Sound & Music

Unit: Mechanical Waves & Sound

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A


Knowledge/Understanding Goals:
- how musical notes are produced and perceived

Skills:
- calculate the frequency of the pitch produced by a string or pipe

Language Objectives:
- Understand and correctly use the terms “resonance,” “frequency,” and “harmonic series.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to the frequencies and pitches (notes) produced by musical instruments.

Labs, Activities & Demonstrations:
- Show & tell: violin, penny whistle, harmonica, boomwhackers.
- Helmholtz resonators—bottles of different sizes/air volumes, slapping your cheek with your mouth open.
- Frequency generator & speaker.
- Rubens tube (sonic flame tube).
- Measure the speed of sound in air using a resonance tube.

Notes:
Sound waves are caused by vibrations that create longitudinal (compressional) waves in the medium they travel through (such as air).
pitch: how “high” or “low” a musical note is. The pitch is determined by the frequency of the sound wave.

resonance: when the wavelength of a half-wave (or an integer number of half-waves) coincides with one of the dimensions of an object. This creates standing waves that reinforce and amplify each other. The body of a musical instrument is an example of an object that is designed to use resonance to amplify the sounds that the instrument produces.
String Instruments
A string instrument (such as a violin or guitar) typically has four or more strings. The lower strings (strings that sound with lower pitches) are thicker, and higher strings are thinner. Pegs are used to tune the instrument by increasing (tightening) or decreasing (loosening) the tension on each string.

The vibration of the string creates a half-wave, i.e., $\lambda = 2L$. The musician changes the half-wavelength by using a finger to shorten the length of the string that is able to vibrate. (A shorter wavelength produces a higher frequency = higher pitch.)

The velocity of the wave produced on a string (such as a violin string) is given by the equation:

$$v_{\text{string}} = \sqrt{\frac{F_T L}{m}}$$

where:

- $f$ = frequency (Hz)
- $F_T$ = tension (N)
- $m$ = mass of string (kg)
- $L$ = length of string (m) = $\frac{\lambda}{2}$

The frequency (pitch) is therefore:

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{1}{2L} \sqrt{\frac{F_T L}{m}} = \sqrt{\frac{F_T}{4mL}}$$
Pipes and Wind Instruments

A pipe (in the musical instrument sense) is a tube filled with air. Something in the design of the mouthpiece causes the air inside the instrument to vibrate. When air is blown through the instrument, the air molecules compress and spread out at regular intervals that correspond with the length of the instrument, which determines the wavelength.

Most wind instruments use one of three ways of causing the air to vibrate:

**Brass Instruments**

With brass instruments like trumpets, trombones, French horns, etc., the player presses his/her lips tightly against the mouthpiece, and the player’s lips vibrate at the appropriate frequency.

**Reed Instruments**

With reed instruments, air is blown past a reed (a semi-stiff object) that vibrates back and forth. Clarinets and saxophones use a single reed made from a piece of cane (a semi-stiff plant similar to bamboo). Oboes and bassoons (“double-reed instruments”) use two pieces of cane that vibrate against each other. Harmonicas and accordions use reeds made from a thin piece of metal.

**Fipples**

Instruments with fipples include recorders, whistles and flutes. A fipple is a sharp edge that air is blown past. The separation of the air going past the fipple causes a pressure difference on one side vs. the other. The pressure builds more on one side, which forces air past the sharp edge. Then the pressure builds on the other side and the air switches back:

The frequency of this back-and-forth motion is what determines the pitch.

Use this space for summary and/or additional notes.
Open vs. Closed-Pipe Instruments

An open pipe has an opening at each end. A closed pipe has an opening at one end, and is closed at the other.

Examples of open pipes include uncapped organ pipes, whistles, recorders and flutes;

Notice that the two openings determine where the nodes are—these are the regions where the air pressure must be equal to atmospheric pressure (i.e., the air is neither compressed nor expanded). Notice also that as with strings, the wavelength of the sound produced is twice the length of the pipe, i.e., \( \lambda = 2L \).

If the pipe is open to the atmosphere at only one end, such as a clarinet or brass instrument, there is only one node, at the mouthpiece. The opening, where the person is blowing into the instrument, is an antinode—a region of high pressure. This means that the body of the instrument is \( \frac{1}{4} \) of a wave instead of \( \frac{1}{2} \), i.e., \( \lambda = 4L \).

This is why a closed-pipe instrument (e.g., a clarinet) sounds an octave lower than an open-pipe instrument of similar length (e.g., a flute).

Use this space for summary and/or additional notes.
The principle of a closed-pipe instrument can be used in a lab experiment to determine the frequency of a tuning fork (or the speed of sound) using a resonance tube—an open tube filled with water to a specific depth.

A tuning fork generates sound waves of a precise frequency at the top of the tube. Because this is a closed pipe, the source (just above the tube) is an antinode (maximum amplitude).

When the height of air above the water is exactly \( \frac{1}{4} \) of a wavelength \( \frac{\lambda}{4} \), the waves that are reflected back have maximum constructive interference with the source wave, which causes the sound to be significantly amplified. This phenomenon is called resonance.

Resonance will occur at any integer plus \( \frac{1}{4} \) of a wave—\( i.e., \) any distance that results in an antinode exactly at the top of the tube \( \frac{\lambda}{4}, \frac{5\lambda}{4}, \frac{9\lambda}{4}, \text{ etc.} \)

The resonance tube lab is a favorite of the College Board, and has appeared in one form or another on several AP Exams.
For an instrument with holes, like a flute or recorder, the first open hole creates a node at that point, which determines the half-wavelength (or quarter-wavelength):

The speed of sound in air is $v_s$ ($343 \text{ m/s}$ at 20°C and 1 atm), which means the frequency of the note (from the formula $v_s = \lambda f$) will be:

$$f = \frac{v_s}{2L} \text{ for an open-pipe instrument (flute, recorder, whistle), and}$$
$$f = \frac{v_s}{4L} \text{ for an closed-pipe instrument (clarinet, brass instrument).}$$

Note that the speed of sound in air increases as the temperature increases. This means that as the air gets colder, the frequency gets lower, and as the air gets warmer, the frequency gets higher. This is why wind instruments go flat at colder temperatures and sharp at warmer temperatures. When this happens, it’s not the instrument that is going out of tune, but the speed of sound!
Helmholtz Resonators (Bottles)

The resonant frequency of a bottle or similar container (called a Helmholtz resonator, named after the German physicist Hermann von Helmholtz) is more complicated to calculate, because it depends on the resonance frequencies of the air in the large cavity, the air in the neck of the bottle, and the cross-sectional area of the opening.

The formula works out to be:

$$f = \frac{v_s}{2\pi} \sqrt{\frac{A}{V_oL}}$$

where:

- $f$ = resonant frequency
- $v_s$ = speed of sound in air (343 m/s at 20°C and 1 atm)
- $A$ = cross-sectional area of the neck of the bottle (m$^2$)
- $V_o$ = volume of the main cavity of the bottle (m$^3$)
- $L$ = length of the neck of the bottle (m)

(Note that it may be more convenient to use measurements in cm, cm$^2$, and cm$^3$, and use $v_s = 34300 \text{ cm/s}$.)

You can make your mouth into a Helmholtz resonator by tapping on your cheek with your mouth open. You change the pitch by changing the size of the opening.

Use this space for summary and/or additional notes.
Frequencies of Music Notes

The frequencies that correspond with the pitches of the Western equal temperament scale are:

<table>
<thead>
<tr>
<th>pitch</th>
<th>frequency (Hz)</th>
<th>pitch</th>
<th>frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>261.6</td>
<td>G</td>
<td>392.0</td>
</tr>
<tr>
<td>D</td>
<td>293.7</td>
<td>A</td>
<td>440.0</td>
</tr>
<tr>
<td>E</td>
<td>329.6</td>
<td>B</td>
<td>493.9</td>
</tr>
<tr>
<td>F</td>
<td>349.2</td>
<td>C</td>
<td>523.2</td>
</tr>
</tbody>
</table>

Note that a note that is an octave above another note has exactly twice the frequency of the lower note.

Harmonic Series

Harmonic series: the additional, shorter standing waves that are generated by a vibrating string or column of air that correspond with integer numbers of half-waves. The natural frequency is called the fundamental frequency, and the harmonics above it are numbered—1\textsuperscript{st} harmonic, 2\textsuperscript{nd} harmonic, etc.) Any sound wave that is produced in a resonance chamber (such as a musical instrument) will produce the fundamental frequency plus all of the other waves of the harmonic series. The fundamental is the loudest, and each harmonic gets more quiet as you go up the harmonic series.
The following diagram shows the fundamental frequency and the first five harmonics produced by a pipe or a vibrating string:

![Diagram](image)

<table>
<thead>
<tr>
<th>Fraction of String</th>
<th>Wavelength</th>
<th>Harmonic</th>
<th>Frequency</th>
<th>Pitch (relative to fundamental)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(2L/1)</td>
<td>0</td>
<td>(f_o)</td>
<td>Fundamental.</td>
</tr>
<tr>
<td>(1/2)</td>
<td>(2L/2)</td>
<td>1(^{st})</td>
<td>(2f_o)</td>
<td>One octave above.</td>
</tr>
<tr>
<td>(1/3)</td>
<td>(2L/3)</td>
<td>2(^{nd})</td>
<td>(3f_o)</td>
<td>One octave + a fifth above.</td>
</tr>
<tr>
<td>(1/4)</td>
<td>(2L/4)</td>
<td>3(^{rd})</td>
<td>(4f_o)</td>
<td>Two octaves above.</td>
</tr>
<tr>
<td>(1/5)</td>
<td>(2L/5)</td>
<td>4(^{th})</td>
<td>(5f_o)</td>
<td>Two octaves + approximately a major third above.</td>
</tr>
<tr>
<td>(1/6)</td>
<td>(2L/6)</td>
<td>5(^{th})</td>
<td>(6f_o)</td>
<td>Two octaves + a fifth above.</td>
</tr>
<tr>
<td>(1/n)</td>
<td>(2L/n)</td>
<td>(n-1)(^{th})</td>
<td>(nf_o)</td>
<td>etc.</td>
</tr>
</tbody>
</table>

Use this space for summary and/or additional notes.
Beats

When two or more waves are close but not identical in frequency, their amplitudes reinforce each other at regular intervals.

For example, when the following pair of waves travels through the same medium, the amplitudes of the two waves have maximum constructive interference every five half-waves (2½ full waves) of the top wave vs. every six half-waves (3 full waves) of the bottom wave.

If this happened with sound waves, you would hear a pulse or “beat” every time the two maxima coincided.

The closer the two wavelengths (and therefore also the two frequencies) are to each other, the more half-waves it takes before the amplitudes coincide. This means that as the frequencies get closer, the time between the beats gets longer.

Piano tuners listen for these beats and adjust the tension of the string they are tuning until the time between beats gets longer and longer and finally disappears.
The Biophysics of Sound

When a person speaks, abdominal muscles force air from the lungs through the larynx.

The vocal cord vibrates, and this vibration creates sound waves. Muscles tighten or loosen the vocal cord, which changes the frequency at which it vibrates. Just like with a string instrument, the change in tension changes the pitch. Tightening the vocal cord increases the tension and produces a higher pitch, and relaxing the vocal cord decreases the tension and produces a lower pitch.

This process happens naturally when you sing. Amateur musicians who sing a lot of high notes can develop laryngitis from tightening their laryngeal muscles too much for too long. Professional musicians need to train themselves to keep their larynx muscles relaxed and use other techniques (such as breath support) to adjust their pitch.
When the sound reaches the ears, it travels through the auditory canal and causes the tympanic membrane (eardrum) to vibrate. The vibrations of the tympanic membrane cause pressure waves to travel through the middle ear and through the oval window into the cochlea.

The basilar membrane in the cochlea is a membrane with cilia (small hairs) connected to it, which can detect very small movements of the membrane. As with a resonance tube, the wavelength determines exactly where the sound waves will vibrate the basilar membrane the most strongly, and the brain determines the pitch (frequency) of a sound based on the precise locations excited by these frequencies.
Homework Problem

A tuning fork is used to establish a standing wave in an open ended pipe filled with air at a temperature of 20°C, where the speed of sound is $343 \text{ m/s}$, as shown below:

The sound wave resonates at the 3rd harmonic frequency of the pipe. The length of the pipe is 33 cm.

1. Sketch the standing wave inside the diagram of the pipe above. (For simplicity, you may sketch a transverse wave to represent the standing wave.)

2. Determine the wavelength of the resonating sound wave.

Answer: 22 cm

3. Determine the frequency of the tuning fork.

Answer: 1559 Hz

4. What is the next higher frequency that will resonate in this pipe?

Answer: 2079 Hz