## Work

**Unit:** Energy, Work & Power

**MA Curriculum Frameworks (2016):** HS-PS3-1

**MA Curriculum Frameworks (2006):** 2.3

**Mastery Objective(s):** (Students will be able to...)

- Calculate the work done when a force displaces an object.

**Success Criteria:**

- Variables are correctly identified and substituted correctly into equation(s).
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

**Tier 2 Vocabulary:** work, energy

**Language Objectives:**

- Explain why a longer lever arm is more effective.

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**Notes:**

*work:* the effort applied over a distance against a force; the accumulation of energy.

If you lift a heavy object off the ground, you are giving the object gravitational potential energy (in the object-Earth system). The Earth’s gravitational field can now cause the object to fall, turning the potential energy into kinetic energy. Therefore, we would say that you are doing work against the force of gravity.

Work is therefore the amount of energy that was added to the object \(W = \Delta E\)\(^*\). (In this case, because the work was turned into potential energy, we would say that \(W = \Delta U\).)

\(^*\) Many texts start with work as the application of force over a distance, and then discuss energy. Those texts then derive the work-energy theorem, which states that the two quantities are equivalent. In these notes, we instead started with energy, and then defined work as the change in energy. This presentation makes the concept of work more intuitive, especially when studying more advanced topics such as thermodynamics.

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Use this space for summary and/or additional notes:
Mathematically, work is also the dot product of the force vector and the displacement vector. Therefore, we can say that:

\[ \Delta E = W = \vec{F} \cdot \vec{d} \]

The dot product is one of three ways of multiplying vectors. The dot product is a scalar (a number without a direction), and is equal to the product of the magnitudes of the force and distance, and the cosine of the angle between them. This means:

\[ W = Fd \cos \theta = F_{\parallel} d \]

Where \( F \) is the magnitude of the force vector \( \vec{F} \), \( d \) is the magnitude of the displacement vector \( \vec{d} \), and \( \theta \) is the angle between the two vectors. Sometimes \( F \cos \theta \) is written as \( F_{\parallel} \), which means “the component of the force that is parallel to the direction of motion.”

Note that when the force and the displacement are in the same direction, the angle \( \theta = 0^\circ \) which means \( \cos \theta = \cos(0^\circ) = 1 \). In this case, \( F_{\parallel} = F \cos \theta = (F)(1) = F \) and the equation reduces to \( W = Fd \).

Work is measured in newton-meters (N∙m), which are equivalent to joules (J).

\[ 1 \text{N} \cdot \text{m} \equiv 1 \text{J} \equiv 1 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \]

**Note that:**

- If the displacement is zero, no work is done by the force. *E.g.*, if you hold a heavy box without moving it, you are exerting a force (counteracting the force of gravity) but you are not doing work.

- If the net force is zero, no work is done by the displacement (change in location) of the object. *E.g.*, if a cart is sliding across a frictionless air track at a constant velocity, the net force on the cart is zero, which means no work is being done.

- If the displacement is perpendicular to the direction of the applied force \( (\theta = 90^\circ \), which means \( \cos \theta = 0 \)), no work is done by the force. *E.g.*, you can slide a very heavy object along a roller conveyor, because the force of gravity is acting vertically and the object’s displacement is horizontal, which means gravity is doing no work, and therefore you do not have to do any work against gravity.
A roller conveyor:

In industry, roller conveyors are often set up at a slight downward angle, which turns the roller conveyor into a low-friction ramp.

**Force vs. Distance Graphs**

If the amount of force is changing as an object moves, work is the area under the graph of force vs. distance:

![Force vs. Distance Graph](image)

In the above example, 18 N·m of work was done as the object moved from 0 m to 3 m, and 9 N·m of work was done as the object moved from 3 m to 6 m, for a total of 27 N·m of work.

Once again, note the general rule that we have a formula for a quantity (work) that equals the quantity on the y-axis (force) times the change in the quantity on the x-axis (distance). Therefore, the value of the quantity (work) equals the area under the graph (force vs. distance)

Use this space for summary and/or additional notes:
Sample Problems:

Q: How much work does it take to lift a 60. kg box 1.5 m off the ground at a constant velocity over a period of 3.0 s?

A: The box is being lifted, which means the work is done against the force of gravity.

\[ W = F \cdot d = F_g \cdot d \]
\[ W = F_g \cdot d = [mg] \cdot d \]
\[ W = [(60)(10)](1.5) \]
\[ W = [600](1.5) = 900 \text{ N} \cdot \text{m} \]

Note that the amount of time it took to lift the box has nothing to do with the amount of work done.

It may be tempting to try to use the time to calculate velocity and acceleration in order to calculate the force. However, because the box is lifted at a constant velocity, the only force needed to lift the box is enough to overcome the weight of the box \((F_g)\).

In general, if work is done to move an object vertically, the work is done against gravity, and you need to use \(a = g = 10 \text{ m/s}^2\) for the acceleration when you calculate \(F = ma\).

Similarly, if work is done to move an object horizontally, the work is not against gravity and either you need to know the force applied or you need to find it from the acceleration of the object using \(F = ma\).

Q: In the picture to the right, the adult is pulling on the handle of the wagon with a force of 150. N at an angle of 60.0°.

If the adult pulls the wagon a distance of 500. m, how much work does he do?

A: \[ W = F \cdot d \]
\[ W = [F \cos \theta] \cdot d \]
\[ W = [(150.) \cos 60.0°](500.) \]
\[ W = [(150.)(0.500)](500. \text{ m}) = 37500 \text{ N} \cdot \text{m} \]