Vectors

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 2 Learning Objectives: N/A

Knowledge/Understanding Goals:

- what a vector is

Skills:

- adding & subtracting vectors

Language Objectives:

- Understand and correctly use the terms "vector," "scalar," and "magnitude."
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

<u>vector</u>: a quantity that has both a magnitude (value) and a direction.

<u>scalar</u>: a quantity that has a value but does not have a direction. (A scalar is what you think of as a "regular" number, including its unit.)

<u>magnitude</u>: the scalar part of a vector (*i.e.,* the number and its units, but without the direction). If you have a force of 25 N to the east, the magnitude of the force is 25 N.

The mathematical operation of taking the magnitude of a vector is represented by two double vertical bars (like double absolute value bars) around the vector. For example, if $\vec{\boldsymbol{F}}$ is 25 N to the east, then $\|\vec{\boldsymbol{F}}\| = 25\,\text{N}$. Note, however, that the College Board generally uses regular absolute value bars for magnitude, *e.g.,* $|\vec{F}| = 25\,\text{N}$.

<u>resultant</u>: a vector that results from a mathematical operation (such as the addition of two vectors).

Use this space for summary and/or additional notes:

<u>unit vector</u>: a vector that has a magnitude of 1.

Unit vectors are typeset as vectors, but with a “hat” instead of an arrow.

The purpose of a unit vector is to turn a scalar into a vector without changing its magnitude (value). For example, if d represents the scalar quantity 25 cm, and $\boldsymbol{\hat{n}}$* represents a unit vector pointing southward, then $d\boldsymbol{\hat{n}}$ would represent a vector of 25 cm to the south.

The letters $\boldsymbol{\hat{i}}$, $\boldsymbol{\hat{j}}$, and $\boldsymbol{\hat{k}}$ are often used to represent unit vectors along the x, y, and z axes, respectively.

Variables that represent vectors are traditionally typeset in ***bold Italics***. Vector variables may also optionally have an arrow above the letter:

$$\boldsymbol{J}, \vec{\boldsymbol{F}}, \boldsymbol{v}$$

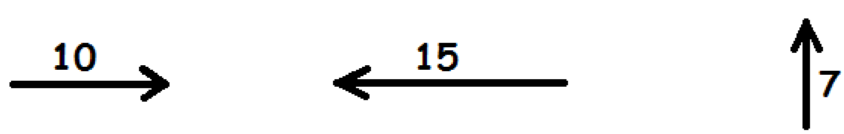
Variables that represent scalars are traditionally typeset in *plain Italics:*

$$V, t, \lambda$$

Note that a variable that represents only the magnitude of a vector quantity is generally typeset as a scalar:

For example, $\vec{\boldsymbol{F}}$ is a vector representing a force of 25 N to the east. (Notice that the vector includes the magnitude or amount ***and*** the direction.) The magnitude would be 25 N, and would be represented by the variable F.

Vectors are represented graphically using arrows. The length of the arrow represents the magnitude of the vector, and the direction of the arrow represents the direction of the vector:



magnitude 10
direction: 0°
(to the right)

magnitude 15
direction: +180°
(to the left)

magnitude 7
direction: +90° (up)

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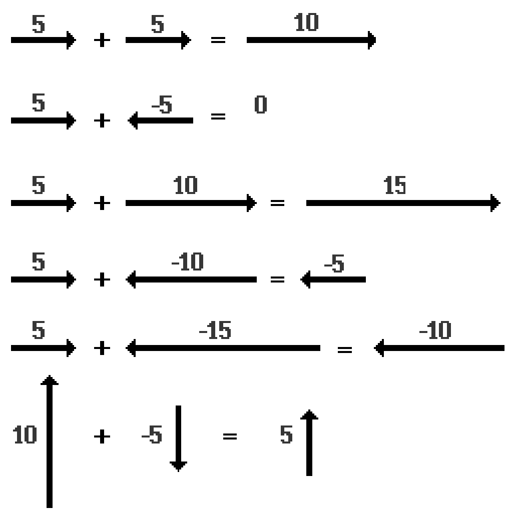
* $\hat{n}$ is pronounced “n hat”

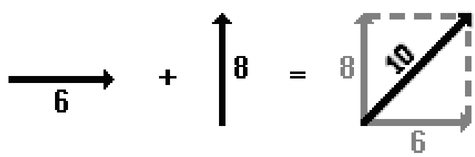
Use this space for summary and/or additional notes:

Adding & Subtracting Vectors

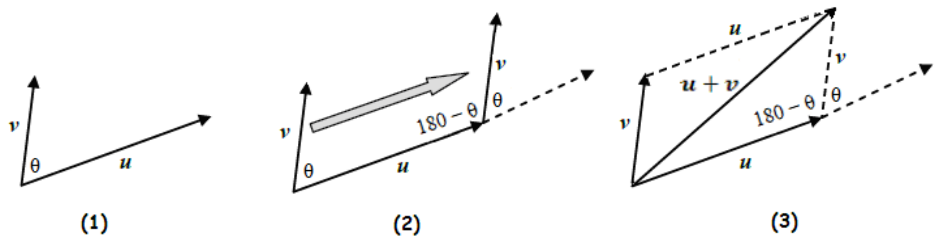
If the vectors have the same direction or opposite directions, the resultant is easy to envision:



If the vectors are not in the same direction, we move them so they start from the same place and complete the parallelogram. If they are perpendicular, we can add them using the Pythagorean theorem:



The same process applies to adding vectors that are not perpendicular:

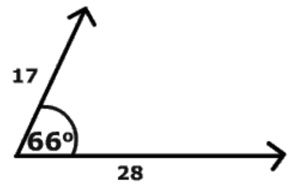


However, the trigonometry needed for the calculations is more involved and is beyond the scope of this course.

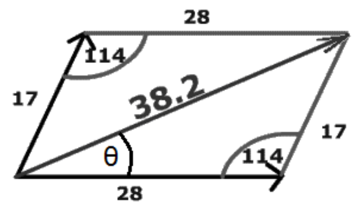
Note that the AP Physics 2 exam requires only a mathematical understanding of vectors at right angles to each other.

Use this space for summary and/or additional notes:

If the vectors have different (and not opposite) directions, the resultant vector is given by the diagonal of the parallelogram created from the two vectors. For example, the following diagram shows addition of a vector with a magnitude of 28 and a direction of 0° added to a vector with a magnitude of 17 and a direction of 66°:



The resultant vector is given by the parallelogram created by the two vectors:



The magnitude can be calculated using the law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$
$$c^2 = 28^2 + 17^2 - 2(28)(17)\cos(114°)$$
$$c^2 = 1460$$
$$c = \sqrt{1460} = 38.2$$

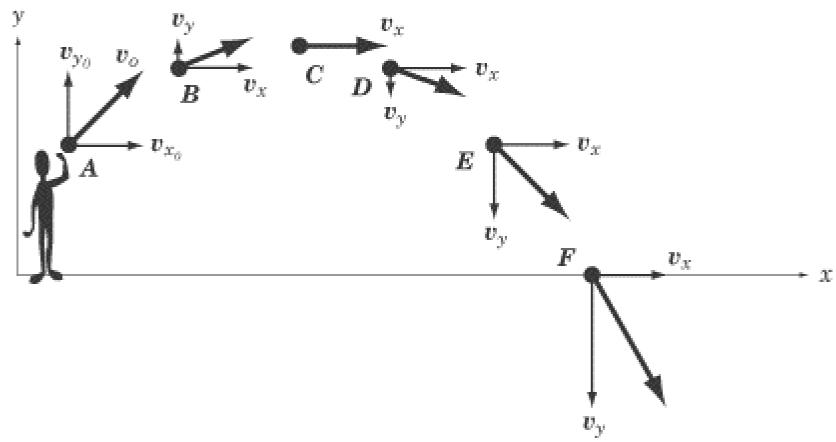
For the direction, use the law of sines:

$$\frac{38.2}{\sin 114°} = \frac{17}{\sin\theta}$$
$$\sin\theta = \frac{17\sin 114°}{38.2} = \frac{(17)(0.914)}{38.2} = 0.407$$
$$\theta = \sin^{-1} 0.407 = 24.0°$$

Thus the resultant vector has a magnitude of 38.2 and a direction of +24.0° (or 24.0° above the horizontal).

Use this space for summary and/or additional notes:

One type of physics problem that commonly uses vectors is two-dimensional projectile motion. If the motion of the projectile is represented by a vector, $\vec{\boldsymbol{v}}$, at angle θ, the vector can be represented as the sum of a horizontal vector $\vec{\boldsymbol{v}}_x$ and a vertical vector $\vec{\boldsymbol{v}}_y$. This is useful because the horizontal vector gives us the component (portion) of the vector in the x-direction, and the vertical vector gives us the component of the vector in the y-direction.



Notice that $\vec{\boldsymbol{v}}_x$ remains constant, but $\vec{\boldsymbol{v}}_y$ changes (because of the effects of gravity).

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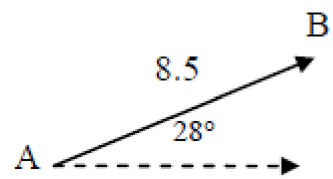
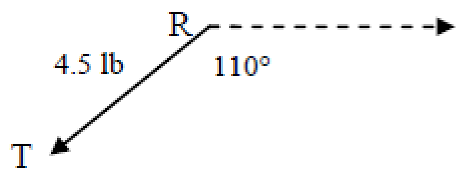
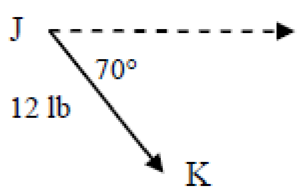
Homework Problems

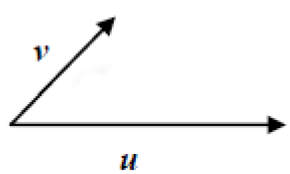
State the magnitude and direction of each of the following:

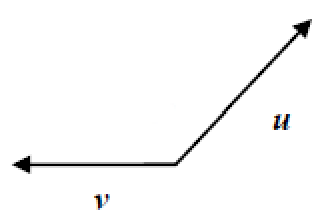
1. 

2. 

3. 

Sketch the resultant of each of the following.

4. 

5. 

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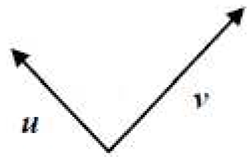
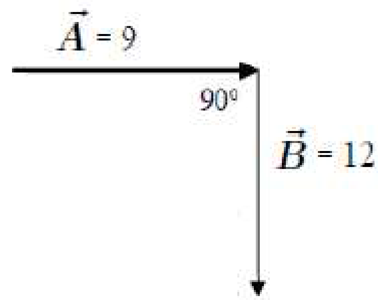
6.



For the following vectors $\vec{A}$ & $\vec{B}$:



7. Determine $\vec{A}+\vec{B}$ (both magnitude and direction)

8. Determine $\vec{A}-\vec{B}$ (both magnitude and direction)

Use this space for summary and/or additional notes: