The Laws of Sines & Cosines

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 2 Learning Objectives: N/A

Knowledge/Understanding Goals:

- the laws of sines and cosines and when they apply

Skills:

- find a missing side or angle of a non-right triangle

Language Objectives:

- Understand and correctly use the terms "sine," "cosine," and "tangent."
- Accurately describe and apply the concepts described in this section, using appropriate academic language.

Notes:

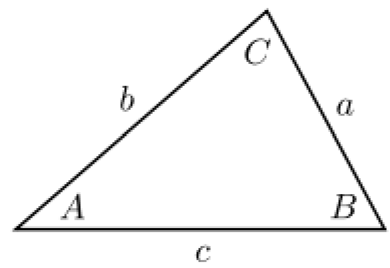
The Law of Sines and the Law of Cosines are often needed to calculate distances or angles in physics problems that involve non-right triangles. Trigonometry involving non-right triangles is beyond the scope of AP Physics 2, and is not tested on the AP exam.

Any triangle has three degrees of freedom, which means it is necessary to specify a minimum of three pieces of information in order to describe the triangle fully.

The law of sines and the law of cosines each relate four quantities, meaning that if three of the quantities are specified, the fourth can be calculated.

Use this space for summary and/or additional notes:

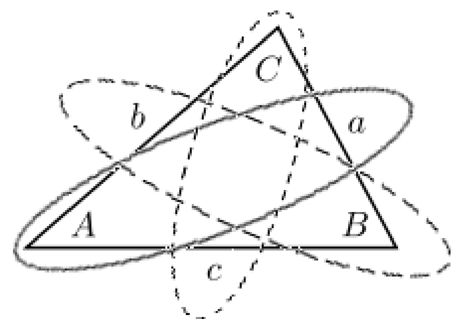
Consider the following triangle *ABC*, with sides *a*, *b*, and *c*, and angles *A*, *B*, and *C*. Angle *A* has its vertex at point *A*, and side *a* is opposite vertex *A* (and hence is also opposite angle *A*).



The Law of Sines

The law of sines states that, for any triangle:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



The four quantities related by the law of sines are two sides and their opposite angles. This means that in order to the law of sines, you need to know one angle and the length of the opposite side, plus any other side or any other angle. From this information, you can find the unknown side or angle, and from there you can work your way around the triangle and calculate every side and every angle.

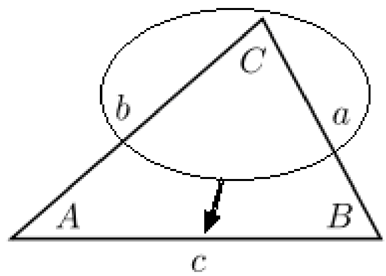
Use this space for summary and/or additional notes:

The Law of Cosines

The law of cosines states that, for any triangle:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

You can use the law of cosines to find any angle or the length of the third side of a triangle as long as you know any two sides and the included angle:



You can also use the law of cosines to find one of the angles if you know the lengths of all three sides.

Remember that which sides and angles you choose to be *a*, *b* and *c*, and *A*, *B* and *C* are arbitrary. This means you can switch the labels around to fit your situation, as long as angle *C* is opposite side *c* and so on.

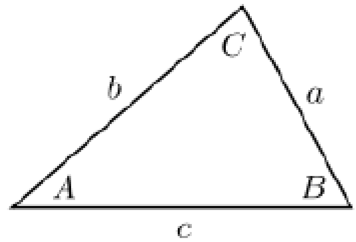
Notice that the Pythagorean Theorem is simply the law of cosines in the special case where *C* = 90° (because cos 90° = 0).

The law of cosines is algebraically less convenient than the law of sines, so a good strategy would be to use the law of sines whenever possible, reserving the law of cosines for situations when it is not possible to use the law of sines.

Use this space for summary and/or additional notes:

Homework Problems

Questions 1–7 are based on the following triangle, with sides *a*, *b* and *c*, and angles *A*, *B* and *C*. Assume that the triangle is ***not*** a right triangle.



Each of these problems requires use of the law of sines and/or the law of cosines. Note that the drawing is not to scale, and that sides *a*, *b* and *c* and angles *A*, *B* and *C* will be different for each problem.

Some problems may also require use of the fact that the angles of a triangle add up to 180°.

1. If $a = 5$, $c = 8$, and $A = 35°$ what is *C*?

   Answer: 66.6°

2. If $a = 7$, $A = 27°$, and $B = 58°$, what is *b*?

   Answer: 13.1

3. If $A = 25°$, $B = 75°$, and $c = 80$ what are *a* and *b*?

   Answer: $a = 34.3$; $b = 78.5$

Use this space for summary and/or additional notes:

4. If $C = 75°$, $b = 13$, and $a = 10$ what is c?

   Answer: 14.2

5. If $A = 30°$, $b = 22$, and $c = 24$ what is a?

   Answer: 12.1

6. If $C = 83°$, $b = 13$, and $c = 15$ what is a?

   Answer: 9.2

7. If $B = 55°$, $b = 20$, and $c = 22$ what is a?

   Answer: 21.3

Use this space for summary and/or additional notes: