldsymbol{\omega}}} \quad \text{and therefore} \quad \vec{\boldsymbol{v}}_T = r\vec{\boldsymbol{\omega}}$$

Sample Problems:

Q: What is the angular velocity ($\frac{\text{rad}}{\text{s}}$) in of a car engine that is spinning at 2400 rpm?

A: 2400 rpm means 2400 revolutions per minute.

$$\left(\frac{2400\,\text{rev}}{1\,\text{min}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\frac{2\pi\,\text{rad}}{1\,\text{rev}} = \frac{4800\pi}{60} = 80\pi\,\tfrac{\text{rev}}{\text{s}} = 251\,\tfrac{\text{rev}}{\text{s}}$$

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Q: Find the total distance traveled in 10 s by a penny sitting on a spinning disc with a radius of 0.25 m that is rotating at a rate of 1 revolution per 2 s.

A: We are looking for the distance around the circle, which is the arc length. (This means we need to work in radians.)

$$s = r\Delta\theta$$

We know that *r* = 0.25 m, but we need to find $\Delta\theta$.

$$\Delta\theta = \omega t$$

We know that *t* = 10 s, but we need to find the angular velocity *ω*.

1 revolution per 2 s is an angular velocity of $\omega = \frac{2\pi}{2} = \pi \, \frac{\text{rad}}{\text{s}}$.

Now we can solve:

$$\Delta\theta = \omega t = (\pi)(10) = 10\pi$$
$$s = r\Delta\theta = (0.25)(10\pi) = 2.5\pi \text{ m} = (2.5)(3.14) = 7.85 \text{ m}$$

Extension

Just as jerk is the rate of change of linear acceleration, angular jerk is the rate of change of angular acceleration. $\vec{\boldsymbol{\zeta}} = \frac{\Delta\vec{\boldsymbol{\alpha}}}{\Delta t}$. (ζ is the Greek letter "zeta". Many college professors cannot draw it correctly and just call it "squiggle".) Angular jerk has not been seen on AP Physics exams.

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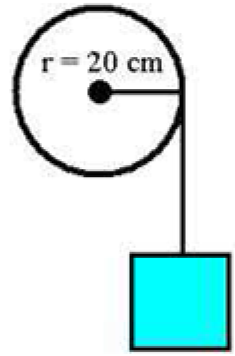
Homework Problems

1. Through what angle must the wheel shown at the right turn in order to unwind 40 cm of string?



Answer: 2 rad

2. Find the average angular velocity of a softball pitcher's arm (in $\frac{\text{rad}}{\text{s}}$) if, in throwing the ball, her arm rotates one-third of a revolution on 0.1 s.

Answer: $20.9\ \frac{\text{rad}}{\text{s}}$

3. A golfer swings a nine iron (radius = 1.1 m) with an average angular velocity of $5.0\ \frac{\text{rad}}{\text{s}}$. Find the tangential velocity of the club head.

Answer: $5.5\ \frac{\text{m}}{\text{s}}$

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