Escape Velocity

**Unit:** Energy, Work & Power

**MA Curriculum Frameworks (2016):** HS-PS2-4

**MA Curriculum Frameworks (2006):** 1.7

**Mastery Objective(s):** (Students will be able to...)

- Calculate the velocity that a rocket or space ship needs in order to escape the pull of gravity of a planet.

**Success Criteria:**

- Variables are correctly identified and substituted correctly into the appropriate equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

**Tier 2 Vocabulary:** escape

**Language Objectives:**

- Explain why we can’t simply use $\frac{mg}{s^2}$ to calculate escape velocity.

**Notes:**

If you want to send a rocket or space ship to explore the rest of the solar system or beyond, the rocket needs enough kinetic energy to escape from the force of Earth’s gravity.

To explain the calculation, we measure height from Earth’s surface and use $\frac{mg}{s^2}$ for the strength of the gravitational field. However, when we calculate the escape velocity of a rocket, the rocket has to go from the surface of the Earth to a point where $g$ is small enough to be negligible.

We can still use the conservation of energy, but we need to calculate the potential energy that the rocket has based on its distance from the center of the Earth instead of the surface of the Earth. (When the distance from the Earth is great enough, the gravitational potential energy becomes zero, and the rocket has escaped.) Therefore, the spaceship needs to turn kinetic energy into this much potential energy.

Use this space for summary and/or additional notes:
To solve this, we need to turn to Newton’s Law of Universal Gravitation. Recall from Universal Gravitation starting on page 283 that:

\[ F_g = \frac{G m_1 m_2}{r^2} \]

The potential energy equals the work that gravity could theoretically do on the rocket, based on the force of gravity and the distance to the center of the Earth:

\[ W = \vec{F} \cdot \vec{d} = F_g h = \left( \frac{G m_1 m_2}{r^2} \right) h \]

Because \( h \) is the distance to the center of the Earth, \( h = r \) and we can cancel, giving the equation:

\[ U_g = \frac{G m_1 m_2}{r} \]

Now, we can use the law of conservation of energy. The kinetic energy that the rocket needs to have at launch needs equals the potential energy that the rocket has due to gravity. Using \( m_1 \) for the mass of the Earth and \( m_2 \) for the mass of the spaceship:

\[
\begin{align*}
\text{Before} & = \text{After} \\
TME_i & = TME_f \\
K_1 & = U_f \\
\frac{1}{2} m_1 v_i^2 & = \frac{G m_1 m_2}{r} \\
v_e^2 & = \frac{2 G m_E}{r} \\
v_e & = \sqrt{\frac{2 G m_E}{r}} \\
\end{align*}
\]

Therefore, at the surface of the Earth, where \( m_E = 5.97 \times 10^{24} \text{ kg} \) and \( r = 6.37 \times 10^6 \text{ m} \), this gives \( v_e = 1.12 \times 10^4 \text{ m/s} = 11200 \text{ m/s} \). (If you’re curious, this equals approximately 25 100 miles per hour.)
Sample Problem:

Q: When Apollo 11 went to the moon, the space ship needed to achieve the Earth’s escape velocity of \(11200 \text{ m/s}\) to escape Earth’s gravity. What velocity did the space ship need to achieve in order to escape the moon’s gravity and return to Earth? (I.e., what is the escape velocity on the surface of the moon?)

A: \(v_e = \sqrt{\frac{2Gm_{\text{moon}}}{d_{\text{moon}}}}\)

\[
v_e = \sqrt{\frac{(2)(6.67 \times 10^{-11})(7.35 \times 10^{22})}{1.74 \times 10^6}}
\]

\(v_e = 2370 \text{ m/s}\)