Gas Laws

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 2 Learning Objectives: 3.4.C.1, 3.4.C.2, 4.C.3.1, 7.A.3.3, 7.B.1.1

Knowledge/Understanding:

- Boyle's Law (pressure *vs.* volume)
- Amontons' Law (pressure *vs.* temperature)
- Charles' Law (temperature *vs.* volume)
- Avogadro's Principle (number of particles *vs.* volume)
- ideal gas law
- combined gas law

Skills:

- Solve problems using the gas laws

Language Objectives:

- Understand and correctly use the terms "pressure," "volume," and "temperature," and "ideal gas."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems using the combined gas law and the ideal gas law.

Labs, Activities & Demonstrations:

- Vacuum pump (pressure & volume) with:
  - balloon (air vs. water)
  - shaving cream
- Absolute zero apparatus (pressure & temperature)
- Can crush (pressure, volume & temperature)

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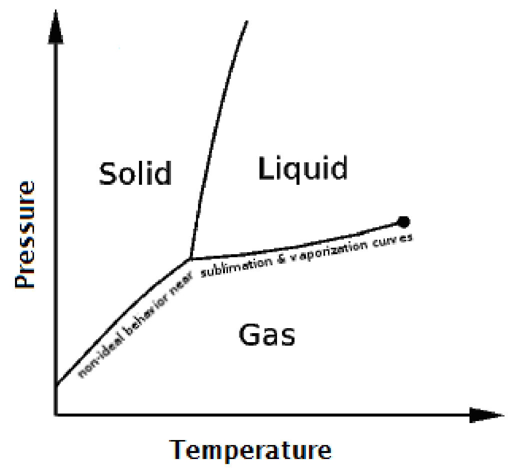
Notes:

ideal gas: a gas that behaves as if each molecule acts independently, according to kinetic-molecular theory. Specifically, this means the molecules are far apart, and move freely in straight lines at constant speeds. When the molecules collide, the collisions are perfectly elastic, which means they bounce off each other with no energy or momentum lost.

Most gases behave ideally except at temperatures and pressures near the vaporization curve on a phase diagram. (*I.e.,* gases stop behaving ideally when conditions are close to those that would cause the gas to condense to a liquid or solid.)



Boyle's Law

In 1662, British physicist and chemist Robert Boyle published his findings that the pressure and volume of a gas were inversely proportional. If temperature and the number of particles of gas are constant, then for an ideal gas:

$$P_1 V_1 = P_2 V_2$$

(Note that by convention, gas laws use subscripts "1" and "2" instead of "i" and "f".)

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Charles' Law

In the 1780s, French physicist Jacques Charles discovered that the volume and temperature of a gas were directly proportional. If pressure and the number of particles are constant, then for an ideal gas:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Amontons' Law

In 1702, French physicist Guillaume Amontons discovered that the pressure and temperature of a gas were directly proportional. If volume and the number of particles are constant, then for an ideal gas:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

(This law is often attributed to the French chemist Joseph Louis Gay-Lussac.)

Avogadro's Principle

In 1811, Italian physicist Amedeo Avogadro (whose full name was Lorenzo Romano Amedeo Carlo Avogadro di Quaregna e di Cerreto) published the principle that equal volumes of an ideal gas at the same temperature and pressure must contain equal numbers of particles:

$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

Because it is usually more convenient to work with moles of gas (n) rather than particles (N), we can rewrite Avogadro's principle as:

$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

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The Combined Gas Law

If each of the above principles is combined, the following relationship is found for an ideal gas:

$$\frac{P_1 V_1}{N_1 T_1} = \frac{P_2 V_2}{N_2 T_2} = \text{constant} = k_B = 1.38 \times 10^{-23}\,\tfrac{\text{J}}{\text{K}}$$

$$\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2} = \text{constant} = R = 8.31\,\tfrac{\text{J}}{\text{mol}\cdot\text{K}}$$

When pressure is in Pa, volume is in m^3, and temperature is in Kelvin, the value of the constant k_B turns out to be $1.38 \times 10^{-23}\,\tfrac{\text{J}}{\text{K}}$. This number is called Boltzmann's constant, named for the German physicist Ludwig Boltzmann. Physicists usually use the variable k_B to represent Boltzmann's constant.

Chemists prefer to use the gas constant, R, which is equal to Boltzmann's constant times the conversion from moles to particles (Avogadro's constant):

$$R = k_B N_A = (1.38 \times 10^{-23}\,\tfrac{\text{J}}{\text{K}})(6.02 \times 10^{23}\,\text{mol}^{-1}) = 8.31\,\tfrac{\text{J}}{\text{mol}\cdot\text{K}}$$

AP Exam problems often use moles, so it is important to understand how to work with both.

Note, however, that in most problems, the number of particles (or moles) of gas remains constant. This means $n_1 = n_2$ and we can cancel it from the equation, which gives:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

This equation is called the "combined gas law", which is used to solve most "before/after" problems involving ideal gases.

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When using the combined gas law, any quantity that is not changing may be cancelled out of the equation. (If a quantity is not mentioned in the problem, you can assume that it is constant and may be cancelled.)

For example, suppose a problem doesn't mention anything about temperature. That means T is constant and you can cancel it. When you cancel *T* from both sides of the combined gas law, you get:

$$\frac{P_1V_1}{\cancel{T_1}} = \frac{P_2V_2}{\cancel{T_2}}$$ which simplifies to $P_1V_1 = P_2V_2$ (Boyle's Law)

Solving Problems Using the Combined Gas Law

You can use this method to solve any "before/after" gas law problem:

1. Determine which variables you have
2. Determine which values are *inital* (#1) *vs.* *final* (#2).
3. Start with the combined gas law and cancel any variables that are explicitly not changing or omitted (assumed not to be changing).
4. Substitute your numbers into the resulting equation and solve. (Make sure all initial and final quantities have the same units, and don't forget that temperatures *must* be in Kelvin!)

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The Ideal Gas Law

The equation that defines either Boltzmann's constant or the gas constant is called the ideal gas law:

$$\frac{PV}{NT} = k_B \quad \text{and} \quad \frac{PV}{nT} = R$$

Multiplying both sides of the first equation by *NT* gives the ideal gas law using particles:

$$PV = Nk_BT$$

Multiplying both sides of the second equation by *nT* gives the more common form of the ideal gas law, using moles:

$$PV = nRT$$

where:

- P = absolute pressure (Pa)
- V = volume (m^3)
- N = number of particles (or molecules) of gas
- n = number of moles of gas
- k_B = Boltzmann's constant ($1.38 \times 10^{-23} \frac{J}{K}$).
- R = gas constant ($8.31 \frac{J}{mol \cdot K}$)
- T = absolute temperature (K)

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Note that the units on both sides of this equation are equivalent to Newton-meters, which are equivalent to joules (energy). This means that the ideal gas law is really just a mathematical statement of the work-energy theorem.

The left side of the equation, PV, represents the work that the gas can do on its surroundings. This is because:

$$P = \frac{F}{A}, \text{ which means } PV = \frac{FV}{A} = Fd = W$$

The right side of the equation, nRT, represents the number of moles (n) times the average kinetic energy of each particle (T), which equals the total kinetic energy of the particles. (The gas constant, R, combines Boltzmann's constant and the conversion from particles to moles. Think of it as the number that makes the units work out correctly.)

In other words, the work that a gas can do equals the total kinetic energy of its particles.

Solving Problems Using the Ideal Gas Law

If a gas behaves according to the ideal gas law, simply substitute the values for pressure, volume, number of moles (or particles), and temperature into the equation. Be sure your units are correct (especially that temperature is in Kelvin), and that you use the correct constant, depending on whether you know the number of particles or the number of moles of the gas.

In chemistry classes, problems often involve different values of the gas constant depending on the units:

$$8.31\,\frac{\text{J}}{\text{mol·K}} = 0.0821\,\frac{\ell\text{·atm}}{\text{mol·K}} = 62.4\,\frac{\ell\text{·Torr}}{\text{mol·K}} = 1.987\,\frac{\text{cal}}{\text{mol·K}} = 1.987\,\frac{\text{BTU}}{\text{lb–mol·°R}}$$

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Sample problems:

Q: A sample of an ideal gas contains 2.10×10^{24} particles, has a pressure of 1.20 bar, and a temperature of 35°C. What is its volume?

