Conservation of Momentum

**Unit:** Momentum

**MA Curriculum Frameworks (2016):** HS-PS2-2

**MA Curriculum Frameworks (2006):** 2.5

**Mastery Objective(s):** (Students will be able to...)
- Solve problems involving collisions in which momentum is conserved, with or without an external impulse.

**Success Criteria:**
- Masses and velocities are correctly identified for each object, both before and after the collision.
- Variables are correctly identified and substituted correctly into the correct part of the equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

**Tier 2 Vocabulary:** momentum, collision

**Language Objectives:**
- Explain what happens before, during, and after a collision from the point of view of one of the objects participating in the collision.

**Labs, Activities & Demonstrations:**
- Collisions on air track.
- “Happy” and “sad” balls knocking over a board.
- Students riding momentum cart.

**Notes:**

**collision:** when two or more objects come together and hit each other.

**elastic collision:** a collision in which the objects bounce off each other (remain separate) after they collide, without any loss of kinetic energy.

**inelastic collision:** a collision in which the objects remain together after colliding. In an inelastic collision, total energy is still conserved, but some of the energy is changed into other forms, so the amount of kinetic energy is different before vs. after the collision.

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Use this space for summary and/or additional notes:
Any macroscopic collision (meaning a collision between objects that are larger than individual atoms or molecules) will convert some of the kinetic energy into internal energy and other forms of energy. This means that no large scale impacts are ever perfectly elastic.

**Conservation of Momentum**

Recall that in physics, if a quantity is “conserved”, that means when some change happens to a system, there is the same amount of that quantity after the change as there was before.

In a closed system, momentum is *conserved*. This means that unless there is an outside force, the combined momentum of all of the objects after they collide is equal to the combined momentum of all of the objects before the collision.

As we did with conservation of energy problems, we can use a momentum bar chart to show changes in momentum before and after a collision.

The following momentum bar chart is for an elastic collision. Imagine that two objects are moving in opposite directions and then collide. Before the collision, the first object has a momentum of +3 N·s, and the second has a momentum of −1 N·s. The total momentum is therefore +3 + (−1) = +2 N·s.

After the collision, the first object has a momentum of +1.5 N·s and the second has a momentum of +0.5 N·s. Because there are no forces changing the momentum of the system, the final momentum *must* also be +2 N·s.
The following momentum bar chart is for an inelastic collision. Imagine that two objects are moving in the same direction, and then collide. Before the collision, the first object has a momentum of +1 N∙s, and the second has a momentum of +3 N∙s. The total momentum is therefore +1 + (+3) = +4 N∙s.

After the collision, the objects stick together, so the total momentum must still be +4 N∙s.

**Solving Conservation of Momentum Problems**

Almost all momentum problems involve the conservation of momentum law:

\[
\sum p_i + \vec{j} = \sum p_f
\]

\[
\sum p_i + \Delta \vec{p} = \sum p_f
\]

The symbol \(\sum\) is the Greek capital letter “sigma”. In mathematics, the symbol \(\sum\) means “summation”. \(\sum \vec{p}\) means the sum of the momentums. The subscript “\(i\)” means initial (before the collision), and the subscript “\(f\)” means final (after the collision). In plain English, \(\sum \vec{p}\) means find each individual value of \(\vec{p}\) (positive or negative, depending on the direction) and then add them all up to find the total.

In plain English, the conservation of momentum law means that the total momentum before a collision, plus any momentum that we add, must be equal to the total momentum after.
For example, if you had a momentum problem with two objects and no added impulse, the law of conservation of momentum becomes:

\[
\text{Before (no impulse)} = \text{After} \\
\sum \mathbf{p}_i + 0 = \sum \mathbf{p}_f \\
\mathbf{p}_{1,i} + \mathbf{p}_{2,i} = \mathbf{p}_{1,f} + \mathbf{p}_{2,f}
\]

Notice that we have two subscripts after each “\(\mathbf{p}\)” and each “\(\mathbf{v}\),” because we have two separate things to keep track of. The “\(i\)” and “\(f\)” mean “initial” and “final,” and the “\(1\)” and “\(2\)” mean object #1 and object #2.

Because \(\mathbf{p} = m\mathbf{v}\), we can replace each \(\mathbf{p}\) with \(m\mathbf{v}\), giving:

\[
\text{Before} = \text{After} \\
\sum \mathbf{p}_i = \sum \mathbf{p}_f \\
\mathbf{p}_{1,i} + \mathbf{p}_{2,i} = \mathbf{p}_{1,f} + \mathbf{p}_{2,f} \\
m_1\mathbf{v}_{1,i} + m_2\mathbf{v}_{2,i} = m_1\mathbf{v}_{1,f} + m_2\mathbf{v}_{2,f}
\]

Note that there are six separate quantities in this problem: \(m_1\), \(m_2\), \(\mathbf{v}_{1,i}\), \(\mathbf{v}_{2,i}\), \(\mathbf{v}_{1,f}\), and \(\mathbf{v}_{2,f}\). A typical momentum problem will give you (or enable you to calculate) five of these, and will ask you for the sixth.

Note also that most momentum problems do not mention the word “momentum.” The problems usually give information about masses and velocities before and after some sort of collision, and it is up to you to realize that the problem involves conservation of momentum.

Most momentum problems involve collisions. Usually, there are two objects initially, and the objects either bounce off each other (elastic collision) or stick together (inelastic collision).
Conservation of Momentum

For an elastic collision between two objects, the problem is exactly as described above. There are six quantities to consider: the two masses, the two initial velocities, and the two final velocities. The equation relating them is:

$$\text{Before} = \text{After}$$

$$m_1 \vec{v}_{1,i} + m_2 \vec{v}_{2,i} = m_1 \vec{v}_{1,f} + m_2 \vec{v}_{2,f}$$

To solve the problem, you need to obtain the quantities given in the word problem and solve for the missing one.

For an inelastic collision between two objects, the objects stick together after the collision, which means there is only one “object” afterwards. The total mass of the object is $$m_T = m_1 + m_2$$, and there is only one “object” with a final velocity.

there are five quantities: the two masses, the two initial velocities, and the final velocity of the combined object. The equation relating them is:

$$\text{Before} = \text{After}$$

$$m_1 \vec{v}_{1,i} + m_2 \vec{v}_{2,i} = m_T \vec{v}_f$$

or

$$m_T \vec{v}_f = m_1 \vec{v}_{1,f} + m_2 \vec{v}_{2,f}$$

To solve this problem, you need to obtain the quantities given in the word problem, add the two masses to find $$m_T$$, and solve for the missing quantity.

Use this space for summary and/or additional notes:
Sample Problems:

Q: An object with a mass of 8.0 kg moving with a velocity of \( +5.0 \text{ m/s} \) collides with a stationary object with a mass of 12 kg. If the two objects stick together after the collision, what is their velocity?

A: The momentum of the moving object before the collision is:

\[
p = m\vec{v} = (8.0)(+5.0) = +40 \text{ N\cdot s}
\]

The stationary object has a momentum of zero, so the total momentum of the two objects combined is +40 N\cdot s.

After the collision, the total mass is 8.0 kg + 12 kg = 20 kg. The momentum after the collision must still be +40 N\cdot s, which means the velocity is:

\[
p = m\vec{v} \quad 40 = 20\vec{v} \quad \vec{v} = +2 \text{ m/s}
\]

Using the equation, we would solve this as follows:

Before \( = \) After
\[
\begin{align*}
p_{1,i} + p_{2,i} & = p_f \\
m_1\vec{v}_{1,i} + m_2\vec{v}_{2,i} & = m_f\vec{v}_f \\
(8)(5) + (12)(0) & = (8 + 12)\vec{v}_f \\
40 & = 20\vec{v}_f \\
\vec{v}_f & = \frac{40}{20} = +2 \text{ m/s}
\end{align*}
\]
Conservation of Momentum

Q: Mr. Stretchy has a mass of 60. kg and is holding a 5.0 kg box as he rides on a skateboard toward the west at a speed of 3.0 \( \text{m/s} \). (Assume the 60. kg is the mass of Mr. Stretchy and the skateboard combined.) He throws the box behind him, giving it a velocity of 2.0 \( \text{m/s} \) to the east. What is Mr. Stretchy’s velocity after throwing the box?

A: This problem is like an inelastic collision in reverse; Mr. Stretchy and the box are together before the “collision” and apart afterwards. The equation would therefore look like this:

\[ m_s \vec{v}_i = m_s \vec{v}_{s,f} + m_b \vec{v}_{b,f} \]

Where the subscript “s” is for Mr. Stretchy, and the subscript “b” is for the box. Note that after Mr. Stretchy throws the box, he is moving one direction and the box is moving the other, which means we need to be careful about our signs. Let’s choose the direction Mr. Stretchy is moving (west) to be positive. Because the box is thrown to the east, this means the final velocity of the box will be:

\[ \vec{v}_{b,f} = -2.0 \text{m/s} \]

Plugging values from the problem into the equation for the law of conservation of momentum, we get:

\[
\begin{align*}
\text{BEFORE} & = \quad \text{AFTER} \\
\vec{p}_i & = \vec{p}_{s,f} + \vec{p}_{b,f} \\
\Rightarrow m_s \vec{v}_i & = m_s \vec{v}_{s,f} + m_b \vec{v}_{b,f} \\
(60 + 5)(+3) & = 60 \vec{v}_{s,f} + (5)(-2) \\
+195 & = 60 \vec{v}_{s,f} + (-10) \\
+205 & = 60 \vec{v}_{s,f} \\
\vec{v}_{s,f} & = \frac{+205}{60} = +3.4 \text{ m/s}
\end{align*}
\]

Use this space for summary and/or additional notes:
Q: A soccer ball that has a mass of 0.43 kg is rolling east with a velocity of $5.0 \frac{m}{s}$. It collides with a volleyball that has a mass of 0.27 that is rolling west with a velocity of $6.5 \frac{m}{s}$. After the collision, the soccer ball is rolling to the west with a velocity of $3.87 \frac{m}{s}$. Assuming the collision is perfectly elastic and friction between both balls and the ground is negligible, what is the velocity (magnitude and direction) of the volleyball after the collision?

A: This is an elastic collision, so the soccer ball and the volleyball are separate both before and after the collision. The equation is:

$$m_s \vec{v}_{s,i} + m_v \vec{v}_{v,i} = m_s \vec{v}_{s,f} + m_v \vec{v}_{v,f}$$

Where the subscript “$s$” is for the soccer ball and the subscript “$v$” is for the volleyball. In all elastic collisions, assume we need to keep track of the directions, which means we need to be careful about our signs. We don’t know which direction the volleyball will be moving after the collision (though a good guess would be that it will probably bounce off the soccer ball and move to the east). So let us arbitrarily choose east to be positive and west to be negative. This means:

<table>
<thead>
<tr>
<th>quantity</th>
<th>direction</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial velocity of soccer ball</td>
<td>east</td>
<td>$+ 5.0 \frac{m}{s}$</td>
</tr>
<tr>
<td>initial velocity of volleyball</td>
<td>west</td>
<td>$- 6.5 \frac{m}{s}$</td>
</tr>
<tr>
<td>final velocity of soccer ball</td>
<td>west</td>
<td>$- 3.87 \frac{m}{s}$</td>
</tr>
</tbody>
</table>

Plugging values from the problem into the equation for the law of conservation of momentum, we get:

Before  =  After

$$m_s \vec{v}_{s,i} + m_v \vec{v}_{v,i} = m_s \vec{v}_{s,f} + m_v \vec{v}_{v,f}$$

$$m_s \vec{v}_{s,i} + m_v \vec{v}_{v,i} = m_s \vec{v}_{s,f} + m_v \vec{v}_{v,f}$$

$$m_s \vec{v}_{s,i} + m_v \vec{v}_{v,i} = m_s \vec{v}_{s,f} + m_v \vec{v}_{v,f}$$

$$(0.43)(5.0) + (0.27)(-6.5) = (0.43)(-3.87) + (0.27) \vec{v}_{v,f}$$

$$2.15 + (-1.755) = -1.664 + 0.27 \vec{v}_{v,f}$$

$$0.395 = -1.664 + 0.27 \vec{v}_{v,f}$$

$$2.059 = 0.27 \vec{v}_{v,f}$$

$$\vec{v}_{v,f} = \frac{2.059}{0.27} = +7.63 \frac{m}{s} \text{ or } 7.63 \frac{m}{s} \text{ to the east.}$$

Use this space for summary and/or additional notes: