Significant Figures

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 2 Learning Objectives: N/A

Knowledge/Understanding:

- Understand how significant figures work.

Skills:

- Perform calculations and round to the "correct" number of significant figures.

Language Objectives:

- Understand and correctly use the term "significant figure".
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

Significant figures are the digits in a number that are not merely placeholders. (*I.e.,* the non-rounded part of the number.)

There are two uses of significant figures, one good and one bad.

The proper use of significant figures is to tell which digits are actually part of the measurement when you record a measurement or data point.

For example, if you record a length as 12 cm, the significant figures indicate that the measurement was recorded to the nearest 1 cm. If you record a length as 12.0 cm, the significant figures indicate that the measurement was recorded to the nearest 0.1 cm.

If you record a length to the nearest 0.1 cm, *you must have a digit holding that place in the number, regardless of whether or not that digit has the value of zero.*

Use this space for summary and/or additional notes:

Significant Figures

The improper use of significant figures is as a crude attempt to imply the uncertainty of a number.

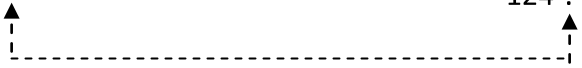
This use of significant figures is popular in high school science classes. It is based on the assumption that every measurement has an uncertainty of ±1 in the last digit. While this assumption is often (though not always) the right order of magnitude, it is rarely a good enough approximation to be useful! For this reason, everyone but high school teachers expresses the uncertainty of a measurement separately from its value, and no one but high school teachers actually use significant figures to express uncertainty.

How Significant Figures Work (as an Approximation)

Note that the rules you learned in chemistry for calculations with significant figures come from the rules for propagating relative error, with the assumption that the uncertainty is ± 1 in the last digit.

For example, consider the following problem:

problem:		“sig figs” equivalent:
123000	± 1000	123 ???.??? ?
0.0075	± 0.0001	0.007 5
+ 1650	± 10	+ 1 65?.??? ?
124650.0075	± 1010.0001	124 ???.??? ?



(Check this digit for rounding)

If we rounded our answer of 124 650 ± 1 010 to 125 000 ± 1 000, it would agree with the number we got by using the rules for significant figures.

Note, however, that our answer of 124 650 ± 1 010 suggests that the actual value is between 123 640 and 125 660, whereas the answer based on significant figures suggests that the actual value is between 124 000 and 126 000. This is why scientists frown upon use of “sig figs” instead of genuine error analysis. In the laboratory, if you insist on using sig figs, it is better to report answers to *one more sig fig* than the problem calls for, in order to minimize round-off answers in your results.

Use this space for summary and/or additional notes:

Significant Figures

As another example, consider the problem:

$$34.52 \times 1.4$$

Assume this means $(34.52 \pm 0.01) \times (1.4 \pm 0.1)$.

The answer (without its uncertainty) is $34.52 \times 1.4 = 48.328$

Because the calculation involves multiplication, we need to add the relative errors, and then multiply the total relative error by the answer to get the uncertainty.

The relative uncertainties are:

$$\frac{0.01}{34.52} = 0.00029$$

$$\frac{0.1}{1.4} = 0.0714$$

Adding them gives: 0.00029 + 0.0714 = 0.0717

Multiplying by our answer gives: 0.0717 × 48.328 = 3.466

The uncertainty is: ±3.466.

Our answer including the uncertainty is therefore 48.328 ± 3.466, which we can round to 48.3 ± 3.5.

Doing the calculation with sig figs would give us an answer of 48, which implies 48 ± 1. Again, this is in the ballpark of the actual uncertainty, but you should notice (and be concerned) that the sig figs method underrepresents the actual uncertainty by a factor of 3.5!

Despite the inherent problems with significant figures, *on the AP Exam, you will need to report calculations using the same sig fig rules that you learned in chemistry*.

Use this space for summary and/or additional notes:

Rules for Using Significant Figures

The first significant digit is where the “measured” part of the number begins—the first digit that is not zero.

The last significant digit is the last “measured” digit—the last digit whose true value is known or accurately estimated (usually ±1).

- If the number doesn’t have a decimal point, the last significant digit will be the last digit that is not zero.
- If the number does have a decimal point, the last significant digit will be the last digit.
- If the number is in scientific notation, the above rules tell us (correctly) that all of the digits before the “times” sign are significant.

For any measurement that does not have an explicitly stated uncertainty value, assume the uncertainty is ±1 *in the last significant digit.*

In the following numbers, the significant figures have been underlined:

- $\underline{13}$,000
- 0.0$\underline{275}$
- 0.0$\underline{150}$
- $\underline{6804.30500}$
- $\underline{6.0} \times 10^{23}$
- $\underline{3400.}$ (note the decimal point at the end)

Use this space for summary and/or additional notes:

Math with Significant Figures

Addition & Subtraction:

Line up the numbers in a column. Any column that has an uncertain digit—a zero from rounding—is an uncertain column. (Uncertain digits are shown as question marks in the right column below.) You need to round off your answer to get rid of all of the uncertain columns.

For example:

problem:	meaning:
123000	123???.????
0.0075	0.0075
+ 1650	+ 165?.????
124650.0075	124???.????

Because we can't know which digits go in the hundreds, tens, ones, and decimal places of ***all*** of the addends, the exact values of those digits must therefore be unknown in the sum.

This means we need to round off the answer to the nearest 1,000, which gives a final answer of 125,000 (which actually means 125,000 ± 1,000).

A silly (but correct) example of addition with significant digits is:

$$100 + 37 = 100$$

Use this space for summary and/or additional notes:

Multiplication, Division, Etc.

For multiplication, division, and just about everything else (except for addition and subtraction, which we have already discussed), round your answer off to the same number of significant digits as the number that has the fewest.

For example, consider the problem 34.52 × 1.4 = 48.328

The number 1.4 has the fewest significant digits (2). Remember that 1.4 really means 1.4 ± 0.1, which means the actual value, if we had more precision, could be anything between 1.3 and 1.5. Using "crank three times," the actual answer could therefore be anything between 34.52 × 1.3 = 44.876 and 34.52 × 1.5 = 51.780.

To get from the answer of 48.328 to the largest and smallest answers we would get from "crank three times," we would have to add or subtract approximately 3.5. (Notice that this agrees with the number we found previously for this same problem by propagating the relative error!) If the uncertainty is in the ones digit (greater than or equal to 1, but less than 10), this means that the ones digit is approximate, and everything beyond it is unknown. Therefore, using the rules of significant figures, we would report the number as 48.

In this problem, notice that the least significant term in the problem (1.4) had 2 significant digits, and the answer (48) also has 2 significant digits. This is where the rule comes from.

A silly (but correct) example of multiplication with significant digits is:

$$147 \times 1 = 100$$

Use this space for summary and/or additional notes:

Mixed Operations

For mixed operations, keep all of the digits until you're finished (so round-off errors don't accumulate), but keep track of the last significant digit in each step by putting a line over it (even if it's not a zero). Once you have your final answer, round it to the correct number of significant digits. Don't forget to use the correct order of operations (PEMDAS)!

For example:

$$137.4 \times 52 + 120 \times 1.77$$
$$(137.4 \times 52) + (120 \times 1.77)$$
$$7,\overline{1}44.8 + 2\overline{1}2.4 = 7,\overline{3}57.2 = 7,400$$

Note that in the above example, we kept all of the digits until the end. This is to avoid introducing small rounding errors at each step, which can add up to enough to change the final answer. Notice how, if we had rounded off the numbers at each step, we would have gotten the "wrong" answer:

$$137.4 \times 52 + 120 \times 1.77$$
$$(137.4 \times 52) + (120 \times 1.77)$$
$$7,\overline{1}00 + 2\overline{1}0 = 7,\overline{3}10 = 7,300 \longleftarrow$$ ☹

However, if we had done actual error propagation (remembering to add absolute errors for addition/subtraction and relative errors for multiplication/division), we would get the following:

$$137.4 \times 52 = 7144.8;\ \text{R.E.} = \frac{0.1}{137.4} + \frac{1}{52} = 0.01996$$
$$\text{partial answer} = 7144.8 \pm 142.6$$

$$120 \times 1.77 = 212.4;\ \text{R.E.} = \frac{1}{120} + \frac{0.01}{1.77} = 0.01398$$
$$\text{partial answer} = 212.4 \pm 2.97$$

The total absolute error is $142.6 + 2.97 = 145.6$

The best answer is therefore 7357.2 ± 145.6. *I.e.,* the actual value lies between approximately 7200 and 7500.

Use this space for summary and/or additional notes: