Solving Problems Symbolically

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 2 Learning Objectives: N/A

Skills:

- Be able to solve problems symbolically (using only variables; no numbers).

Language Objectives:

- Accurately describe and apply the concepts described in this section, using appropriate academic language.

Notes:

In solving physics problems, we are more often interested in the relationship between the quantities in the problem than we are in the numerical answer.

For example, suppose we are given a problem in which a person with a mass of 65 kg accelerates on a bicycle from rest $(0\,\frac{\text{m}}{\text{s}})$ to a velocity of $10\,\frac{\text{m}}{\text{s}}$ over a duration of 12 s and we wanted to know the force that was applied.

We could calculate acceleration as follows:

$$v - v_o = at$$
$$10 - 0 = a(12)$$
$$a = \frac{10}{12} = 0.8\overline{3}\,\frac{\text{m}}{\text{s}^2}$$

Then we could use Newton's second law:

$$F = ma$$
$$F = (65)(0.8\overline{3}) = 54.2\,\text{N}$$

We have succeeded in answering the question. However, the question and the answer are of no consequence. Obtaining the correct answer shows that we can manipulate two related equations and come out with the correct number.

Use this space for summary and/or additional notes:

However, if instead we decided that we wanted to come up with an expression for force in terms of the quantities given (mass, initial and final velocities and time), we would do the following:

We know that $F = ma$. We are given *m*, but not *a*, which means we need to replace a with an expression that includes only the quantities given.

We find an expression that contains *a*:

$$v - v_o = at$$

We recognize that $v_o = 0$, and we rearrange the rest of the equation:

$$v - v_o = at$$
$$v - 0 = at$$
$$v = at$$
$$a = \frac{v}{t}$$

Finally, we replace *a* in the first equation:

$$F = ma$$
$$F = (m)(\frac{v}{t})$$
$$F = \frac{mv}{t}$$

If the only thing we want to know is the value of *F* in one specific situation, we can substitute numbers at this point. However, we can also see from our final equation that increasing the mass or velocity will increase the value of the fraction on the right, which means the force would increase. We can also see that increasing the time would decrease the value of the fraction on the right, which means the force would decrease.

Solving the problem symbolically gives a relationship that holds true for all problems of this type in the natural world, instead of merely giving a number that answers a single pointless question. This is why the College Board and many college professors insist on symbolic solutions to equations.

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Homework Problems

1. Given $a = 2bc$ and $e = c^2 d$, write an expression for *e* in terms of *a*, *b*, and *d*.

2. Given $w = \frac{3}{2}xy^2$ and $z = \dfrac{q}{y}$:

   a. Write an expression for *z* in terms of *q*, *w*, and *x*.

   b. If you wanted to maximize the value of the variable *z* in question #2 above, what adjustments could you make to the values of *q*, *w*, and *x*?

   c. Changing which of the variables *q*, *w*, or *x* would give the largest change in the value of *z*?

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