Pressure-Volume Diagrams

Unit: Heat & Thermodynamics

NGSS Standards: HS-PS2-6

MA Curriculum Frameworks (2006): N/A

AP Physics 2 Learning Objectives: 5.B.5.6, 5.B.7.2, 5.B.7.3, 7.A.3.2, 7.A.3.3

Knowledge/Understanding:

- understand and interpret P-V diagrams

Skills:

- determine changes in heat, work, internal energy and entropy from P-V diagrams

Language Objectives:

- Understand and correctly use the terms "isochoric," "isobaric," and "adiabatic."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to the action of heat engines.

Notes:

P-V diagram: a graph that shows changes in pressure *vs.* changes in volume.

From mechanics, recall that work is the force that it took to move an object a given distance:

$$W = \int \vec{\boldsymbol{F}} \bullet d\vec{\boldsymbol{x}} = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}} = F\Delta x$$

If the force is applied by a gas that is expanding, then the change in volume caused by the pressure is responsible for the work:

$$W = -\int P dV = -P\Delta V$$

Because the integral of a function gives the area under its curve, this means that if we plot a graph of pressure *vs.* volume, *the area under the graph is the amount of work done*. Note that direction matters—work done on the system decreases the volume (compresses the gas). When the gas expands (the volume increases), it is able to push on (do work on) its surroundings.

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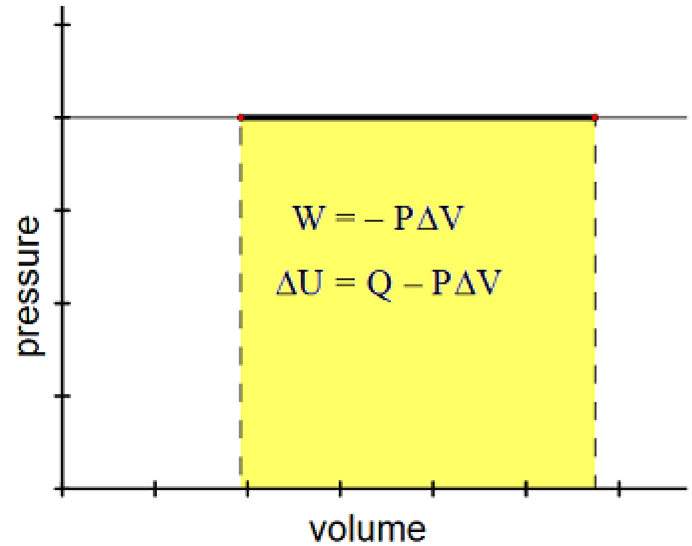
We will look at the effects of changes in pressure *vs.* volume in four types of systems: isobaric (constant pressure), isochoric (constant volume), isothermal (constant temperature), and adiabatic (no heat loss).

Isobaric

From Greek "iso" (same) and "baric" (heavy). An isobaric change is one in which pressure remains constant, but volume and temperature may vary.

Some examples include a weighted piston, a flexible container in earth's atmosphere, or a hot air balloon.



$$\Delta W = -P\Delta V$$
$$\Delta U = \Delta Q + \Delta W = \Delta Q + (-P\Delta V)$$
$$\Delta U = \Delta Q - P\Delta V$$

We can rearrange the above to show that $\Delta Q = \Delta U + P\Delta V$. We can then rewrite ΔU as $\frac{3}{2}Nk_B\Delta T$ and $P\Delta V$ as $Nk_B\Delta T$ (from the ideal gas law), which yields:

$$\Delta Q = \frac{3}{2}Nk_B\Delta T + Nk_B\Delta T = \frac{5}{2}Nk_B\Delta T = \frac{5}{2}nR\Delta T$$

Recall that the reason for the negative sign in the equation $W = -P\Delta V$ is because if work is done on the system $(\Delta W > 0)$, then the work compresses the gas $(\Delta V < 0)$. If work is done on the surroundings by the system $(\Delta W < 0)$, it is done by the gas expanding $(\Delta V > 0)$. Thus ΔW and ΔV must have opposite signs.

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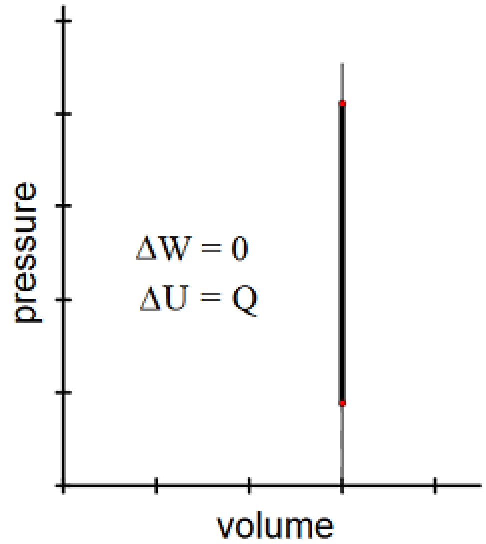
Isochoric



From Greek "iso" (same) and "khoros" (volume). An isochoric change is one in which volume remains constant, but pressure and temperature may vary.

An example is any rigid, closed container, such as a thermometer.

$$\Delta W = 0$$
$$\Delta U = \Delta Q + \Delta W = \Delta Q + 0$$
$$\Delta U = \Delta Q$$

Note that because work is done by the force from the expanding gas, if there is no volume change, then the gas does not expand, and therefore does no work.

Another way to think of a constant volume change is that if you add heat to a rigid container of gas, none of the energy can be converted to work, so all of it must be converted to an increase in internal energy (*i.e.,* an increase in temperature).

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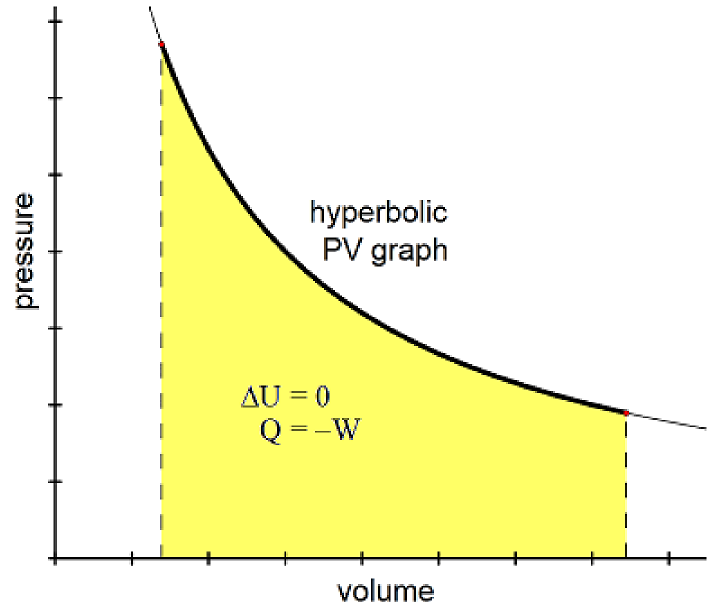
Isothermal

Constant temperature. From Greek "iso" (same) and "thermotita" (heat). An isothermal change is one in which temperature remains constant, but pressure and volume may vary.

An example is any "slow" process, such as breathing out through a wide open mouth.

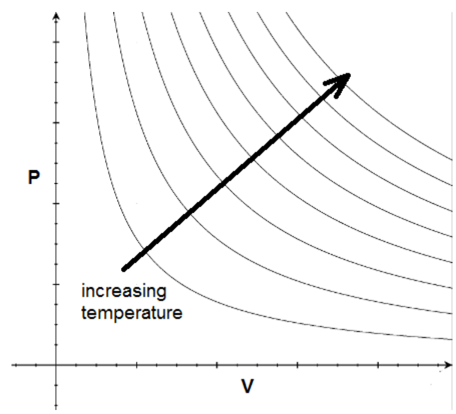


$$\Delta U = \frac{3}{2} N k_B \Delta T$$

Because $\Delta T = 0$ (definition of isothermal), this means $\Delta U = 0$

$$\Delta U = 0 = \Delta Q + \Delta W$$
$$\Delta W = -\Delta Q$$

<u>isotherm</u>: a line of constant temperature on a graph.



Note that isotherms are hyperbolas. As the temperature increases, the hyperbola moves up and to the right:

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Adiabatic

An adiabatic process is one in which there is no heat exchange with the environment.

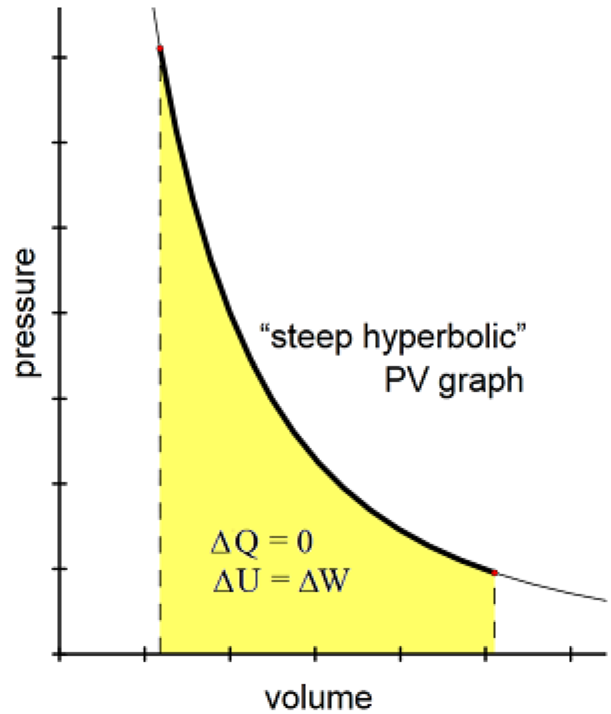
An example is any "fast" process, such as forcing air out through pursed lips or a bicycle tire pump.

$$Q = 0$$

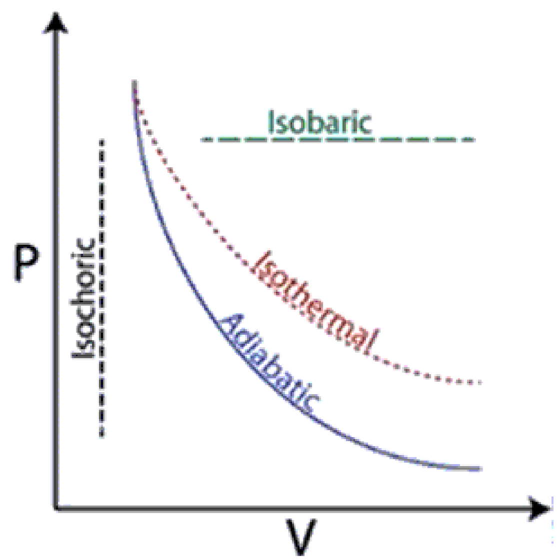
$$\Delta U = Q + W = 0 + W$$

$$\Delta U = W$$

Note that adiabatic expansion results in a drop in temperature, and adiabatic compression results in a rise in temperature.



These four processes look like the following on a PV diagram:



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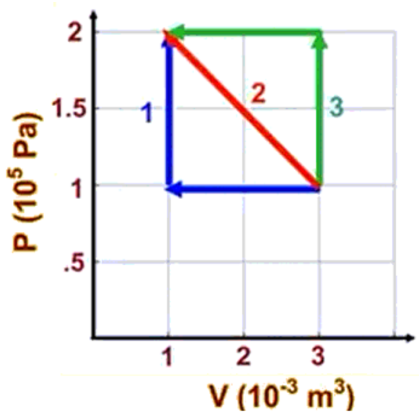
Sample Problem

Q: Calculate the work done along each of paths 1, 2, and 3.



A: Process #1 is isobaric (constant pressure), then isochoric (constant volume).

For the isobaric part of the process:

$$W = -P\Delta V$$
$$W = -(1\times10^5)(3\times10^{-3} - 1\times10^{-3})$$
$$W = -(1\times10^5)(2\times10^{-3})$$
$$W = -2\times10^2 = 200\text{ J}$$

For the isochoric process, there is no change in volume, which means the gas does no work (because it cannot push against anything). Therefore $W = 0$.

The total work for process #1 is therefore 200 J.

We can also remember that work is the area under a PV graph, which gives:

$$W = -(1\times10^5)(2\times10^{-3}) = -200\text{ J}$$

For process #2, the area is the 200 J square plus the area of the triangle, which is $\frac{1}{2}bh = \frac{1}{2}(2\times10^{-3})(1\times10^5) = -100\text{ J}$. Therefore, −200 J + (−100 J) = −300 J.

For process #3, the area under the curve is $W = -(2\times10^5)(2\times10^{-3}) = -400\text{ J}$.

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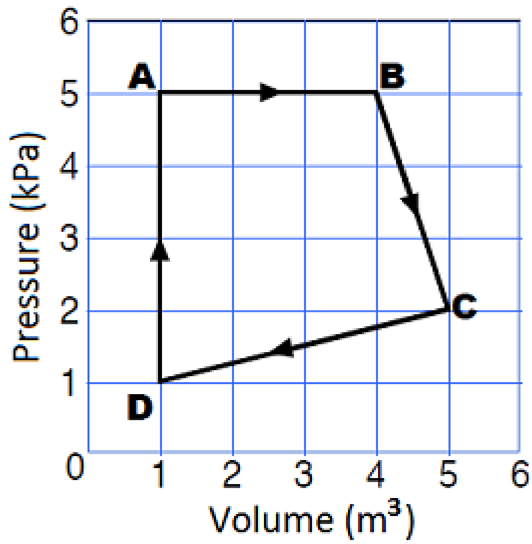
Homework

Problems #1–8 refer to the following PV diagram, in which 2 moles of gas undergo the pressure and volume changes represented by the path from point A to B to C to D and back to A.



1. Which thermodynamic process takes place along the path from point A to point B?

2. Which thermodynamic process takes place along the path from point D to point A?

3. How much work is done as the gas undergoes a change along the curve from point B to point C? (Remember to use a positive number for work done on the gas by the surroundings, and a negative number for work done by the gas on the surroundings.)

   Answer: +3500 J

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4. How much work is done as a gas undergoes a change along the curve from point C to point D?

   Answer: −6000 J

5. How much ***net*** work is done on or by the gas as it undergoes a change along the curve from point A to B to C to D and back to A?

   Answer: +12 500 J

6. What is the temperature of the 2 moles of gas at point A?

   Answer: 300.8 K

7. What is the change in internal energy of the gas during the process from point D to point A?

   Answer: 5998 J

8. How much work is done on or by the gas during the process from point D to point A?

   Answer: zero

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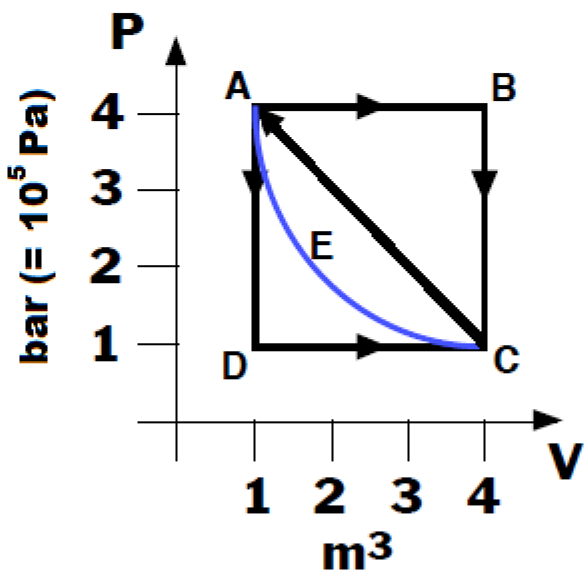
Problems #9–13 refer to the following diagram:



9. For which process(es) is $\Delta Q = \frac{5}{2} nRT$? Show calculations to justify your answer.

Answer: A → B and D → C

10. For which process(es) is no work done? Explain.

Answer: A → D and B → C

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11. Which thermodynamic process takes place along path E?

12. Does path A → D → C → E → A require more or less work than path A → D → C → A? Explain.

13. Calculate the work done by the gas in processes A → B → C → A and A → D → C → A.

Answer: A → B → C → A: 450 000 J
A → D → C → A: −450 000 J

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