## Thermal Expansion

**Unit:** Heat & Thermodynamics  
**NGSS Standards:** HS-PS2-6  
**MA Curriculum Frameworks (2006):** N/A  
**AP Physics 2 Learning Objectives:** N/A

### Knowledge/Understanding:
- thermal expansion in solids, liquids and gases

### Skills:
- calculate changes in length and volume for solids and liquids  
- calculate changes in volume for gases

### Language Objectives:
- Understand and correctly use the term “thermal expansion.”  
- Accurately describe and apply the concepts described in this section using appropriate academic language.  
- Set up and solve word problems relating to thermal expansion of solids, liquids and gases.

### Labs, Activities & Demonstrations:
- balloon with string & heat gun  
- brass ball & ring  
- bi-metal strip

### Notes:
Note that thermal expansion of solids and liquids is not covered on the AP Physics 2 exam. (However, expansion of gases is covered.)

**expand:** to become larger  
**contract:** to become smaller

**thermal expansion:** an increase in the length and/or volume of an object caused by a change in temperature.
When a substance is heated, the particles it is made of gain kinetic energy and momentum, which makes them move faster and farther. This causes the particles to move farther apart, which causes the substance to expand.

Solids tend to keep their shape when they expand. (Liquids and gases do not have definite shape to begin with.)

A few materials are known to contract with increasing temperature over specific temperature ranges. One well-known example is liquid water, which contracts as it heats from 0°C to 4°C. (Water expands as the temperature increases above 4°C.)
Thermal Expansion of Solids and Liquids

Thermal expansion is quantified in solids and liquids by defining a coefficient of thermal expansion. The changes in length or volume are given by the following equations:

\[
\text{Length: } \Delta L = \alpha L_i \Delta T \\
\text{Volume: } \Delta V = \beta V_i \Delta T
\]

where:
\[
\Delta L = \text{change in length (m)} \quad \Delta V = \text{change in volume (m}^3\text{)} \\
L_i = \text{initial length (m)} \quad V_i = \text{initial volume (m}^3\text{)} \\
\alpha = \text{linear coefficient of thermal expansion (°C}^{-1}\text{)} \\
\beta = \text{volumetric coefficient of thermal expansion (°C}^{-1}\text{)} \\
\Delta T = \text{temperature change}
\]

Values of \(\alpha\) and \(\beta\) at 20°C for some solids and liquids:

<table>
<thead>
<tr>
<th>Substance</th>
<th>(\alpha) (°C(^{-1}))</th>
<th>(\beta) (°C(^{-1}))</th>
<th>Substance</th>
<th>(\alpha) (°C(^{-1}))</th>
<th>(\beta) (°C(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>aluminum</td>
<td>(2.3 \times 10^{-5})</td>
<td>(6.9 \times 10^{-5})</td>
<td>gold</td>
<td>(1.4 \times 10^{-5})</td>
<td>(4.2 \times 10^{-5})</td>
</tr>
<tr>
<td>copper</td>
<td>(1.7 \times 10^{-5})</td>
<td>(5.1 \times 10^{-5})</td>
<td>iron</td>
<td>(1.18 \times 10^{-5})</td>
<td>(3.33 \times 10^{-5})</td>
</tr>
<tr>
<td>brass</td>
<td>(1.9 \times 10^{-5})</td>
<td>(5.6 \times 10^{-5})</td>
<td>lead</td>
<td>(2.9 \times 10^{-5})</td>
<td>(8.7 \times 10^{-5})</td>
</tr>
<tr>
<td>diamond</td>
<td>(1 \times 10^{-6})</td>
<td>(3 \times 10^{-6})</td>
<td>mercury</td>
<td>(6.1 \times 10^{-5})</td>
<td>(1.82 \times 10^{-4})</td>
</tr>
<tr>
<td>ethanol</td>
<td>(2.5 \times 10^{-4})</td>
<td>(7.5 \times 10^{-4})</td>
<td>silver</td>
<td>(1.8 \times 10^{-5})</td>
<td>(5.4 \times 10^{-5})</td>
</tr>
<tr>
<td>glass</td>
<td>(8.5 \times 10^{-6})</td>
<td>(2.55 \times 10^{-5})</td>
<td>water (liq.)</td>
<td>(6.9 \times 10^{-5})</td>
<td>(2.07 \times 10^{-4})</td>
</tr>
</tbody>
</table>
**Thermal Expansion**

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**expansion joint:** a space deliberately placed between two objects to allow room for the objects to expand without coming into contact with each other.

Bridges often have expansion joints in order to leave room for sections of the bridge to expand or contract without damaging the bridge or the roadway:

![Expansion Joint Image]

Railroad rails are sometimes welded together in order to create a smoother ride, which allows for higher speeds. Unfortunately, if expansion joints are not placed at frequent enough intervals, thermal expansion can cause the rails to bend and buckle, resulting in derailments:

![Railroad Images]

Use this space for summary and/or additional notes.
bimetal strip: a strip made from two metals with different coefficients of thermal expansion that are bonded together. When the strip is heated or cooled, the two metals expand or contract different amounts, which causes the strip to bend. When the strip is returned to room temperature, the metals revert back to their original lengths.

Use this space for summary and/or additional notes.
Sample Problems:

Q: Find the change in length of an 0.40 m brass rod that is heated from 25°C to 980°C.

A: For brass, $\alpha = 1.9 \times 10^{-5}$ °C$^{-1}$.

$$\Delta L = \alpha L_i \Delta T$$

$$\Delta L = (1.9 \times 10^{-5})(0.40)(955)$$

$$\Delta L = 0.0073 \text{ m}$$

Q: A typical mercury thermometer contains about 0.22 cm$^3$ (about 3.0 g) of mercury. Find the change in volume of the mercury in a thermometer when it is heated from 25°C to 50.°C.

A: For mercury, $\beta = 1.82 \times 10^{-4}$ °C$^{-1}$.

$$\Delta V = \beta V_i \Delta T$$

$$\Delta V = (1.82 \times 10^{-4})(0.22)(25)$$

$$\Delta V = 0.00091 \text{ cm}^3$$

If the distance from the 25°C to the 50°C mark is about 3.0 cm, we could use this information to figure out the bore of the thermometer:

$$V = \pi r^2 h$$

$$0.00091 = (3.14)r^2(3.0)$$

$$r^2 = 9.66 \times 10^{-5}$$

$$r = \sqrt{9.66 \times 10^{-5}} = 0.0098 \text{ cm}$$

The bore is the diameter, which is twice the radius, so the bore of the thermometer is 0.020 cm or about 0.20 mm.
Homework Problems

You will need to look up coefficients of thermal expansion in Table H of your reference tables on page 583.

1. A brass rod is 27.50 cm long at 25°C. How long would the rod be if it were heated to 750°C in a flame?

Answer: 27.88 cm

2. A steel bridge is 625 m long when the temperature is 0°C.
   a. If the bridge did not have any expansion joints, how much longer would the bridge be on a hot summer day when the temperature is 35°C?
      (Use the linear coefficient of expansion for iron.)

      Answer: 0.258m

   b. Why do bridges need expansion joints?
3. A 15.00 cm long bimetal strip is aluminum on one side and copper on the other. If the two metals are the same length at 20.0°C, how long will each be at 800.°C?

Answers: aluminum: 15.269 cm; copper: 15.199 cm

4. A metal rod expands from length $L_i$ to length $L_f$ when it is heated from temperature $T_i$ to $T_f$. Write an expression for the coefficient of thermal expansion, $\alpha$, of this metal.

Answer: $\alpha = \frac{L_f - L_i}{L_i(T_f - T_i)}$

5. A glass volumetric flask is filled with water exactly to the 250.00 mL line at 50.°C. What volume will the water occupy after it cools down to 20.°C?

Answer: 248.45 mL
Thermal Expansion of Gases

ideal gas: a gas that behaves as if each molecule acts independently, according to kinetic-molecular theory. Most gases behave ideally except at temperatures and pressures near the vaporization curve on a phase diagram. (*I.e.*, gases stop behaving ideally when conditions are close to those that would cause the gas to condense to a liquid or solid.)

![Phase diagram showing solid, liquid, and gas phases]

For an ideal gas, the change in volume for a change in temperature (provided that the pressure and number of molecules are kept constant) is given by Charles’ Law:

\[
\frac{V_i}{T_i} = \frac{V_f}{T_f}
\]

where volume can be any volume unit (as long as it is the same on both sides), but temperature *must be* in Kelvin.
Sample Problem:

Q: If a 250 mL container of air is heated from 25°C to 95°C, what is the new volume?

A: Temperatures must be in Kelvin, so we need to convert first.

\[25°C + 273 = 298 \text{ K}\]

\[95°C + 273 = 368 \text{ K}\]

\[
\frac{V_i}{T_i} = \frac{V_f}{T_f}
\]

\[
\frac{250}{298} = \frac{V_f}{368}
\]

\[V_f = 308.7 = 310 \text{ mL}\]

Because we used mL for \(V_1\), the value of \(V_2\) is therefore also in mL.
Homework Problems

1. A sample of argon gas was cooled, and its volume went from 380. mL to 250. mL. If its final temperature was −45.0°C, what was its original temperature?

Answer: 347 K or 74°C

2. A balloon contains 250. mL of air at 50°C. If the air in the balloon is cooled to 20.0°C, what will be the new volume of the air?

Answer: 226.8mL

3. A sample of gas with a volume $V_i$ is heated from temperature $T_i$ to $T_f$. (Assume both temperatures are in Kelvin.) Write an expression for the final volume of the gas.

Answer: $V_f = V_i \frac{T_f}{T_i}$