Escape Velocity

Unit: Work, Energy & Momentum

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 2.1, 2.2, 2.3

AP Physics 1 Learning Objectives: N/A

Knowledge/Understanding Goals:
- how fast a rocket or space ship needs to travel to escape Earth’s (or any other planet’s) gravity
- how escape velocity relates to gravitational potential energy, kinetic energy, and Newton’s Law of Universal Gravitation

Language Objectives:
- Understand and correctly use the term “escape velocity.”
- Set up and solve word problems involving escape velocity.

Notes:

Escape Velocity

If you want to send a rocket or space ship to explore the rest of the solar system or beyond, the rocket needs enough kinetic energy to escape from the force of Earth’s gravity.

To explain the calculation, it is necessary to understand that our formula for gravitational potential energy is actually a simplification. Because the force of gravity attracts objects to the center of the Earth, the equation should be:

\[ U_g = mg\Delta h = mg(h - h_o) \]

where \( h_o \) is the distance to the center of the Earth (radius of the Earth), and \( h \) is the total distance between the object and the center of the Earth. Normally, we calculate gravitational potential energy based on a difference in height between some height above the ground \((h)\) and the ground \((h = 0)\).

However, when we consider the problem of escaping from the gravity of the entire planet, we need to consider the potential energy relative to the center of the Earth, where the total force of gravity would be zero.
To do this, the rocket’s kinetic energy \( \left( \frac{1}{2} m v^2 \right) \) must be greater than or equal to the Earth’s gravitational potential energy \( (mgh) \). This means \( \frac{1}{2} m v^2 = F_g h \). We can write this in terms of Newton’s Law of Universal Gravitation:

\[
\frac{1}{2} m v^2 = F_g h = \left( \frac{G m_1 m_2}{d^2} \right) h
\]

However, because we need to escape the gravitational pull of the entire planet, we need to measure \( h \) from the center of the Earth, not the surface. This means that at the surface of the earth, \( h \) in the above equation is the same as \( r \) in Newton’s Law of Universal Gravitation. Similarly, the mass of the spaceship is one of the masses (let’s choose \( m_2 \)) in Newton’s Law of Universal Gravitation. This gives:

\[
F_g h = F_g r = \frac{G m_1 m_2}{r^2} r = \frac{G m_1 m_2}{r}
\]

\[
\frac{1}{2} m_2 v_e^2 = \frac{G m_1}{r}
\]

\[
\frac{1}{2} v_e^2 = \frac{G m_1}{r}
\]

\[
v_e = \sqrt{\frac{2 G m_{\text{planet}}}{r}}
\]

At the surface of the Earth, where \( m_{\text{planet}} = 5.97 \times 10^{24} \text{ kg} \) and \( r = 6.37 \times 10^6 \text{ m} \), \( v_e = 1.12 \times 10^4 \frac{\text{m}}{\text{s}} = 11200 \frac{\text{m}}{\text{s}} \). (This equals approximately 25 100 miles per hour.)
Sample Problem:

Q: When Apollo 11 went to the moon, the space ship needed to achieve the Earth’s escape velocity of $11200 \text{ m/s}$ to escape Earth’s gravity. What velocity did the space ship need to achieve in order to escape the moon’s gravity and return to Earth? \textit{(i.e., what is the escape velocity on the surface of the moon?)}

A: $v_e = \sqrt{\frac{2G m_{\text{moon}}}{d_{\text{moon}}}}$

$$v_e = \sqrt{\frac{(2)(6.67 \times 10^{-11})(7.35 \times 10^{22})}{1.74 \times 10^6}}$$

$$v_e = 2370 \text{ m/s}$$