Fluid Motion & Bernoulli's Law

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 2 Learning Objectives: 5.B.10.1, 5.B.10.2, 5.B.10.3, 5.F.1.1

Knowledge/Understanding:

- Bernoulli's Law/Equation

Skills:

- Solve fluid flow problems using Bernoulli's Equation

Language Objectives:

- Understand and correctly use the term "flow."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems using Bernoulli's equation.

Labs, Activities & Demonstrations:

- Blow across paper (unfolded & folded)
- Blow between two empty cans.
- Ping-pong ball and air blower (without & with funnel)
- Venturi tube
- Compressed air blowing across Styrofoam
- Leaf blower & large ball

Notes:

flow: the net movement of a fluid

velocity of a fluid: the average velocity of a particle of fluid as the fluid flows past a reference point.

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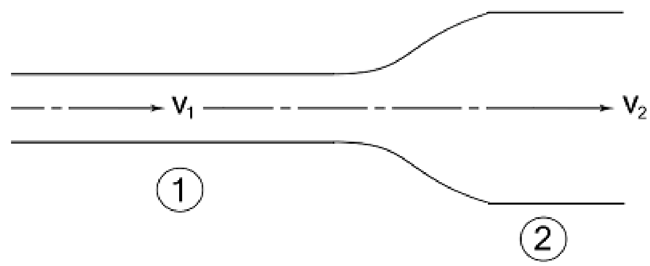
When a fluid flows through a pipe, the total volume of fluid per unit of time $\left(\frac{\text{m}^3}{\text{s}}\right)$ is equal to the velocity $\left(\frac{\text{m}}{\text{s}}\right)$ times the cross-sectional area (m^2). If the total flow remains the same, but the, but the diameter of the pipe changes:



then the cross-sectional area (A) times the fluid velocity (v) at point 1 equals the cross-sectional area times the fluid velocity at point 2:

$$A_1 v_1 = A_2 v_2$$

According to the Dutch-Swiss mathematician Daniel Bernoulli, the pressures in a moving fluid are caused by:

- The pressure exerted by the fluid, P (This is the pressure we would measure with a pressure gauge.)
- The hydrostatic pressure, $P_H = \rho \text{gh}$.
- The dynamic pressure, $P_D = \frac{1}{2}\rho v^2$, which results from the force that the moving fluid particles exert on the other fluid particles around them. (Note that $\frac{1}{2}\rho v^2$ has units of pressure: $\left(\frac{\text{kg}}{\text{m}^3}\right)\cdot\left(\frac{\text{m}}{\text{s}}\right)^2 = \frac{\text{kg}}{\text{m}\cdot\text{s}^2} = \text{Pa}$.)

A change in any of these pressures affects the others, which means:

$$P_{ext} + P_H + P_D = \text{constant}$$

or, more familiarly:

$$P + \rho g h + \tfrac{1}{2}\rho v^2 = \text{constant}$$

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$

This equation is Bernoulli's equation.

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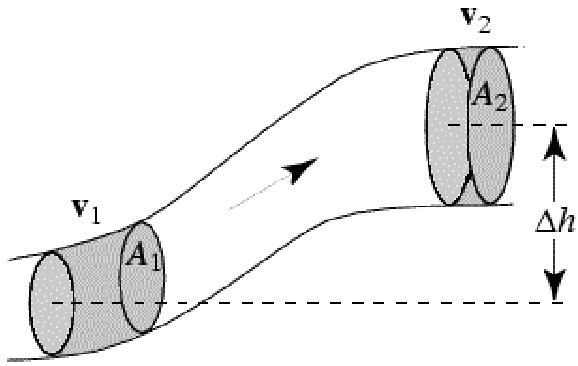
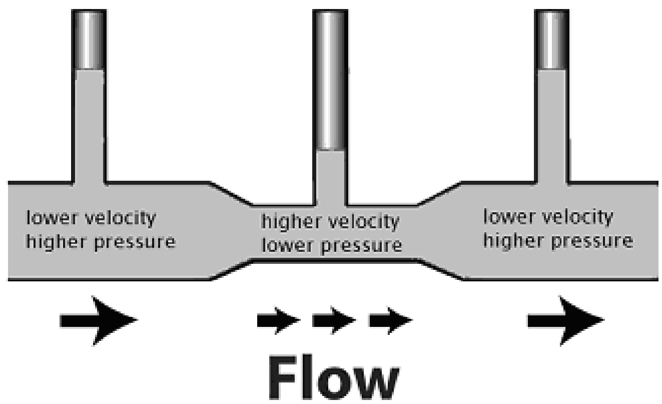
If the velocity and height of the fluid are changing, as in the following diagram, then the pressure must also change as a result:



In this situation, Bernoulli's equation becomes:

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$

In Bernoulli's equation, increasing the fluid velocity (v) increases the $\frac{1}{2}\rho v^2$ term. Consider the following example:



This pipe is horizontal, which means h is constant; therefore $\rho g h$ is constant. This means that if $\frac{1}{2}\rho v^2$ increases, then pressure (P) must decrease so that $P + \rho g h + \frac{1}{2}\rho v^2$ remains constant.

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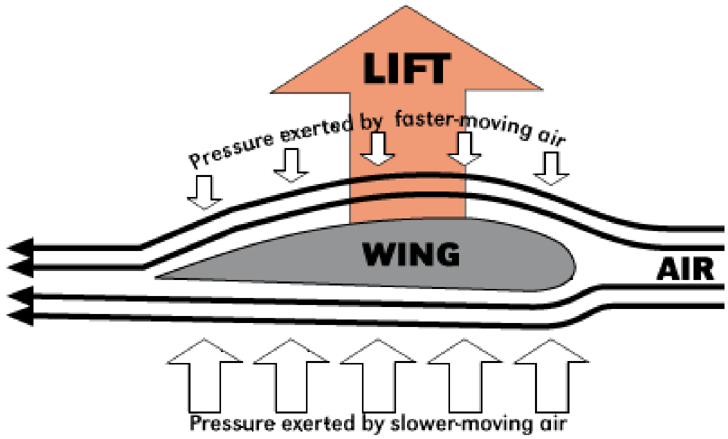
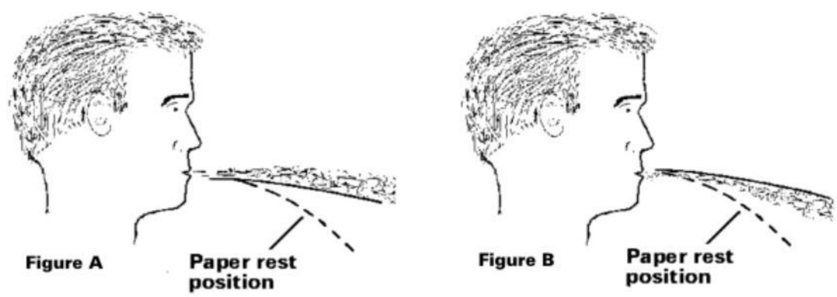
This decrease in pressure caused by an increase in fluid velocity explains one of the ways in which an airplane wing provides lift:



Of course, most of an airplane's lift comes from the fact that the wing is inclined with an angle of attack relative to its direction of motion.

A common demonstration of Bernoulli's Law is to blow across a piece of paper:



The air moving across the top of the paper causes a decrease in pressure, which causes the paper to lift.

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Sample Problems:

Q: A fluid in a pipe with a diameter of 0.40 m is moving with a velocity of $0.30\,\frac{\text{m}}{\text{s}}$. If the fluid moves into a second pipe with half the diameter, what will the new fluid velocity be?

A: The cross-sectional area of the first pipe is:

$A_1 = \pi r^2 = (3.14)(0.20)^2 = 0.126\ \text{m}^2$

The cross-sectional area of the second pipe is:

$A_2 = \pi r^2 = (3.14)(0.10)^2 = 0.0314\ \text{m}^2$

$A_1 v_1 = A_2 v_2$

$(0.126)(0.30) = (0.0314)v_2$

$v_2 = \boxed{1.2\,\frac{\text{m}}{\text{s}}}$.

Q: A fluid with a density of $1250\,\frac{\text{kg}}{\text{m}^3}$ has a pressure of 45 000 Pa as it flows at $1.5\,\frac{\text{m}}{\text{s}}$ through a pipe. The pipe rises to a height of 2.5 m, where it connects to a second, smaller pipe. What is the pressure in the smaller pipe if the fluid flows at a rate of $3.4\,\frac{\text{m}}{\text{s}}$ through it?

A:

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$

$$45\,000 + (1250)(10)(0) + (\tfrac{1}{2})(1250)(1.5)^2 = P_2 + (1250)(10)(2.5) + (\tfrac{1}{2})(1250)(3.4)^2$$

$$45\,000 + 1406 = P_2 + 31\,250 + 7225$$

$$46\,406 = P_2 + 38\,475$$

$$P_2 = \boxed{7\,931\ \text{Pa}}$$

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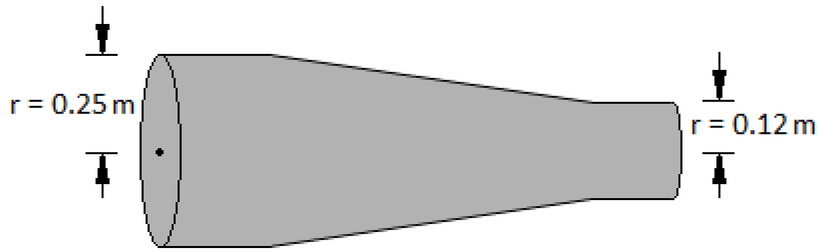
Homework Problems

1. A pipe has a radius of 0.25 m at the entrance and a radius of 0.12 m at the exit, as shown in the figure below:



If the fluid in the pipe is flowing at $5.2\,\frac{\text{m}}{\text{s}}$ at the inlet, then how fast is it flowing at the outlet?

Answer: $22.6\,\frac{\text{m}}{\text{s}}$

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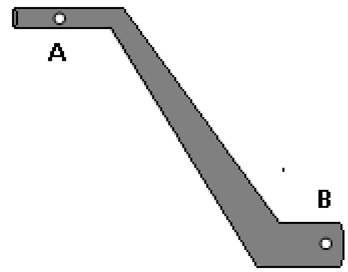
2. At point A on the pipe to the right, the water's speed is $4.8\,\frac{\text{m}}{\text{s}}$ and the pressure is 52.0 kPa. The water drops 14.8 m to point B, where the pipe's cross sectional area is twice that at point A.



   a. Calculate the velocity of the water at point B.

      Answer: $2.4\,\frac{\text{m}}{\text{s}}$

   b. Calculate the pressure at point B.

      Answer: 50.6 kPa

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