Linear Momentum

Unit: Work, Energy & Momentum

NGSS Standards: HS-PS2-2


Knowledge/Understanding Goals:
- definition of momentum

Skills:
- calculate the momentum of an object
- solve problems involving the conservation of momentum

Language Objectives:
- Understand and correctly use the terms “collision,” “elastic collision,” “inelastic collision,” “inertia” and “momentum.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving collisions and momentum.

Labs, Activities & Demonstrations:
- Collisions on air track.
- “Happy” and “sad” balls knocking over a board.
- Students riding momentum cart.

Notes:
collision: when two or more objects come together and hit each other.

elastic collision: a collision in which the objects bounce off each other after they collide, without any loss of kinetic energy.

inelastic collision: a collision in which the objects remain together after colliding.
In an inelastic collision, total energy is still conserved, but some of the energy is changed into other forms, so the amount of kinetic energy is different before vs. after the collision.

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Any macroscopic collision (meaning a collision between objects that are larger than individual atoms or molecules) will convert some of the kinetic energy into internal energy and other forms of energy. This means that no large scale impacts are ever perfectly elastic.

In the 17th century, the German mathematician Gottfried Leibnitz recognized the fact that in some cases, the mass and velocity of objects before and after a collision were related by kinetic energy ($\frac{1}{2}mv^2$, which he called the “quantity of motion”); in other cases, however, the “quantity of motion” was not preserved but another quantity ($mv$, which he called the “motive force”) was the same before and after. Debate about “quantity of motion” and “motive force” continued through the 17th and 18th centuries.

We now realize that the two quantities are different, but related. “Quantity of motion” is what we now call “kinetic energy”; “motive force” is what we now call “momentum”.

While total energy is always conserved in a collision, kinetic energy is not; during the collision, kinetic energy may be converted to other forms of energy, such as heat. Momentum, however, is always conserved in a collision, regardless of what happens to the energy.

**momentum** ($\vec{p}$): the amount of force that a moving object could transfer in a given time in a collision. (Formerly called “motive force”.)

Momentum is a vector quantity given by the formula:

$$\vec{p} = m\vec{v}$$

and is measured in units of N·s, or $\frac{kg·m}{s}$.

Note that an object at rest ($\vec{v} = 0$) has a momentum of zero.

*Momentum is the quantity that is transferred in all collisions. Any problem that involves one or more collisions is a momentum problem.*

The net force on an object is its change in momentum with respect to time:

$$\vec{F}_{net} = \frac{Δ\vec{p}}{Δt} = \frac{d\vec{p}}{dt}$$

Use this space for summary and/or additional notes.
inertia: an object’s ability to resist the action of a force.

Inertia and momentum are related, but are not the same thing; an object has inertia even at rest. An object’s momentum changes if either its mass or its velocity changes, but an the inertia of an object can change only if its mass changes.

**Conservation of Momentum**

In a closed system, momentum is **conserved**. This means that unless there is an outside force, the combined momentum of objects after they collide is equal to the combined momentum of the objects before the collision.

**elastic collision**: a collision in which the objects are separate both before and after the collision.

In the following momentum bar chart, imagine that two objects are moving in opposite directions and then collide. Before the collision, the first object has a momentum of +3 arbitrary units, and the second has a momentum of −1. The total momentum is therefore +3 + (−1) = +2.

After the collision, the first object has a momentum of +1.5 and the second has a momentum of +0.5. Because there are no forces changing the momentum of the system, the final momentum **must** also be +2.

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**inelastic collision**: a collision in which the objects are joined (or are a single object) either before or after the collision.

Examples of inelastic collisions include a person catching a ball and a bullet being fired from a gun. (The bullet and gun are together initially and separate after the “collision”.)

**Solving Momentum Problems**

Almost all momentum problems involve the conservation of momentum law:

\[
\sum p_i = \sum p_f
\]

The symbol \( \sum \) is the Greek capital letter “sigma”. In mathematics, the symbol \( \sum \) means “summation”. \( \sum \dot{p} \) means the sum of the momentums. The subscript “\(i\)” means initial (before the collision), and the subscript “\(f\)” means final (after the collision). In plain English, \( \sum \dot{p} \) means find each individual value of \( \dot{p} \) (positive or negative, depending on the direction) and then add them all up to find the total. The conservation of momentum law means that the total before a collision must be equal to the total after.

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For example, if you had a momentum problem with two objects, the law of conservation of momentum becomes:
\[
\begin{align*}
\text{before} & = \text{after} \\
\vec{p}_{1,i} + \vec{p}_{2,i} & = \vec{p}_{1,f} + \vec{p}_{2,f}
\end{align*}
\]

Notice that we have two subscripts after each “\(p\)”, because we have two separate things to keep track of. The “\(i\)” and “\(f\)” mean “initial” and “final,” and the “1” and “2” mean object #1 and object #2.

Because \(\vec{p} = m\vec{v}\), we can replace each \(\vec{p}\) with \(m\vec{v}\).

For our momentum problem with two objects, this becomes:
\[
\begin{align*}
\text{before} & = \text{after} \\
m_1\vec{v}_{1,i} + m_2\vec{v}_{2,i} & = m_1\vec{v}_{1,f} + m_2\vec{v}_{2,f}
\end{align*}
\]

Note that there are six separate quantities in this problem: \(m_1, m_2, \vec{v}_{1,i}, \vec{v}_{2,i}, \vec{v}_{1,f}, \text{ and } \vec{v}_{2,f}\). A typical momentum problem will give you (or enable you to calculate) five of these, and will ask you for the sixth.

Note also that most momentum problems do not mention the word “momentum.” The problems usually give information about masses and velocities before and after some sort of collision, and it is up to you to realize that the problem involves conservation of momentum.

Most momentum problems involve collisions. Usually, there are two objects initially, and the objects either bounce off each other (elastic collision) or stick together (inelastic collision).

For an elastic collision between two objects, the problem is exactly as described above: there are six quantities to consider: the two masses, the two initial velocities, and the two final velocities. The equation relating them is:
\[
\begin{align*}
\text{before} & = \text{after} \\
m_1\vec{v}_{1,i} + m_2\vec{v}_{2,i} & = m_1\vec{v}_{1,f} + m_2\vec{v}_{2,f}
\end{align*}
\]

To solve the problem, you need to obtain the quantities given in the word problem and solve for the missing one.
For an inelastic collision between two objects, the objects stick together after the collision, which means there is only one “object” afterwards. The total mass of the object is \( m_T = m_1 + m_2 \), and there is only one “object” with a final velocity.

There are five quantities: the two masses, the two initial velocities, and the final velocity of the combined object. The equation relating them is:

\[
\begin{align*}
\text{before} & = \text{after} \\
m_1 \vec{v}_{1,i} + m_2 \vec{v}_{2,i} &= m_T \vec{v}_f
\end{align*}
\]

To solve this problem, you need to obtain the quantities given in the word problem, add the two masses to find \( m_T \), and solve for the missing quantity.
Sample Problems

Q: What is the momentum of a 15 kg object moving at a velocity of \(+3.0 \text{ m/s}\) ?

A: 
\[
\vec{p} = m\vec{v} \\
\vec{p} = (15 \text{ kg})(+3.0 \text{ m/s}) = 45 \frac{\text{kg}\cdot\text{m}}{\text{s}} = +45 \text{ N} \cdot \text{s}
\]

The answer is given as \(+45 \text{ N} \cdot \text{s}\) because momentum is a vector, and we indicate the direction as positive or negative.

Q: An object with a mass of 8.0 kg moving with a velocity of \(+5.0 \text{ m/s}\) collides with a stationary object with a mass of 12 kg. If the two objects stick together after the collision, what is their velocity?

A: The momentum of the moving object before the collision is:
\[
\vec{p} = m\vec{v} = (8.0)(+5.0) = +40 \text{ N} \cdot \text{s}
\]

The stationary object has a momentum of zero, so the total momentum of the two objects combined is \(+40 \text{ N} \cdot \text{s}\).

After the collision, the total mass is \(8.0 \text{ kg} + 12 \text{ kg} = 20 \text{ kg}\). The momentum after the collision must still be \(+40 \text{ N} \cdot \text{s}\), which means the velocity is:
\[
\vec{p} = m\vec{v} \quad 40 = 20\vec{v} \quad \vec{v} = +2 \frac{\text{m}}{\text{s}}
\]

Using the equation, we would solve this as follows:
\[
\begin{align*}
\text{before} & \quad = \quad \text{after} \\
\vec{p}_{1,i} + \vec{p}_{2,i} & \quad = \quad \vec{p}_f \\
m_1\vec{v}_{1,i} + m_2\vec{v}_{2,i} & \quad = \quad m_f\vec{v}_f \\
(8)(5) + (12)(0) & \quad = \quad (8 + 12)\vec{v}_f \\
40 & \quad = \quad 20\vec{v}_f \\
\vec{v}_f & \quad = \quad \frac{40}{20} = +2 \frac{\text{m}}{\text{s}}
\end{align*}
\]
Q: Mr. Stretchy has a mass of 60 kg and is holding a 5.0 kg box as he rides on a skateboard toward the west at a speed of 3.0 \( \frac{m}{s} \). (Assume the 60 kg is the mass of Mr. Stretchy and the skateboard combined.) He throws the box behind him, giving it a velocity of 2.0 \( \frac{m}{s} \) to the east.

What is Mr. Stretchy’s velocity after throwing the box?

A: This problem is like an inelastic collision in reverse; Mr. Stretchy and the box are together before the “collision” and apart afterwards. The equation would therefore look like this:

\[ m_s \vec{v}_s = m_s \vec{v}_{s,f} + m_b \vec{v}_{b,f} \]

Where the subscript “s” is for Mr. Stretchy, and the subscript “b” is for the box. Note that after Mr. Stretchy throws the box, he is moving one direction and the box is moving the other, which means we need to be careful about our signs. Let’s choose the direction Mr. Stretchy is moving (west) to be positive. Because the box is thrown to the east, this means the final velocity of the box will be \( \vec{v}_{b,f} = -2.0 \frac{m}{s} \)

Plugging values from the problem into the equation for the law of conservation of momentum, we get:

\[
\begin{align*}
\text{before} &= \text{after} \\
\vec{p}_i &= \vec{p}_{s,f} + \vec{p}_{b,f} \\
m_f \vec{v}_i &= m_s \vec{v}_{s,f} + m_b \vec{v}_{b,f} \\
(60 + 5)(+3) &= 60 \vec{v}_{s,f} + (5)(-2) \\
+195 &= 60 \vec{v}_{s,f} + (-10) \\
+205 &= 60 \vec{v}_{s,f} \\
\vec{v}_{s,f} &= \frac{+205}{60} = +3.4 \frac{m}{s}
\end{align*}
\]

Use this space for summary and/or additional notes.
Q: A soccer ball that has a mass of 0.43 kg is rolling east with a velocity of \( \frac{5.0}{s} \). It collides with a volleyball that has a mass of 0.27 that is rolling west with a velocity of \( \frac{6.5}{s} \). After the collision, the soccer ball is rolling to the west with a velocity of \( \frac{3.87}{s} \). Assuming the collision is perfectly elastic and friction between both balls and the ground is negligible, what is the velocity (magnitude and direction) of the volleyball after the collision?

A: This is an elastic collision, so the soccer ball and the volleyball are separate both before and after the collision. The equation is:

\[
m_s \vec{v}_{s,i} + m_v \vec{v}_{v,i} = m_s \vec{v}_{s,f} + m_v \vec{v}_{v,f}
\]

Where the subscript “s” is for the soccer ball and the subscript “v” is for the volleyball. In all elastic collisions, assume we need to keep track of the directions, which means we need to be careful about our signs. We don’t know which direction the volleyball will be moving after the collision (though a good guess would be that it will probably bounce off the soccer ball and move to the east). So let us arbitrarily choose east to be positive and west to be negative. This means:

<table>
<thead>
<tr>
<th>quantity</th>
<th>direction</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial velocity of soccer ball</td>
<td>east</td>
<td>(+\frac{5.0}{m} )</td>
</tr>
<tr>
<td>initial velocity of volleyball</td>
<td>west</td>
<td>(-\frac{6.5}{m} )</td>
</tr>
<tr>
<td>final velocity of soccer ball</td>
<td>west</td>
<td>(-\frac{3.87}{m} )</td>
</tr>
</tbody>
</table>

Plugging values from the problem into the equation for the law of conservation of momentum, we get:

\[
\begin{align*}
\vec{p}_{s,i} + \vec{p}_{v,i} &= \vec{p}_{s,f} + \vec{p}_{v,f} \\
m_s \vec{v}_{s,i} + m_v \vec{v}_{v,i} &= m_s \vec{v}_{s,f} + m_v \vec{v}_{v,f} \\
(0.43)(5.0) + (0.27)(-6.5) &= (0.43)(-3.87) + (0.27) \vec{v}_{v,f} \\
2.15 + (-1.755) &= -1.664 + 0.27 \vec{v}_{v,f} \\
0.395 &= -1.664 + 0.27 \vec{v}_{v,f} \\
2.059 &= 0.27 \vec{v}_{v,f} \\
\vec{v}_{s,f} &= \frac{+2.059}{0.27} = +\frac{7.63}{m} \text{ or } 7.63 \frac{m}{s} \text{ to the east}
\end{align*}
\]

Use this space for summary and/or additional notes.
Homework Problems

1. A large 10. kg sports ball is caught by a 70. kg student on the track team. If the ball was moving at \(4.0 \text{ m/s}\) and the student was stationary before catching the ball, how fast will the student be moving after catching the ball?

Answer: \(0.5 \text{ m/s}\)

Use this space for summary and/or additional notes.
2. An 0.010 kg bullet is fired down the barrel of a gun, which is pointed to the left. The bullet accelerates from rest to a velocity of 400 m/s. What velocity does the 1.5 kg gun acquire as a result of this impulse? (Note that neither the bullet nor the gun was moving before the “collision.”)

Answer: 2.7 m/s to the right

Use this space for summary and/or additional notes.
Although blank momentum bar charts are not provided for the remaining problems, feel free to draw them if you would find them helpful.

3. A 730 kg Mini runs into a stationary 2500 kg sport utility vehicle. If the Mini was moving at $10. \text{ m/s}$ initially, how fast will it be moving after making a completely inelastic collision with the SUV?

Answer: $2.3 \text{ m/s}$

4. A 6.0 kg bowling ball moving at $3.5 \text{ m/s}$ to the right makes a collision, head-on, with a stationary 0.70 kg bowling pin. If the ball is moving $2.77 \text{ m/s}$ to the right after the collision, what will be the velocity (magnitude and direction) of the pin?

Answer: $6.25 \text{ m/s}$ to the right

5. A pair of 0.20 kg billiard balls make an elastic collision. Before the collision, the 4-ball was moving $0.50 \text{ m/s}$ to the right, and the 8-ball was moving $1.0 \text{ m/s}$ to the left. After the collision, the 4-ball is now moving at $1.0 \text{ m/s}$ to the left. What is the velocity (magnitude and direction) of the 8-ball after the collision?

Answer: $0.50 \text{ m/s}$ to the right
6. A pair of billiard balls, each with mass \( m \), make an elastic collision. Before the collision, the 4-ball was moving with a velocity of \( \vec{v}_{4,i} \), and the 8-ball was moving with a velocity of \( \vec{v}_{8,j} \). After the collision, the 4-ball is now moving with a velocity of \( \vec{v}_{4,f} \). What is the velocity of the 8-ball after the collision? (You may use your work from problem #5 above to guide your algebra.)

Answer: \( \vec{v}_{8,f} = \vec{v}_{4,j} + \vec{v}_{8,j} - \vec{v}_{4,f} \)

7. A 75 kg astronaut on a space walk pushes to the right on a 1000. kg satellite. If the velocity of the satellite after the push is 0.75 \( \text{m/s} \), what is the velocity of the astronaut?

Answer: 10 \( \text{m/s} \) to the left