A: First, we need to declare our variables.

Note that for the units to work out correctly (because of the units of Boltzmann's constant), pressure must be in Pa, volume must be in m^3, and temperature must be in Kelvin.

$P = 1.20\,\text{bar} \times \frac{100\,000\,\text{Pa}}{1\,\text{bar}} = 120\,000\,\text{Pa}$

$V = V$ (because we don't' know it yet)

$N = 2.1\times10^{24}$

$k_B = 1.38\times10^{-23}\,\frac{\text{m}^2\cdot\text{kg}}{\text{s}^2\cdot\text{K}}$

$T = 35°\text{C} + 273 = 308\,\text{K}$

Then we substitute these numbers into the ideal gas law and solve:

$$PV = Nk_BT$$
$$(120\,000\,\text{Pa})V = (2.10\times10^{24})(1.38\times10^{-23}\,\tfrac{\text{kg}\cdot\text{m}^2}{\text{s}^2\cdot\text{K}})(308\,\text{K})$$
$$120\,000\,V = 8926$$
$$V = 0.0744\,\text{m}^3$$

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Q: A gas has a temperature of 25°C and a pressure of 1.5 bar. If the gas is heated to 35°C, what will the new pressure be?

A: 1. Find which variables we have.

We have two temperatures (25°C and 35°C), and two pressures (1.5 bar and the new pressure that we're looking for).

2. Find the action being done on the gas ("heated"). Anything that was true about the gas *before* the action is time "1", and anything that is true about the gas *after* the action is time "2".

Time 1 ("before"):	**Time 2 ("after")**:
$P_1 = 1.5$ bar	$P_2 = P_2$
$T_1 = 25°\text{C} + 273 = 298$ K	$T_2 = 35°\text{C} + 273 = 308$ K

3. Set up the formula. We can cancel volume (V), because the problem doesn't mention it:

$$\frac{P_1 \cancel{V_1}}{T_1} = \frac{P_2 \cancel{V_2}}{T_2} \text{ which gives us } \frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ (Amontons' Law)}$$

4. Plug in our values and solve:

$$\frac{1.5\,\text{bar}}{298\,\text{K}} = \frac{P_2}{308\,\text{K}} \rightarrow \boxed{P_2 = 1.55\text{ bar}}$$

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Homework Problems

For these problems, you may either convert kilograms to grams to moles (as you did in chemistry) and use $PV = nRT$, or you may convert kilograms to molecules directly using the atomic mass, and use $PV = Nk_BT$.

1. 1. A sample of 0.032 kg of oxygen (atomic mass 16.0 amu; formula O_2) at 50.0°C and 98 600 Pa occupies what volume?

   Answer: 0.0272 m^3

2. When the pressure in a certain gas cylinder with a volume of 0.0045 m^3 reaches 500. bar, the cylinder is likely to explode. What would the pressure in this cylinder be if it contained 3.2 kg of argon gas (atomic mass 40.0 amu) at 25.0°C? Would it explode?

   Answer: 440 bar; No.

3. A sample of oxygen gas occupies a volume of 0.250 m^3 at a pressure of 1.75 bar. What volume will it occupy at 2.50 bar?

   Answer: 0.175 m^3

4. H_2 gas was cooled from 150.°C to 50.°C. Its new pressure is 75 000 Pa. What was its original pressure?

   Answer: 98 220 Pa

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5. A sample of air has a volume of 60.0 mL at 1.00 bar and 0.0°C. What volume will the sample have at 55.0°C and 0.95 bar?

   Answer: 75.88 mL

6. 6. A scuba diver's 10. L air tank is filled to a pressure of 210 bar at a dockside temperature of 32.0°C. The water temperature is 8.0°C, the diver is swimming at a depth of 11.3 m, and the barometric (atmospheric) pressure is 1.013 bar. The density of sea water is $1025\frac{\text{kg}}{\text{m}^3}$.

   a. What is the total pressure on the diver?

      214 808 Pa   or   2.14 bar

   b. How many liters of air does the diver use?

      899.9 L

   c. If the diver uses air at the rate of 8.0 L/min, how long will the diver's air last?

      112.5 min

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