Mixed Series & Parallel Circuits

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3


Skills:
- analyze circuits by replacing networks of resistors with a single resistor of equivalent resistance.

Language Objectives:
- Set up and solve word problems involving electrical circuits with some components in series and others in parallel with each other.

Labs, Activities & Demonstrations:
- Light bulb mystery circuits.

Notes:

Mixed Series and Parallel Circuits (Resistors Only)

If a circuit has mixed series and parallel sections, you can determine the various voltages, currents and resistances by applying Kirchhoff’s Rules and/or by “simplifying the circuit.” Simplifying the circuit, in this case, means replacing resistors in series or parallel with a single resistor of equivalent resistance.

For example, suppose we need to solve the following mixed series & parallel circuit for voltage, current, resistance and power for each resistor:

![Mixed Series & Parallel Circuits Diagram]

Use this space for summary and/or additional notes.
Because the circuit has series and parallel sections, we cannot simply use the series and parallel rules across the entire table.

<table>
<thead>
<tr>
<th></th>
<th>R₁</th>
<th>R₂</th>
<th>R₃</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage (V)</td>
<td></td>
<td></td>
<td>40 V</td>
<td></td>
</tr>
<tr>
<td>Current (I)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resistance (R)</td>
<td>25 Ω</td>
<td>40 Ω</td>
<td>35 Ω</td>
<td></td>
</tr>
<tr>
<td>Power (P)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We can use Ohm’s Law \( V = IR \) and the power equation \( P = VI \) on each individual resistor and the totals for the circuit (columns), but we need two pieces of information for each resistor in order to do this.

Our strategy will be:

1. Simplify the resistor network until all resistances are combined into one equivalent resistor to find the total resistance.
2. Use \( V = IR \) to find the total current.
3. Work backwards through your simplification, using the equations for series and parallel circuits in the appropriate sections of the circuit until you have all of the information.

**Step 1:** If we follow the current through the circuit, we see that it goes through resistor R₁ first. Then it splits into two parallel pathways. One path goes through R₂ and the other goes through R₃.

There is no universal shorthand for representing series and parallel components, so let’s define the symbols “—” to show resistors in series, and “∥” to show resistors in parallel. The above network of resistors could be represented as:

\[ R₁ — (R₂ ∥ R₃) \]

Now, we simplify the network just like a math problem—start inside the parentheses and work your way out.
Step 2: Combine the parallel 40Ω and 35Ω resistors into a single equivalent resistance:

\[
\frac{1}{R_{\text{total}}} = \frac{1}{40} + \frac{1}{35}
\]

\[
\frac{1}{R_{\text{total}}} = 0.0250 + 0.0286 = 0.0536
\]

\[
R_{\text{total}} = \frac{1}{0.0536} = 18.6 \Omega
\]

Now, our circuit is equivalent to:

\[\begin{align*}
\text{25Ω} & \quad \text{40V} & \quad \text{18.6Ω}
\end{align*}\]

Step 3: Add the two resistances in series to get the total combined resistance of the circuit:

\[
18.6 + 25 = 43.6 \Omega
\]

Step 4: Now that we know the total voltage and resistance, we can use Ohm’s Law to find the total current:

\[
V = IR
\]

\[
40 = I(43.6)
\]

\[
I = \frac{40}{43.6} = 0.916 \text{ A}
\]

While we’re at it, let’s use \( P = VI = (40)(0.916) = 36.6 \text{ W} \) to find the total power.
Now we have:

<table>
<thead>
<tr>
<th></th>
<th>R₁</th>
<th>R₂</th>
<th>R₃</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>25</td>
<td>40</td>
<td>35</td>
<td>643.6Ω</td>
</tr>
<tr>
<td>Current</td>
<td>0.916 A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resistance</td>
<td>25</td>
<td>40</td>
<td>35</td>
<td>43.6Ω</td>
</tr>
<tr>
<td>Power</td>
<td>36.6 W</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now we work backwards.

The next-to-last simplification step was:

\[
\begin{align*}
25 \Omega & \quad 18.7 \Omega \\
40 \Omega & \quad 40 \text{ V}
\end{align*}
\]

The 25 Ω resistor is R₁. All of the current goes through it, so the current through R₁ must be 0.916 A. Using Ohm’s Law, this means the voltage drop across R₁ must be:

\[
V = IR
\]

\[
V = (0.916)(25) = 22.9 \text{ V}
\]

and the power must be:

\[
P = VI
\]

\[
P = (22.9)(0.916) = 21.0 \text{ W}
\]

This means that the voltage across the parallel portion of the circuit (R₂ || R₃) must be 40 – 22.9 = 17.1 V.
We can use this and Ohm’s Law to find the current through one branch:

\[ V_{40} = V_{35} = 40 - V_1 = 40 - 22.9 = 17.1V \]

\[ V_{40} = I_{40}R_{40} \]

\[ I_{40} = \frac{V_{40}}{R_{40}} = \frac{17.1}{40} = 0.428 \text{ A} \]

We can use Kirchhoff’s Junction Rule to find the current through the other branch:

\[ I_{total} = I_{40} + I_{35} \]

\[ 0.916 = 0.428 + I_{35} \]

\[ I_{35} = 0.488 \text{ A} \]

This gives us:

<table>
<thead>
<tr>
<th></th>
<th>R₁</th>
<th>R₂</th>
<th>R₃</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage (V)</td>
<td>22.9 V</td>
<td>17.1 V</td>
<td>17.1 V</td>
<td>40 V</td>
</tr>
<tr>
<td>Current (I)</td>
<td>0.916 A</td>
<td></td>
<td></td>
<td>0.916 A</td>
</tr>
<tr>
<td>Resistance (R)</td>
<td>25 Ω</td>
<td>40 Ω</td>
<td>35 Ω</td>
<td>43.6 Ω</td>
</tr>
<tr>
<td>Power (P)</td>
<td>21.0 W</td>
<td></td>
<td></td>
<td>36.6 W</td>
</tr>
</tbody>
</table>
Finally, because we now have voltage, current and resistance for each of the resistors $R_2$ and $R_3$, we can use $P = VI$ to find the power:

\[ V_2 = I_2R_2 \]
\[ V_3 = I_3R_3 \]
\[ V_2 = (0.428)(10) \]
\[ V_3 = (0.428)(30) \]
\[ V_2 = 4.28 \text{ V} \]
\[ V_3 = 12.84 \text{ V} \]
\[ P_2 = V_2I_2 \]
\[ P_3 = V_3I_3 \]
\[ P_2 = (4.28)(0.428) \]
\[ P_3 = (12.84)(0.428) \]
\[ P_2 = 1.83 \text{ W} \]
\[ P_3 = 5.50 \text{ W} \]

<table>
<thead>
<tr>
<th></th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_4$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage (V)</td>
<td>22.9 V</td>
<td>17.1 V</td>
<td>17.1 V</td>
<td>40 V</td>
</tr>
<tr>
<td>Current (I)</td>
<td>0.916 A</td>
<td>0.428 A</td>
<td>0.488 A</td>
<td>0.916 A</td>
</tr>
<tr>
<td>Resistance (R)</td>
<td>25 Ω</td>
<td>40 Ω</td>
<td>35 Ω</td>
<td>43.6 Ω</td>
</tr>
<tr>
<td>Power (P)</td>
<td>21.0 W</td>
<td><strong>7.32 W</strong></td>
<td><strong>8.34 W</strong></td>
<td>36.6 W</td>
</tr>
</tbody>
</table>

Alternately, because the total power is the sum of the power of each component, once we had the power in all but one resistor, we could have subtracted from the total to find the last one.
Homework Problems

1. What is the equivalent resistance between points A and B?

```
/\       1 kΩ
A--------B
|       |
| 1 kΩ   |
|       |
| 1 kΩ   |
/\       1 kΩ
```

Answer: 750 Ω

2. What is the equivalent resistance between points A and B?

```
/\       2 kΩ
A-------B
|       |
| 5 kΩ   |
|       |
| 1 kΩ   |
\       /\ 100 Ω
```

Answer: 1511 Ω or 1.511 kΩ

Use this space for summary and/or additional notes.
3. What is the equivalent resistance between points A and B?

Answer: 80.5 Ω
4. Fill in the table for the circuit below:

![Circuit Diagram]

<table>
<thead>
<tr>
<th></th>
<th>R₁</th>
<th>R₂</th>
<th>R₃</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage (V)</td>
<td></td>
<td></td>
<td></td>
<td>12 V</td>
</tr>
<tr>
<td>Current (I)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resistance (R)</td>
<td>220 Ω</td>
<td>130 Ω</td>
<td>470 Ω</td>
<td></td>
</tr>
<tr>
<td>Power (P)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use this space for summary and/or additional notes.