Universal Gravitation

Unit: Dynamics (Forces) & Gravitation

NGSS Standards: HS-PS2-4

MA Curriculum Frameworks (2006): 1.7

AP Physics 1 Learning Objectives: **2.B.2.1**, 2.B.2.2, 3.A.3.3, **3.C.1.1**, **3.C.1.2**, 3.G.1.1

Skills:

- solve problems involving Newton's Law of Universal Gravitation

Language Objectives:

- Understand and correctly use the terms "gravity" and "gravitation."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to Newton's Law of Universal Gravitation.

Notes:

Gravity is a force of attraction between two objects because of their mass. The cause of this attraction is not currently known. More mass causes a stronger force, but the force gets weaker as the object gets farther away.

This means that the gravitational pull of a single object is directly proportional to its mass, and inversely proportional to its distance, *i.e.*,

$$F_g \propto \frac{m}{r}$$

(The symbol $\propto$ means "is proportional to". We use "*r*" for radius because gravitational fields act in all directions, which means we should use spherical coördinates.)

If we have two objects, "1" and "2":

$$F_{g,1} \propto \frac{m_1}{r_1} \quad \text{and} \quad F_{g,2} \propto \frac{m_2}{r_2}$$

Use this space for summary and/or additional notes.

Because each object is pulling on the other one, $r_1 = r_2$ and the total force is therefore:

$$F_g \propto \frac{m_1}{r_1} \cdot \frac{m_2}{r_2} = \frac{m_1\, m_2}{r_1\, r_2} = \frac{m_1\, m_2}{r^2}$$

Finally, if we are using MKS units, the masses are in kilograms, the distance is in meters. If we want the force in newtons to be correct, we have to multiply by the appropriate conversion factor, which turns out to be $6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$. (The units are chosen because they cancel the m^2 and kg^2 from the formula and give newtons, which is the desired unit. Thus the formula becomes:

$$F_g = (6.67\times10^{-11}\ \tfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2})\frac{m_1 m_2}{r^2}$$

The number $6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$ is called the universal gravitation constant, and is represented by the symbol "G". Thus we have Isaac Newton's Law of Universal Gravitation in equation form:

$$F_g = \frac{G m_1 m_2}{r^2}$$

If we divide this expression by the mass of the object being attracted (m_2), we get the equation:

$$\frac{F_g}{m_2} = \frac{G m_1 \cancel{m_2}}{r^2 \cancel{m_2}} = \frac{G m_1}{r^2}$$

Recall from the section on Gravitational Fields on page 215 that $\vec{\boldsymbol{g}} = \frac{\vec{\boldsymbol{F}}_g}{m}$.

This means that:

$$\vec{\boldsymbol{g}} = \frac{G m_1}{r^2}$$

In the above expression, G is $6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$, m_1 is the mass of the Earth (5.97×10^{24} kg) and r is the radius of the Earth $(6.37\times10^{6}\ \text{m})$. If you plug these numbers into the formula, you get $\vec{\boldsymbol{g}} = 9.81\ \frac{\text{m}}{\text{s}^2} \approx 10\ \frac{\text{m}}{\text{s}^2}$ as expected.

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Sample Problems:

Q: Find the force of gravitational attraction between the Earth and a person with a mass of 75 kg. The mass of the Earth is 5.97×10^{24} kg, and its radius is 6.37×10^{6} m.

A: $$F_g = \frac{Gm_1m_2}{r^2}$$
$$F_g = \frac{(6.67\times10^{-11})(5.97\times10^{24})(75)}{(6.37\times10^{6})^2}$$
$$F_g = 736\ \text{N}$$

This is the same number that we would get using $F_g = mg$, with $g = 9.81\ \frac{\text{m}}{\text{s}^2}$.

Our approximation of $g = 10\ \frac{\text{m}}{\text{s}^2}$ gives $F_g = 750\ \text{N}$, which is within 2%.

Q: Find the acceleration due to gravity on the moon.

A: $$g_{moon} = \frac{Gm_{moon}}{r_{moon}^2}$$
$$g_{moon} = \frac{(6.67\times10^{-11})(7.35\times10^{22})}{(1.74\times10^{6})^2} = 1.62\ \frac{\text{m}}{\text{s}^2}$$

Q: If the distance between an object and the center of mass of a planet is tripled, what happens to the force of gravity between the planet and the object?

A: Starting with $F_g = \frac{Gm_1m_2}{r^2}$, if we replace r with 3r, we would get:

$$F_g' = \frac{Gm_1m_2}{(3r)^2} = \frac{Gm_1m_2}{9r^2} = \frac{1}{9}\cdot\frac{Gm_1m_2}{r^2}$$

Thus F_g' is $\frac{1}{9}$ of the original F_g.

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Homework Problems

You will need planetary data from Table S and Table T on page 608 of your Reference Tables. The value of the universal gravitation constant G may be found in Table B on page 600 of your Reference Tables.

1. Find the force of gravity between the earth and the sun.

   Answer: 3.52×10^{22} N

2. Find the acceleration due to gravity (the value of g) on the planet Mars.

   Answer: $3.73 \frac{m}{s^2}$

3. A mystery planet in another part of the galaxy has an acceleration due to gravity of $5.0 \frac{m}{s^2}$. If the radius of this planet is 2.0×10^6 m, what is its mass?

   Answer: 3.0×10^{23} kg

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4. A person has a mass of 80. kg.

   a. What is the weight of this person on the surface of the Earth?

   Answer: 785 N

   b. What is the weight of the same person when orbiting the Earth at a height of 4.0×10^{6} m above its surface? (*Hint: Remember that gravity acts from the center of the Earth. It may be helpful to draw a sketch.*)

   Answer: 296N

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