Buoyancy

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 2 Learning Objectives: 1.E.1.1, 1.E.1.2, 3.4.C.1, 3.4.C.2

Knowledge/Understanding:

- Buoyancy & Archimedes' Principle

Skills:

- Calculate the buoyant force on an object

Language Objectives:

- Understand and correctly use the terms "displace" and "buoyant" or "buoyancy."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to buoyancy.

Labs, Activities & Demonstrations:

- Upside-down beaker with tissue
- Ping-pong ball or balloon under water
- beaker floating in water
    - right-side-up with weights
    - upside-down with trapped air
- Spring scale with mass in & out of water on a balance
- Cartesian diver
- Aluminum foil & weights
- Cardboard & duct tape canoes

Notes:

displace: to push out of the way

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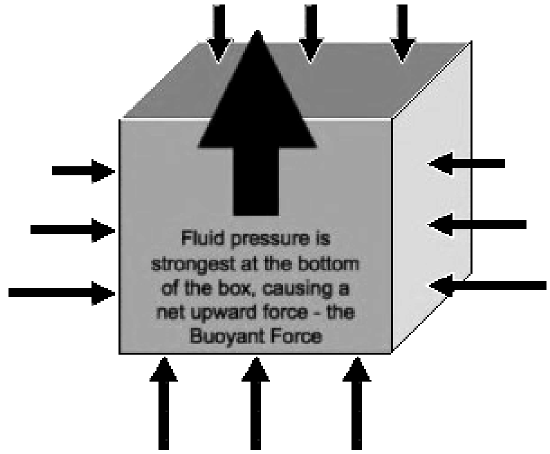
buoyancy: a net upward force caused by the differences in hydrostatic pressure at different levels within a fluid:



The hydrostatic pressure is stronger at the bottom of the object than at the top, which causes a net upward force on the object.

When an object displaces a fluid:

1. The volume of the fluid displaced equals the volume of the submerged part of the object: $V_d = V_{submerged}$
2. The weight of the fluid displaced equals the buoyant force (F_B).
3. The net force on the object, if any, is the difference between its weight and the buoyant force: $F_{net} = F_g - F_B$.

The buoyant force is caused by the difference in hydrostatic pressure from the bottom of the object to the top, *i.e.,* $F_B = PA_d$. However, the hydrostatic pressure is a function of the depth and the area is a function of the shape of the object. By manipulating the equations and recognizing that area times height equals volume, we get the equation for the buoyant force:

$$F_B = \rho V_d g$$

Where F_B is the buoyant force, ρ is the density of the fluid that the object is submerged in, V_d is the volume of the fluid that has been displaced, and g is the strength of the gravitational field (which is the same as the acceleration due to gravity, $10\,\frac{m}{s^2}$).

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For those who want a calculus-based explanation, the hydrostatic force over the surface of the object is given by combining the hydrostatic equation:

$$P_H = \int \rho g \, dh$$

with the equation for pressure, rearranged to solve for force:

$$F = PA$$

We recognize that because the area varies with depth, this results in a double surface integral over the area of the object (*dA*) and the height (*dh*):

$$F_B =$$

Conveniently, is simply the volume of the part of the object that is submerged, which is equal to the volume of fluid displaced (item #1 above). Therefore:

$$F_B = \rho g$$

This results the familiar equation for the buoyant force:

$$F_B = \rho V_d g$$

If the object floats, there is no net force, which means the weight of the object is equal to the buoyant force. This means:

$$F_g = F_B$$
$$mg = \rho V_d g$$

Cancelling *g* from both sides gives $m = \rho V_d$, which can be rearranged to give the equation for density:

$$\rho = \frac{m}{V_d}$$

If the object sinks, the weight of the object is greater than the buoyant force. This means $F_B = \rho V_d g$, $F_g = mg$, and the weight of the submerged object is $F_{net} = F_g - F_B$.

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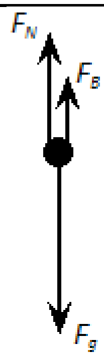
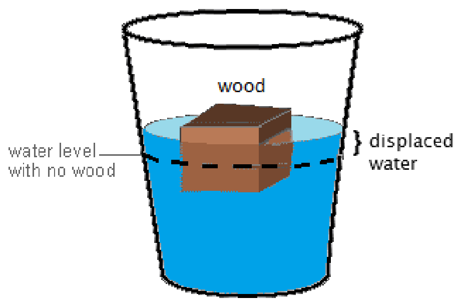


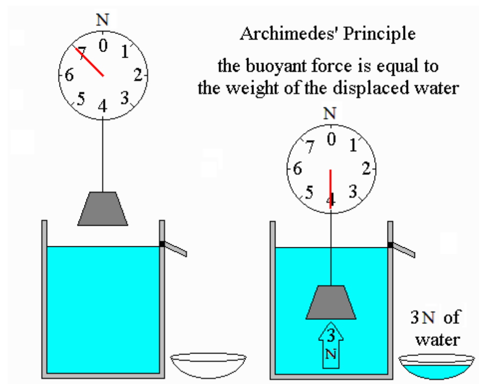
Note that if the object is resting on the bottom of the container, the net force must be zero, which means the normal force and the buoyant force combine to supply the total upward force. *I.e.,* for an object resting on the bottom:

$$F_{net} = 0 = F_g - (F_B + F_N)$$

which means:

$$F_g = F_B + F_N$$





This concept is known as Archimedes' Principle, named for the ancient Greek scientist who discovered it.

The buoyant force can be calculated from the following equation:

$$F_B = m_d g = \rho V_d g$$

where:

F_B = buoyant force
m_d = mass of fluid displaced by the object
g = strength of gravitational field = acceleration due to gravity ($10\,\frac{\text{m}}{\text{s}^2}$ on Earth)
ρ = density of the fluid applying the buoyant force (*e.g.,* water, air)
V_d = volume of fluid displaced by the object

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Sample Problems:

Q: A cruise ship displaces 35 000 tonnes of water when it is floating. (1 tonne = 1 000 kg) If sea water has a density of $1\,025\,\frac{\text{kg}}{\text{m}^3}$, what volume of water does the ship displace? What is the buoyant force on the ship?

A: $\rho = \frac{m}{V}$

$1025\,\frac{\text{kg}}{\text{m}^3} = \frac{35\,000\,000\,\text{kg}}{V}$

$V = 34\,146\,\text{m}^3$

$F_B = \rho V_d g$

$F_B = (1025\,\frac{\text{kg}}{\text{m}^3})(34\,146\,\text{m}^3)(10\,\frac{\text{m}}{\text{s}^2})$

$F_B = \boxed{3.5\times10^8\ \text{N}}$

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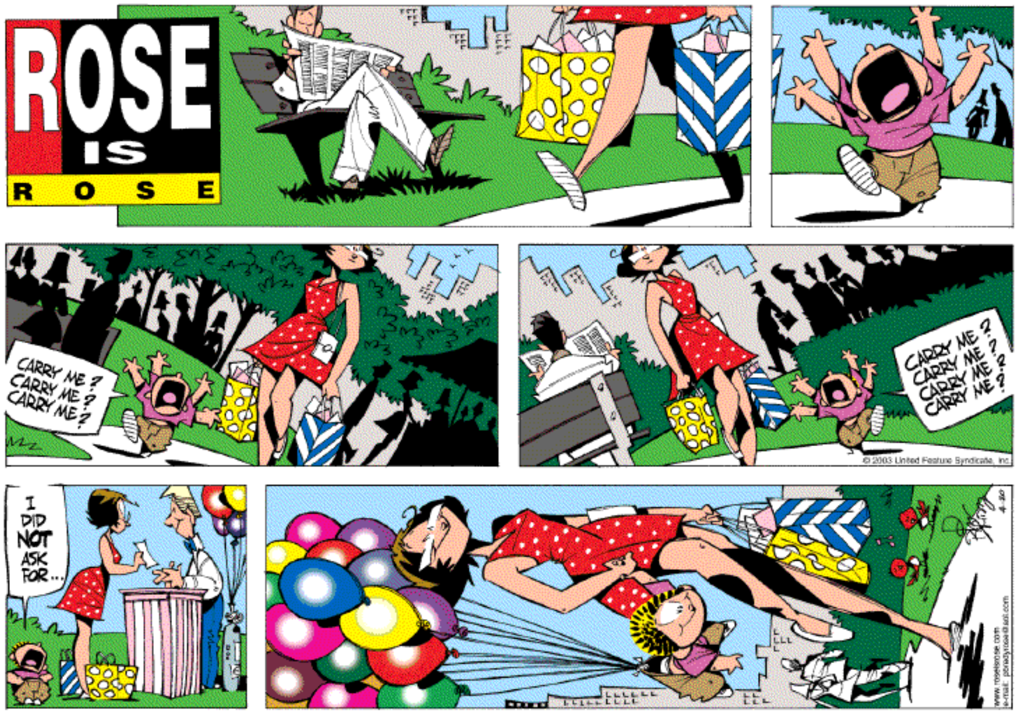
Q: Consider the following cartoon:



Given the following assumptions:

- The balloons are standard 11″ balloons, meaning that they have a diameter of 11 inches (28 cm), which equals a radius of 14 cm = 0.14 m.
- The temperature is 20°C. At this temperature, air has a density of $1.200\,\frac{\text{kg}}{\text{m}^3}$, and helium has a density of $0.166\,\frac{\text{kg}}{\text{m}^3}$.
- Pasquale is probably about four years old. The average mass a four-year-old boy is about 16 kg.
- The mass of an empty balloon plus string is 2.37 g = 0.00237 kg

How many balloons would it actually take to lift Pasquale?

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A: In order to lift Pasquale, $F_B = F_g$.

$$F_g = mg = (16)(10) = 160\text{ N}$$

$$F_B = \rho_{air} V_d g = (1.2)\, V_d\ (10)$$

Because $F_B = F_g$, this means:

$$160 = 12\, V_d$$

$$V_d = 13.\overline{3}\text{ m}^3$$

Assuming spherical balloons, the volume of one balloon is:

$$V = \frac{4}{3}\pi r^3 = (\frac{4}{3})(3.14)(0.14)^3 = 0.0115\text{ m}^3$$

Therefore, we need $\dfrac{13.\overline{3}}{0.0115}$ = 1 160 balloons to lift Pasquale.

However, the problem with this answer is that it doesn't account for the mass of the helium, the balloons and the strings.

Each balloon contains $0.0115\text{ m}^3 \times 0.166\frac{\text{kg}}{\text{m}^3} = 0.00191\text{ kg} = 1.91\text{ g}$ of helium.

Each empty balloon (including the string) has a mass of 2.37 g = 0.00237 kg, so the total mass of each balloon full of helium is
1.91 g + 2.37 g = 4.28 g = 0.00428 kg.

This means if we have *n* balloons, the total mass of Pasquale plus the balloons is $16 + 0.00428n$ kilograms. The total weight of Pasquale plus the balloons is therefore this number times 10, which equals 160 + 0.0428*n.*

The buoyant force of one balloon is:

$$F_B = \rho_{air} V_d g = (1.2)(0.0115)(10) = 0.138\text{ N}$$

Therefore, the buoyant force of *n* balloons is 0.138*n* newtons.

For Pasquale to be able to float, $F_B = F_g$, which means

$$0.138n = 0.0428n + 160$$

$$0.0952n = 160$$

n = $\boxed{\text{1 680 balloons}}$

Use this space for summary and/or additional notes.

Homework Problems

1. A block is 0.12 m wide, 0.07 m long and 0.09 m tall and has a mass of 0.50 kg. The block is floating in water with a density of $1000\,\frac{\text{kg}}{\text{m}^3}$.
    a. What volume of the block is below the surface of the water?

       Answer: $5\times10^{-4}\ \text{m}^3$

    b. If the entire block were pushed under water, what volume of water would it displace?

       Answer: $7.56\times10^{-4}\ \text{m}^3$

    c. How much additional mass could be piled on top of the block before it sinks?

       Answer: 0.256 kg

2. The SS United Victory was a cargo ship launched in 1944. The ship had a mass of 15 200 tonnes fully loaded. (1 tonne = 1 000 kg). The density of sea water is $1025\,\frac{\text{kg}}{\text{m}^3}$. What volume of sea water did the SS United Victory displace when fully loaded?

   Answer: 14 829 m3

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3. An empty box is 0.11 m per side. It will slowly be filled with sand that has a density of $3500\,\frac{\text{kg}}{\text{m}^3}$. What volume of sand will cause the box to sink in water? (Assume water has a density of $1000\,\frac{\text{kg}}{\text{m}^3}$. You may neglect the weight of the box.)

   Answer: $3.80\times10^{-4}\ \text{m}^3$

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